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Evaluation of Complex Heat Transfer Coefficients for Passive Heating Concepts

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Passive heating concepts namely Trombe wall, Water wall and Trans wall have been analysed to obtain overall heat transfer coefficients for average values and for time-dependent variations. The numerical values have been obtained and tabulated for various wall thicknesses.

NOMENCLATURE

- A area (m^2)
- A constant (°C)
- average solair temperature (°C) a_0
- A_1 constant (°C/m)
- nth harmonic in the Fourier expansion of solair temperature (°C)
- Awl area of first water column (m²)
- area of second water column (m²) A_{w2} C
- specific heat (J/Kg °C)
- specific heat of air (J/Kg °C) C_a
- C_w specific heat of water (J/Kg °C)
- dwi width of first water column facing glazing (m)
- width of second water column facing the room (m)
- ho heat transfer coefficient from front surface of the wall to ambient (W/m² °C)
- heat transfer coefficient from blackened surface of the h. wall to the enclosed air $(W/m^2 \circ C)$
- h' heat transfer coefficient from blackened surface of plate to water (W/m²°C)
- heat transfer coefficient from enclosed air to glazing ha $(W/m^2 \circ C)$
- h'z heat transfer coefficient from water to the concrete layer $(W/m^2 \circ C)$
- h_3 heat transfer coefficient from glazing to ambient $(W/m^2 °C)$
- h4 heat transfer coefficient from interior surface of the wall to the room $(W/m^2 \circ C)$
- radiative heat transfer coefficient from blackened surh. face of the wall to the glazing $(W/m^2 \circ C)$
- heat transfer coefficient from concrete wall to room in hR the case of water wall $(W/m^2 \circ C)$
- h_{i1} heat transfer coefficient from front surface of the trap to first water column (W/m² °C)
- h_{12} heat transfer coefficient from trap to second water column (W/m² °C)
- heat transfer coefficient from first water column to the h_{wa} ambient (W/m² °C)
- thermal conductivity (W/mk) K
- L thickness of the wall and trap material (m)
- M_{w1} mass of water in first water column (kg)
- M_{w2} mass of water in second water column (kg)

- m_a mass flow rate of air (kg/sec)
- n number of harmonics (integer)
- reflectance (dimensionless)
- average solar insolation (W/m²) Sa
- nth harmonic of solar insolation in its Fourier series S" expansion (W/m²)
- T temperature of the wall and the trap (°C)
- time (sec)
- T_{a} ambient temperature (°C)
- average value of ambient temperature (°C) T_{a0}
- nth harmonic in the Fourier series expansion of ambient Tan temperature (°C)
- T_R room temperature (°C)
- T_{sa} solair temperature (°C)
- T_{w1} temperature of first water column (°C)
- T_{w^2} temperature of second water column (°C)
- coordinate (m)
- $2\pi/T$ ω
- λ, constant (°C)
- X'n constant (°C)
- transmittance of glazing (dimensionless) τ
- absorptance of the blackened surface of the wall (dimensionless)
- density (Kg/m³)
- $(E_{bl}-E_{bl-1})/E_b$ emissive power of a black body source with $E_{b0} = 0$ and $E_{bn} = E_b$ (dimensionless)
 - wavelength (μm) 2 extinction coefficient of trap material for solar radiation μ_i in the wavelength region between j and j-1
 - fractional absorption of solar radiation in water in the wavelength region between j and j-1 (dimensionless)
 - extinction coefficient of water for solar radiation in the wavelength region between j and j-1.
 - phase of the nth harmonic of heat transfer coefficient σ. (radians)

1. INTRODUCTION

PASSIVE heating concepts for building have been a subject of several studies in recent years [1-6]. Using numerical simulation analysis [7,8], the year long performance of storage walls has been well characterized. Simple empirical methods [9,10] have also been presented for estimating the annual solar heating performance of a building using either thermal storage wall or direct gain concept.

The periodic technique based on Fourier series rep-

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resentation of temperature and heat flux has been extensively applied [11,12] to rate solar passive heating concepts. This rating has often been expressed in terms of the heat flux coming into a room assumed to be maintained at a constant temperature. In this approach each time either the geometry or the location of the building changes, one has to perform a fresh set of detailed numerical calculations. In this paper, the passive heating concepts, namely Trombe wall with or without vents, Water wall and Trans wall, have been characterized in terms of overall complex heat transfer coefficients, appropriate for periodic variations. The heat transfer coefficients, which are independent of the meteorological parameters and the area of the wall, can be used to quickly evaluate the effect of passive concepts on building response; the heat flux in an air-conditioned room (with constant temperature of room air) can be expressed as a simple function on these coefficients and Fourier coefficients of the solair temperature at the outer interface of the wall. For some concepts in which the heat collection and the subsequent heat transfer mechanism is different than an opaque wall, an equivalent solair temperature has to be redefined to take advantage of the overall definition of heat transfer coefficient for the calculation of heat flux.

