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THE PASSIVE SOLAR HEATED SCHOOL IN WALLASEY. III

MODEL STUDIES OF THE THERMAL RESPONSE OF A PASSIVE SCHOOL BUILDING

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SUMMARY

Using information on the dimensions and materials of a room in the Wallasey School, estimates are presented of its response to heat inputs due to the lighting system and the sun. The daily variation due to these influences is calculated using transient and harmonic approaches.

KEY WORDS Solar heating Passive solar design

1. INTRODUCTION

In the first of these articles (Davies, 1986) a description was given of the thermal features of the Wallasey School. This article presents an account of how these features may be expected to function so as to produce an appropriate internal climate.

For most of the year, the inside temperature T_i, required for comfort purposes and broadly in the range of perhaps 17 to 24°C, is higher than the outside temperature T_{ao}; the long-term value of T_{ao} in midwinter is $4\frac{1}{2}$° C. The temperature elevation that the building establishes has the form of the ratio (heat gains—units W)/(heat losses—units W/K) and this permits us to express the daily mean internal temperature in terms of a number of factors.

$$\underbrace{T_i}_{\text{comfort}} = \underbrace{T_{ao}}_{\text{climate}} + \frac{\underbrace{\text{(solar gain)}}_{\text{climate, fabric, site}} + \underbrace{\text{(supplementary heating)}}_{\text{fabric, usage}} + \underbrace{\text{(metabolic heat)}}_{\text{usage}}}{\underbrace{\text{conduction loss}}_{\text{mainly fabric}} + \underbrace{\text{ventilation loss}}_{\text{climate, fabric, site, usage}}}$$

These factors are now better understood by engineers and architects than during the early years of operation of the school but it is useful to discuss them (Section 2). In Section 3 an account of their pattern of interaction is presented and this allows us to solve the set of heat transfer equations so as to estimate the temperature response of the building in specific circumstances.

In a further article an account is given of observational techniques used in an investigation of part of the building and some of the observed results can be compared with the model estimates of this article.

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2. FACTORS AFFECTING THE TEMPERATURE RESPONSE OF THE BUILDING

2.1. *The solar flux on a vertical wall*

The amount of solar radiation falling on a vertical surface is made up of direct, diffuse and ground-reflected components.

The direct component depends on the orientation of the wall, the season of the year and the time of day. For energy conservation considerations, which are largely based on time-averaged analyses, the variation over the day is less important than the total incident energy H. The variation is important in discussing swings of temperature.

Values of the total incident energy on vertical walls during very sunny days are given in Figure 3 of Loudon (1970). The units are MJ/m^2 per day. For a south-facing wall, H has roughly equal maxima in March and late September, and minima in December and June. The June minimum comes about since the sun is high in the sky much of the time and so 'sees' a reduced area of wall.

The variation of H over the year is relatively small. If the wall deviates much from south-facing, the difference between the March maximum and June minimum grows less, and the December minimum decreases. For a north-facing wall, there is a weak maximum in June and a very low minimum in December. On a horizontal surface the total flux in June is around ten times the December minimum.

Values of the total incident energy H on vertical walls on averagely sunny days in the U.K. are given in Table 4.3 of UKISES (1976). In average conditions around half the energy falling on surfaces is direct, and half diffuse. There is little variation in H on a south-facing wall during the summer months, and again the variation over the year is less than for any other orientation.

It follows therefore that to make best use of solar gain the glazing should be near south-facing. Small deviations from south make little difference to overall gain. The further west of south the wall faces, however, the later in the day the time of peak temperature will fall.

Since the sun is always low in sky in winter (about 14° maximum in Wallasey in December), nearby trees, buildings etc. may cause severe shading problems in winter. To make use of solar gain an open unobstructed site is needed and this implies low building density. O'Cathain (1981) has explored this consideration in connection with passive solar housing at Milton Keynes, U.K.

2.2. *The solar wall*

The glazing in the near-south-facing wall of the Wallasey School is usually called the solar wall, and this term will be used here.

Most of the short-wave radiation incident normally and up to about 60° on a sheet of glass is transmitted, the remainder being reflected. As incidence angles approach 90°, the reflected fraction increases.

For clear glass most of the radiation passing through the first surface passes on and is ultimately transmitted through the second surface to the other side. A little is absorbed and thus warms the glass. Absorbing glasses absorb higher fractions. The net solar flux that penetrates to the interior thus consists of a transmitted fraction, together with that part of the absorbed energy which is lost to the interior by convection and long-wave radiation. Work at the U.K. Building Research Establishment at Garston suggests that $S = 0{\cdot}76$ of the total daily mean flux may serve as short-wave heat gain to the interior. The IHVE 1970 Guide (IHVE, 1971) (Table A6.12) lists values for other glasses. They were evaluated in connection with summer overheating.

For double glazing these processes take place at both surfaces, and values of the solar gain factor are less. The values are likely to be affected by details of construction and paintwork etc. of the double glazing.

Higher values of S are to be expected in winter than in summer. In midsummer the sun's rays hit the glass at 90° at the first incidence and around 60° at midday; much of the direct component is reflected away. In winter the rays are more nearly normally incident. The diffuse and reflected components will not be subject to this seasonal variation.

The mechanism has been examined in some detail by the present author. See Davies (1980) Tables 14 (single glazing) and 15 (double glazing) where the variation of S with month is given.

Part of the Wallasey solar wall consists of double glazing with a section of pinboard spaced so as to provide a cavity of some 8 cm width inside the inner leaf. The cavity is sealed on its vertical surfaces and at the bottom; its top is open. A sheet of aluminium is suspended in the cavity, largely filling its superficial area; the sheet may have its bright or its black-painted side facing the sun, according to season. Of the solar radiation which reaches the aluminium panel part is immediately reflected out again. Of the absorbed fraction—the larger fraction in winter—some heats the air in the cavity and this will then be perceived in the room as a convective gain to the air. Some will be radiated and convected to the pinboard, conducted through the pinboard and then lost by further convection and radiation at the visible surface to the room; this process may be complicated by any drawings, etc. which are attached to the pinboards. The aluminium sheets, however, sometimes reach 70°C—too hot to touch. There must be a large loss by convection and long-wave radiation back to the inner leaf of the solar wall and much of this must constitute a loss of heat. The solar gain factor for the area occupied by the pinboard must thus be lower than the double glazing values, but it is not possible to estimate values reliably.

There are further difficulties in arriving at an effective S value for the whole solar wall opposite a classroom. The single glazed window is not vertical; in the closed position and as seen from outside it overhangs the vertical. It is contained in a frame some 64 cm deep with dirty white painted surfaces, both on the exposed sides and the sides facing into the cavity of the solar wall. There are vertical metal plates associated with the top and bottom 'anti-burglar' bars. All these factors must serve to reduce the S value for the single glazed window but it is not known by how much.

Most of the visible sections of double glazing have an inner leaf of diffusing glass. It is not known what effect this might have on the solar gain factor but it might be expected to be relatively slight.

There are also the constructional features of the solar wall: the frames and the walkways which affect the view of one leaf as seen by the other.

Finally, it may be noted that breakages (other than single cracks) in the inner or outer leaf will allow some untoward ventilation of the solar wall cavity. The author has seen as many as 20 such breakages simultaneously at the gymnasium and occasionally there have been instances of pairs of aligned holes in both leaves. The additional loss of heat must depend fairly strongly on wind conditions.

It is concluded that although a solar wall is a simple heat gathering device, the efficiency of the Wallasey wall is difficult to estimate reliably.

2.3. *The greenhouse effect*

The heat convected away from the warm glazing is received immediately by the air. The heat radiated away is received by the solid bounding surfaces of the room and internal furnishings. Since the absorptivity to long-wave radiation of most materials is around 0·9, most long-wave radiation will be absorbed without much internal reflection.

The directly transmitted short-wave radiation is also absorbed at internal surfaces, though a few reflections may occur if the surfaces are light coloured. A small fraction of the reflected radiation may be retransmitted through the window.

The absorbed radiation, long and short wave, heats up the solid material and the heat is variously conducted into the material, lost by convection to the air and by long-wave radiation to other parts of the enclosure. The long-wave radiation will be absorbed by the glass, but glass does not transmit this thermal radiation. Accordingly the glass traps the radiation within the enclosure and so establishes a higher temperature inside than out. This is the greenhouse effect.

Heat will be lost from the enclosure by the normal process of conduction, convection and radiation through both opaque and transparent parts of the structure; heat is also lost by ventilation. The Wallasey School presents no special features in this respect.

2.4. *Supplementary heating*

Solar gain is sufficient to reach design temperature indoors only over a fraction of the heating season. Supplementary heating of some sort must be supplied and is supplied in the school by the lighting system. Its form is unimportant as far as solar gain is concerned. If the heating device is of low power, the body heat of the occupants may contribute significantly to the total heat input.

2.5. *Reduction of heat losses*

In order to restrict the amount of supplementary heating, the opaque parts of the fabric are well insulated and ventilation rates can be restricted.

The supplementary heat needed depends too on the conductance through the glazing. This is normally large ($U_s \sim 3$ W/m^{2}K). (It could be reduced by using special reflecting glass or by incorporating moveable insulation, such as curtaining, for use during the 16 hours of heat loss per day in winter).

2.6. *Smoothing the variation of temperature*

Surfaces presented to the interior should be massive so as to restrain the rate of change of temperature which follows the ingress of large solar gains. The action of the heat storage material can be viewed from two angles:

(i) It will restrain the swings of temperature which result from the large periodic solar heat inputs which occur during a succession of sunny days.

(ii) It will carry the effects of one day's large solar gain through to the following sunless day.

The insulation mentioned above must be placed outside these heat storage surfaces. It will be noted that the effect of the insulation is to keep the mean temperature of the storage surfaces around mean indoor air temperature, which is beneficial in preventing condensation and pattern staining.

2.7. *Prevention of summer overheating*

Although the action of south-facing glazing is to collect most of the available winter sunshine and to reject much summer sunshine, overheating remains a problem to be tackled. The potential passive means available are by preventing its ingress by reflectors and variable insulation in the solar wall, and by ensuring cross-ventilation at night. As mentioned by Davies (1986) night ventilation in the school is not as effective as the architect had hoped.

2.8. *Controls*

The shutters adjacent to the concrete mass walls and the aluminium reflectors should be reversed twice a year, but this is not always done.

Daily control is effected by the use of the lights and ventilators. In winter, the light switches are normally left in the 'on' position and they are switched off during the night by a time clock; the on and off times are set by the caretaker in accordance with the weather and local needs. They have been switched off completely during the Christmas holidays, and then run for 24 hours a day for a few days to restore comfort levels. (It has been remarked that the lighting system provides an illuminated football pitch on the playing fields outside and this may have led to some of the breakages mentioned above). The lights can of course be switched off during the day if they are not needed.

The openable windows in the classroom and the ventilators in the masonry walls provide a ready way of ventilating the building and their use is understood.

3. MATHEMATICAL MODELLING

The climate of the Wallasey School depends more upon the weather (air temperature, wind speed, solar radiation) and upon the way the building is used than does the climate of most public buildings. Two important questions arise:

(i) Do the constructional features discussed in the last section, together with the way in which they are operated, serve to impose an elevation on the mainly hostile outdoor climate, so as to achieve a daily mean indoor climate which can be described as 'equitable'? All seasons of the year require consideration.

(ii) Do the constructional features, again as operated, prove sufficient to prevent excessive daily variations about the mean indoor condition?

The questions can be tackled either by examination of the behaviour of a theoretical model of the building, or by observing it, and both methods have been used. Further, it is important to know whether the users themselves, teachers and children, are satisfied with the temperature field (or any other environmental factor) which the fabric leads to, and this question too has been examined. This article is concerned with modelling.

3.1. *Basis of modelling*

Much of the thermal detail described in the last section can be represented in a mathematical model. It would be very satisfactory to use measured hourly values of the exciting variables and compare predicted and observed values of temperatures. This path has not been followed, however. The difficulties of modelling the solar wall have already been mentioned. The other main technical factor was the absence of observed values for continuous ventilation rate. The techniques of the 1960s did not permit this. A few spot measurements were made using nitrous oxide as a tracer gas and these tests indicated that the ventilation rate could be as low as 0·2 air changes per hour. On the other hand, they could amount to 10 air changes per hour or more. Now the time-averaged response in particular depends very strongly on ventilation. The considerable uncertainty about solar gains and greater uncertainty about a major element of heat loss make a direct comparison of model and observed results hardly possible.

Instead, the behaviour of a quite simple model as it responds to simple idealized inputs (steady state, periodic and transient) will be evaluated and placed alongside the corresponding quantities found from observation. The model may be expected to provide some semi-quantitative insight into the behaviour of the space.

A downstairs room, most typical of teaching spaces generally, will be used for this exercise but some simple properties of a larger portion of the building will be given first.

