Washington Energy Extension Service

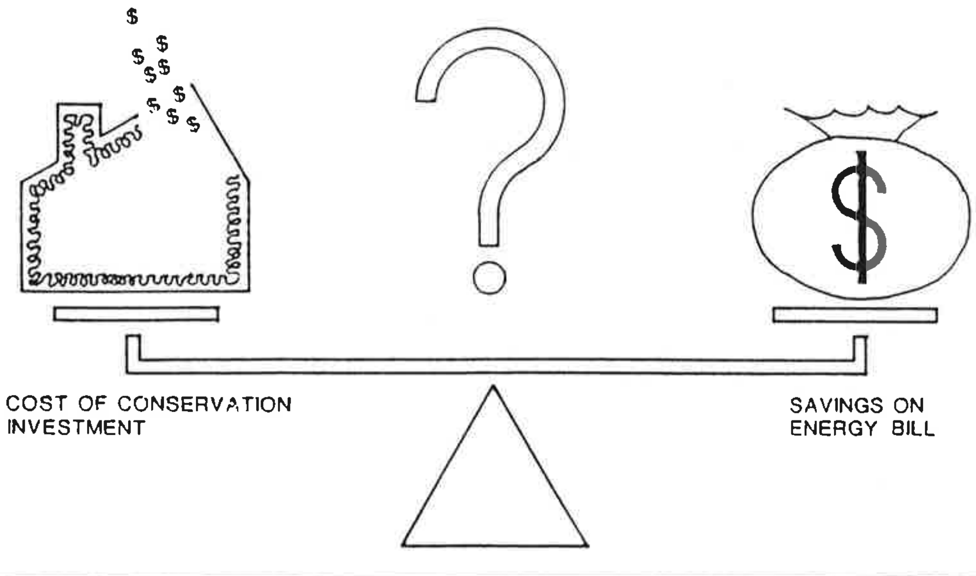
ECONOMICS OF ENERGY CONSERVATION INVESTMENTS

EY 0030
August 1985

Is home energy conservation a good investment? Although often asked, this question is usually very difficult to answer. Admittedly, energy prices have been rising fast; however, to save energy involves various kinds of investments. Which ones are worthwhile? This factsheet will offer two approaches to economic analysis - "payback" and "life cycle costing", that can help simplify this decision making process.

Payback analysis, by far the simpler approach, is discussed first and may be all that is needed to evaluate certain investments. Life cycle costing, a more comprehensive analysis tool, is subsequently introduced and two methods of evaluation are examined in greater detail - the Present Worth (PW) method and Internal Rate of Return (IRR) method. Discussion is also provided on the types of benefits and costs associated with energy conservation investments and how they are affected by certain economic factors.

Life cycle economic evaluations, as will be discovered, depend upon numerous estimates and assumptions. To the extent that most future benefits and costs are unknown, a high element of guesswork is inherent in the evaluation process. It is important to acknowledge this uncertainty and interpret results accordingly. The economic methods discussed below can serve a useful purpose, if care is taken to use only the best available data. Small differences among competing investments, however, are probably not critical and examination of non-quantifiable factors may be more significant.



Washington Energy Extension Service is funded by the U.S. Department of Energy and the Bonneville Power Administration

PAYBACK ANALYSIS

Payback analysis is perhaps the most widely used technique for comparing energy conservation investments. It refers to the time required for energy savings to recover or "payback" the initial investment. Payback is an important consideration primarily for investors seeking a rapid turnover of investment funds or if the investment has a highly uncertain life expectancy. "Simple payback" is defined as the initial cost of an investment divided by its first year savings. For example, an investment of $100 that saves $20 during the first year would have a simple payback of 5 years ($100/$20 per year = 5 years). Since only initial cost and first year energy savings are evaluated, simple payback analysis is most useful for comparing investments that have few annual benefits and costs other than energy savings to consider. In actuality, the payback period will be somewhat less than that calculated since simple payback analysis also neglects the affect of future increases in energy rates.

