Hybrid Simulation of a Ventilation System based on Natural Convection

Part 1—Proposal of Hybrid Simulation Process and Attempt with Flat Roof Model with Four Vertical Walls

by Roya Taheri*, Yasuyuki Miyagawa**, Ken'Ichi Kimura*** and Tatsuaki Tanaka****

Key Words: Hybrid Simulation, Natural Ventilation System, Calculation, Measurement, Convective Heat Transfer Coefficient, Flat Roof Model, Law of Similarity, Rayleigh Number

Synopsis: A hybrid simulation implying a simultaneously performed simulation of calculation and experiment is proposed. This research is about the simulation of a ventilation system based on natural convection produced by the warm inside wall surface heated up by the conducted heat from the outside wall surface which itself has been warmed up by the solar radiation. Calculations are made with the heat transfer of a building and at the same time a model experiment, both being combined in a hybrid simulation on the real time basis. The convective heat transfer rate is obtained from the experiment.

Nomenclature

- $g$: gravity acceleration constant 9.8 [m/s²]
- $Gr$: Grashof number $(=\frac{g\beta\Delta T L^3}{v^2})$ [-]
- $I_i(n)$: intensity of solar radiation of exterior surface $i$ at time $n$ [kcal/m²·h]
- $l$: length [m]
- $n$: time step [h]
- $Nu$: Nusselt number $(=\frac{a_d}{\lambda})$ [-]
- $Pr$: Prandtl number $(=\frac{v}{\alpha})$ [-]
- $q$: rate of heat flow [kcal/m²·h]
- $q_{ci}(n)$: rate of convective heat flow from the $i$ part of the inside surface at time $n$ [kcal/m²·h]
- $Ra$: Rayleigh number $(=\frac{g\beta\Delta T L^4}{(\nu\alpha)})$ [-]
- $Y_{i}(j)$: thermal response factors of exterior wall or roof relating sol-air temperature to the heat flow in the $i$ part of the inside surface [kcal/m²·h·°C]

* Former Graduate Student in Doctor Course at Department of Architecture, Waseda University, Member
** Section of Environment, Ohbayashi-Gumi Ltd. Technical Research Institute, Member
*** Department of Architecture, School of Science and Engineering, Waseda University, Member
**** Technical Administration Department, Ohbayashi-Gumi Ltd., Member
**Fig. 1 Examples of natural ventilation**

\[ Z(i) \]: thermal response factor of exterior wall or roof relating \( i \) part of inside surface temperature to the heat flow in the \( i \) part of the inside surface

\[ \alpha \]: convective heat transfer coefficient \([\text{kcal/m}^2\cdot\text{h} \cdot \text{°C}]\)

\[ \alpha_a \]: outside film coefficient \([\text{kcal/m}^2\cdot\text{h} \cdot \text{°C}]\)

\[ \alpha_r \]: radiative heat transfer coefficient among \( i \) and \( k \) parts of the inside room surface \([\text{kcal/m}^2\cdot\text{h} \cdot \text{°C}]\)

\[ \beta \]: volume expansivity \([\text{°C}^{-1}]\)

\[ \delta \]: distance between two points \([\text{m}]\)

\[ \delta_1 \]: 1 mm distance between first measurement point and surface \([\text{m}]\)

\[ \delta_2 \]: 1 mm distance between first and second measurement points \([\text{m}]\)

\[ \delta_n \]: 1 mm width of the inner layer of the model wall or roof \([\text{m}]\)

\[ \Delta T \]: temperature difference \([\text{°C}]\)

\[ \gamma \]: specific weight \([\text{kg/m}^3]\)

\[ \lambda \]: thermal conductivity \([\text{kcal/m} \cdot \text{h} \cdot \text{°C}]\)

\[ \nu \]: coefficient of kinematic viscosity \([\text{m}^2/\text{h}]\)

\[ \phi \]: slope tilt angle \([\text{°}]\)

\[ \theta_0 \]: temperature of the room-side surface of the wall or roof \([\text{°C}]\)

\[ \theta_{0i}(n) \]: temperature of the \( i \) part of the room-side surface of the wall or roof at time \( n \) \([\text{°C}]\)

\[ \theta_{1i}, \theta_{2i} \]: temperature of points at 1 mm and 2 mm distance from room-side surface of wall or roof \([\text{°C}]\)

\[ \theta_a(n) \]: outside ambient air temperature at time \( n \) \([\text{°C}]\)

\[ \theta_{ai}(n) \]: sol-air temperature for the \( i \) part of the outside wall or roof surface at time \( n \) \([\text{°C}]\)

**Introduction**

Natural ventilation in a building is sometimes caused by the natural convection along the inside surface of higher temperature. In a natural ventilation system as shown in Fig. 1 (a)–(d), outside air comes in from the bottom entrance and flows out of the top opening. The walls and roof absorb intense solar radiation as a result of which the inside room surface temperature increases.

