NUMERICAL ANALYSIS OF AIRFLOW AND POLLUTION IN BUILDINGS

H. Matsumoto
Tohoku University
Sendai, 980, Japan

F. Hasegawa
Tohoku Institute of Technology
Sendai, 982, Japan

Y. Utsumi
Miyagi National College of Technology
Natori, 981-12, Japan

1. Introduction

In an air-conditioning system, it is necessary to remove or to supply heat in order to maintain a comfortable temperature level, but it is also necessary to supply the room with a proper amount of fresh air. Therefore the indoor airflow is an important factor in the investigation of thermal comfort, ventilation efficiency, air pollution and energy conservation.

Model experiments and numerical analysis are known to predict indoor airflow and distribution of air pollutants. In the model experiments, the accuracy of prediction is affected by the level of the measuring techniques. Even though accurate and highly responsive instruments, such as an infrared gas detector or a flame ionization detector (FID), are used, it is difficult to analyze the detailed structure of the turbulent diffusion.

Through a model experiment, Yoshizawa (1), Ito (2), and Hayakawa (3) showed the distribution and the variation of pollutant concentration. Kobayashi (4) and Tanaka (5) conducted a model experiment in which the relation between the fluctuation of concentration and the airflow structure was thoroughly measured.

On the other hand, though there are several studies on the numerical methods, few methods dealing with turbulence, in which for example, the fluctuation of concentration is considered as the root mean square (6), have been carried out.

This paper describes a numerical method of airflow and air pollution in buildings by the finite element method, and shows the mechanism for the diffusion of air pollutants by computer simulations.

2. Nomenclature

ρ :density
u_i, u_i' :average and turbulence of air velocity
P :pressure
C :average of gas concentration
k :turbulent kinetic energy

ε :turbulent dissipation rate
ν :molecular viscosity
ν_t :eddy viscosity
G :generation rate of gas
Sc :Schmidt number
σc :turbulent Schmidt number
κ :Karman's constant
D :molecular diffusivity
g :gravity
n_j :j-th component of unit normal vector
λ :penalty parameter
u_* :friction velocity
Re :Reynolds number
Pec :Peclet number
Cav :average concentration of the field
C95 :95% cumulative concentration of the field
$-\overline{u'v'}$:Reynolds stress
M_A :Molecular weight of gas A
$C_\mu, C_{D1}, C_{D2}, \sigma_k, \sigma_\varepsilon$:constants of two-equation turbulence model
As :universal constant

3. Governing equations

Air pollutants diffusing in a room generally consist of various components, however, in this paper they are limited to gases such as CO_2, NO_2, etc. and the focus is on the two-component diffusion of a pollutant of interest in the room air as a medium. The basic equations are based on the following three assumptions, 1) the fluid is incompressible, 2) absorption is neglected, and 3) pressure diffusion and heat diffusion are neglected. A two-equation model (k-ε model) (7) is adopted as a mathematical model of turbulent flow. The governing equations of steady flow are given in tensor form, using Einstein's summation convention.

$$u_{j},_{j} = 0 \tag{1}$$

$$u_j u_i,_j + \Pi,_i - ((\nu + \nu_t)(u_i,_j + u_j,_i)),_j - F_i = 0 \tag{2}$$

$$u_j C,_j - ((D + \nu_t/\sigma c)C,_j),_j - G = 0 \tag{3}$$

$$u_j k,_j - ((\nu + \nu_t/\sigma_k)k,_j),_j - \nu_t S + \varepsilon = 0 \tag{4}$$

$$u_j \varepsilon,_j - ((\nu + \nu_t/\sigma_\varepsilon)\varepsilon,_j),_j - C_{D1}(\varepsilon/k)\nu_t S - C_{D2}\varepsilon^2/k = 0 \tag{5}$$

$$\nu_t = C k^2/\varepsilon \tag{6}$$

$$\Pi = P/\rho + 2k/3 \tag{7}$$

$$S = (u_i,_j + u_j,_i)u_i,_j \tag{8}$$

where C_μ, C_{D1}, C_{D2}, σ_k, σ_ε and σ_c are constants.