Comparing energy conservation investments according to simple payback is often desirable to give a general idea of what energy savings might be in relation to initial cost. Typically, low cost investments are associated with rapid payback times and high cost investments with longer paybacks. However, shorter payback periods do not necessarily indicate the most economically profitable investment since benefits or costs occurring after the payback period are not evaluated. Ranking investments by true economic efficiency, requiring the use of life cycle costing methods, would probably yield a slightly different order since the useful life of the investment is evaluated. When one recognizes the limitations of payback analysis, a list of energy conservation projects by payback period can serve as a useful reference for prioritizing investments. Estimated paybacks for typical home energy improvements are provided in Table 1.

An example of how simple payback may be used would be the comparison of attic and wall insulation, both having similar useful lives and virtually no annual benefits and costs other than energy savings to consider. A comparison is useful in this hypothetical case since it is assumed the homeowner is able to afford only one investment or the other. If the attic insulation costs $500 and saves $100 the first year, its payback is 5 years. If the wall insulation costs $1,000 and saves $125 the first year, its payback is 8 years. It may then be concluded that the attic insulation is a better investment. Although yearly savings are less ($100 vs. $125), the percentage "return" on the investment is greater (100/500 = 20% vs. 125/1000 = 12.5%). Payback analysis can also be used to determine the optimum level of insulation for greatest savings. For example, a higher level of attic insulation costing $720 may save $120 the first year for a 6 year payback. For the highest return, then, it would be better to invest only at the $500 level. Such prioritizing is not meant to suggest that wall insulation or higher levels of attic insulation is not a good investment. Payback merely ranks investments based on initial savings so that limited funds may be invested in the most economical projects first. If affordable it may be most economical to invest in numerous types of energy conservation options.

IMMEDIATE PAYBACK: NO COST ACTIONS

Lower space heating thermostat (at night and whenever house unoccupied)
Lower water heater thermostat to 120°F
Install shower flow restrictor (often free or low-cost from utilities)
Install gaskets behind electric outlet and switch plates (often free or low-cost from utilities)

LESS THAN 2 YEARS PAYBACK

Automatic setback thermostat (minimum 8 hour setback of 10°F)
Do-it-yourself weatherstripping
Do-it-yourself caulking
Do-it-yourself storm windows ($<\$1/ft^2$, e.g. flexible vinyl glazing)
Sheetmetal fireplace cover
Oil furnace annual tune-up
Hot water tank and hot water pipe insulation

2 - 5 YEARS PAYBACK

Attic insulation to R-30
Underfloor insulation to R-19 (over unheated spaces)
Furnace duct or boiler pipe insulation in unheated spaces
Do-it-yourself storm windows ($<\$3/ft^2$, e.g. rigid acrylic glazing)
Do-it-yourself insulated window covers (Minimum R-3 and $<\$5/ft^2$)
Low cost fireplace modifications (e.g. flue top damper, inexpensive glass doors)
Passive solar design in new construction
Solar heating of pools

GREATER THAN 5 YEARS PAYBACK

Insulated window covers (Minimum R-3 and $>\$5/ft^2$)
Commercially installed storm windows or insulated glass
Wall insulation
Replace conventional oil burner with flame retention burner
Fireplace inserts and woodstoves
Solar or wood domestic water heating
Energy efficient appliances (especially refrigerators, freezers and A/C's)

SIMPLE PAYBACK is assumed for the above calculations and refers to the initial cost of the investment divided by the first year's energy savings. For discussion of payback's uses and limitations as an economic criteria, see the appropriate section in the text.