2. ANALYSIS

2.1. Trombe wall without vents

Corresponding to the configuration given in Fig. 1a, the heat transfer through the glazed mass wall can be assumed to be one dimensional governed by the following heat conduction equation, the periodic solution of which is written as

$$T(x,t) = A_0 + A_1 x + R_e \sum_{n=1}^{\infty} (\lambda_n e^{\beta_n x} + \lambda'_n e^{-\beta_n x}) e^{in\omega t}, \quad (1)$$

where

$$\beta_n = (1+i) \left\{ \frac{n\omega\rho c}{2K} \right\}^{1/2}.$$
 (2)

The unknown constants A_0 , A_1 , λ_n and λ'_n are to be

determined by the application of appropriate boundary conditions which can be written as

$$-K\frac{\partial T}{\partial x}\Big|_{x=0} = h_0[T_{sa} - T(x,t)|_{x=0}]$$
(3)

and

$$-K\frac{\partial T}{\partial x}\Big|_{x=L} = h_4[T(x,t)]_{x=L} - T_R].$$
(4)

In equation (3), the solair temperature is an equivalent environmental temperature which takes into account the mechanism of radiation absorption and subsequent heat transfer from the hot surface.

The right hand side in equation (4) represents the heat flux, \dot{Q} , coming into the room. Substituting for T from equation (1) in equations (3) and (4) and solving the resultant algebraic equations one obtains the expressions for the unknown constants and the heat flux. The expression for the heat flux, \dot{Q} , can be expressed in the form

$$\dot{Q} = U_0(a_0 - T_R) + R_e \sum_{n=1}^{\infty} U_n a_n e^{in\omega t},$$
(5)

where a_0 and a_n are Fourier coefficients of the time variation of solair temperature, i.e.,

$$T_{sa} = a_0 + R_e \sum_{n=1}^{\infty} a_n e^{in\omega t}$$
(6)

and U_0 and U_n defined as the average overall heat transfer coefficients and the complex heat transfer coefficient for the harmonic are given by the expressions.

$$U_{0} = \left[\frac{1}{h_{0}} + \frac{L}{K} + \frac{1}{h_{4}}\right]^{-1}$$
(7)

(8)

and $U_n =$

$$\frac{Kh_0h_4\beta_n}{\left[(h_0h_4+K^2\beta_n^2)\sinh(\beta_nL)+(h_0+h_4)K\beta_n\cosh(\beta_nL)\right]},$$

The overall heat transfer coefficient from the collecting



Fig. 1. Schematics of the systems.

surface to the ambient is in equation (7). In fact, this again consists of 3 parts, viz the conductance of the air gap between the hot surface and the glazing, the conduction effect of the glazing and the heat loss from the outermost surface of the glazing. As with a flat plate collector, an overall heat transfer coefficient is assigned to single or double glazed surfaces [11,12].

2.2. Trombe wall with vents

For Trombe wall with vents (Fig. 1b), the heat transfer from the hot surface consists of two parts (i) heat losses to the ambient through glazing and (ii) heat transfer to the moving air in the air gap, in addition to heat conduction through the wall. The heat flux, therefore, into the room consists of two parts i.e.

$$\dot{Q} = \dot{Q}_k + \dot{Q}_c, \tag{9}$$

where

$$\dot{Q}_k = h_R[T(x,t)|_{x=L} - T_R]$$
 (10)

and

$$\dot{Q}_c = \frac{\dot{m}_a c_a}{A} \left[\theta_{\text{out}} - T_R \right]. \tag{11}$$

 θ_{out} is the temperature of hot air, entering the room through the upper vent. After rearrangement, the expression for heat flux can be put in the form,

$$\dot{Q} = U_0 \left[G_1 (G_2 S_0 + T_{a0}) - T_R \right] + R_e \sum_{n=1}^{\infty} U_n (G_n S_n + T_{an}) e^{in\omega t}.$$
 (12)

As with equation (5), U_0 and U_n may be termed as overall average and associated *n*th harmonic heat transfer coefficients respectively, and $G_1(G_2S_0+T_{a0})$ and $(G_nS_n+T_{an})$ as the modified solair temperatures, expressions for various parameters being given in Appendix A.

2.3. Water wall

The water wall consists of water filled in drums as the storage media (Fig. 1c). The water wall may have a concrete structure at its back surface facing the room. Following the analysis given in Nayak *et al.* [11], the expression for the heat flux, when there is no concrete wall, can be written in the same form as equation (5)with,

$$U_0 = \left[\frac{1}{h_0} + \frac{1}{h_1'} + \frac{1}{h_4}\right]^{-1}$$
(13)

and

$$U_{n} = \left[\frac{h_{0}h'_{1}h_{4}}{h_{0}h'_{1} + h_{0}h_{4} + h'_{1}h_{4} + (h'_{1} + h_{4})(M_{w}C_{w}in\omega/A)}\right].$$
(14)

In equation (13) h_0 is the overall heat transfer coefficient from the heat collection surface to the ambient, h'_n is the coefficient of heat transfer from hot surface to water and h_4 is the overall heat transfer coefficient from water to the room air.

If the water wall is followed by a thin concrete wall,

 U_0 and U_n assume the following expressions

$$U_0 = \left[\frac{1}{h_0} + \frac{1}{h_1'} + \frac{1}{h_4'} + \frac{L}{K} + \frac{1}{h_R}\right]^{-1}$$
(15)

and

$$U_n = h_R [F_1 e^{\beta_n L} + F_2 e^{-\beta_n L}].$$
(16)

In equation (15) h'_4 is the heat transfer coefficient from water to the concrete surface in contact with water, where F_1 and F_2 are given in Appendix B.