3.2. *Losses from the central section*

The central section of the building containing the classrooms, apart from the most easterly classroom, is comparatively simple in form. The various areas constituting its envelope, their assumed U values and the resulting conductances are listed in Table I. (No east end walls are to be considered.) A thickness of $12\frac{1}{2}$ cm (5 inches) of expanded polystyrene is assumed to provide a U value of 0·24 W/m^{2}K. The U value of the glazing is necessarily nominal. The conduction of the solar wall provides much the largest heat loss, though the other subtotals are not negligible. The overall U value for the length is 1·0 W/m^{2}K, by no means a remarkable standard of insulation and much in excess of the current U.K. requirements as laid down in FF regulations.

3.3. *Radiative heat transfer*

The radiative transfer between the surfaces of an enclosed space depends upon the emittances ε_1, ε_2, of surfaces 1 and 2 having areas A_1, A_2, respectively. It also depends on the pattern of view factors between the several surfaces. For surface 1, an emittance conductance of size $A_1\varepsilon_1/(1-\varepsilon_1)$ acts between its temperature node at T_1 and its radiosity node B_1. The driving potential is the difference $\sigma T_1^4 - B_1$ where σ is the Stefan–Boltzmann constant. The conductance between the radiosity nodes B_1 and B_2 is $A_1F_{12} = A_2F_{21}$ where F_{12} is the fraction of radiation leaving surface 1 isotropically and intercepted by surface A_2.

To make radiant heat transfer in circuit notation compatible with convective and conductive transfer, the driving potential must be made linear in differences in T (rather than differences in T^4), and this is reasonable when temperature differences are fairly small as in a room. In this case the emittance conductance for surface 1 becomes replaced by $A_1h_r\varepsilon_1/(1-\varepsilon_1)$, and the radiosity node has the value of the blackbody equivalent temperature T'_1; if ε_1 is unity, $T'_1 = T_1$; if ε_1 is zero so that A_1 reflects all radiation falling on it and emits none, $T'_{1/2}$ is determined by the other surface temperatures alone. h_r is around 5·7 W/m^{2}K at room temperatures.

3.4. *A model for heat exchange in a classroom*

The pattern of heat exchanges in a downstairs classroom is indicated in circuit form in Figure 1. It shows four types of solar wall construction (the diffusing area, the pinboard area, the openable windows and the transparent section at floor level). It also indicates the main bounding elements, ceiling, the sloping part of the

Table I. Conductances of the central section

Element	Area, m^2	U-value, W/m^2K	AU, W/K
Solar wall: central length of 35·5 m			
double glazed area	262·2	2·8	734·2
single glazed area	32·6	5·7	185·8
	294·8		920·0
North wall ground floor:			
west of toilets	11·1	0·24	2·7
solar wall of toilets	26·9	2·8	75·3
east of toilets	39·2	0·24	9·4
beam adjacent to intermediate floor	28·1	0·24	6·7
	105·3		94·1
North wall first floor:			
insulated area	67·1	0·24	16·1
louvred area	18 1	2·1	38·0
	85·2		54·1
Ground floor	394·4	0·34	134·1
Roof:			
solid area	424·9	0·24	102·0
skylights*	1·5	5·7	8·5
	426·4		110·5
Totals	1306·1		1312·8

* These areas have since been double glazed.

ceiling, east, west and north walls, door and internal furniture, and the 'corridor'. It omits further detail such as the presence of the blackboard, glazed/opaque/glazed parts of the classroom north wall and the full length wall bench and pinboard in the east wall, all of which will affect the room response to some extent (and will be included.)

Each surface j is represented by a temperature T_j which represents its mean temperature (obtainable in principle by multiple measurements and averaging). Convective exchange takes place between T_j and the mean air temperature T_{ai} in the room (T_{ac} in the corridor). The heat transfer coefficient h_c depends upon temperature difference and surface orientation. It is often taken to be around 3 W/m^2K at vertical walls and a larger or smaller value at horizontal surfaces, according to the direction of heat flow. The small differences, however, between air and surface temperatures that may exist for some periods may lead to low values. If there are n identifiable surfaces in the room, there will be n convective conductances of type $A_j h_c$.

There will also be n equivalent radiation nodes, T'_j, denoted by dots in Figure 1, n emittance conductances and a possible total of $\frac{1}{2}n(n-1)$ view factor linkages between them. (The conductances between the sections of solar wall are zero, since one cannot intercept radiation from another.)

For the sake of clarity, the convective and radiative linkages are omitted from Figure 1.

The walls have conduction and storage properties as denoted. Furnishings can be represented by an area alone. The air capacity is usually taken to be negligible and this is reasonable in a small room.

The ventilation loss is an exchange between T_{ai} and the ambient air temperature T_{ao}. Heat lost from the building envelope via the solar wall and the corridor north wall may be taken to be lost to T_{ao}.

The double glazed translucent sections of the solar wall will be modelled in a straightforward manner, though the performance must be affected very considerably by the horizontal surfaces at ground level and near

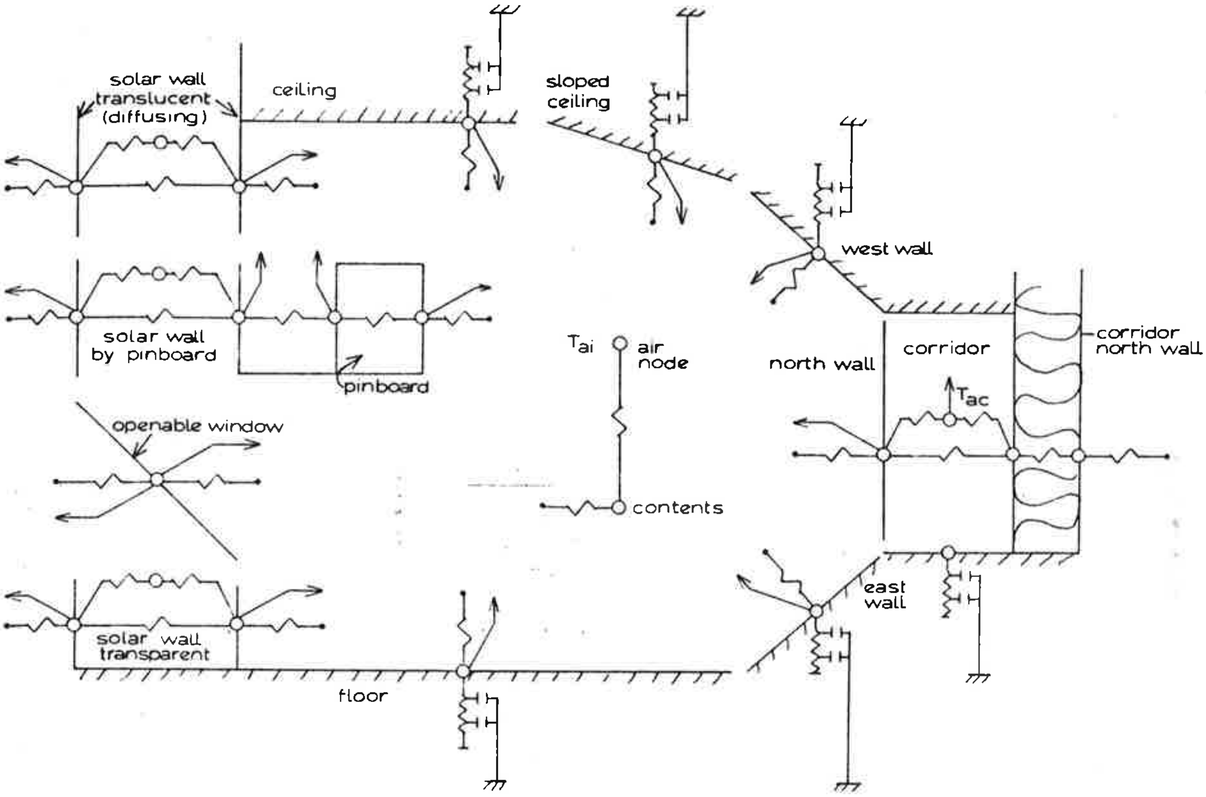

Figure 1. A diagrammatic representation of the principal heat transfer processes within an enclosure. The arrow headed connections indicate a link with inside or ambient air temperature. The dots denote blackbody equivalent nodes associated with radiating surfaces; these nodes are interconnected if the corresponding surfaces can exchange radiation

the ceiling, which redistribute light falling on them. The single glazed windows set at a varying depth in partially reflecting frames, with incident radiation intercepted by the anti-burglar bars, and usually dirty, defy modelling. Single glazing would normally admit more gain than double glazing, but the housing and orientation of these windows must reduce the gain somewhat and for the present purpose, the single glazed area will be simply lumped with the translucent glazed area.

An attempt has been to model the pinboard area separately.

The floor is modelled as such. There are problems in evaluating the conductance between the floor surface and T_{ao}. Indeed it could be argued that the long-term heat transfer from the floor takes place under the action of the yearly mean subground temperature of 10°C.

The ceiling and east and west walls are all heat storing volumes limited by notional adiabatic planes. The horizontal and sloping portions of the ceiling, which provide different storage are modelled separately. The east and west walls are lumped. The north wall, however, is lightweight. It may be expected to heat up relatively quickly under the action of solar gain and so provide a means of raising air temperature.

Since much of the radiation entering falls downward onto the desks and chairs, an attempt will be made to include its effect, at any rate qualitatively, in the model. There are two reasons for this. The hottest location in the room must be the location of heat input. From that point the heat diffuses to locations of lower temperature and is finally lost. Solar gain is short-wave gain, absorbed at solid surfaces. These surfaces will in consequence be the 'hottest' part of the enclosure—hotter than the air for example. However, four of the six bounding surfaces are massive, and will respond only slowly to large solar inputs. The air response due to contact with them will be correspondingly slow. Now radiation falling on lightweight surfaces will cause a comparatively rapid change in surface, and so air, temperatures. The light walls in the classroom are the north wall, and the pinboard attached to the solar wall. However, all internal furnishings, desks, tables, curtains, the blackboard, etc. are also relatively lightweight, and of an area comparable with the north wall; their presence must substantially affect the air temperature.

The second reason is that the temperature of the contents, an equilibrium temperature resulting from the air and the radiant field within the enclosure, is the most representative single environmental measure with which to describe the enclosure, and it will be used in this way. Globe temperature is probably the most closely associated objective measure. Other temperatures (floor, etc.) may be useful in that the observational study provides measures for them.

The various solid surfaces and the air (node 17) in the model (Figure 2) are given odd numbered suffixes and the corresponding blackbody equivalent nodes are given even numbered suffixes. The delta configuration of conductances between the even numbered nodes will be simplified as indicated by Davies (1983) to a star configuration, and this provides node 18, mean radiant temperature.

The translucent and transparent areas of the solar wall admit short-wave radiation which physically falls on the internal surfaces and furnishings. Most of the surfaces are light coloured, so the radiation suffers multiple reflections before it is finally absorbed. If the reflections took place diffusely, the exchange could be described by the delta configuration of conductance, and so by the star configuration. Short-wave values of emittance for the emittance conductances, however, are appropriate.

The radiation falling directly on the north wall for example is to be treated as though totally absorbed at the

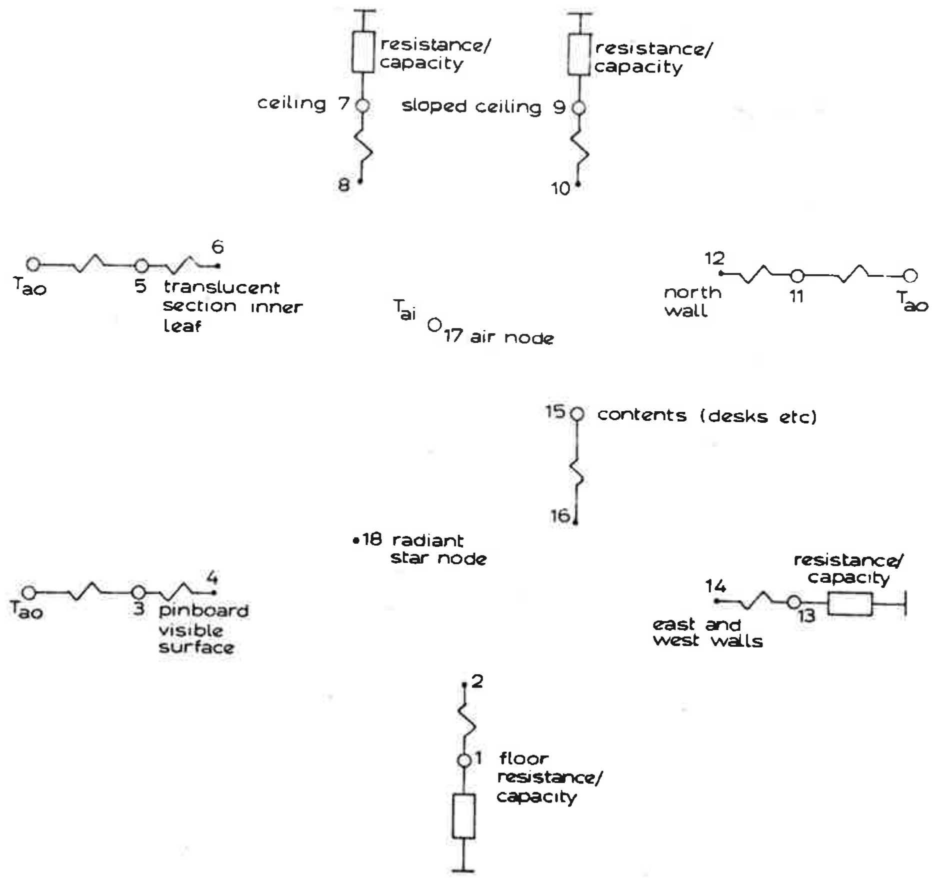

Figure 2. A simplification of Figure 1. Seven separate bounding surfaces are represented together with their blackbody equivalent nodes. The room contents are included, together with air temperature. The interconnections between the blackbody nodes are simplified to individual connections with a radiation star point. This circuit is used to estimate the response to strong sunshine

blackbody equivalent node, number 12. Short-wave radiation, however, is eventually absorbed at the solid surfaces where it serves as pure heat sources. It does not interact with other heat transfer processes.