Table 1. Energy Conservation Investment Priorities

An example of the limitations of payback analysis would be the comparison of an oil furnace annual tune-up with an automatic setback thermostat. The annual tune-up may cost $70 and save $70 the first year for a one year payback. Likewise the thermostat may cost $100 and save $50 the first year for a 2 year payback. According to these figures then, the tune-up offers greater savings. While this is true for the first year only, the economics change considerably during later years. Note that the benefits of a tune-up last only one year and that the benefits of the thermostat may last more than 15 years. Thus, the thermostat will pay for itself many times over compared to the tune-up due to its significantly longer useful life. This fact is not accounted for by simple payback analysis and points to the need for an analysis technique that is able to consider all benefits and costs over the useful life of an investment.

LIFE CYCLE COSTING

In response to this need, life cycle costing is a way of evaluating investments based on all associated benefits and costs occurring throughout the useful life of the investment. This includes initial costs plus the benefits and costs of ownership. Due to this added complexity, life cycle costing methods are more difficult to use than payback analysis although their applicability is much greater. Unlike the speculative investor interested in rapid payback and resale value, the homeowner is generally more concerned with long-term economic efficiency based on overall ownership costs. Life cycle costing methods provide this evaluation tool.

Discounting

The central aspect of all life cycle costing methods is a term known as "discounting". Discounting refers to converting benefits or costs that accrue at different points in time to a time equivalent basis. Discounting is necessary to account for the inflationary nature of money, investment risk and the amount of interest that may be earned as a result. For example, a dollar earned next year is worth less than a dollar earned this year. To account for this difference future dollars must be discounted by the anticipated inflation rate before being compared to present earnings. Usually, cash flows are converted to equivalent present values. They may also be converted to equivalent future values at some future year or to equivalent annual values based on the useful life of the investment. For use in discounting formulas, present values are abbreviated as "P", future values as "F" and annual values as "A".

The discount rate used, expressed as a percentage, is selected to reflect the investor's time preference for money, which may correspond to a wide range of investment opportunities. While one investor may require only a 5% return, another may demand a 10% return on investment funds. Whatever the rate selected, it should take into consideration the fact that energy savings "income" is not taxable as is income from many other investments. Also it is necessary to distinguish between "nominal" and "real" discount rates. Nominal rates include the effect of inflation whereas real rates do not.

As an example, suppose one has the opportunity to earn 12% on a typical financial investment with taxable earnings. What equivalent discount rate could be used to evaluate an energy conservation investment? Assuming a 25% tax bracket, actual earnings from the typical investment are 25% less, or 9%. Thus one could use a 9% nominal discount rate. It is more common however to use real discount rates so that future cash flows need not include inflation. If inflation is forecasted to average 5% over the life of the investment, the approximate real discount rate to use would be 4%, or the after tax interest rate (9%) minus the inflation rate (5%). More precisely, the rate would be 3.8%, or (1.09/1.05 - 1) 100%.

Present Worth Method

The Present Worth (PW) method, also referred to as the Net Present Value (NPV) method, uses discounting to convert cash flows associated with an investment over its useful life to an equivalent present value for comparison with other investment alternatives. The greater the net present worth, the more profitable the investment. To convert future amounts (F) to present amounts (P), it is necessary to know the specific "P/F" discount factor for the time period and discount rate under consideration. To convert annual amounts (A) to present amounts (P), the appropriate "P/A" discount factor is required. Expressed mathematically:

$$P = (P/F)\ F \text{ and}$$
$$P = (P/A)\ A,$$

where F and A are known and P/F and P/A depend on the assumed discount rate and investment life. Discount factors may be mathematically calculated, but typically are taken from standard tables such as appear at the back of this factsheet (Tables 4A - 4H). As commonly occurs, when comparing a conservation investment to the base line case of investing nothing, net benefits of the project, e.g. energy savings, may simply be compared to the initial cost of the investment. A present worth in excess of the initial cost would then indicate a profitable investment. The PW method is specifically useful for determining the optimum investment amount when various levels of related conservation measures are being considered, e.g. R19 vs. R30 attic insulation. However, PW cannot easily rank investments as with the internal rate of return method, discussed in the next section.