The heat transfer on the inside surface in the form of heat convection produces the driving force of natural ventilation. It is necessary, therefore, to examine the nature of natural convection within the boundary layer of the surface in order to estimate the ventilation rate of such a system.

In a factory where the driving force of natural ventilation is caused by the existence of high temperature equipment, the convection pattern of room air may be regarded as identical to that of the scaled down model\(^{12-13}\). In this case the convective heat transfer of the boundary layer may be presumed to have no effect on the ventilation and radiation exchange and it is quite different from the above case.

Simple methods with air have been used in experiments, such as the method which ignores the laminar boundary layer and only concerns the turbulent zone of the middle of the room\(^ {14}\), or the method in which heat transfer near the wall surface is treated with an estimated heat transfer coefficient\(^ {15}\). But even in these methods it is very difficult to simulate the inside room surface.
temperature with a scaled down model. Especially in the case of natural ventilation caused by the rising air stream along the inside wall surface where natural convection takes place, experiments would be very troublesome if air were to be used to simulate the behavior of a boundary layer in which the Gr number dominates and the effect of length changes by the power of 3. On the other hand simultaneous prediction of air movement and temperature distribution in a room with a complex shape and arrangement of openings, by numerical calculation is extremely complicated and time consuming. The calculation models presented so far are restricted to the rooms of simple shape and configuration. It is difficult to set up models of complicated shape and arrangement of openings, by numerical calculation in which calculations are made for the heat transfer of a building and determine the values of convective heat transfer coefficient along the inside surfaces, and to observe the air movement pattern. The results of both calculation and experiment are combined on a real time basis.

1. Use of water as fluid media for experimental model

Recently simulations of natural convection in two or three dimensions based on Navier-Stokes equations are often made and verifications by visual experiments using water have also been attempted. In these cases the surrounding inside walls of the room have set temperatures and the shape is limited to a simple rectangular space. The results obtained show the agreement of conventional theories and experiments, thus proving the reliability of using water.

To simulate the thermal behavior in a wall boundary layer, the wall surface temperature within the system could be used as a variable reference for each hour. Using water as fluid media for the boundary layer, an experiment with a scaled down model would be easier and at the same time the radiation exchange among inside room surfaces could be taken into account by calculation.

The dimensionless numbers dominating the system of natural convection are 1) \( Ra \), when \( Pr \) is large, and 2) \( Gr \) and \( Pr \), when \( Pr \) is small. To preserve the accuracy of the law of similarity in the case of natural convection, the simultaneous coincidence of \( Ra \) and \( Pr \), or \( Gr \) and \( Pr \) is necessary. It is known that if \( Pr \) is greater than about 0.5, the dependence of the heat transfer on the \( Pr \) number can be ignored. Such a convection is usually described by the following equation

\[ Nu = c \cdot Ra^b \]  

where \( c \) and \( b \) are constants which change with slope angle of the plate and with the flow range from laminar to turbulent.

In the case of natural convection, heat transfer and air movement are inseparable, and in the range where heat transfer is independent of \( Pr \), when \( Ra \) is the same both in a scaled down model and in a real size, the fluid movement pattern will also be almost the same. If a substance with a large \( Pr \) like water is to be used, the only condition for similarity is the identity of the \( Ra \) number. Namely,

\[ \frac{Ra_1}{Ra_2} = \left( \frac{g_1 \cdot \rho_1}{g_2 \cdot \rho_2} \right) \left( \frac{\Delta \theta_1 / \Delta \theta_2}{(L_1 / L_2)^3} \right) \]

\[ \left/ \left( \frac{\nu_1 / \rho_1 }{\nu_2 / \rho_2} \right) \right. \left( \lambda_1 / \lambda_2 \right) \right. = 1 \]  

From equation (2) in the case of 20°C water and air, the following relationship can be obtained.

\[ \Delta \theta_1 / \Delta \theta_2 = \left( 1.06 \times 10^{-4} \right) \]  

Using this equation the model scale \( L_1 / L_2 \), and the temperature ratio \( \Delta \theta_1 / \Delta \theta_2 \) can be optionally selected for experiments.