Accordingly the modelling of solar gains must proceed in two stages. First the short-wave solar radiation entering the room is to be redistributed using the radiation star network and short-wave emittance conductances. The receiving nodes 1, 3, 5, 7, 9, 11, 13 and 15 have to be taken to be at zero temperature and the energy flows to them constitute pure heat sources. (Any short-wave radiation lost from the enclosure as short-wave energy could be allowed for at this point.)

In the model to be evaluated, it will be assumed that the short-wave energy entering through the diffusing glass above the pinboards enters very largely diffusely, and that it falls upon the remaining surfaces in an amount proportional to the view factor F_{5j} from the diffusing area 5 to the various receiving areas j. The approximate values for F_{ij} given by Davies (1984) will be used. (Since the radiation probably enters, not completely diffusely, but with a forward component, this procedure probably underestimates the radiation falling on the north wall, but the choice of a non-unity value of ε may correct to some degree the error this assumption introduces).

In the second stage, internal hot body sources—the lights (which provide very little short-wave energy) and the occupants—output radiantly to the blackbody equivalent nodes, and convectively to the air. Solar gain absorbed in the glass provides a hot body source at the inner leaf (node 5). Over the pinboard section, a subsidiary calculation enables the heat absorbed in the glass and in the aluminium sheet to be treated as heat sources to node 3 (the visible pinboard surface) and node 17 (air). Long-wave exchange is now to be described by the star network, but of course using long-wave values for emittance.

The conductances depend upon room dimensions and assumptions about the heat transfer processes and choice of heat transfer coefficient to be attached to them. Details are given in the appendices.

The thermal model of Figure 2 is required to provide an answer to the problem: given the conductances C_{ij} between the nodes, given ambient temperature T_{ao} and perhaps ground temperature T_g, and given the heat inputs at each of the inside nodes, find the temperature at each of the nodes.

Heat flow continuity at each of the 18 nodes leads to a set of equations of the form

$$\mathbf{GT} = \mathbf{Q}$$

or

$$\begin{bmatrix} g_{11} & g_{12} & g_{13} & \cdots \\ g_{21} & g_{22} & g_{23} & \cdots \\ g_{31} & g_{32} & g_{33} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \end{bmatrix}$$

The diagonal terms g_{11}, g_{22} etc. denote the sums of conductances attached to node 1, node 2 etc. including the links with the reference temperature and T_{ao} and T_g if applicable. For example, the floor surface has internal links with its blackbody equivalent node ($C_{1,\,2}$) and with the air node ($C_{1,\,17}$). In steady-state conditions it has a linkage with the mean ground temperature notated here as $C_{1,\mathrm{r}}(0)$, where the subscript r denotes the real part of a complex conductance, and (0) denotes the harmonic, here the zeroth or steady state. For the first harmonic (period 24 hours), there are real and imaginary components $C_{1,\mathrm{r}}(1)$ and $C_{1,\mathrm{i}}(1)$. Thus in handling the first harmonic of excitation

$$g_{11} = C_{1,\,2} + C_{1,\,17} + C_{1,\mathrm{r}}(1) + C_{1,\mathrm{i}}(1)$$

The complex conductances are only associated with the volumes which provide storage, namely the floor and ceiling and east/west surfaces.

The off-diagonal terms are such that

$$g_{ij} = -C_{ij}$$

g_{ij} is zero for a pair of nodes if there is no direct heat transfer between them in the model. Thus $g_{17,\,18} = 0$ for example, since no heat can be exchanged directly between the fictitious mean radiant temperature node and the air.

The vector of terms $q_1, q_2 \ldots$ denotes the heat flow to the nodes indicated. q_j includes the term $C_{0,\,j}T_{ao}$ for nodes 3, 5, 11 and 17 which link directly with T_{ao}. In order to find the effect of a fixed ground temperature, q_1 must include the term $C_{1,r}(0)T_g$.

Thus we have a set of 18 simultaneous equations, which can be solved routinely. Since storage is involved, a computer procedure handling complex quantities is needed, and the resulting temperatures T_1, T_2, etc. have real and imaginary components. In evaluating this model, the steady state and 10 harmonics were used in order to estimate the variation in temperature throughout the day that may result from strong sunshine.

3.5. *Questions to be asked of the model*

In the U.K. winter daily mean ambient temperatures are typically around 5°C, and frequently go down to −1°C. The preferred comfort temperature in the Wallasey School is about 19°C. Thus the sources of heat in conjunction with the building have to achieve an elevation of up to 20K. In summer conditions the desirable elevation may be only a degree or two. The purpose of evaluating a simple thermal model of the building is to examine whether the available sources of heat and the means of control enable comfort temperatures to be achieved. Thus we require answers to the following questions:

(a) What temperature might be established in the school in the absence of any heat source or solar gain? The conventional answer is that ambient air temperature T_{ao} is established, but the constant ground temperature of 10°C might have some effect.
(b) What temperature elevation may the lights, when left on for 24 hours a day, set up?
(c) What time-averaged elevation might the average irradiance in December and maximum irradiance in September set up? Solar gain is least effective in December in meeting heat needs. The solar gain on a south-facing wall is largest in March and September, but overheating is unlikely to cause problems in March.
(d) What time-averaged elevation might result from the presence of a class of children during part of the day?
(e) What daily variation in temperature is likely to result from strong September sunshine?
(f) What daily variation in temperature is likely to result from the large daily variation in ambient temperature that sometimes occurs in midsummer?
(g) What daily variation in temperature is likely in winter due to regular intermittent use of the lighting system for heating purposes?
(h) The observational study provides values, among other quantities, for the inside and outside air temperatures and the heat inputs that establish the difference. On a daily basis this information can be processed so as to provide values for the air node conductance:

$$C = \text{air node conductance} = \frac{\text{daily heat input at the air node}}{\text{daily mean temperature difference}}$$

How may C be expected to vary with ventilation rate and choice of convective heat transfer coefficient?

3.6. *The effect of ground temperature*

It is usual to assume that the heat loss to the ground is lost to a sink temperature equal to ambient, but this assumption has been challenged recently (Spooner, 1982). The estimates of temperature within the room as a function of ambient temperature and ventilation rate, with a fixed ground temperature of 10°C, are shown in Table II. When $T_{ao} = 10$°C all temperatures must be 10°C. The warm earth temperature might lead to ground floor temperatures being half a degree or more warmer than conventional estimates suggest.

3.7. *Elevation due to the lighting system*

There are 6 lamp fittings which have been variously provided with 150 or 200 W bulbs, and also a blackboard lamp with a 100 W bulb. A total of 1200 W will be assumed. According to Henderson and Marsden (1972, p. 126), bulbs output 6 per cent of their energy as visible radiation, 75 per cent as long wave, and 19 per cent by convection. The bulbs are contained within luminaires however, and these fractions will be assumed to

Table II. Contents temperatures resulting from an assumed ground temperature of 10°C

Air temperatures °C		Ventilation rate (air changes/h) 0	1	4
0	floor	1·6	1·1	0·7
	contents	1·4	0·9	0·5
5	floor	5·8	5·6	5·4
	contents	5·7	5·5	5·2
10	floor	10·0	10·0	10·0
	contents	10·0	10·0	10·0
15	floor	14·2	14·4	14·6
	contents	14·3	14·5	14·8

change to 5, 50 and 45 per cent, respectively. Thus $0{\cdot}45 \times 1200 = 540$ W will be assumed to act at node 17, the short wave will be lumped with the long wave, and the resulting flux is assumed distributed between the various nodes in proportion to their area. (For this purpose, 'floor area' was taken as actual floor area, less the area of furnishings).

The elevations at the various surfaces and air are shown in Table III. The temperature of the glass is the lowest estimated temperature since the resistance between the glass and the ambient temperature is the lowest such resistance.

3.8. *Elevation due to solar gain*

Solar intensity and duration are lowest in December and conditions are predominantly cloudy. An average December total irradiance on a south-facing vertical wall might amount to around 2 MJ/m^2 day and this value will be used. The handling of this radiation by the pinboard and translucent portions of the solar wall is explained in Appendix II. It is sufficient to remark here that the input over the pinboard area is treated as input at nodes 3 and 13. The input from the translucent area is taken to be input as short-wave radiation at nodes 2, 8, 10 and 12. It is distributed between them in proportion to the radiation conductance from node 6 (the blackbody equivalent node) to nodes 2, 8, 10, 12, 14 and 16 as calculated by the author's approximate view-factor expression. (Logically this expression would impose a heat flow from node 6 to node 4, but this is physically not possible. Since the diffusing glass of the translucent section probably has a sizeable forward diffusing component, the intensity on the north wall may be underestimated and so the 'contribution' to node 4 is taken to act at node 10. This is a totally *ad hoc* arrangement.) This short-wave inputs at the equivalent nodes were then transformed to thermal inputs at the surfaces. The fraction of radiation absorbed in the inner and

Table III. Temperature elevations resulting from 1200 W lighting system, running continuously

Ventilation rate, air changes/h	Floor	Pinboard	Glass	Ceiling, walls	North walls	Contents	Air
0	13·4	12·1	7·4	13·8	13·4	13·9	14·0
0·5	10·4	9·4	5·7	10·7	10·4	10·8	10·6
1	8·5	7·7	4·7	8·8	8·6	8·9	8·5
2	6·4	5·8	3·5	6·6	6·5	6·7	6·2
4	4·4	4·0	2·5	4·6	4·5	4·7	3·9

outer leaves of the translucent section is taken to act at node 5, but its contribution is very small. The estimated elevations are shown in Table IV.

To compute the mean temperature rise due to strong sunshine, we use the hourly values for direct solar gain upon a south-facing wall in September (IHVE, 1971):

time (h)	06, 18	07, 17	08, 16	09, 15	10, 14	11, 13	12
W/m^2	0	90	255	425	570	655	685

This leads to an irradiance of about 16·8 MJ/m^2 day. The estimated mean elevations are given in Table V. They show the importance of ventilation control in sunny weather.

3.9. *Elevation due to occupants*

The elevations to be expected from the body heat of the occupants can be estimated by scaling the results for the lighting system.

The heat output from a child depends on his/her size and state of activity. The input to the classroom further depends on the number of children present in any class and the effective length of occupation, and these are known to vary enormously.

However, a high value for the daily input might be obtained as the product of 30 children present, each with an output of 80 W, for 5 hours per day. (The school day is 7 hours, but allowance must be made for lunch-time and breaks.)

Averaged over the day, this amounts to 500 W. This would normally be negligible, but it is comparable with the output from the lighting system as normally used. In round terms, the elevation due to the occupants might be a little less than half that the lights supply, when used full time. This density of occupation does not occur often.

3.10. *Daily variation due to solar gains*

The peak temperatures reached in rooms with large unobstructed south-facing glazed walls can become unacceptably high during sunny periods. The variation in temperature can be restrained by the presence of sufficient thermal storage and it is useful to see how the storage provided by the concrete ceiling and floor of the classroom, together with its double thickness brick east and west walls, serves to control solar gains.

Table IV. Mean temperature elevations resulting from average December solar radiation

Ventilation rate, air changes/h	Floor	Pinboard	Glass	Ceiling, walls	North walls	Contents	Air
0	3·5	3·7	1·9	3·6	3·6	3·7	3·5
0·5	2·8	3·1	1·5	2·9	2·8	2·9	2·7
1	2·3	2·6	1·3	2·4	2·3	2·4	2·2
2	1·8	2·2	1·0	1·8	1·8	1·8	1·6
4	1·3	1·7	0·7	1·3	1·3	1·3	1·0

Table V. Mean temperature elevations resulting from strong September sunshine

Ventilation rate, air changes/h	Floor	Pinboard	Glass	Ceiling, walls	North walls	Contents	Air
0	24·7	26·0	13·6	25·4	24·8	25·4	24·7
1	16·1	18·4	8·9	16·6	16·2	16·6	15·1
5	8·0	11·1	4·5	8·2	8·1	8·2	5·9
10	5·7	9·0	3·2	5·9	5·8	5·9	3·3
20	4·3	7·8	2·5	4·5	4·4	4·4	1·8

Calculations of this sort can be performed using harmonic anlaysis, response factors or finite difference methods. The first method will be used here. Hourly values of irradiance on a south-facing wall are chosen for the month of September, as quoted above in Section 3.8. The daily total of radiation on the wall amounts to some 16·8 mJ/m^2 day. The profile is then represented by its steady-state term and 10 harmonics. The real and imaginary components of the surface admittances for each harmonic were computed as indicated in the appendices. The remainder of the thermal details are as shown in Figure 2.