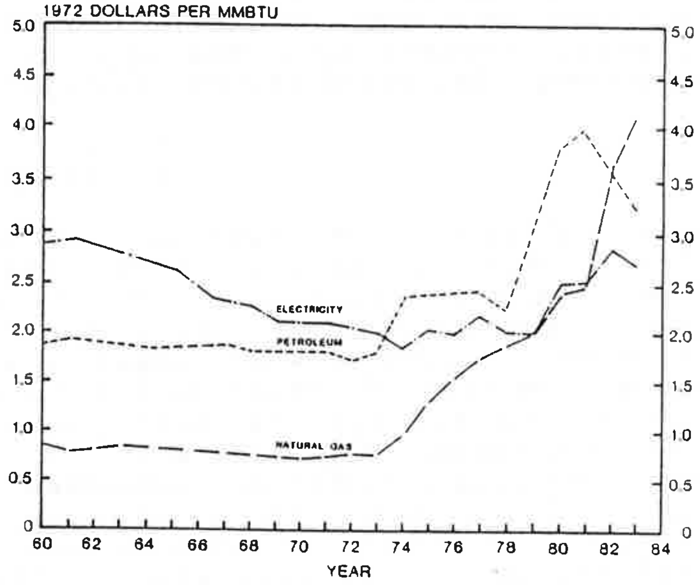
To demonstrate use of the P/F and P/A factors, the following sample problem will be useful. Suppose that it has been suggested that an existing oil burner be replaced with a flame-retention burner. The improvement in seasonal efficiency is estimated to be 11% at a cost of $600. Is this a good investment assuming a present annual heating bill of $500, a 15 year useful life and an 8% real discount rate? Note that the discount rate is normally selected based on the investor's individual preference but for present purposes has been chosen arbitrarily. Assuming no rate increases, the energy savings each year, in real or constant dollars (excluding inflation) is 11% of $500, or $55. If a nominal discount rate were used, the energy savings each year would have to be escalated to remain consistent.

Referring to Table 4F, the P/A factor for a discount rate i = 8% and a time period n = 15 years is 8.559. Then the present worth of the annual savings,

$$P = (P/A)\ A = (8.559)\ \$55 = \$471$$

and compared to the initial investment of $600 results in a negative net present worth ($471 - $600 = -$129) and is not economically justified on this basis. Possible situations that would improve the present worth of this investment would be a higher present fuel bill, an extended useful life (n>15 years), a reduced discount rate (i<8%) or a possible reduction in the initial investment. Other factors that were not accounted for may also affect the overall economics, such as lower maintenance costs or fuel prices escalating faster than the the average inflation rate.

Accordingly, this problem might be modified. Suppose the annual heating bill is $700, and it is estimated that fuel prices escalate at a real rate (above inflation) of 2% per year. In this case the overall time period and discount rate remain as before but the annual benefit is 11% of $700 or $77. Furthermore, this $77 is escalated each year at a rate of 2%. Rather than calculating the benefit each year and multiplying each number by the appropriate P/F factor and summing the resulting present worth amounts, a simpler method is possible. For a present benefit (or cost) that is assumed to escalate at a fixed rate throughout the time period, a "P/A*" discount factor is used that accounts for such changes. These factors are also given in standard tables for various discount and escalation rates (Tables 5A - 5H). Escalation rates, also expressed as a percent, are abbreviated as "e".



ENERGY PRICES (WASHINGTON STATE)
1972 $/10⁶ BTU

Referring to Table 5F, for i = 8%, the P/A* factor for n = 15 years and e = 2% is 9.787. Thus the present worth of the annual savings in this case,

$$P = (P/A^*)\ A = (9.787)\ \$77 = \$754$$

and compared to the initial investment of $600 results in a positive net present worth ($754 - $600 = +$154) and is therefore a good investment earning better than 8% higher than the rate of inflation. This problem also demonstrates how the overall economics of certain investments may be significantly altered by relatively small changes in one's initial assumptions.