In the case that the buoyancy effect caused by the difference of density between a component and medium is not negligible, Rodi (8) proposed those equations in which production terms based on buoyancy force are added to the transport equations of k and ε, however, these production terms are neglected in this paper and Boussinesq approximation is used as the effect of buoyancy only in a transport equation. The penalty function method is also adopted to treat incompressibility so that the continuum equation is eliminated from the original equations, i.e., penalty function for pressure is given (9)

$$\Pi = -\lambda u_{j,j} \tag{9}$$

So momentum equation is given as

$$u_j u_{i,j} - \lambda (u_{j,j}) - ((\nu + \nu_t)(u_{i,j} + u_{j,i}))_{,j} - F_i = 0 \tag{10}$$

where λ is penalty parameter.

Boundary conditions are considered as

$$\begin{aligned}
&u_i = \hat{u}_i \quad \text{on } \Gamma_1, \quad t_i = ((\nu+\nu_t)(u_{i,j} + u_{j,i}))u_j = \hat{t}_i \quad \text{on } \Gamma 2 \\
&C = \hat{C} \quad \text{on } \Gamma 3, \quad r = (D + \nu_t/\sigma c)C_{,j} n_j = \hat{r} \quad \text{on } \Gamma 4 \\
&k = \hat{k} \quad \text{on } \Gamma 5, \quad s = (\nu + \nu_t/\sigma_k)k_{,j} n_j = \hat{s} \quad \text{on } \Gamma 6 \\
&\varepsilon = \hat{\varepsilon} \quad \text{on } \Gamma 7, \quad h = (\nu + \nu_t/\sigma_\varepsilon)\varepsilon_{,j} n_j = \hat{h} \quad \text{on } \Gamma 8
\end{aligned} \tag{11}$$

where ^ indicates a specified value. Using empty set ϕ and total boundary Γ, the relationship among boundaries Γ_1, Γ_2, and Γ_3 can be written as

$$\begin{aligned}
&\Gamma_1 \cap \Gamma_2 = \Gamma_3 \cap \Gamma_4 = \Gamma_5 \cap \Gamma_6 = \Gamma_7 \cap \Gamma_8 = \phi \\
&\Gamma_1 \cup \Gamma_2 = \Gamma_3 \cup \Gamma_4 = \Gamma_5 \cup \Gamma_6 = \Gamma_7 \cup \Gamma_8 = \Gamma
\end{aligned} \tag{12}$$

Practical boundary conditions for u, k, and ε considering turbulent boundary layer theory and local isotropy near the wall, will be given (10)

$$u/u_* = (1/\kappa)\log_e(u_* z/\nu) + As \tag{13}$$

$$k = u_*^2/C_\mu^{1/2} \tag{14}$$

$$\varepsilon = u_*^3/(\kappa z) \tag{15}$$

where u_* means friction velocity, κ means Karman's constant, and As varies with the state of wall surface.

4. Numerical approach

Governing equations (3),(4),(5) and (10) are discretized by the finite element method, and governing functions will be given as approximate interporate functions of unknown velocity u_i, concentration C, turbulent kinetic energy k, and turbulent dissipation rate ε.

$$u_i = \Phi_\alpha u_{i\alpha}, \quad C = \Phi_\alpha C_\alpha, \quad k = \Phi_\alpha k_\alpha, \quad \varepsilon = \Phi_\alpha \varepsilon_\alpha \tag{16}$$