Thus for each harmonic, heat inputs and conductances are known and so the real and imaginary components of temperature $T_{n,r}$ and $T_{n,i}$ for each location. The daily profiles of temperature elevation can then be computed in the usual way:

$$T_t = \sum_{n=0}^{10} [T_{n,r} \cos(2\pi nt/24) + T_{n,i} \sin(2\pi nt/24)]$$

This is illustrated for three ventilation rates in Figure 3.

As noted earlier, the ventilation rate has a major effect on the general temperature levels in the room. It has less effect on the daily variation in temperature. The floor is the most stable and the contents the least. The temperature variations are noted in Table VI.

3.11. *Daily variation due to ambient air temperature*

During June the temperature of the ambient air may vary throughout the day by ± 7 K about its mean value. The variation is sufficiently smooth that for the present purpose its variation can be represented by its first harmonic only.

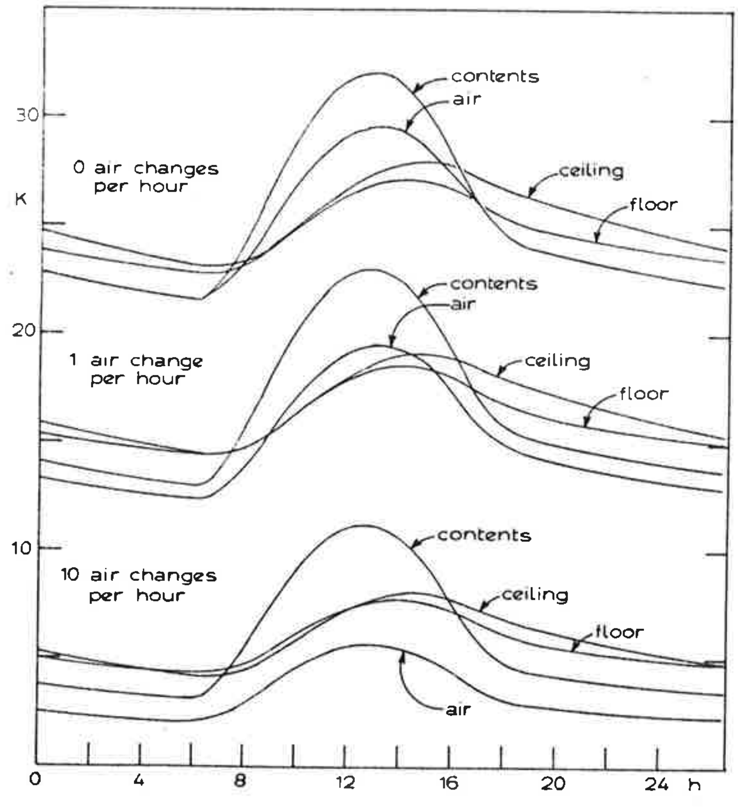

Figure 3. The estimated response of the enclosure to strong September sunshine. The temperatures indicated are the elevation above mean ambient temperature. (Actual inside temperature would be higher by an amount of about 13°C which is the daily mean temperature during September.) The Figure illustrates the very dominant effect of ventilation on the general levels of temperature

Table VI. Daily temperature variation (maximum – minimum) due to solar radiation of about 17 MJ/m² day incident on the solar wall

Ventilation rate, air changes/h	Floor	Ceiling	Contents	Air
0	4·4	4·9	10·5	8·0
1	4·2	4·8	10·0	7·2
5	3·8	4·2	8·8	5·1
10	3·5	4·0	8·1	3·7
20	3·1	3·6	7·4	2·4

The magnitude and time of maximum are shown in Table VII. The variation at the massive surfaces is relatively small and the time of maximum is around 3 hours after the maximum in ambient temperature. The temperatures of the non-capacitative elements are larger and more nearly in phase with ambient. All temperatures increase with ventilation rate.

3.12. *Daily variation due to switching the lights on and off*

In winter, the lights provide the only assured source of heat. They can be switched on and off by a time clock set by the caretaker and, as a result, the building is subject to step excitation at regular intervals.

The harmonics approach to modelling the response is possible, but transient methods seem better suited and two forms of transient modelling have been used, one of which lumps together the whole of the thermal capacity, but treats it as a distributed resistive/capacitative quantity, whereas the other method simply treats the mass of the enclosure as made up of two separate capacities but with no resistive component.

3.12.1. *A single resistance/capacity model for the classroom.* A solution to the following problem appears frequently in mechanical engineering heat transfer texts: a slab of material of thickness X is initially at constant temperature T_{in}, one surface is adiabatic, the other is wetted by a fluid, with a film coefficient of h. At time $t = 0$, the fluid temperature changes to a value T_u and stays there. The subsequent temperature T_s of the surface is then given as

$$\theta = \frac{T_s - T_u}{T_{in} - T_u} = \sum_{n=1}^{\infty} \frac{2B}{B + B^2 + u_n^2} \exp\left(-\frac{u_n^2 kt}{\rho c X^2}\right) \qquad (1)$$

where $B = hX/k$ and u_n is the nth solution of the eigenvalue equation $\tan u = B/u$.

θ proves to be a function of two dimensionless variables which are best chosen to be the Biot number B, together with time, expressed as

$$\xi_2 = \frac{h^2 t}{k \rho c}$$

In the early stages of cooling or heating θ is independent of B; after a value of the Fourier number, $kt/\rho c X^2$ equal to 0·15 or so, θ becomes dependent additionally on B. These matters are discussed by Davies (1978) whose Table 4 lists values of ξ_2 for certain values of θ and B.

Table VII. Temperature variation due to a sinusoidal variation on -5 to $+5$ K in ambient temperature, and time lag (hours)

Ventilation rate, air changes/h	Floor K	Floor h	Ceiling K	Ceiling h	Contents K	Contents h	Air K	Air h
0	0·4	3¼	0·4	4¼	0·9	1½	0·9	1½
1	0·6	3¼	0·7	4	1·4	1¼	1·8	¾
5	1·2	3	1·3	4	2·8	1	4·2	½
10	1·5	2¾	1·7	3¾	3·6	1	5·8	½
20	1·8	2¾	2·1	3¾	4·4	1	7·3	¼

This solution does not appear to have been used in connection with building thermal response when step changes are imposed on the structure, yet it is reasonably well suited to estimating the response to the intermittent output from the lighting system and will be used here. The formulation has to be generalized, however. First, the single heat loss mechanism h above has to be extended to express the convection, radiation, conduction and ventilation mechanisms that actually lead to heat loss from the floor, walls and ceiling. Secondly, the 'fluid temperature change' mentioned above has to be generalized to express the fact that the change in the classroom temperature comes about, not because of any temperature change as such, but due to a change in heat input, which results from switching on the lights. Thirdly, the algebra is best handled in matrix form. Fourthly, the thermal capacity of the simple slab has to be generalized to include the thermal capacity of all the massive surfaces (exact equivalence is not possible here).

If the storage provided by the walls, floor and ceiling is taken to be lumped, an analysis based on the above solution is comparatively simple. If however, two separate heat stores are to be included, the equation for the eigenvalues and the equation for the temperature distribution become more complicated. The present author (Davies, 1984) has shown how such a set can be evaluated. The model involves a massive outside wall and is not relevant to the Wallasey School, but the complication is sufficient to discourage any attempt to develop a transient solution for the model of Figure 2. The simpler model of Figure 4 will be used.

To effect the first three of these generalizations, we note the elementary heat balance equation at the surface of the simple slab:

$$-k\frac{\partial T_s}{\partial x} = h(T_s - T_u)$$

If the slab has an area A, this can be written

$$-\frac{\partial T_s}{\partial(x/X)} = \frac{Ah}{Ak/X}(T_s - T_u) = B(T_s - T_u) \qquad (2)$$

$$= \frac{\text{conductance between the } T_s \text{ node and the reference temperature}}{\text{slab conductance}}(T_s - T_u)$$

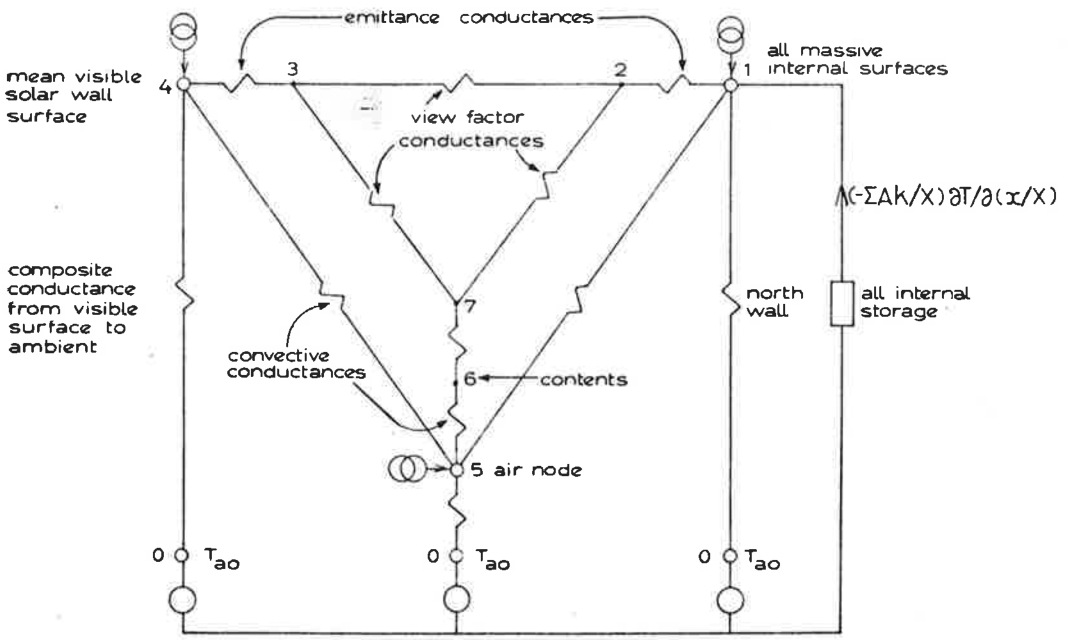

Figure 4. A further simplification in which the entire mass is represented as a slab of finite thickness in order to calculate the transient response

Turning to the thermal circuit expressed as in Figure 4, the heat flow arriving at node 1 by conduction from the various massive surfaces is

$$q = -k'A'\frac{\partial T'}{\partial x} - k''A''\frac{\partial T''}{\partial x} - \ldots$$

which can be written

$$q = -\frac{k'A'}{X'}\frac{\partial T'}{\partial(x/X')} - \frac{k''A''}{X''}\frac{\partial T''}{\partial(x/X'')} - \ldots$$

and this is approximately

$$q = -\left(\sum \frac{kA}{X}\right)\frac{\overline{\partial T_1}}{\partial(x/X)}$$

where summation takes place over all surfaces providing thermal storage and the right term denotes the mean gradient. Then at node 1 we have

$$q_1 = -\left(\sum \frac{kA}{X}\right)\frac{\partial T_1}{\partial(x/X)} + (T_0 + T_1)C_{01} + (T_2 - T_1)C_{12} + (T_5 - T_1)C_{15} = 0$$

Thus the term g_{11} in the **G** matrix of the last section becomes

$$\sum \frac{kA}{X}\frac{\partial}{\partial(x/X)} + C_{01} + C_{12} + C_{15}$$

and the term e_1 in the excitation matrix becomes simply q_1. The equation

$$\mathbf{GT} = \mathbf{e}$$

cannot now be solved directly because of the presence of the operator term $\sum(kA/X)\partial/\partial(x/X)$. The set of equations amounts to a differential equation, which we have to write down.

We formally solve for T_1:

$$|\mathbf{G}|T_1 = |\mathbf{E}_1|$$

$|\mathbf{E}_1|$ denotes the determinants of the matrix formed when the **e** vector elements replace column 1 in the **G** matrix, as usual:

$$|\mathbf{E}_1| = \begin{vmatrix} q_1 & -C_{12} & & \\ q_2 & C_{12}+C_{23} & -C_{23} & \\ q_3 & -C_{23} & C_{23}+C_{34} & \cdot \\ \cdot & & -C_{34} & \cdot \\ \cdot & & & \cdot \end{vmatrix}$$

This constitutes, of course, a 7×7 array. All elements in $|\mathbf{E}_1|$ are known and its value (and other determinant values) were found using the NAG library routine FO3AAA.