2.4. Trans wall

Trans wall, a translucent storage wall introduced by Fuchs and McClelland [9] and shown conceptually in Fig. 1d, has been studied earlier by Sodha *et al.* [13]. Since solar radiation is absorbed throughout the translucent media, the governing heat conduction equation for the temperature distribution is

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{K} \frac{\partial T}{\partial x} = \rho c \frac{\partial T}{\partial t}$$
(17)

where I(x,t), the solar intensity at any point inside the translucent material is given by the expression [14],

$$I(x,t) = \tau_g(1-r)S(t)\sum_{j=1}^{5} \frac{E_{bj} - E_{bj-1}}{E_b} e^{-\mu_j x}, \quad (18)$$

with I(x, t) given by equation (18), the solution of equation (18) is obtained as,

$$T(x,t) = A_0 + A_1 x - \frac{S_0 \tau_g (1-s)}{K} \sum_{j=1}^{5} \frac{E_{bj} - E_{bj-1}}{\mu_j E_b}$$
$$\times e^{-\mu_j x} v_j e^{-\eta_j d_{w_1}} + R_e \sum_{n=1}^{\infty} \left\{ \lambda_n e^{\beta_n x} + \lambda'_n e^{-\beta_n x} - \frac{S_n \tau_g (1-s)}{K} \sum_{j=1}^{5} \frac{\mu_j}{\mu_j^2 - \beta_n^2} \frac{E_{bj} - E_{bj-1}}{E_b} + v_j e^{-\eta_j d\omega_1} \cdot e^{-\mu_j x} \right\} e^{in\omega t}.$$
(19)

With appropriate boundary conditions given as

$$K \frac{\partial T}{\partial x}\Big|_{x=0} = h_{t1} [T(x,t)|_{x=0} - T_{w1}(t)], \qquad (20)$$

$$-K\frac{\partial T}{\partial x}\Big|_{x=L} = h_{12}[T(x,t)]_{x=L} - T_{w2}(t)], \qquad (21)$$

$$\frac{M_{w1}C_{\omega}}{A_{\omega 1}}\frac{\partial T_{w1}}{\partial t} = h_{t1}[T(x,t)|_{x=0} - T_{w1}(t)] + \tau_g(1-s)S(t)\left\{1 - \sum_{j=1}^{5} v_j e^{-\eta_j d_{w1}}\right\} - h_{wa}[T_{w1}(t) - T_a(t)]$$
(22)

and

$$\frac{M_{w2}C_{\omega}}{A_{w2}} \frac{\partial T_{w2}}{\partial t} = h_{t2}[T(x,t)|_{x=L} - T_{w2}(t)] + \tau_g(1-s)S(t)\sum_{j=1}^5 v_j e^{-\eta_j d_{w1}} \{1-v_j e^{-\eta_j d_{w2}}\} \cdot \frac{E_{bj} - E_{bj-1}}{E_b} e^{-\mu_j L} - h_4[T_{w2}(t) - T_R].$$
(23)

| | HTC (W/m ³ °C) | Specific heat (J/kg °C) | K (W/m K) | Density (kg/m ³) | | Other |
|---|--|-------------------------------|--------------|---------------------------------|--|--|
| $ \begin{array}{c} h_{0} \\ h_{1} \\ h'_{1} \\ h'_{2} \\ h'_{2} \\ h_{3} \\ h_{4} \\ h_{r} \\ h_{R} \\ h_{r1} \\ h_{r2} \\ h$ | 6.0 3.2 206.5 2.9 206.5 21.96 8.29 4.9 8.29 200 200 6.0 | | | | \dot{m}_a S T_R $	au_g$ $lpha_g$ | 0.104 kg/sec 0.04 20°C 0.9 0.9 |
| Concrete Trap Air Water | | 795.5 1500 1008 4200 | 0.72 0.2 | 1858 1190 | | |

| Table 1. Values of different heat transfer coefficients | s (HTC) and thermophysical parameters |
|---|---------------------------------------|
|---|---------------------------------------|

After involved but simple algebra, the flux entering into the room can be expressed in the form,

$$\dot{Q} = h_4 S' [G'_1 (G'_2 S_0 + T_{a0}) - T_R] + h_4 h_{wa} S'' [G'_n S_n - T_{an}]$$
(24)

where $h_4S'(=U_0)$ and $h_4h_{wa}S''(=U_n)$ are the overall average and *n*th harmonic complex heat transfer coefficients and the equivalent solair temperatures for Trans wall have the following Fourier coefficients

$$a_{0} = G'_{1}(G'_{2}S_{0} + T_{a0})$$
$$a_{n} = (G'_{n}S_{n} - T_{an})$$

The expressions for G_1 , G_2 , G_n , S' and S'' are given in Appendix C.

3. RESULTS AND DISCUSSIONS

Various values of heat transfer coefficients and thermophysical properties of the materials used in the calculation of overall heat transfer coefficients are given in Table 1. The values of extinction coefficients and absorption coefficients for Transwall are given in Table 2.