Φ is adopted as the weighted function and the governing equations are integrated over the element domain. As the final step, governing equations are transformed into the weighted residual equations in weak form, after using the Green-Gauss theory. For example,

$$\int_{\Omega e}(\Phi_\alpha\Phi_\beta\Phi_{\gamma,j})d\Omega u_{\beta j}u_{\gamma i}+\int_{\Omega_e}\{(\nu+\nu_t)(\Phi_{\alpha,j}\Phi_{\beta,i}+\Phi_{\alpha,k}\Phi_{\beta,k}\delta_{ij})-\lambda\Phi_{\alpha,i}\Phi_{\beta,j}\}d\Omega u_{\beta j}$$
$$-\int_{\Gamma e}\{\Phi_\alpha(\nu+\nu_t)(u_{i,j}+u_{j,i})n_j+\Phi_\alpha\lambda u_{j,j}n_i\}d\Gamma = 0 \qquad (17)$$

$$\int_{\Omega e}(\Phi_\alpha\Phi_\beta\Phi_{\gamma,j})d\Omega u_{\beta j}C_\gamma+\int_{\Omega e}\{(D+\nu_t/\sigma_c)\Phi_{\alpha,j}\Phi_{\beta,j}\}d\Omega C_\beta-\int_{\Omega e}(\Phi_\alpha G)d\Omega$$
$$-\int_{\Omega e}\{\Phi_\alpha(D+\nu_t/\sigma_c)C_{,j}n_j\}d\Gamma = 0 \qquad (18)$$

where n_j is the j-th component of unit vector n and Γe is the boundary of e-th element. Finite element equations are given by superposition of the above equations and are solved with specified boundary conditions. We can easily integrate each term by numerical integration, however, reduced integration in which the number of integral points is one point less is adopted for penalty term (second term of eqn.(10)), in order to avoid the "locking phenomena". The procedure to arrive at the solution is already reported in the reference (11). In the case that the buoyancy effect caused by the difference of concentration is negligible, the calculation of concentration is executed after that of u, k, and ε.

In the first half, the momentum equation is solved by the Newton-Raphson method, and the approximate solutions of k and ε by the modified Newton-Raphson method with the calculated mean velocity. Consequently, this iteration is continued until the residual of finite element equations becomes small enough.

5. Mechanism for the diffusion by computer simulations

There are many factors which influence the distribution of concentration in the air-conditioned room. In this chapter, two factors that will considerably affect the concentration are discussed. The first factor is a flow pattern caused by the various ventilation systems. The second factor is the location of gas generation. The mechanisms for the diffusion are clarified by computer simulations.

5.1. Comparison of numerical analysis with the model experiment

The accuracy of the calculated results is compared with the data of model experiments and is discussed. A room model (3mX2.5mX2m) with an outlet and an inlet is used, and fresh air is blown through the chamber from the inlet. Air velocity is measured by a hot wire anemometer and a ultra-sonic anemometer, and the concentration of methane as a tracer gas is measured by FID.

The finite element model for the calculation has 144 elements and 169 nodes. The constants of the two-equation model are as follows;

$$C_\mu=0.09,\ C_{D1}=1.44,\ C_{D2}=1.92, \sigma_k=1.0, \sigma_\varepsilon=1.3, \sigma c=1.0 \qquad (19)$$

The comparison between the calculation and the model experiment is shown in Fig.1. The velocity and the concentration are expressed as nondimensional values relative to the standard values. There seems to be no significant difference between the results of the calculation and the experiment.