Because of the operator term, $|\mathbf{G}|$ cannot be evaluated as it stands and it has to be split into two parts:

$$|\mathbf{G}| = |\mathbf{G}'| + |\mathbf{G}''|$$

where

$$|\mathbf{G}'| = \begin{vmatrix} \sum(kA/X)\partial/\partial(x/X) & -C_{12} & & \\ & C_{12}+C_{23} & -C_{23} & \cdot \\ & -C_{23} & C_{23}+C_{34} & \cdot \\ & & -C_{34} & \cdot \\ & & & \cdot \end{vmatrix}$$

$$|\mathbf{G}''| = \begin{vmatrix} C_{01}+C_{12}+C_{15} & -C_{12} & & \\ & C_{12}+C_{23} & -C_{23} & \\ & -C_{23} & C_{23}+C_{34} & \cdot \\ & & -C_{34} & \cdot \\ & & & \cdot \end{vmatrix}$$

We denote the determinant of the cofactor of g_{11} as:

$$|g_{11}| = \begin{vmatrix} C_{12}+C_{23} & -C_{23} & & \\ -C_{23} & C_{23}+C_{34} & -C_{34} & \\ & -C_{34} & C_{04}+C_{34}+C_{45} & \cdot \\ & & -C_{45} & \cdot \\ & & & \cdot \end{vmatrix}$$

These elements are those of **G**″ that remain when the first row and the first column are omitted. $|g_{12}|$ is similarly the determinant of the array formed by missing the first row and the second column of **G**″, and so on.

Then

$$|\mathbf{G}| = \left(\sum \frac{Ak}{X} \frac{\partial}{\partial(x/X)}\right)|g_{11}| + (C_{01}+C_{12}+C_{15})|g_{11}| + (-C_{12})|g_{12}| + \ldots$$

Also

$$|\mathbf{E}_1| = q_1|g_{11}| + q_2|g_{12}| + \ldots$$

If these forms for $|\mathbf{G}|$ and $|\mathbf{E}_1|$ are substituted into the formal solution for T_1, we have after rearrangement

$$-\frac{\partial T_1}{\partial(x/X)} = \frac{(C_{01}+C_{12}+C_{15})+(-C_{12})|g_{12}|/|g_{11}|+\ldots}{\sum Ak/X} \times \left\{T_1 - \frac{q_1+q_2|g_{12}|/|g_{11}|+\ldots}{(C_{01}+C_{12}+C_{15})+(-C_{12})|g_{12}|/|g_{11}|+\ldots}\right\}$$

This equation is formally similar to equation (2). The quantity

$$C_e = (C_{01}+C_{12}+C_{15})+(-C_{12})|g_{12}|/|g_{11}|+\ldots$$

is the resultant or equivalent conductance between the mean slab surface temperature node (node 1) and the reference temperature.

The quantity

$$T_u = \frac{q_1+q_2|g_{12}|/|g_{11}|+\ldots}{(C_{01}+C_{12}+C_{15})+(-C_{12})|g_{12}|/|g_{11}|+\ldots}$$

is the ultimate temperature reached (assuming that T_0 and T_g are zero), and has the form of a 'flux temperature' (Davies, 1981). If the heat were input only at T_1 itself, the final temperature reached by the slab combination would be

$$T_u = q_1/C_e$$

and this is obvious on physical grounds. A heat input at, say, node 3 alone generates a value

$$T_u = \frac{q_3|g_{13}|/|g_{11}|}{C_e}$$

It is obvious on physical grounds that a heat flux, if applied at node 3, will generate a lower final temperature than if applied at node 1, and this analysis evaluates the scaling factor in terms of the network conductances. More generally T_u is a linear function of the several heat inputs. The Biot number for use in the Groeber solution is

$$B = C_e/\sum Ak/X$$

and eigenvalues $u_1, u_2, \ldots$ follow from this.

The non-dimensionalized time is

$$\xi_2 = \frac{h^2 t}{k\rho c} = \frac{(Ah)^2 t}{(Ak/X)A\rho cX}$$

and this will be generalized to include the total slab conductance $\Sigma Ak/X$ as above, together with the total thermal capacity $\Sigma A\rho cX$. The quantity Ah is the heat loss conductance, here represented by C_e. So

$$\xi_2 = \frac{(C_e)^2 t}{(\Sigma Ak/X)(\Sigma A\rho cX)}$$

Thus direct use can be made of equation (1) to find mean surface temperature T_1 as a function of time, following a step change in heat input. Once T_1 is known, the other temperatures follow by routine solution of the simultaneous equations

$$\begin{bmatrix} C_{12}+C_{23} & -C_{23} & & \\ -C_{23} & C_{23}+C_{34} & -C_{34} & \\ & -C_{34} & C_{04}+C_{34}+C_{45} & . \\ & & -C_{45} & . \\ & & & . \end{bmatrix} \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \\ . \end{bmatrix} = \begin{bmatrix} q_2+C_{12}T_1 \\ q_3 \\ q_4 \\ q_5+C_{15}T_1 \\ . \end{bmatrix}$$

The values for the conductances to be used in this model are listed at the ends of Appendices I and III.

Finally some times associated with transient change may be noted. (i) When a step in the temperature of the fluid within a slab takes place, cooling at the surface at first takes place at a rate independent of the thickness of the slab and the state persists for time given rather roughly by the relation $kt/\rho cX^2 = 0{\cdot}155$ (Davies, 1978, section 6.6). In the present context the time is $t = 0{\cdot}155(\Sigma A\rho cX)/(\Sigma Ak/X)$ or 2 hours. This value is determined almost entirely by the slab properties alone and does not depend on the film conductance. The generalization of this idea from the case of the homogeneous slab to the equivalent slab is somewhat speculative and it is only expected to provide a rough guide for the duration of early cooling.

(ii) After cooling has progressed for some while, the further rate of cooling at the surface is exponential. This phase starts roughly after a time given by $kt/\rho cX^2 \simeq 0{\cdot}5$ (Davies, 1978, section 6.8). So $t = 0{\cdot}5(\Sigma A\rho cX)/(\Sigma Ak/X)$, or about 6 hours. This value is determined mainly by the slab properties, but it depends a little on the film conductance. Again, it is only a rough estimate.

(iii) The response time t_r, or time constant at a point in a system, is by definition the time taken for the response at the point to change by $1-(1/e) = 0{\cdot}632$ of the ultimate change at the point.

For a simple pure-capacity–resistance system t_r = capacity × resistance. The building construction under discussion here is not a pure capacity; it has conducting as well as storage properties, so that the wall material is not at uniform temperature. The Biot number provides the appropriate comparison between the slab resistance X/k and the film resistance $1/h$:

$$B = \frac{hX}{k} = \frac{\text{slab resistance}}{\text{film resistance}}$$

If the slab has a sufficiently low Biot number, by the time the surface temperature has fallen to 1/e of its initial value, the higher eigenfunction contributions have died out and cooling is expressed by the first eigenvalue alone. In this case we have two versions for the response time:

$$t_r = \frac{\rho cX^2}{u_1^2 k} = \frac{\rho cX}{h}$$

In the form appropriate to the current application, these can be written as

$$t_r = \frac{\sum A\rho cX}{u_1^2 \sum Ak/X} = \frac{\sum A\rho cX}{C_e}$$

These forms apply for Biot values less than about 0·4. t_r thus depends only on the thermal capacity properties of the slab and not on its resistance; it depends strongly on the film conductance, C_e in this context. As B increases, the expression breaks down and for values of B greater than about 4 a different form is appropriate (Davies, 1978, Table 4):

$$t_r = 1{\cdot}562\,\frac{k\rho c}{h^2}$$

These times describe the scale of the cooling or heating process; they do not describe exact events.

Values for t_r are listed in Table VIII. The Biot number is less than 0·4. The two versions for t_r are in good agreement but they are based on lumped measures and exact agreement is not to be expected.

The lights in the classroom are switched on and off by the time clock at predetermined times. To estimate the response due to such treatment we can use the superposition principle. Suppose that up to $t = 0$, the wall temperature is zero everywhere and that at $t = 0$, the heat source imposing a set of inputs is switched on and remains on indefinitely. The surface and air temperature can be calculated as indicated above. Suppose that at time t_1, equal 'negative' heat sources are switched on, and stay on indefinitely. The change in temperature due to this excitation is the same as the earlier excitation, except that it is negative, and occurs with a time lag t_1 later. By the principle of superposition, the total change in the sum of the constituent changes; the total heat input is zero, and the total temperature change is the sum of the separate changes. At time t_2, heat is swtiched on, at t_3 negative heat is switched on, etc. In this way the effects of any pattern of heat input can be estimated.

The results of such computations are shown in Figure 5(a). The room is supposed to be at zero temperature up to $t = 0$. At $t = 0$ a heat source of 1200 W, as assumed in section 3.7, was supposed switched on for a period of 16 hours, and then off for 8 hours, day after day. The Figure shows the estimated build up of temperature. Attention may be drawn to certain features.

(i) The final temperatures that the node representing the lumped surface temperature would reach if the heat were sustained without interruption are shown in Table III. In the eventual steady cyclic state reached after a long while, the mean temperature will be around 16/24 or 2/3 of this value.

(ii) After a time t_r, the surface temperature reaches $1-(1/e) = 0{\cdot}632$ of its ultimate rise. After $2\,t_r$, $3t_r$ and $4t_r$, the figures are 0·835, 0·950 and 0·982, respectively. With a response time of approaching 5 days (zero ventilation rate), most of the temperature build-up is achieved during the first ten days. With the higher ventilation rate, the steady-state mean temperature is lower than in the zero case, and is reached rather earlier.

(iii) Since 540 of the total 1200 W is supposed input into the air, the air temperature is substantially higher than the mean surface temperature during the heating phase. During cooling the air temperature must be a little lower than the surface temperature since it lies on a path between the heat store and the heat sink. The value of $T(\text{surface}) - T(\text{air})$ is larger for the higher ventilation rate than the zero ventilation rate, as expected.

(iv) The initial response of the slab surface, whether during heating or cooling, is faster than exponential. According to the theory for a slab, its course for the first couple of hours should be independent of the thickness of the material. The exponential phase should be being established after some 6 hours. These are in any case imprecise concepts, and they will be made more imprecise in this connection since simple slab parameters have to be replaced by lumped parameters. They are unlikely to be seriously misleading however.

Table VIII. Response time of the mean internal surface temperature: heat input 1200 W

	$h_c = 3\,\text{W/m}^2\text{K}$				$h_c = 0{\cdot}5\,\text{W/m}^2\text{K}$			
Ventilation rate, air changes/h	B	$\dfrac{\Sigma A\rho cX}{u_1^2\Sigma Ak/X}$, days	$\dfrac{\Sigma A\rho cX}{C_e}$, days	Final temperature, K	B	$\dfrac{\Sigma A\rho cX}{u_1^2\Sigma Ak/X}$, days	$\dfrac{\Sigma A\rho cX}{C_e}$, days	Final temperature, K
0·0	0·081	6·6	6·4	14·9	0·070	7·6	7·4	17·2
0·4	0·100	5·3	5·2	11·9	0·086	6·2	6·0	12·8
1·0	0·127	4·3	4·1	9·2	0·102	5·3	5·1	9·9
2	0·165	3·3	3·1	6·8	0·117	4·6	4·4	7·8
4	0·227	2·5	2·3	4·6	0·133	4·1	3·9	6·2
10	0·342	1·7	1·5	2·7	0·148	3·7	3·5	4·9

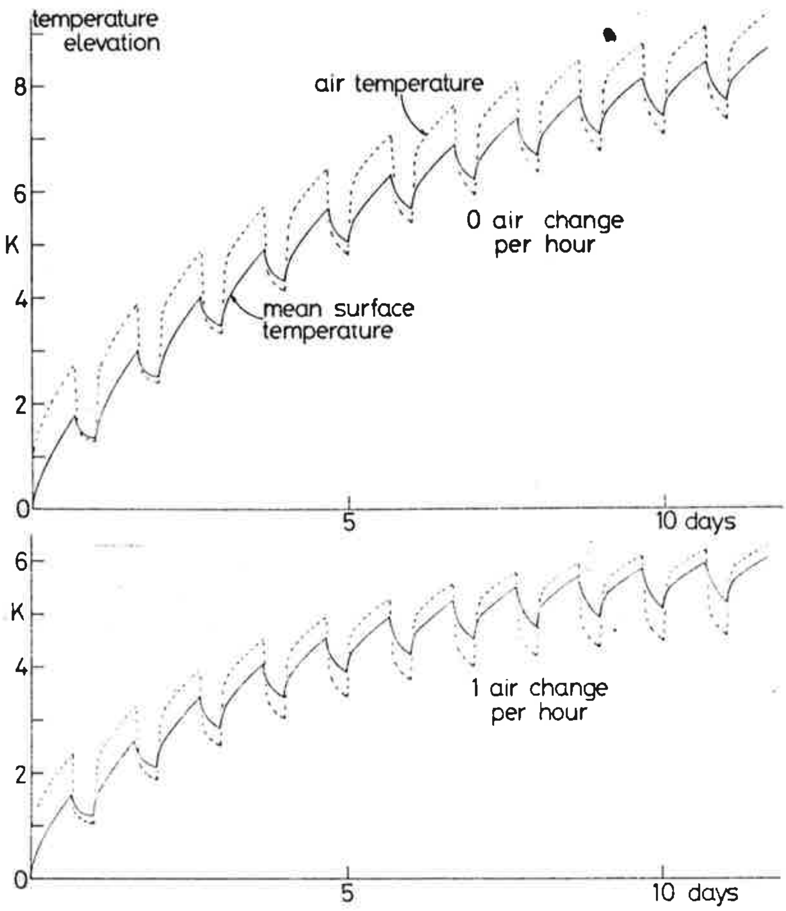

Figure 5. The estimated responses of surface and air temperature to the lighting system being switched on for 16 hours and off for 8 hours a day. Input 1200 W

(v) The initial response of the air is taken to be instantaneous. The air has its own response time given by $\rho c V/\Sigma A h$, where ρc is the volumetric specific heat of air of about 1200 J/m^{3}K. V is the room volume of 151·8 m^3, ΣA is the total room surface area of 189 m^2 and h is the assumed convective heat transfer coefficient of say 3 W/m^{2}K. The response time is thus about 5 min. There is therefore a rapid if not instantaneous change in air temperature from heated to cooled values.