Internal Rate of Return Method

As demonstrated above the PW method calculates the economic worth of an investment based on a preselected discount rate. The actual "rate of return" of the investment is not known except that it may fall above or below the assumed discount rate. Rate of return is defined as the interest rate, expressed as a percentage, for which discounted life time costs and savings are equal and is a measure of economic efficiency. The Internal Rate of Return (IRR) method, also referred to as the Return on Investment (ROI), or Rate of Return (ROR) method is used to determine the actual rate of return on an investment by a structured process of trial and error. It is particularly useful for evaluating conservation projects of different useful lives competing for the same budget where the optimal size of each project has already been determined.

This method also relies on discounting life cycle costs and another sample problem will serve to demonstrate its use. Suppose that most low cost, short payback conservation projects have already been undertaken, and consequently attic insulation is being considered in addition to the flame-retention burner mentioned in the previous section. The insulation is estimated to cost $800 with annual fuel savings of 10%. Useful life is assumed to be greater than 30 years. Using the IRR method the rate of return of each investment can be calculated and compared.

To begin the trial and error method, a discount rate near the expected rate of return is selected and net present worth calculated as above. A positive present worth indicates a discount rate that is too small and a negative value one that is too large. Successive rates are selected until a net present worth of zero is bracketed by two rates. At this point the actual rate of return corresponding to a net present worth of zero must be estimated by a process known as "interpolation". Interpolation starts by calculating a fractional amount. The fraction is obtained by dividing the positive net present worth by the sum of both positive and negative values (no minus signs used). The next step is to multiply this fraction by the difference in the two rates. Finally, this amount is added to the smaller rate.

Assuming a constant annual benefit of 10% of $500, or $50, a real discount rate of 6% is chosen first. With A = $50, i = 6%, and n = 30 years, and referring to Table 4E,

$$P = (P/A)\ A = (13.77)\ \$50 = \$689$$

for a net present worth of $689 - $800 = -$111. Since this is negative, a smaller discount rate is needed. Choosing 4%, and referring to Table 4C,

$$P = (P/A)\ A = (17.29)\ \$50 = \$865$$

for a net present worth of $865 - $800 = +65. Since this is positive, the actual rate of return must fall somewhere between 4%

and 6%. To interpolate this rate,

the "fractional amount" = 65/(65+111) = 0.37,
the "rate difference" = 6% - 4% = 2%,

and the internal rate of return is thus,

4% + (0.37) 2% = 4.7%.

The flame retention burner may be evaluated in the same way. With A = \$55, i = 6%, and n = 15 years (Table 4E),

P = (P/A) A = (9.71) \$55 = \$534

for a net present worth of \$534 - \$600 = -\$66. Choosing 4% (Table 4C),

P = (P/A) A = (11.12) \$55 = \$612

for a net present worth of \$612 - \$600 = +\$12. By interpolating, the actual rate of return is calculated as above,

4% + [12/(12 + 66)] (6% - 4%) = 4.3%.

Although the insulation yields a higher rate of return, it must still meet the investor's minimum acceptable rate of return. If the minimum acceptable rate is 5%, neither investment is justifiable and other alternatives should be investigated. Likewise, if the minimum acceptable rate is 4%, both investments could be made provided that funds are available.

To summarize this analysis, the insulation investment costs slightly more and offers slightly lower annual savings than the new burner yet it has a higher rate of return due to its relative longevity. Another way of explaining this is that in order for the burner to continue providing energy savings for as long as the insulation, a replacement burner would be required after 15 years thus reducing its economic appeal. It should be noted that since these particular investments have such closely comparable rates of return (4.3% vs. 4.7% above inflation) a more thorough look may provide information that would easily influence one's decision if a choice must be made. For example, if the oil burner is going to need replacement in several years anyway, then the incremental cost of a flame-retention burner over a conventional burner could be used for evaluation purposes rather than its full cost. It is likely that the burner investment would prove preferable under these circumstances.