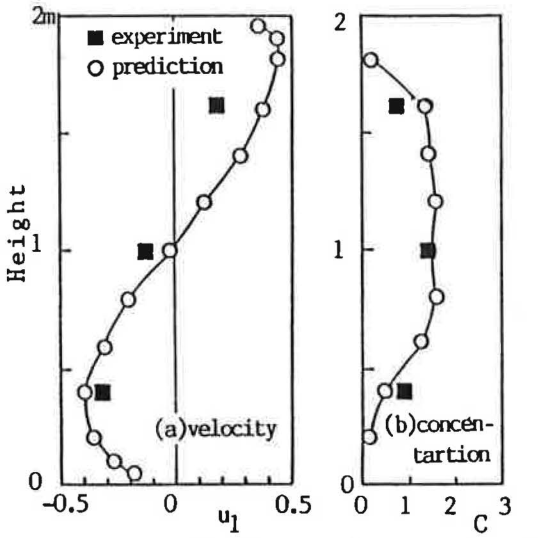


Fig.1 Prediction and experiment

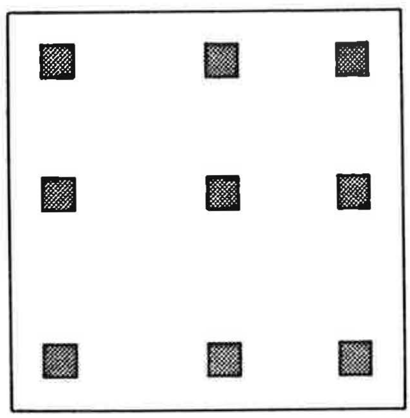


Fig.2 Location of gas generation

5.2. Flow pattern and distribution of concentration

The numerical calculations of the representative eight ventilation systems, which are simplified to two-dimensional models and in which the location of gas generation is set in the middle of the space, are carried out. Fig.2 shows the flow patterns and the distributions of concentration. The concentration is relatively low in the types of Pattern-1,2,3,5,7 and 8 in which air is circulated in the room. But, in the types of Pattern-4 and 6, the concentration is considerably higher.

5.3. Mechanism for the diffusion

The relation between the distribution of concentration and the velocity at the location of gas generation is shown. Nine points (Fig.3) are chosen as the location of gas generation. Fig.4 shows the cumulative relative frequency of the concentration for Pattern-1 and Pattern-4. The two types differ considerably in the distribution of concentration.

Fig.5 shows the relation between the concentration and the velocity of all types. Two mechanisms for the diffusion induced by the convection and the turbulent diffusion are recognized. One is that the turbulent diffusion exerts an influence on the concentration where the air velocity is small, and the other is that the convection dominated diffusion where the air velocity is relatively large.

Additionally the concentration in the room is approximately predicted by the velocity in the location of gas generation. The equations in this case are as follows;