With these values of the wall properties, the numerical values of the average and other harmonic for overall heattransfer coefficients for various passive heating concepts discussed in this paper are given in Tables 3-7. The *n*th harmonic heat transfer coefficient U_n is complex as $U_n = A_n \exp(i\sigma_n)$, A_n and σ_n being given in the tables. On the average, the overall U-value for the Trombe wall with vents is higher than other passive components i.e. Trombe wall without vents, Water wall and Trans wall. Also there is little change in its value with increasing wall thickness for the case of vents, essentially because the convective heat transfer starts dominating and the phase lag attenuation goes on reducing as the wall thickness increases.

Average heat flux into a room of constant temperature environment depends on the equivalent average solair temperature and the overall heat transfer coefficient. These are given in Tables 8 and 9 for different heat capacities. On average it is seen that though the solair temperature for the water wall and Trombe wall is identical, the water wall has larger U-value. This results in the transfer of greater heat flux into the room.

Looking only at the average values, the Trombe wall with vents gives the best performance; however the timedependent behaviour shows that the storage capacity of this type of Trombe wall is negligible. The time-dependent behaviour of the heat flux (a measure of the overall performance) is shown in Figs 2 and 3 corresponding to the solair temperature variations for Trombe wall, Water wall and Trans wall given in Figs 4 and 5. It is observed that the solair for the Trombe wall without vents and the water wall has maximum amplitude; this, along with the appropriate damping effect of various harmonics of heat transfer coefficients, yields positive flux for the water wall throughout 24 hr.

Table 2. Value of extinction and absorption coefficients in trap (translucent) material for solar radiation in different wavelength regions

| Wavelength region | $F_{i} = F_{i} / F_{i}$ | ,,, | ν | n |
|-------------------------------|---|-------|-------|-------|
| travelength region | \square_{bj} \square_{bj-1} \square_b | ~ | | |
| $0 \leq \lambda \leq 0.36$ | 0.08137 | 0.0 | 0.237 | 0.032 |
| $0.36 \leq \lambda \leq 1.06$ | 0.66880 | 0.725 | 0.193 | 0.450 |
| $1.06 \leq \lambda \leq 1.3$ | 0.08620 | 3.82 | 0.167 | 3.00 |
| $1.3 \leq \lambda \leq 1.6$ | 0.06120 | 9.45 | 0.179 | 35.0 |
| $1.6 \leq \lambda \leq 0.0$ | 0.10244 | 0.0 | 0.224 | 255.0 |

262

| | n | | | | | | | | | | | |
|------|----------------|------|---------|--------|--------|---------|---------|---------|--|--|--|--|
| L(m) | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | |
| 0.10 | A _n | 2.35 | 1.88 | 1.202 | 0.82 | 0.596 | 0.453 | 0.356 | | | | |
| 0.10 | σ_n | 0 | 5.4502 | 4.8701 | 4.4914 | 4.2034 | 3.9633 | 3.7528 | | | | |
| 0.15 | σ_n | 2.02 | 1.191 | 0.613 | 0.37 | 0.244 | 0.17 | 0.123 | | | | |
| 0.15 | | 0 | 4.9225 | 4.2186 | 3.7359 | 3.3433 | 3.0033 | 2.6997 | | | | |
| 0.00 | σ_{a} | 1.77 | 0.74 | 0.326 | 0.174 | 0.103 | 0.065 | 0.426 | | | | |
| 0.20 | | 0 | 4.4600 | 3.6116 | 2,9925 | 2,4814 | 2.0380 | 1.6417 | | | | |
| 0.05 | σ_{r} | 1.58 | 0.47 | 0.177 | 0.083 | 0.043 | 0.025 | 0.015 | | | | |
| 0.25 | | 0 | 4.02730 | 3.0043 | 2.2448 | 1.6176 | 1.0724 | 0.5843 | | | | |
| 0.00 | σ_{π} | 1.42 | 0.3034 | 0.0959 | 0.039 | 0.018 | 0.0094 | 0.00513 | | | | |
| 0.30 | n | 0 | 3.5999 | 2.3939 | 1.4968 | 0.75416 | 0.10721 | 5.8100 | | | | |

Table 3. Amplitude and phase of the heat transfer coefficient of Trombe wall without vent

Table 4. Trombe wall with vents ($\dot{m}/A = 0.104$ kg/sec)

| | n | | | | | | | | | | | |
|------|------------|------|--------|--------|--------|--------|--------|--------|--|--|--|--|
| | | 0 | I | 2 | 3 | 4 | 5 | 6 | | | | |
| 0.05 | A_n | 4.65 | 4.49 | 3.982 | 3.50 | 3.14 | 2.88 | 2.70 | | | | |
| 0.05 | σ_n | 0 | 6.0885 | 5.9606 | 5.9031 | 5.8907 | 5.9016 | 5.9229 | | | | |
| 0.10 | A_n | 4.51 | 3.8048 | 3.014 | 2.64 | 2.48 | 2.39 | 2.36 | | | | |
| 0.10 | σ_n | 0 | 5.9686 | 5.9287 | 5.9775 | 6.0328 | 6.0783 | 6.1131 | | | | |
| 0.15 | A, | 4.43 | 3.24 | 2.65 | 2.497 | 2.456 | 2.445 | 2.441 | | | | |
| 0.15 | σ, | 0 | 5.9535 | 6.0265 | 6.1005 | 6.1445 | 6.1686 | 6.1817 | | | | |
| 0.00 | A, | 4.35 | 2.9264 | 2.5985 | 2.5521 | 2.5385 | 2.5238 | 2.5055 | | | | |
| 0.20 | σ_n | 0 | 6.0059 | 6.1115 | 6.1592 | 6.1760 | 6.1812 | 6.1827 | | | | |
| 0.25 | An | 4.30 | 2.805 | 2.648 | 2.62 | 2.586 | 2.55 | 2.517 | | | | |
| 0.25 | σ_n | 0 | 6.0683 | 6.1538 | 6.172 | 6.1741 | 6.1742 | 6.1751 | | | | |