2. Process of hybrid simulation

In order to establish \( \Delta \theta = \Delta \theta_1 \), the scale of the model for the experiment should be \( L_1 / L_2 = 1/5.18 \) according to the above conditions in equation (3). Here \( \Delta \theta \) is the difference between wall or roof surface temperature and room temperature. Inside a steel framed glass water tank of the size 1.8×1.6×1.8 (height) m, a scaled natural ventilation model was placed as shown in Figs. 2-4. Experiments were made for two cases; case 1: using a flat roof model, case 2: using a sloped roof model.
Section of the flat roof model is shown in Fig. 5. The walls of the models are made of double-layer acrylic plates between which water passes through and each wall of the four orientations in the case 1, is divided into 3 parts. The water passing through the wall is heated up by a computer programmed controller with a step control system to keep the surface temperatures of roof and each part of each wall as specified. The inside surface temperatures can be calculated from the following heat balance equation at the inside surfaces of the real exterior envelope of the building, taking into account radiation exchange among inside surfaces of the building in reference to Fig. 6, and applying thermal response factors to the heat transfer across the building enclosure.

\[
\begin{align*}
\sum_{j=0}^{\infty} Y_i(j)\theta_{oi}(n-j) &= \sum_{j=0}^{\infty} Z_i(j)\theta_{oi}(n-j) \\
+ \sum_{j=0}^{\infty} \alpha_{oi} [\theta_{oi}(n) - \theta_{oi}(n)] + q_{oi}(n)
\end{align*}
\]

where sol-air temperature is used as excitation in time series expressed as

\[
\theta_{oi}(n) = \theta_{oi}(n) + a_{oi}I(n)/a_o
\]

For experimental models in water, radiation exchange among interior surfaces can be ignored because water absorbs long wavelength radiation. In calculation, the model surface temperatures contain the radiation component. Therefore these surface temperatures controlled by computer can be considered to produce the same heat transfer as that of a real building.

\( Y_i(j) \) and \( Z_i(j) \) are thermal response factors of the exterior envelope components. As a result of heat transfer across the sunlit roof and walls, the inside surface temperature is raised and convective heat transfer \( q_{oi}(n) \) takes place along the inside surfaces. The heat transfer then at the inside
Hybrid Simulation of a Ventilation System based on Natural Convection (Part 1)

surfaces becomes the driving force of natural ventilation and \( q_{ct}(n) \) can be regarded as equal to the conductive heat flux across the acrylic plates to be expressed by

\[
q_{ct}(n) = \left( \frac{\lambda_n}{\delta_n} \right) (\theta_n - \theta_0(n)) 
\]

Referring to Fig. 7, the convective heat transfer in the laminar range of the boundary layer may be treated as heat conduction to be expressed by the following:

\[
(\lambda_n/\delta_n)(\theta_n - \theta_i) = (\lambda_n/\delta_i)(\theta_i - \theta_o) 
\]

On the other hand, the surface temperature \( \theta_{ai}(n) \) is calculated by a digital computer using the values obtained from experiments with on-line connection from

\[
\theta_{ai}(n) = \sum_{j=0}^{\infty} Y_i(j)\theta_{ai}(n-j) - \sum_{j=1}^{\infty} Z_i(j)\theta_{ai}(n-j) + \sum_{k=1}^{\infty} \alpha_{ai} \theta_{ai}(n) - \theta_{ai}\lambda_n/\delta_n 
\]

Fig. 8 shows the flow chart explaining the whole process of hybrid simulation. In the beginning the surface temperatures are computed by finite difference method. Then the results are put into the response factor equation (4) as initial conditions, and new surface temperatures as well as heat transfers are calculated. The new results are then used to set the initial conditions of the experimental temperatures and heat flows. The above process is applied to eliminate as much as possible the effect of the initial assumptions on the hybrid simulation.

Here it is necessary to set the surface temperatures of nine hours before the start of the experiment. These temperatures are calculated using the finite difference method, where the walls and the roof thickness is divided into 4 sections, with 6 equations including boundary heat balance to be solved simultaneously.