DETERMINING BENEFITS AND COSTS

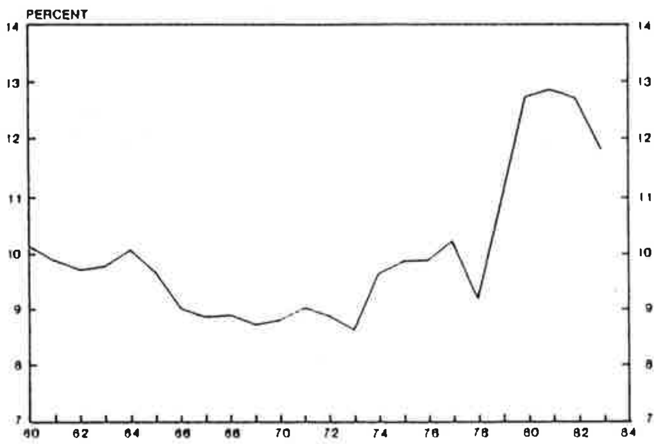
As shown in Table 1, homebuilt storm windows have a payback of 2 - 5 years compared to commercially installed units with a payback of 5 - 20 years. Does this fact make the homebuilt windows a better investment? It is important to remember that payback and life cycle cost comparisons of energy conservation investments are only as useful as the accuracy of the values assigned to benefits and costs. Furthermore, not all benefits and costs may even be quantifiable and able to be included in the economic analysis. If the commercially installed storm windows are more "attractive" to the buyer, how is this benefit evaluated? This section will attempt to introduce the many kinds of benefits and costs generally associated with energy conservation and how they may be affected by various economic factors.

Investor Perspective

From the perspective of the investor, quantifiable or "direct" benefits and costs are most simply the net energy savings and initial outlay, respectively. In addition, other benefits or costs associated with operation, maintenance, repair and replacement should be considered for most major investments. Non-quantifiable, or "indirect" benefits and costs generally relate to comfort, appearance, and feelings of independence, security or prestige. The storm window question used above is a good example of how indirect benefits may overrule economic concerns. Although homebuilt storms will save as much energy as commercial units, appearance or other operable features of the more expensive designs may outweigh this cost advantage. The effect on resale value of a residence may also be an important criteria for selecting energy conservation investments.

Regional Perspective

The Northwest Power Planning Council now recognizes conservation as the preferred alternative for meeting increasing electric power demands, due to its significant cost advantage. The direct benefit of conservation to the region then is lower energy prices for all users as higher cost power is subsequently avoided. Indirect benefits of conservation to the region primarily include improvements in environmental quality from reduced use of thermal generation, and stimulation of numerous local economies involved in the energy conservation and home improvement markets. Possible indirect costs of certain conservation actions relate to building aesthetics which may negatively affect some individuals or communities.



PERCENT OF INCOME SPENT ON ENERGY
(WASHINGTON STATE)