Table 5. Water wall without a concrete slab

| | | п | | | | | | | | | | | |
|------------|----------------|-----------|------------------|-------------------|-------------------|-------------------|-------------------|-------------------|--|--|--|--|--|
| M_w (kg) | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
| 50 | A_n | 3.41 0 | 2.316 | 1.431 | 1.004 | 0.768 4.93918 | 0.620 | 0.520 4.86503 | | | | | |
| 100 | $A_n \sigma_n$ | 3.41 0 | 1.431 5.14478 | 0.7678 4.93918 | 0.5192 4.86503 | 0.3914 4.82726 | 0.3138 4.80443 | 0.2620 4.78915 | | | | | |
| 150 | $A_n \sigma_n$ | 3.41 0 | 1.0042 5.9088 | 0.519 4.86503 | 0.348 4.81469 | 0.2619 4.78915 | 0.2097 4.77384 | 0.175 4.76362 | | | | | |
| 200 | $A_n \sigma_n$ | 3.41 0 | 0.7678 4.9392 | 0.3914 4.8272 | 0.2619 4.7892 | 0.1966 4.7700 | 0.1574 4.7585 | 0.131 4.7508 | | | | | |
| 250 | $A_n \sigma_n$ | 3.41 0 | 0.620 4.8949 | 0.314 4.80443 | 0.209 4.7738 | 0.157 4.7585 | 0.126 4.74929 | 0.105 4.74315 | | | | | |

Table 6. Water wall with a concrete layer of thickness 0.1 m

| | | n | | | | | | | | | | | |
|----------------------------|----------------|------|---------|---------|---------|---------|---------|---------|--|--|--|--|--|
| <i>M</i> _w (kg) | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
| 50 | A_n | 2.26 | 0.8678 | 0.4114 | 0.2380 | 0.1519 | 0.1033 | 0.0735 | | | | | |
| 50 | σ, | 0 | 4.68986 | 4.10009 | 3.69385 | 3.37274 | 3.10431 | 2.87136 | | | | | |
| 100 | A _n | 2.26 | 0.5382 | 0.2391 | 0.1338 | 0.0836 | 0.0560 | 0.03944 | | | | | |
| 100 | σ_n | 0 | 4.49300 | 3.9524 | 3.5660 | 3.2590 | 3.0021 | 2.7784 | | | | | |
| 150 | An | 2.26 | 0.3853 | 0.1675 | 0.0927 | 0.05748 | 0.0383 | 0.2689 | | | | | |
| 150 | σ., | 0 | 4.4060 | 3.8925 | 3.5163 | 3.2161 | 2.9642 | 2.7444 | | | | | |
| 200 | Å, | 2.26 | 0.2990 | 0.1286 | 0.0708 | 0.0437 | 0.0291 | 0.0204 | | | | | |
| 200 | σ_{n} | 0 | 4.3578 | 3.8603 | 3.49004 | 3.19364 | 2.9445 | 2.72687 | | | | | |
| 250 | A. | 2.26 | 0.24404 | 0.1044 | 0.0573 | 0.0353 | 0.02345 | 0.01642 | | | | | |
| 250 | σ_n | 0 | 4.3272 | 3.8402 | 3.4738 | 3.1798 | 2.9325 | 2.7161 | | | | | |

| | n | | | | | | | | | | | |
|-----------------------------|--------------|------|--------|---------|--------|--------|---------|--------|--|--|--|--|
| L(m) - | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | |
| 0.005 | A_n | 3.1 | 0.639 | 0.262 | 0.138 | 0.0845 | 0.0564 | 0.0401 | | | | |
| 0.005 A 0.01 A 0.02 A | σ_n | 0 | 1.3507 | 0.9169 | 0.6747 | 0.5234 | 0.4212 | 0.3478 | | | | |
| 0.01 | A_n | 2.87 | 0.517 | 0.1824 | 0.089 | 0.0521 | 0.034 | 0.024 | | | | |
| 0.01 | σ_n | 0 | 1.1338 | 0.6686 | 0.4469 | 0.3173 | 0.2304 | 0.1665 | | | | |
| 0.02 | A_n | 2.51 | 0.352 | 0.107 | 0.0497 | 0.0283 | 0.0182 | 0.0126 | | | | |
| 0.02 | σ_n | 0 | 0.8585 | 0.4010 | 0.1901 | 0.0575 | 6.2398 | 6.1559 | | | | |
| 0.02 | A, | 2.23 | 0.2569 | 0.074 | 0.033 | 0.0186 | 0.0117 | 0.008 | | | | |
| 0.03 | σ_{n} | 0 | 0.6751 | 0.2046 | 6.2479 | 6.0754 | 5.9320 | 5.8046 | | | | |
| 0.04 | A, | 2.01 | 0.198 | 0.0542 | 0.0239 | 0.013 | 0.00798 | 0.0053 | | | | |
| 0.04 | σ_n | 0 | 0.5216 | 0.00857 | 5.9972 | 5.7681 | 5.5705 | 5.3931 | | | | |
| 0.05 | A_n | 1.82 | 0.158 | 0.0416 | 0.0177 | 0.0093 | 0.0054 | 0.0035 | | | | |
| 0.03 | σ_n | 0 | 0.3750 | 6.0800 | 5.7174 | 5.4266 | 5.1757 | 4.9525 | | | | |