The possibility of low convective coefficients is discussed below. Table VIII also gives response times for a value of $h_c = 0{\cdot}5$ W/m^{2}K. The low value makes little difference at zero ventilation rate. Since convection is a feeble mechanism for moving heat at the lower h_c value, ventilation makes little difference to the response time of the structure, but the response time of the air itself becomes larger.

3.12.2. *A two-capacity model for the room.* Although it is mathematically laborious to evaluate the transient response of an enclosure consisting of two separate thermal capacities, one of which is considered to have distributed thermal resistance/capacity properties, it is very simple to evaluate the response if the two capacities are taken to be lumped. It is worth examining the behaviour of the classroom by considering the total capacity to be located as a lump in the floor, and a lump in the ceiling. The wall capacity can be taken as included partly in the ceiling and partly in the floor.

The thermal model to be evaluated is shown in Figure 6. The node T_3 represents the superposition of the room air temperature and radiation star nodes—nodes 17 and 18 in the 18 node model. R_1 represents the resistance which results from the parallel addition of the convective transfer from ceiling to air and radiant

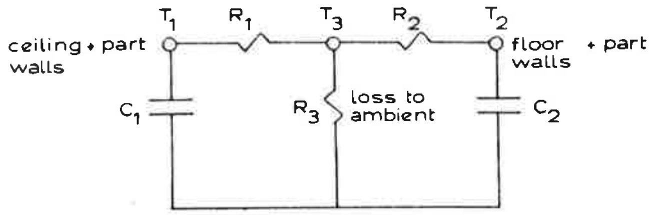

Figure 6. A two-capacity model of the room to estimate how internal differences in temperature in the enclosure die away following the switching off of the lights. The ceiling responds more rapidly than the floor

transfer from ceiling to starpoint; R_2 acts similarly for the floor. R_3 denotes the combined losses due to ventilation, and other external losses on non-capacitative paths. (The loss of heat through the ground floor should be neglected, since we are concerned here with transient excitation of diurnal periodicity. The ground floor resistance is only important in steady-state conditions.)

The input/output quantities (T_1, q_1), (T_2, q_2) are related by the matrix equation

$$\begin{bmatrix} T_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ C_1 \mathrm{D} & 1 \end{bmatrix} \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/R_3 & 1 \end{bmatrix} \begin{bmatrix} 1 & R_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ C_2 \mathrm{D} & 1 \end{bmatrix} \begin{bmatrix} T_2 \\ q_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} T_2 \\ q_2 \end{bmatrix}$$

where D denotes d/dt.

The two capacities are taken to be adiabatic on their far surfaces, so $q_1 = q_2 = 0$ and the element in the overall matrix, $g_{21} = 0$. Upon multiplying out, we have

$$A_0 + A_1 \mathrm{D} + A_2 \mathrm{D}^2 = 0$$

where

$$A_0 = 1$$
$$A_1 = (C_1 + C_2)R_3 + C_1 R_1 + C_2 R_2$$

and

$$A_2 = C_1 C_2 [(R_1 + R_2)R_3 + R_1 R_2]$$

Now the temperature response at some particular node i following a step in thermal excitation, such as results when the lights are switched on or off, can be expressed as the sum of two exponential components:

$$T(t) = B_{1i} \exp(-t/t_{d1}) + B_{2i} \exp(-t/t_{d2})$$

where t_{d1} and t_{d2} are longer and shorter decay times. On substitution we have

$$t_{d1} = [A_1 + (A_1^2 - 4A_0 A_2)^{1/2}]/(2A_0)$$
$$t_{d2} = [A_1 - (A_1^2 - 4A_0 A_2)^{1/2}]/(2A_0)$$

The decay times are somewhat involved functions of enclosure parameters. When, however, the ventilation rate is very low, so that numerically the external resistance R_3 is large in relation to the internal resistances R_1 and R_2, we have

$$t'_{d1} = (C_1 + C_2)R_3 = (\text{total capacity}) \times (\text{external resistance})$$

as R_3 tends to infinity. t_{d1} denotes the decay time of the enclosure as a whole and is the response time or time constant as usually understood. It describes the external draining away of heat from the enclosure.

The shorter time constant tends toward

$$t'_{d2} = [C_1 C_2/(C_1 + C_2)](R_1 + R_2)$$

t_{d2} is concerned with any internal heat flows during the transient change. If the surface temperatures T_1 and T_2 have different values at time $t = 0$, as is the case prior to switching off the lights since the ceiling is warmer than the floor, the initial effect must be a comparatively sharp drop in the higher value and a much slower decline in the lower value, and the time scale of this essentially internal exchange of heat from T_1 to T_2 is described by the shorter decay time t_{d2}. After a time interval something in excess of t_{d2}, the internal transient changes die away and the slower overall decline continues.

To find the order of size of the decay times we use the following numerical values: lumped convective and radiant internal film coefficient between a room surface and the index node, T_3, 9 W/m²K; conductances between index node and ambient:

via the pinboard section of the solar wall	5·96 W/K
via the translucent section of the solar wall	64·99 W/K
via the north wall	4·56 W/K
by ventilation	50·6 n W/K

where n is the number of air changes per hour; R_1 and R_2 are based on the floor area, and on the wall plus ceiling area, respectively; floor capacity, 23×10^6 J/K; remaining capacity 18×10^6 J/K.

The estimates are given in Table IX. The longer response time, that for the enclosure as a whole, again appears to be around several days; the two-capacity model suggests that values are more sensitive to ventilation rate than does the resistance/capacity model. The shorter response time, expressing internal settlement, is of order of some hours, but is less than a day.

The table indicates also that these response times can be satisfactorily estimated from the asymptotic values given above.

3.13. *The air node conductance*

The air node conductance is defined as the conductance the enclosure provides to the steady state flow of heat supplied to the air node, node 17, as the heat is lost to the ambient state at zero through the floor, south wall, north wall and by ventilation. The relevant equations are

$$\mathbf{G}\mathbf{T} = \mathbf{q}$$

or

$$\begin{bmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,16} & g_{1,17} & g_{1,18} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,16} & g_{2,17} & g_{2,18} \\ \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot \\ g_{16,1} & g_{16,2} & & g_{16,16} & g_{16,17} & g_{16,18} \\ g_{17,1} & g_{17,2} & & g_{17,16} & g_{17,17} & g_{17,18} \\ g_{18,1} & g_{18,2} & & g_{18,16} & g_{18,17} & g_{18,18} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ T_{16} \\ T_{17} \\ T_{18} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ q_{17} \\ 0 \end{bmatrix}$$

The air node conductance will be denoted $C^*_{0,17}$ to distinguish it from $C_{0,17}$, the ventilation conductance. $C_{0,17}$ is a constituent part of $C^*_{0,17}$, but $C^*_{0,17}$ contains the effect of all heat transfer mechanisms, internal and external.

Consideration of the array of terms shows that

$$C^*_{0,17} = \frac{q_{17}}{T_{17}} = \frac{|G|}{|g_{17,17}|}$$

Table IX. Response time for the two-capacity model

Ventilation rate, air changes/h	Longer response time, t_{d1}, days	Shorter response time, t_{d2}, h	$\frac{t_{d1}}{t'_{d1}}$	$\frac{t_{d2}}{t'_{d2}}$
0	6·9	10	1·07	1·00
0·4	5·5	10	1·09	0·99
1·0	4·3	10	1·11	0·99
2	3·2	10	1·16	0·99
4	2·2	9·9	1·26	0·98
10	1·3	9·8	1·55	0·97

$|G|$ is the determinant of the left hand (18 × 18) array of terms. $|g_{17,17}|$ is the determinant of the array of terms formed by omitting the 17th row and the 17th column from this array.

In fact it is obvious on physical grounds that if $(C^*_{0,17})_0$ represents this conductance when the ventilation rate is zero, its general value is simply

$$C^*_{0,17} = (C^*_{0,17})_0 + C_{0,17}$$

since $C_{0,17}$ is a conductance in parallel with the composite conductance.

A deeper question is the value of $(C^*_{0,17})_0$ itself. All the linkages with the air node are conductances formed as the products of areas multiplied by a heat transfer coefficient h_c.

Now h_c for laminar flow is proportional to $\theta^{\frac{1}{4}}$ where θ is the temperature difference between the surface concerned and the air. If θ is zero, h_c itself is zero. (As θ tends to zero, the boundary layer thickness, normally thought of as perhaps a few centimetres thick at most, becomes 'large', comparable with the linear dimensions of the room. The simple theory no longer holds, and we cannot use it to compute h_c; h_c may not reach zero. It is apparent however that it can be lower than the value of 2·8 or 3 W/m²K that is often assumed. It may be mentioned that some adjustment to simple theory is needed, since $\theta^{\frac{1}{4}}$ cannot strictly be evaluated when θ is negative, whereas θ may readily change sign.)

The value of the air node conductance $C^*_{0,17}$ or C_{an} as a function of the heat transfer coefficient is shown in Table X.

When h_c is zero C_{an} is zero. As h_c tends to infinity, C_{an} tends to the conductance of 144 W/K as provided by the external conductances, mainly through the solar wall.

A ventilation rate of n air change per hour simply leads to an increase in C_{an} to 50·6 n W/K. This follows from first principles. (It is also given by the determinant ratio, which provides a check on the determinant computation.)

4. DISCUSSION OF THE MODEL ESTIMATES

The estimates of temperature arrived at in the model study represent the outcome of a chain of operations which include a great many assumptions. The only ingredients to the calculations that can be regarded as certain are the room internal dimensions. The conductivity, density, specific heat and long-wave emittance of the wall materials are unlikely to be seriously in error. As will appear in the next article, however, the inside convective heat transfer coefficient, often assumed to be 3 W/m²K, may differ very considerably from this value. An attempt has been made to compute the solar gain factor for the solar wall. That for the translucent section may be considerably affected by the internal construction of the solar wall and the nature of the inner leaf. The area of the openable windows is simply lumped with it. The pinboard area is further complicated by the presence of a cavity, closed at the bottom, open but constricted at the top, and the author does not regard the solar gain values as very reliable.

The losses of heat by conduction to the floor and through the classroom north wall, are, for different reasons, not estimated with precision. In all the analyses involving heat storage, assumptions have had to be made regarding the effective position of the adiabatic surfaces. It is clear that for the ground floor there can be no adiabatic surface: for calculations based on admittance, this does not matter—the ground floor is effectively very thick for higher harmonics; the assumption is somewhat dubious in the single storage node transient

Table X. Value of the air node conductance $C^*_{0,17}$ or C_{an} (zero ventilation rate) in the 18 node model

h_c, W/m²K	0·01	0·02	0·05	0·1	0·2	0·5	1	2	3
C_{an}, W/K	1·9	3·8	8·8	15·9	26·6	44·8	59·0	72·1	79·4
h_c, W/m²K	5	7	10	∞					
C_{an}, W/K	88·9	95·4	102	144					

approach of section 3.12.1, and is sufficiently serious to reduce the status of the two storage node analysis of section 3.12.2 to that of a rather qualitative calculation.

Finally, the ventilation rate at any time can only be guessed; since ventilation plays an important part in the steady-state component of temperature in a room, ventilation rate is treated in this article as an independent variable.

One may further ask what measure of temperature is the most appropriate to judge thermal comfort. Air temperature is often used. In an enclosure sometimes experiencing high levels of solar radiation, the 'contents' temperature, which includes the effect of radiation from all sources, may be more suitable. Globe temperature is the nearest observable version of contents temperature and globe temperature correlates a little better with reported thermal comfort than does air temperature.

With these comments on the uncertainty of the estimated values, we can attempt to provide answers to the questions put to the model.

In cold weather, the loss of heat to the ground at a temperature of 10°C might be to increase inside temperature by up to a degree (Table II). The lighting system, when on full time in an unoccupied building, might achieve elevations approaching 10 K (Table III). Average midwinter sunshine may provide $2\frac{1}{2}$ K deviation (Table IV). (The effect of occupation must be to increase ventilation rate somewhat, but this is compensated to some extent by the additional heat from the children.) An elevation of $\frac{1}{2}+10+2\frac{1}{2}=13$ K on the average winter temperature of $4\frac{1}{2}$°C gives a value for the mean indoor temperature of $17\frac{1}{2}$°C. This value includes the low temperatures during the night. Daytime values will be higher due to solar gain, and to occupancy. The model study thus suggests that temperatures might be within the comfort band of 17°C to 21°C during average winter conditions.