when $U > 0.3$,

$$\begin{aligned} C_{av} &< 0.3 \\ C_{95} &< 0.5 \end{aligned} \qquad (20)$$

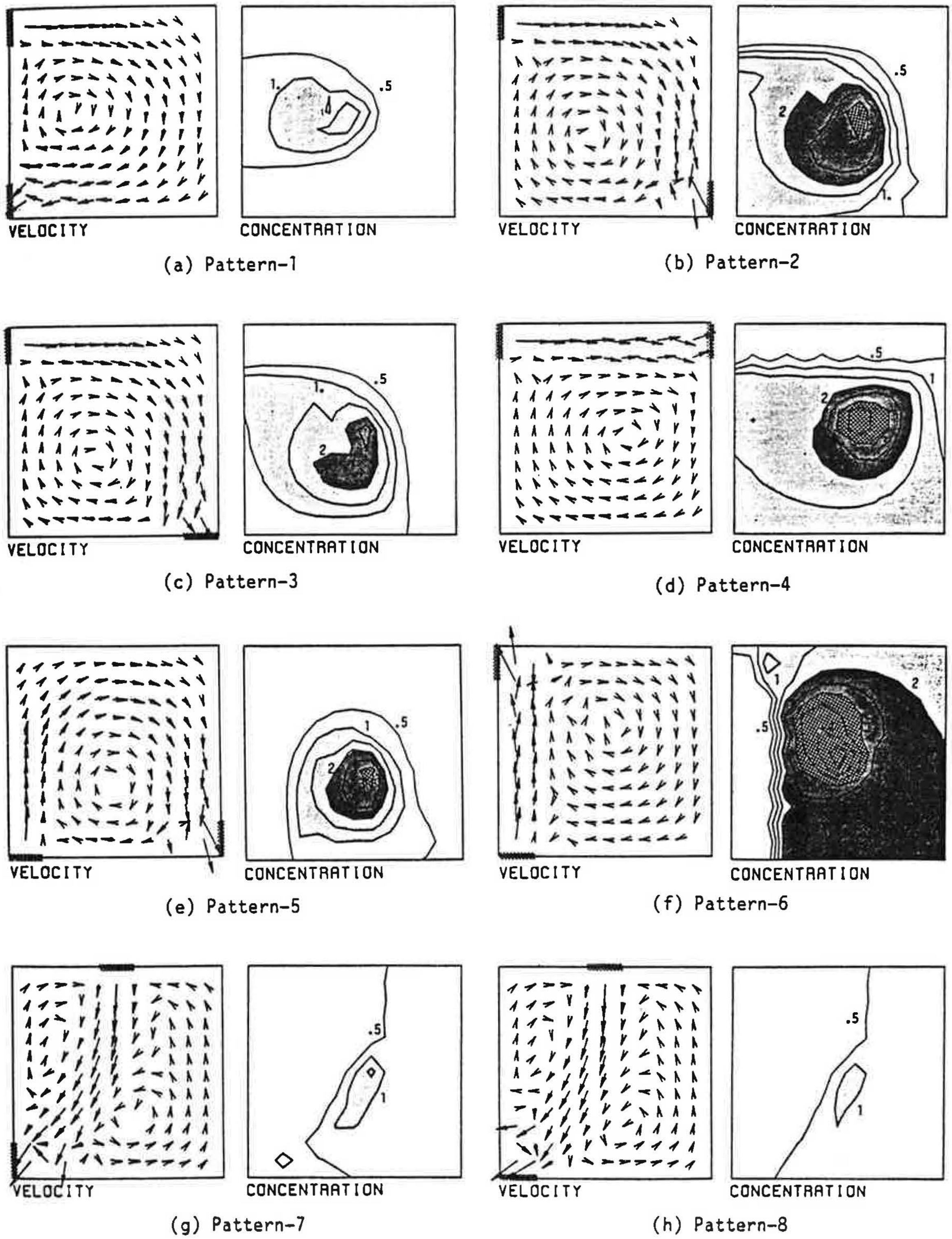
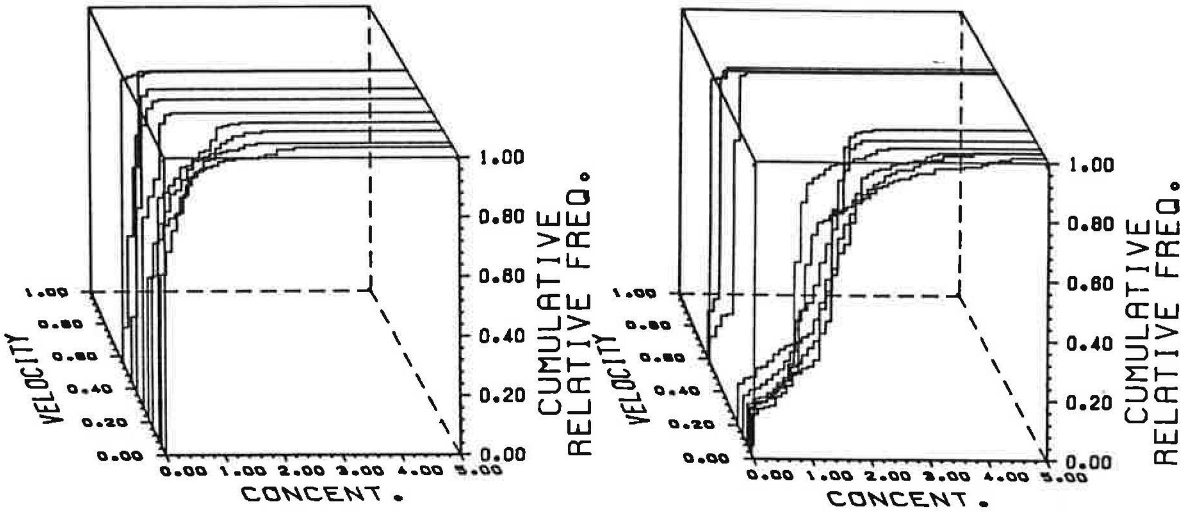


Fig.3 Flow patterns and concentrations

when $U < 0.3$,

$$
\begin{aligned}
Cav &< -5 \cdot U + 2 \\
C_{95} &= -8.9 \cdot U + 3.1 ,
\end{aligned}
\qquad (21)
$$

where Cav and C_{95} respectively are the average concentration of the field and the 95% cumulative concentration.