The climatic conditions of the calculation are the same as that of the response factor method used during the experiment process.

Assuming a constant room temperature of 26°C, the calculation is repeated for four days with the same conditions until stable results are obtained. The surface temperatures thus obtained are put into equation (8) with the assumption of room temperatures of 27, 26 and 25°C respectively for upper, middle and lower parts of the wall and 27°C for the roof and calculated for 24 hours, the results being shown in Fig. 9. The results of this response factor method calculation is used to set the surface temperatures of 9 hours before as well as the first hour of the experiment corresponding to 9 a.m.
Fig. 9 Calculated surface temperatures with assumed room air temperature for setting up initial conditions of hybrid simulation, against given direct normal radiation and outside air temperature.

Here in order to find the rate of convective heat flow $q_{ci}(n)$, the convective heat transfer coefficient was assumed to be 3 kcal/m²·h·°C for the wall and 1 kcal/m²·h·°C for the roof. During the experiment the $q_{ci}(n)$ is obtained from the model measurement.

The measured inside surface temperature is compared each time to the calculated temperature and the power released by the heater is adjusted so that the measured inside surface temperature can become equal to the calculated temperature.

During the experiment process in the case of the flat roof model, as the model room temperature could not be controlled, in order to converge the measured room temperature to the calculated one, each time an average was taken bringing the model room air temperature $\theta_{rn}$ closer to the actual room temperature $\theta_{ra}$, as expressed by,

$$\theta_{rn}(n) = (\theta_{ra}(n) + \theta_{rn}(n))/2 \quad \cdots \cdots \quad (9)$$

However later on it became clear that the convergence of $\theta_{ra}$ and $\theta_{rn}$ was not necessary, as the room temperature measured directly from the experiment was needed to calculate the conductive heat transfer coefficient. In the next experiment with the sloped roof model to be described in the accompanied paper (Part 2), the mentioned calculation was omitted, thus improving the process of the experiment. Anyhow, Eq. (9) had no effect on the experiment results.

To convert $q_{ci}$, rate of convective heat flow, of water to the air, equation (6) is used. As $\Delta\theta$ in water and air are equal, from the ratio $q_{ci}$ of water to air the following relationship can be used:

$$q_{cia} = q_{ciw} (\lambda_a/\lambda_w) (\delta_a/\delta_w) = 0.0083$$

3. Results of hybrid simulation with a flat roof model with four vertical walls

In the case of a building with 20 cm thick concrete walls and a flat roof, the experimental temperature results for every part of inside surface are shown in Fig. 10, for a summer day in Tokyo from 9 to 18 o'clock. At the beginning of the experiment, as it was summer and the tank water temperature could not be decreased easily, to
facilitate and speed up the experiment process, it was decided to increase the calculated temperatures by increasing the room temperature of the initial calculation of response factor method, for nine o'clock.

The allowable range of convergence of calculated and experimental temperatures was set ±1°C, because a smaller range would take a much longer time for convergence.

It is necessary to obtain the heat flux as conductive heat flow within the boundary layer which is about 2 mm for the vertical wall of the model in water. The convective heat transfer coefficient can be obtained from the following equation, regarding heat convection as heat conduction of fluid in reference to Eq. 6.

\[
\alpha = \frac{\lambda_{e} / \delta_{e}}{(\theta_{e} - \theta_{r}) / (\theta_{0} - \theta_{r})} \quad \cdots \cdots (10)
\]

\[
\alpha = \frac{1}{\delta_{e} / (\theta_{e} + \theta_{r})} \cdot \frac{(\theta_{0} - \theta_{r})}{(\theta_{0} - \theta_{r})} \quad \cdots \cdots (11)
\]

The convective heat transfer coefficients can be obtained by measurement of heat conduction at the surface by two methods. One method uses the heat conduction through the acrylic plate and is obtained from the equation (6) and the following equation,

\[
\alpha = \frac{q_{e} / (\theta_{e} - \theta_{r})}{(\theta_{0} - \theta_{r})} \quad \cdots \cdots (12)
\]

The results are shown in Fig. 11. The other method uses the heat conduction in the boundary layer between the surface and 1 mm distance and between the surface and 2 mm distance from the wall, and employs equations (10) and (11) respectively. The results of the second method are shown in Fig. 12 for vertical south wall and ceiling.