The ability of the fabric to cope with conditions more severe than average could be a topic for further study. Records of ambient temperature and sunshine hours are available for the site for a period of 50 years, and will be used in the energy study. This approach will not be followed here since the actual daily values for temperature during two winter periods have been observed and will be presented in the observational study. It is sufficient to make two qualitative remarks. First, a value of 2 MJ/m^2 day of solar radiation was taken as 'average'. It could be as much as 9 MJ/m^2 day on a particularly bright day and the Wallasey construction is well suited to storing the additional heat, so that its effect may be felt over the next few days. Secondly, there is a tendency in winter for cold days to be somewhat sunnier than average. The correlations between ambient temperature and solar radiation for the Wallasey area, based on the 50 years data mentioned above are: November −0·178, December −0·139, January −0·147, February −0·075. Both these factors suggest that the Wallasey construction might continue to achieve comfort levels in winter conditions more severe than average. It must be emphasized, however, that this would only be possible when ventilation rates were low.

The calculation on the response of the classroom to switching lights on and off indicates that if the light were on for 16 hours out of 24, for example, the elevation the lights would achieve is simply 2/3 that of their 24 hour value (see Figure 5). The response time of the building (estimated as more than 4 days) is so long that temperatures only vary a little about the mean value during the periods of lights-on and lights-off. It is clear too that the lights have to be switched on for a period of a few days to bring the fabric temperature up to comfort values after being swtiched off for some length of time over Christmas. By the same token the lighting system is not capable of warming the building itself quickly. It may be remarked, however, that because of the radiant output of the lights, switching them on in winter has an immediate and perceptible effect on the contents temperature and they may thus be seen to be an immediate, if small, source of heat by the occupants.

In considering the question of the possibility or likelihood of overheating during sunny periods, we have to take account of the effects of both changing ambient temperature and solar gains, which lead to the variation of relevant temperatures during the hours of occupancy. It will be clear from Figure 3 that if the classroom were unventilated during a very sunny day the contents temperature might reach some 32 K above ambient, or 32 $+13\frac{1}{2}$, about 45°C with a September daily mean temperature of 13·5°C. A calculation of the profile of temperatures between suntimes of 08.00 and 18.00 hours suggests that a steady ventilation rate of about 10 air changes per hour would be needed to maintain just tolerable conditions within the room. Details are shown in Table XI. Since the classroom faces 16° west of south the sun has its maximum intensity between one and one and a half hours after solar noon. Air temperature has a mean value of 13·5°C during September (IHVE 1971,

Table XI. Estimated temperature in the classroom during hours of occupation due to sunny conditions in September and an air change rate of 10 air changes per hour

Suntime, h	8	9	10	11	12	13	14	15	16	17	18
BST*	9	10	11	12	13	14	15	16	17	18	19
Contents temperature											
T (solar)	3·3	4·6	6·4	8·2	9·8	10·8	11·2	10·9	10·0	8·6	6·8
T (ambient)	−0·9	−0·5	0·0	0·5	0·9	1·3	1·6	1·8	1·8	1·7	1·6
T (contents)	15·9	17·6	19·9	22·2	24·2	25·6	26·3	26·2	25·3	23·8	21·9
Air temperature											
T (solar)	2·1	2·6	3·3	4·2	4·9	5·4	5·6	5·6	5·3	4·8	4·0
T (ambient)	−1·0	−0·3	0·5	1·2	1·8	2·4	2·7	2·9	2·9	2·6	2·3
T (air)	14·6	15·8	17·3	18·9	20·2	21·3	21·8	22·0	21·7	20·9	19·8

* British Summer Time, not allowing for longitude.

Table A6.21) and a nearly sinusoidal of variation about it of 5·5 K; 5·0 K is used in Table XI. The deviations from mean ambient temperature due to both solar gain and varying ambient ~~are~~ shown at hourly intervals. The resultant temperature is their sum, together with the mean value of 13·5°C.

The estimates suggest that with this ventilation rate a contents temperature of about 26°C might be reached. Other temperatures are of course lower. The air temperature reaches nearly 22°C but the cooling effect of air movement causing evaporation loss will reduce the effective temperature.

The temperature elevation ΔT (solar) due to solar radiation varies strongly with ventilation rate, and if wind conditions were insufficient to provide sufficient ventilation, serious overheating might occur. (ΔT (ambient), the variation due to daily variation in ambient temperature, actually increases with ventilation rate, but to a much lesser extent that does ΔT (solar).)

Clearly, any number of daily profiles of temperature such as Table XI illustrates could be computed, for average or extreme conditions of sunshine, ambient temperatures or ventilation rate for all months of the year, but it would not serve a useful purpose. Table XI indicates that as far as model estimates are concerned, serious overheating is unlikely to occur often but in a combination of high air temperatures, sunny days and calm conditions, temperatures might become excessively hot. This is true of many buildings. In the next article an account will be given of the actual temperatures encountered during a 19 month period of observation.

APPENDIX I: SOME ENCLOSURE CONDUCTANCES

Values for the conductances to be used in estimating the response of the classroom are noted in this and the following appendices.

Information on dimensions is taken from the drawings of the Borough Architect and by site measurement. The leading room dimensions are: height 9 ft 3 in = 2·82 m, depth (north to south) 22 ft 8 in = 6·91 m, width (east to west) 26 ft 6 in = 8·08 m. Part of the ceiling is obscured by a sloping portion and certain dimensions have to be amended. The areas assumed are listed in Table XII.

The conductance between the area i and the air is

$$C_{i,17} = A_i h_c$$

and h_c will be assumed to be 3 W/m^{2}K. The conductance between area i and the adjacent blackbody equivalent node $i+1$ is

$$C_{i,i+1} = A_i(\varepsilon/(1-\varepsilon))h_r$$

where h_r = 5·7 W/m^{2}K, and ε is the long- or short-wave surface emittance.

The delta connections between nodes 2, 4, 6, 8, 10, 12 and 14 will be replaced by a star network (Davies, 1983); the connections with the star node, node 18, are accordingly

$$C_{j,18} = (A_{j-1}/\beta_j)h_r$$

Table XII.

location		area m^2
1	Floor	55·82
3	Solar wall pinboard area	5·39
5	Solar wall translucent area	15·81
7	Ceiling, horizontal portion	35·07
9	Ceiling, sloping portion	22·14
11	North wall	17·21
13	East wall and west wall	2 × 18·80
15	Contents (assumed)	10

where

$$\beta_j = 1 - f_j - 3{\cdot}53\,(f_j^2 - \tfrac{1}{2}f_j) + 5{\cdot}04\,(f_j^3 - \tfrac{1}{4}f_j)$$

and

$$f_j = A_j/(\text{total enclosure area})$$

Only qualitative account can be taken of the wooden furniture. It will be taken to consist of an area of 10 m^2, spherical in form, at the room centroid. The solid angle subtended by the window wall at the centre is given by

$$\phi = 4\tan^{-1}\left\{\frac{(w/2)(h/2)}{(d/2)[(w/2)^2 + (h/2) + (d/2)^2]^{1/2}}\right\} = 0{\cdot}0929 \simeq 0{\cdot}09$$

The view factor conductance between the whole of the window and the contents is accordingly around 10 m^2 × 0·09 × 5·7. The conductances $C_{j,16}, j = 2, 4, \ldots, 14$ were formed in this way. These conductances can only be rough estimates. Since the values are small compared with the conductances to node 18, this may not matter too much.

Finally there are the linkages between nodes 1, 3, . . . , 17 with the externally imposed temperatures, T_{ao}, T_{ground} and $T_{reference}$; ($T_{reference}$ is zero by definition).

If the heat loss from the floor, node 1, is taken to move between T_1 and T_{ao}, the conductance can be estimated from Macey's formula:

$$C_{0,1} = \frac{2kw}{\pi}\ln\left(1 + \frac{2d}{t}\right)$$

Here, k is the conductivity of the concrete or the earth infill below the concrete, w is the room east/west dimension, d is the north/south dimension, and t is the wall thickness of the wall on the south side and of an identical wall on the north side.

To apply this approximately to the classroom, we consider the dimension d to comprise the classroom depth itself, together with the corridor, 9·1 m altogether, and suppose that the north wall too has a thickness of 0·62 m, as does the south wall. The ground conductivity will be taken as 1 W/m^2K. With w = 8·08 m, the conductance is 17·6 W/K. The part of this to apply to the classroom will be taken as 12·9 W/K, so $C_{0,1}$ = 12·9 W/K.

There are evident uncertainties and approximations in this value. However, there is the question whether Macey's formula is even appropriate in principle. The earth forms a large thermal capacity, whose temperature a few metres down scarcely varies from the yearly mean value of about 10°C. This, rather than the varying value for T_{ao} serves as the sink temperature for floor losses. But if T_g is the sink temperature the heat flow lines must be effectively shorter than Macey's construction assumes, and they are not necessarily circular. The value of $C_{0,1}$ = 12·9 is thus almost certainly too low, but no guidance about the correction seems available.

The value for $C_{0,1}$ for steady cyclic response will be considered later.

The values for the pinboard and translucent wall links with T_{ao} too will be considered later.

The value for $C_{0,7}$, the steady-state link between the massive inside surfaces and T_{ao}, is zero. Again the values for $C_{0,7}$ (harmonics) will be considered later.

The value of the conductance between the room north wall and ambient is made up of the corridor resistance (assumed to be 0·18 m^2 K/W, a conventional value) and of the well-insulated corridor north wall. $C_{0,9}$ will be taken to be 4·7 W/K. (There is often an air movement in one or other direction along the corridor. Since this is an internal movement, we have to assume that, on average, it does not affect the corridor losses.)

The contents are not in contact with ambient, so $C_{0,15} = 0$.

The ventilation loss, due to n air changes per hour is conventionally taken as

$$C_{0,13} = \frac{(n\,\mathrm{h}^{-1}) \times (1200\,\mathrm{J/m^3K}) \times (151{\cdot}8\,\mathrm{m^3})}{(3600\,\mathrm{s/h})}$$

These values were also used, with change of notation, to perform the calculations on transient response (see Figure 4):

$$C_{01} = 4{\cdot}7;\ C_{04} = 126{\cdot}53;\ C_{12} = 167{\cdot}8 \times 5.7 \times \varepsilon/(1-\varepsilon);\ C_{15} = 167{\cdot}8 \times 3$$
$$C_{23} = (21{\cdot}1 - 10 \times 0{\cdot}0921) \times 5{\cdot}7;\ C_{34} = 21{\cdot}1 \times 5{\cdot}7 \times \varepsilon/(1-\varepsilon);\ C_{45} = 21{\cdot}1 \times 3$$
$$C_{27} = 10 \times (1 - 0{\cdot}0921) \times 5{\cdot}7;\ C_{37} = 10 \times 0{\cdot}0921 \times 5{\cdot}7$$
$$C_{67} = 10 \times 5{\cdot}7 \times \varepsilon/(1-\varepsilon) \times 5{\cdot}7;\ C_{56} = 10 \times 3$$

all with units W/K.

APPENDIX II: THE CONDUCTANCES AND SOLAR GAIN FACTOR OF THE SOLAR WALL

The conductance of the solar wall was found routinely by summing the resistances between the various paths from the visible parts of the wall to the ambient air node. An attempt has been made to allow for the complications of the window geometry (see Table XIII).

The solar gain factor S of a window area is the fraction of solar gain incident on the window during (in the present application) one day that serves to heat the enclosure. For single glazing, the outwardly reflected radiation makes zero contribution, the transmitted fraction makes its full contribution and, of the absorbed fraction (comparatively small for clear glass), the part that is lost inwards by convection and the long-wave radiation contributes to the heat gain.

For double glazing the situation is a little more complicated; radiation absorbed in the inner leaf is more effective in heating the enclosure than heat absorbed in the outer leaf and appropriate scaling factors have to be used. Month by month daily mean S values for single glazing S_s and for double glazing S_d are given by Davies

Table XIII. Conductance of solar wall

	Area, m^2	Transmittance, W/m^2K	Conductance, W/K
Pinboard area			
Area of aluminium inserts	4·27	1·2	5·12
Remaining pinboard area	1·12	1·5	1·68
			6·8
Translucent area			
Figured glass above pinboards	5·99	4·3	25·76
Figure glass below pinboards	1·39	4·3	5.98
Clear glass below pinboards	2·19	4·3	9·42
Double glazing above window	1·28	4.3	5·50
Single glazing above window	1·04	18·2	18·93
Window—nominal aperture	2·85	4·5*	12·82
Window—superficial area	2·68	13·7†	36·72
Below window	1·07	4.3	4.60
			119·7

* Radiation to outside.
† Convection to outside.

(1980). The heat flow was taken to be received at the environmental temperature node. For the present purpose all heat is taken to be input at one or other of the nodes of Figure 2. Over the translucent section of the wall the absorbed fraction is taken to act at node 5, and the transmitted fraction acts at the blackbody equivalent nodes. Its manner of distribution between them is discussed elsewhere in this paper. Absorption and transmission solar gain fractions α' and τ' are needed and they were found as indicated by Davies (1980).