(a) Pattern-1 (b) Pattern-4

Fig.4 Cumulative relative frequency of concentrations

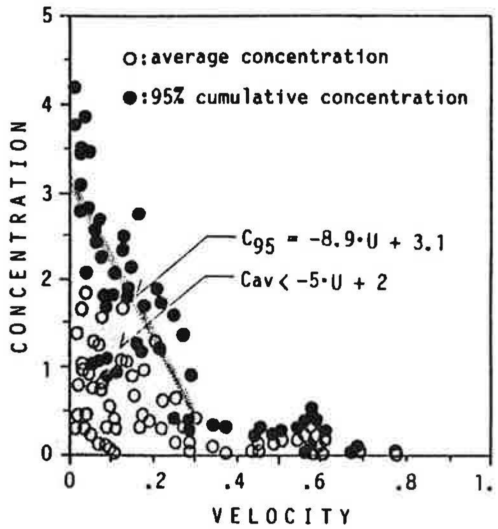


Fig.5 Relation between velocity and concentration

6. Conclusions

The numerical predicting method of the airflow and the distribution of concentration by the finite element method are described. The relation between the ventilation system and the distribution of concentration, and two mechanisms for the diffusion induced by the convection and for the turbulent diffusion are investigated by means of computer simulations. In the conclusion, the concentration in the room is approximately predicted in terms of the velocity at the location of gas concentration.

7. References

(1) S.Yoshizawa, Y.Kobayashi, T.Irie, T.Hosokawa, and S.Ozao: Ununiform Distribution for the Concentration of Contaminants in Room: Report of the Annual Meeting 1973: Architectural Institute of Japan

(2) H.Itoh, S.Yoshizawa, T.Irie, and T.Hosokawa: Mechanism for the Diffusion of Air Pollutants in room Part 1: Report of the Annual Meeting 1975: Architectural Institute of Japan

(3) K.Hayakawa, H.Harada, S.Hujii, T.Kubo, and A.Yamamoto: Basic Study on the Estimation of the Concentration of Aerosols Part 6: Report of the Annual Meeting 1979: Architectural Institute of Japan

(4) N.Kobayashi, S.Murakami, and K.Tokunaga: Study on Model Experiment for the Diffusion in Room: Report of the Annual Meeting 1976: Architectural Institute of Japan

(5) T.Tanaka and S.Murakami: Study on the Diffusion and Fluctuation of Matters in Room: Report of the Annual Meeting 1979: Architectural Institute of Japan

(6) S.Murakami, T.Tanaka and S.Kato: Numerical Simulation of Air Flow and Gas Diffusion in Room Model - Correspondence between Numerical Simulation and Model Experiment -: The Fourth International Symposium on the Use of Computers for Environmental Engineering Related to Buildings: 1983

(7) B.E.Launder and D.B.Spalding: The Numerical Computation of Turbulent Flows: Computer methods in Applied Mechanics and Engineering, Vol.3: 1974

(8) W.Rodi: Turbulence Models and Their Application in Hydraulics: IAHR Publication: 1980

(9) T.J.R.Hughes, W.K.Liu and A.Brooks: Finite Element Analysis of Incompressible Viscous Flows be the Penalty Function Formulation: Journal of Computational Physics, Vol.30: 1979

(10) B.A.Devantier and B.E.Larock: Computation Coupled Turbulent Flow and Sediment Transport: Proceedings of 4th Transactions of International Symposium on Finite Element Methods in Flow Problems: 1982

(11) H.Matsumoto, F.Hasegawa, and Y.Utsumi: Numerical Analysis of Room Air Distribution by the Finite Element Method: Architectural Institute of Japan, Vol.352: 1985