The convection of air within an enclosed space is called natural convection, while the convection in the free space is called free convection. As there is a constant ventilation between the doors and the top opening, the conditions of convection in this experiment is comparable to that of the free convection as well. The free convection thus described is usually calculated using Ra number by equation (1) and the following equation.

\[
\alpha = Nu(\lambda / d) \quad \cdots \cdots (13)
\]

In equation (1) it is generally accepted that in the case of the vertical plane for $Ra < 10^5$, $c\approx 0.56$ and $b=1/4$, and for $Ra > 10^5$, $c\approx 0.13$ and $b=1/3$; for the roof $c\approx 0.27$ and $b=1/4$ [9].

The measured temperature difference between the inside surface and the room was put into equation (1). The convective heat transfer coefficient in the experiment with water was obtained from equation (13) and converted to the value of air as shown in Fig. 13.

Comparisons were made between the convective heat transfer coefficients obtained by the above three methods. The difference between the heat transfer coefficient shown in Figs. 11 and 12 and those obtained by using $Ra$ in Fig. 13 was found to be quite small for vertical walls. While the difference for the flat roof was found considerable and the reason may be explained as follows. In
the case of a room with a flat roof, a part of the heat from the walls is transferred to the ceiling along with the rising air, thus making the inside roof surface temperature higher. Moreover, a continuous ventilation from the doors to the top opening makes the room air temperature decrease. Thus the temperature difference between ceiling surface and room air becomes greater and a higher heat transfer coefficient is obtained from the formula of free convection using $Ra$ number.

As shown in Fig. 11, the convective heat transfer coefficient may become negative. For example, in the morning when surface temperature of the west wall is lower than the room air temperature, the heat may flow from the middle of the room towards the surface in the reverse direction to that of the conduction heat flow within the surface boundary layer, as shown in Fig. 14. The convective heat transfer coefficient in such a case is negative.

On the other hand, convective heat transfer coefficient for free convection obtained by $Ra$ number is always positive regardless of the directional condition of the heat flow.

It can be concluded therefore, that the $Ra$ number with the constants commonly used could not be simply applied to the natural heat convection within a combination of free and enclosed space with a complicated system of heat flows.

Conclusion

A method of hybrid simulation for a natural ventilation system is proposed and actual attempts were made with scaled down models using water as fluid media to verify this method to be possible. The major findings obtained from the experiments were as follows:

1) Inside surface temperatures in all wall parts and roof could be set to the values calculated on-line within a range of $\pm 1^\circ\text{C}$, which shows that the experimental set is acceptable considering the accuracy level of the experiment.

2) The convective heat transfer coefficient obtained through the hybrid simulation method presented, is considered applicable to a building with a configuration and characteristics similar to those employed in this paper. As the calculation is made with the assumption of the thermal conductivity to be constant, when the surface temperature is low, the conductive heat transfer coefficient may have a decrease of up to about 5% caused by the temperature difference. This error can be neglected.

3) There was a general agreement on convective heat transfer coefficients obtained by different methods. The convective heat transfer coefficient of roof obtained from heat conduction across the acrylic plate and from the heat conduction between surface and 1mm and 2mm distances from the surface, were found to be almost identical, but the coefficient of roof obtained for free convection using $Ra$ number was larger than the above coefficients. The reason was described by the rising warm air along the walls to the ceiling and the decrease of room temperature caused by the continuous natural ventilation.
4) Where the direction of heat transfer within the surface boundary layer is opposite to that of the heat transfer between the surface and the middle of the room, the convective heat transfer coefficient becomes negative. This phenomenon cannot be explained by the results of the heat transfer coefficient obtained using $Ra$ number.

5) From the conclusion 3) and 4), it can be concluded that in a natural convection within a combination of free and enclosed space with a complicated system of heat flows, the $Ra$ number with the constants commonly used cannot be applied completely.

In such a case the hybrid simulation method can provide acceptable results.

References

2) T. Nishioka: Research about utilizing an air inlet in floor, to provide ventilation in a high temperature factory No. 5 (in Japanese), Proceedings of AIJ, (1980-9), pp. 283, 284
4) T. Shoda and H. Tsuchiya: The research about the methods of the model experiment of the room inside air distribution (in Japanese), Trans. SHASE, No. 17 (1981-10), p. 4
10) Same as 9), p. 29

(Received July 7, 1986)