Table 7. Amplitude and phase of the heat transfer coefficient of trans wall with $M_{w1} = M_{w2} = 100 \,\text{kg}$ and $d_{w1} = d_{w2} = 0.1 \,\text{m}$

4. CONCLUSIONS

Explicit expressions for the average overall heat transfer coefficients and the associated complex heat transfer coefficients for various harmonics have been obtained for the Trombe wall, Water wall and Trans wall passive heating concepts. The performance of buildings incorporating any of these can be easily evaluated by using the values of overall heat transfer coefficients tabulated in this paper. The expressions for an equivalent solair temperature for the Trombe wall with vents, Trans wall and Water wall are also obtained in the process.





Fig. 2. Hourly variations of the flux entering into the room through different systems for heat capacity equal to $221.6 \text{ kJ}/^{\circ}\text{C}$.

Fig. 3. Hourly variations of the flux entering into the room through different systems for heat capacity equal to $443.2 \text{ kJ}/^{\circ}C$.



Fig. 4. Hourly variations of the solair temperatures for different systems for heat capacity equal to 221.6 $\,kJ/^{\circ}C.$

| Evaluation |
|--------------|
| of |
| Complex |
| Heat |
| Transfer |
| Coefficients |
| for |
| Passive |
| Heating |
| Concepts |

| Total heat capacity $(M_P C_p/m^2)$ of the wall $(kJ/^{\circ}C)$ | Trombe wall without vents | | | Trombe wall with vents (natural convection) | | | Water wall | | | Trans wall | | |
|---|---------------------------|--------------------------------|---------------------------------------|---|--------------------------------|---------------------------------------|-------------------------------------|--------------------------------|---------------------------------------|-------------------|---------------------|---------------------------------------|
| | <i>U</i> ₀ (W/m² °C) | <i>T</i> _{s0} (°C) | Q ₀ (W/m ²) | <i>U</i> ₀ (W/m ² °C) | <i>T</i> _{s0} (°C) | Q ₀ (W/m ²) | <i>U</i> ₀ (₩/m ² °C) | <i>T</i> _{s0} (°C) | Q ₀ (W/m ²) | $(W/m^2 \circ C)$ | <i>T</i> ₅₀ (°C) | Q ₀ (W/m ²) |
| 221.6 | 2.02 | 44.4 | 49.3 | 4.43 | 33.0 | 57.6 | 3.42 | 44.4 | 83.45 | 1.82 | 30.0 | 18.2 |
| 443.2 | 1.42 | 44.4 | 34.6 | 4.26 | 31.8 | 50.2 | 3.42 | 44.4 | 83.45 | 1.82 | 33.0 | 23.7 |
| 664.8 | 1.10 | 44.4 | 26.8 | 4.17 | 30.7 | 44.6 | 3.42 | 44.4 | 83.45 | 1.82 | 34.0 | 25.5 |

Table 8. Average solair, heat transfer coefficients and flux for different passive heating concepts for different heat capacities (kJ/°C)

Table 9. Average solair, heat transfer coefficients and flux for different passive heating concepts for different wall thicknesses

| Total thickness | Trombe wall without vents | | | Trombe wall with vents | | | Water wall | | | Trans wall* | | |
|---------------------------|---------------------------|--------------------------------|---------------------------------------|------------------------|---------------------|---------------------------------------|-------------------------------------|--------------------------------|---------------------------------------|-----------------------------|--------------------------------|---------------------------------------|
| of the wall (L) (m) | U₀ (W/m² °C) | <i>T</i> _{s0} (°C) | Q ₀ (W/m ²) | U₀ (₩/m² °C) | <i>T</i> ₅₀ (°C) | Q ₀ (W/m ²) | <i>U</i> ₀ (₩/m ² °C) | <i>T</i> _{s0} (°C) | Q ₀ (W/m ²) | U₀ (₩/m ² °C) | <i>T</i> _{s0} (°C) | Q ₀ (W/m ²) |
| 0.15 | 2.02 | 44.4 | 49.3 | 4.43 | 33.0 | 57.6 | 3.42 | 44.4 | 83.45 | 1.82 | 30.0 | 23.7 |
| 0.30 | 1.42 | 44.4 | 34.6 | 4.26 | 31.8 | 50.2 | 3.42 | 44.4 | 83.45 | 1.82 | 34.0 | 25.5 |
| 0.45 | 1.10 | 44.4 | 26.8 | 4.17 | 30.7 | 44.6 | 3.42 | 44.4 | 83.45 | 1.82 | 35.3 | 28.0 |

* (I) $d_{w1} = 0.1 \text{ m}$; $d_{w2} = 0.0 \text{ m}$; trap thickness = 0.05 m. (II) $d_{w1} = 0.15 \text{ m}$; $d_{w2} = 0.10 \text{ m}$; trap thickness = 0.05. (III) $d_{w1} = 0.3 \text{ m}$; $d_{w2} = 0.10 \text{ m}$; trap thickness = 0.05 m.