If a beam of radiation is incident upon a sheet of glass, a fraction ρ is ultimately reflected, a fraction α is absorbed and a fraction τ transmitted. If two parallel sheets of glass intercept the beam:

fraction reflected as short-wave radiation from the system $= \rho + \dfrac{\rho\tau^2}{1-\rho^2}$

fraction absorbed in the outer leaf $= \alpha + \dfrac{\alpha\rho\tau}{1-\rho^2}$

fraction absorbed in the inner leaf $= \dfrac{\alpha\tau}{1-\rho^2}$

fraction transmitted as short-wave radiation through the system $= \dfrac{\tau^2}{1-\rho^2} = \tau'$

The fractions add up to unity, since $\rho + \alpha + \tau = 1$.

If r_{cav} and r_{out} denote the cavity resistance (due to convection and short-wave radiation) and the outside film resistance, the fraction of radiation absorbed by the outer leaf can be lumped with that at the inner leaf:

$$\alpha' = \left(\alpha + \frac{\alpha\rho\tau}{1-\rho^2}\right)\left(\frac{r_{out}}{r_{cav}+r_{out}}\right) + \frac{\alpha\tau}{1-\rho^2}$$

The action of the pinboard section requires separate consideration; see Figure 7, which indicates a section, not to scale, through the wall, including the pinboard. The heat absorbed in the two leaves can be lumped as indicated above. The transmitted fraction will be assumed to be completely absorbed at the outward-facing surface of the pinboard. (Most of it is actually absorbed at the aluminium sheets; this does not affect the absorption part of the argument but the presence of the aluminium sheets will affect the transmittance of the wall.)

Air from the room will convect downward on the inner leaf of the solar wall (the outer leaf of the cavity) and upward at the pinboard surface, so transferring some of the heat to the air. The conductances are denoted as

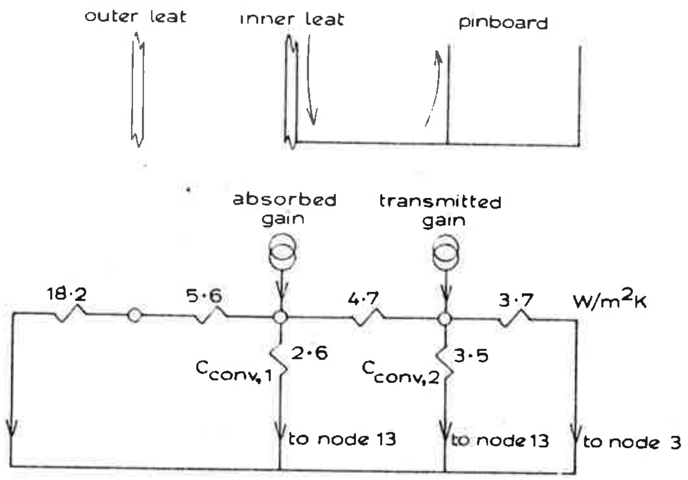

Figure 7. To illustrate calculation of the solar gain factor for the wall opposite the pinboard areas

$C_{conv,1}$ and $C_{conv,2}$. Since the pinboard may be much hotter than the inner leaf these two may differ somewhat and their values can be estimated using the standard theory for natural convection in a laminar boundary layer on a vertical wall. The heat transfer coefficient h is related to the Prandtl number Pr and Grashof number Gr as

$$\frac{hl}{k} = \left(\frac{512}{2430}\right)^{\frac{1}{4}} Pr^{\frac{1}{2}} \left(Pr + \frac{20}{21}\right)^{-\frac{1}{4}} Gr$$

Noting that $Pr = 0{\cdot}714$ and substituting for Gr, we have

$$h_c = 0{\cdot}596 \left(\frac{k\rho c}{\sqrt{(c/g)}}\right)^{\frac{1}{2}} (\beta\theta)^{\frac{1}{4}}$$

where θ is the temperature difference between the surface concerned and the room air temperature. Substituting appropriate values for k, ρ and c, at typical temperatures in the situation it turns out that h at the solar wall inner leaf is around 2·6 W/m²K and that at the pinboard surface, h is perhaps 3·5 W/m²K.

The transmittance through the 4 cm wooden pinboard will be assumed to be (0·15 W/m K)/(0·04 m) = 3·75 W/m²K. Long-wave radiation between the pinboard and the inner leaf leads to a transmittance of 4·76 W/m²K; the conventional value for the solar wall cavity is 5·56 W/m²K and for the outer film transmittance, 18·2 W/m²K.

If radiation of intensity I strikes the outer surface of the solar wall the fraction $\alpha' I$ acts as a heat source at the inner leaf, and the fraction $\tau' I$ acts as a heat source at the pinboard outer surface (or the aluminium sheet). This is then redistributed to node 17 (the air) and node 3 (the visible pinboard surface) as indicated in Figure 7.

The cavity between the inner leaf and the pinboard is about 12 cm wide and this is sufficient to allow the assumption of two independent boundary layers on the two surfaces. Unfortunately, near the top of the cavity, the horizontal window frame member blocks the greater part of the cavity, and the fittings to support the aluminium sheets are located near the frame. The details of the fixings of the five sheets are by no means identical. This complicates the heat transfer process shown in Figure 7. It must reduce the efficiency of the collecting device, but it does not seem possible to make allowances for this quantitatively.

The values for the solar gain factors used in the translucent section are

	December	September
transmission factor τ'	0·766	0·630
absorption factor α'	0·031	0·031

APPENDIX III: CONDUCTANCES RESULTING FROM THERMAL STORAGE

The conductances associated with steady-state transfer were evaluated as discussed in Appendix I. When temperatures vary, however, the storage provided by the four massive surfaces of the room (floor, ceiling and east and west walls), provides conductances which are much larger than the steady-state conductances, and their values tend to dominate the estimated response.

To estimate the response due on a very sunny day, the harmonic method will be used. The incident solar radiation, values for which are given in section 3.8, as expressed in terms of Fourier harmonics up to the 10th:

$$I_t = I_0 + \sum_{n=1}^{10} I_n \cos 2\pi \frac{nt}{24}$$

with t in hours. The coefficients have values

I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}
194·8	−320·0	164·9	−31·1	−20·4	11·3	4·6	−3·4	−2·9	1·9	1·8

The original intensities were those on a south-facing wall, symmetrical about midday and so have no components to be multiplied by $\sin 2\pi nt/24$. The components reproduce the original intensities well: the maximum error is an estimated −1·4 W/m² at midnight.

Table XIV. Thermal constants assumed for heat storage

Material	Conductivity, W/mK	Density, kg/m³	Specific heat, J/kg K
Concrete	1·40	2100	840
Brickwork	0·84	1700	800
Plaster	0·50	1300	1000
Plasterboard	0·16	950	840
Hardboard	0·094	750	1760
Wood	0·125	610	1760

Table XV. Thermal storage provided by classroom surfaces

	Area, m^2	y_{sr}, W/m^2K	y_{si}, W/m^2K	$\left(\sum \frac{X}{K}\right)^{-1}$, W/m^2K	$\Sigma(\rho c X)$ $\times 10^{-3}$, J/m^2K	Ay_{sr}, W/K	Ay_{si}, W/K	$A\left(\sum \frac{X}{K}\right)^{-1}$, W/K	$A\Sigma\rho c X$ $\times 10^{-6}$, J/K
Ceiling									
main surface	35·07	4·75	12·39	12·25	201·6	166·7	434·6	429·6	7·071
addition due to beam*	(2·67)	(4·75)	(12·39)	1·89	582·1	12·7	33·1	5·0	1·554
sloping area (plasterboard)	22·14	1·49	2·52	3·11	51·2	33·0	55·8	68·9	1·134
East wall, plastered brick									
backed by corridor	2·33	6·79	5·79	3·70	292·8	15·8	13·5	8·6	0·682
backed by study	4·08	5·57	7·64	5·95	176·2	22·7	31·2	24·3	0·719
with hardboard, backed by corridor	2·74	6·15	3·98	3·29	296·9	16·9	10·9	9·0	0·814
with hardboard, backed by study	3·82	5·86	5·42	4·96	180·4	22·4	20·7	18·9	0·689
with cupboard, backed by corridor	2·44	2·04	2·11	1·07	357·2	5·0	5·1	2·6	0·872
with cupboard, backed by study	3·39	2·07	2·09	1·20	240·7	7·0	7·1	4·1	0·816
	18·8								
West wall, plastered brick									
backed by storeroom	8·56	6·69	5·59	3·40	320·0	57·3	47·9	29·1	2·739
backed by stairwell	3·05	6·79	5·79	3·70	292·8	20·7	17·7	11·3	0·893
with cupboard or blackboard	5·49	2·39	1·44	1·57	341·5	13·1	7·9	8·6	1·875
door	1·70	1·46	3·00	2·50	53·6	2·5	5·1	4·2	0·091
	18·8								
Floor	55·82	10·06	9·83	5·60	420·0	561·6	548·5	312·6	23·444
								936·8	43·393

* The presence of the beam was ignored for harmonics other than the first.

The conductances provided by the massive surfaces are the product of their area and their surface admittance, appropriate to the harmonic concerned. This was found using the scheme set out by the author (Davies, 1982).

Consider a parallel sided slab of thickness X, conductivity k, density ρ, specific heat c, subjected to sinusoidal excitation of period P. The temperature variations T_1 and T_2 at the two surfaces and the heat flux variations q_1 and q_2 are related as

$$\begin{bmatrix} T_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} \cosh(\tau + j\tau) & \sinh(\tau + j\tau)/\mathbf{a} \\ \sinh(\tau + j\tau)\mathbf{a} & \cosh(\tau + j\tau) \end{bmatrix} \begin{bmatrix} T_2 \\ q_2 \end{bmatrix}$$

τ is the cyclic thickness, $(\pi\rho c X^2/Pk)^{\frac{1}{2}}$ and is dimensionless. $\mathbf{a}$ is the characteristic admittance for the material; it has magnitude $(2\pi k\rho c/P)^{\frac{1}{2}}$ with units of W/m^{2}K. It also has phase $-1/8$th of a cycle. (Physically this means that if the surface of a semi-infinite slab undergoes sinusoidal temperature variation, the heat flux associated with the swing of temperature has its maximum 1/8th of a cycle (3 hours in 24 hours) ahead of the temperature maximum). P has a fundamental value of 24 hours; for the second and third harmonics, P is 12 and 8 hours, respectively, etc.

Since heat flux and temperature must both be continuous across the interface between two adjacent slabs (e.g. a layer of plaster on brickwork) the overall properties of the combination can be found by evaluating the products of the matrices of the separate slabs.

The product required from this operation is the surface admittance—the ratio of heat flux to temperature at the exposed surface $y = q_1/T_1$. It is complex, corresponding to the resistive and capacitative properties of the wall materials, and is conveniently represented as $y = y_r + j\,y_i$, where y_r and y_i are the real and imaginary parts. In order to find y however, an assumption has to be made about the boundary condition at the extreme surface. For the single slab above, the assumption amounts to deciding whether surface 2 is an isothermal surface ($T_2 = 0$) or an adiabatic surface ($q_2 = 0$).

y has to be evaluated for all surfaces whose materials provide significant storage. A difficulty arises here in connection with the outer boundary condition. The ceiling of the classroom is the floor of the artroom and is of solid concrete. We assume that the temperature history in both rooms is substantially the same, so that temperature rise and fall on the upper and lower surface are in step. In this case the central plane is an adiabatic surface and this provides the boundary condition.

For the floor no adiabatic surface exists. Since the surface admittance eventually becomes independent of thickness, the floor for this purpose can be treated as infinitely thick.

The boundary conditions for the east and west walls are more arbitrary. The room adjoins on its east side a small room which we can assume undergoes thermal behaviour similar to that in the classroom. This leads to an adiabatic surface at mid-plane. The adjoining room is served by a short corridor and its wall forms the remainder of the classroom east wall. The corridor is not sunlit. Its surface undergoes small periodic temperature variations and we have to assume an adiabatic surface not at mid-plane, but near the corridor surface. Further complications on the east wall are the presence of a layer of hardboard adhering to the plastic surface, and beneath it a bench/cupboard extending the full north–south length of the wall. These factors make it necessary to compute the area-admittance of the east wall as the sum of 6 separate parts.

Some comparable considerations are needed for the west wall. A staircase runs up the far side of the west wall and so a little of the wall is backed by a 'corridor'. The remainder of the wall is backed by a store room, on the other side of which is a room comparable with the classroom. The heat loss from the store room must be very small and so the far surface of the brickwork of the west wall has been treated as adiabatic. Allowance has been made for the door into the store room. Also, the west wall carries a blackboard unit, the board being spaced some 10 cm from the wall, so making a cavity. A shallow cupboard beneath the blackboard has a similar thermal effect, and allowance has been made for these features. (The transmission matrix for a cavity is simply

$$\begin{bmatrix} 1 & r \\ 0 & 1 \end{bmatrix}$$

where r is the thermal resistance of the cavity.)

Table XIV lists the assumed value for the thermal properties of the wall materials. Table XV gives values for the area-admittance for $P = 24$ hours for the various items which were included in storage calculations.

Table XV also lists the values for the thermal capacities and conductances used in the calculation of transient response: total assumed capacity = 43·39 MJ/K; total wall conductance = 936·8 W/K.

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