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N. K. Bansal et al.



Fig. 5. Hourly variations of the solair temperature for different systems for heat capacity equal to 443.2 kJ/°C.

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(A2)

APPENDIX A

 $U_0 = h_4 P_{12} P_{14} - P_{17}$

 $G_1 = (P_{19} + h_4 P_{12} P_{14} P_{16}) / U_0$

- $P_{19} = W(S_2 S_1 P_{11} P_{13})$ (A6)
- $P_{18} = -S_1 W P_{10} P_{13}$ (A7) (A1)
 - $P_{17} = W[S_3 + S_1(1 P_{12}P_{13})]$ (A8)
 - $P_{16} = -P_{11}/P_{12}$ (A9) (A3)
- $G_2 = (P_{10} + P_{12}P_{14}P_{15}h_4)/(P_{19} + P_{12}P_{14}P_{16})$ $P_{15} = -P_{10}/P_{12}$ $U_n = Z_{11} + h_4 (Z_n e^{\beta_n L} + Z_7 e^{-\beta_n L})$ (A10) (A4)
 - $G_n = [Z_{10} + h_4(Z_0 e^{\beta_n L} + Z_6 e^{-\beta_n L})]/U_n$ $P_{14} = K/h_4$ (A5) (A11)

266

$$P_{13} = L + \frac{K}{h_4}$$
(A12)

$$P_{12} = -h_4(P_9 - P_7)/H$$
(A13)

$$P_{11} = -h_4 P_8/H$$
(A14)

$$P_{10} = -h_4 \alpha_g \tau_g/H$$
(A15)

$$P_9 = P_6(h_1 + h_\gamma P_2)$$
(A16)

$$P_8 = h_1 P_5 + h_\gamma (P_3 + P_2 P_5)$$
(A17)

$$W$$

$$P_6 = \frac{W}{1 - Gh_2 P_2} \tag{A18}$$

$$P_{5} = \frac{Gh_{2}P_{3}}{1 - Gh_{2}P_{2}}$$
(A19)
$$G(h + h_{2}P_{2})$$

$$P_4 = \frac{G(n_1 + n_2 F_1)}{1 - Gh_2 P_2} \tag{A20}$$

$$P_{3} = \frac{h_{3}}{h_{2} + h_{3} + h_{\gamma}}$$
(A21)

$$P_2 = \frac{h_2}{h_2 + h_3 + h_\gamma} \tag{A22}$$

$$P_{1} = \frac{17}{h_{2} + h_{3} + h_{7}}$$
(A23)

$$H = h_{4}K + P_{7}(K + h_{4}L)$$
(A24)

$$S_{1} = h_{1} + h_{2}P_{1} + h_{2}P_{2}P_{4}$$
(A25)

$$S_{2} = h_{2}(P_{2}P_{5} + P_{3})$$
(A26)

$$S_{3} = h_{2}(P_{2}P_{6} - 1) - h_{1}$$
(A27)

$$S_3 = h_2(P_2P_6 - 1) - h_1$$
(A27)
$$G = \frac{1 - W}{h_1 + h_2}$$
(A28)

$$W = \frac{m_a C_a}{A(h_1 + h_2)} \left[1 - \exp\left\{ \frac{-(h_1 + h_2)A}{\dot{m}_a C_a} \right\} \right]$$
(A29)
$$Z_{11} = W(Z_6 + Z_8)$$
(A30)

$$Z_{10} = WS_1(Z_7 + Z_9) + S_2 W$$
(A31)
$$Z_{-} = -\frac{Z_4 Z_7}{2}$$
(A32)

$$Z_9 = -\frac{Z_4}{Z_3}$$
(A32)
$$Z_8 = -\frac{Z_4}{Z_6}$$
(A33)

$$Z_{8} = \frac{Z_{3}}{Z_{3}}$$
(A34)
$$Z_{7} = \frac{Z_{3}P_{8}}{Z_{5}}$$
(A34)

$$Z_6 = \frac{Z_3 \tau_g \alpha_g}{Z_5} \tag{A35}$$

$$Z_5 = Z_2 Z_3 - Z_1 Z_4 \tag{A36}$$

$$Z_4 = (h_4 - K\beta_n) e^{-\beta_n L}$$
(A37)

$$Z_4 = (h_4 + K\beta_n) e^{\beta_n L}$$
(A38)

$$Z_3 = (h_4 + K\beta_n) e^{\rho_n E}$$
 (A38)
 $Z_2 = (P_7 + K\beta_n)$ (A39)

$$Z_1 = (P_7 - K\beta_n)$$
 (A40)

APPENDIX B

$$h_e = \left(\frac{1}{h_0} + \frac{1}{h_1}\right)^{-1}$$
(B1)

$$R_1 = \frac{h_2}{h_e + h_2 + M_w C_w in\omega}$$
(B2)
$$h_e$$
(B2)

$$R_2 = \frac{n_e}{h_e + h_2 + M_w C_w in\omega} \tag{B3}$$

$$R_3 = h_2 - K\beta_n \tag{B4}$$

$$R_4 = h_2 + K\beta_n \tag{B5}$$

$$R_4 = h_2 + K\beta_n \tag{B}$$

$$R_{5} = (h_{2} - K\beta_{n}) - \left\lfloor \frac{h_{2}^{2}}{h_{e} + h_{2} + M_{w}C_{w}in\omega} \right\rfloor$$
(B6)

$$R_6 = (h_2 + K\beta_n) - \left[\frac{h_2^2}{h_e + h_2 + M_w C_w in\omega}\right]$$
(B7)

$$R_{7} = \frac{h_{e}h_{2}}{h_{e} + h_{2} + M_{w}C_{w}in\omega}$$
(B8)
$$R_{e} = (h_{e} + K\beta) e^{\beta_{e}L}$$
(B9)

$$R_{8} = (n_{R} + K \beta_{n}) e^{-\beta_{n}L}$$

$$R_{9} = (h_{R} - K \beta_{n}) e^{-\beta_{n}L}$$
(B10)

$$R_{10} = (h_R + K\beta_n) e^{\beta_n L} R_6 - (h_R - K\beta_n) e^{-\beta_n L} R_5$$
 (B11)

$$F_1 = \frac{R_7 R_9}{R_{10}} \tag{B12}$$

$$F_2 = \frac{R_7 R_8}{R_{10}}$$
(B13)

$$\begin{array}{l} \textbf{APPENDIX C} \\ S' = S_{15} \\ G'_1 = S_{13}/S_{15} \\ G'_2 = E_9/S_{13} \\ S''' = y_{36} \\ G'_n = \frac{S_1 EE(J)}{S''h_{wa}} \\ S_1 = \tau(1-\gamma)/K \\ S_2 = \frac{\tau}{h_4} \\ S_3 = \frac{h_{12}}{h_8} \\ S_4 = \frac{\tau(1-\gamma)h_{12}}{K \cdot h_8} \\ S_5 = \frac{\tau(1-\gamma)}{h_8} \\ S_6 = \frac{h_4}{h_8} \\ S_7 = S_3h_{10} \\ S_8 = \frac{(h_{c1} - B_6)}{B_2} \\ S_9 = (h_{c1} + S_8h_{c2}) \\ S_{10} = K - S_8h_5 \\ S_{11} = S_9h_{10} + S_{10}S_7 \\ S_{12} = S_{10}S_6 \\ S_{13} = \frac{S_{11}}{B_{11}} \\ S_{14} = \frac{S_{12}}{B_{11}} \\ S_{15} = 1 + S_{14} \\ SS = \tau(1-\gamma) \\ h_5 = h_{c2}L + K \\ h_6 = h_{c1} + h_{wa} \\ h_7 = h_{c2} + h_4 \\ h_8 = h_{c2}L \\ h_9 = \frac{h_6}{h_{c1}} \\ E_1 = \sum_{j=1}^5 \frac{E_{bj} - E_{bj-1}}{E_b} e^{-\mu_j L} v_j e^{-\eta_j d_{w1}} \end{array}$$

267

$$\begin{split} E_{3} &= \sum_{j=1}^{3} \frac{E_{yj} - E_{yj-1}}{E_{b}} e^{-p_{j}^{j} q_{j}^{j}} e^{-p_{j}^{j} d_{k+1}^{j} q_{j}^{j}} \\ E_{4} &= \sum_{j=1}^{3} \frac{E_{yj} - E_{yj-1}}{B_{b}^{j} h_{b}^{j} h_{b}^{j}} v_{j} e^{-q_{j}^{j} d_{k}} \\ E_{5} &= \sum_{j=1}^{3} v_{j} e^{-q_{j}^{j} d_{k}} \\ E_{5} &= \sum_{j=1}^{3} v_{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{1}{h_{b}^{j}} \left[\frac{(1-2)h_{b}E_{1}}{K} + (1-3)(E_{2} - E_{3}) - \frac{(1-2)h_{2}t_{k}}{K} \\ &+ \frac{h_{2}(1-E_{3})}{h_{1}} \right] \\ E_{5} &= \sum_{j=1}^{3} v_{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{1}{h_{b}^{j}} \left[\frac{(E_{4} - E_{4j-1})}{K} \right) v_{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{h_{c}^{j} (E_{4} - E_{4j-1})}{K} v_{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{h_{c}^{j} (E_{4} - E_{4j-1})}{K} v_{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{h_{c}^{j} (E_{4} - E_{4j-1})}{K} v_{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{h_{c}^{j} (1-2)h_{2}E_{1}}{E_{b}} v_{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{h_{c}^{j} (1-2)h_{c}^{j} h_{j}^{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{h_{c}^{j} (1-2)h_{c}^{j} e^{-q_{j}^{j} d_{k}} \\ &= \frac{h_{c}^{j} h_{c}^{j} e^{-q_{j}^{j} d_{k}} \\ \\ &= \frac{h_{c}^{j} h_{c}^{j} e^{-q_{$$

268

 E_9