Factors Affecting Benefits and Costs

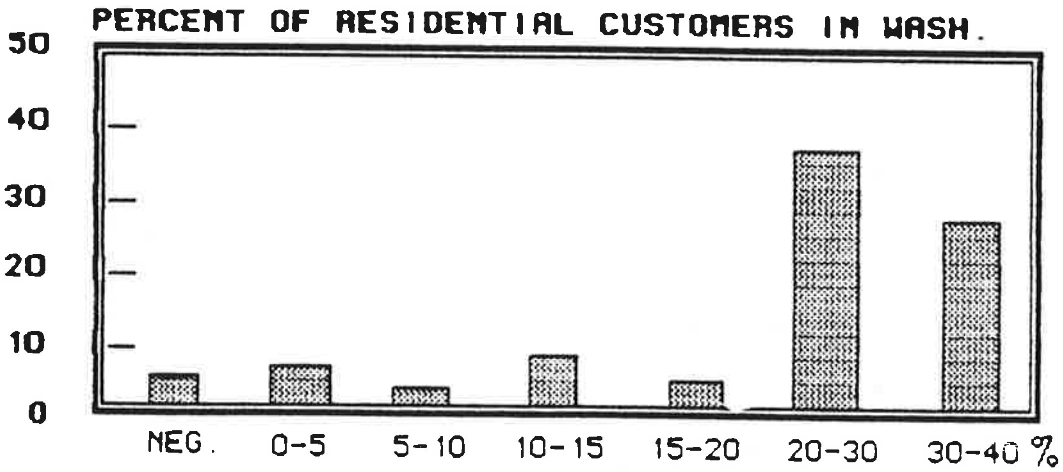
Of many economic factors affecting benefits and costs used in life cycle costing techniques, the most obvious perhaps are the discount rate and time period of evaluation. As observed in the sections describing these techniques a higher discount rate and shorter economic life adversely affect the present worth of an investment while a lower discount rate and longer life will show improvement. The selection of a discount rate is normally guided by the level of return on alternative investment opportunities or the cost of borrowing money. Higher discount rates may be suitable for high risk projects or to account for the fact that conservation investments are generally less "liquid" (easily cashed out) than other investments. Risk may be treated in other ways, such as basing benefit and cost estimates on probabilities of occurrence or incorporating contingency cash flows. It is important that fuel escalation rates and cash flow estimates that inflate over time be consistent (real or nominal) with the discount rate chosen.

The selection of a time period of evaluation is based either on the useful life of the investment or on the specific time perspective of the investor. "Useful" life refers to the time period during which the conservation investment is able to provide energy savings. "Economic" life, often used interchangeably, actually refers to the time period that the investment remains the least costly means of providing a particular conservation saving. In practice, useful life is easier to predict and is more often used. If the investor's interest in a conservation application is limited to the planned time of occupancy or, as with a speculative builder, to the time period between initial property development and resale, the time period of evaluation may be much shorter than the investment's useful life. In such cases, long term conservation benefits to society are usually foregone, as many new homes and buildings are still built with insufficient regard for energy conservation.

Other important factors affecting benefits and costs are financial incentives and salvage values. Financial incentives offered by a variety of government and private sources may reduce the initial cost of a conservation investment significantly. Cash grants are available from various utilities for residential solar hot water heating systems. A trial two year market incentive program initiated by BPA in 1985 offers rebates of $200 or $500 for residential installations of either solar or heat pump water heaters in selected counties and Public Utility Districts throughout the region. Tax incentives include a federal program of tax credits for conservation and alternative energy investments (due to expire after 1985). Property tax exemptions and liberal depreciation allowances for conservation investments also serve to reduce annual tax obligations. Low interest loans and loan buydowns, sometimes available from lending institutions, utilities or public assistance programs, result in reduced borrowing costs. Energy conservation investments that are uneconomical without financial incentives may become cost effective if subsidies are included in the economic evaluation.

Salvage value is the value of a capital asset remaining either at the end of the study period, or at the end of its useful life. It may be determined by estimating resale value of the asset, net the cost of removal, whenever the asset ceases to be evaluated or used. If an existing investment is being compared to a new one that would replace it, the current salvage (or resale) value of the existing investment may be subtracted from the first cost of the new alternative. A home or building utilizing energy conservation investments at time of resale will only provide salvage benefits to the seller if the remaining energy savings contained in the investment can be reflected in a higher resale price, or if the conservation devices can be removed and sold or used in another application. Since these possibilities are uncertain, it is often difficult to estimate salvage value with a high level of reliability. The importance of salvage value pertains particularly to short evaluation periods and to assets with a long useful life. In most cases when the useful life of an asset is exhausted, salvage values will be minimal.

The uncertainty relating to estimating salvage values applies to most cash flows that occur in the future. Since economic evaluations of energy conservation investments are only as accurate as the values used in formulating the analysis, particular attention should be given to making valid assumptions and realistic estimates. Analytical techniques for evaluating uncertainty that may be applied in complex economic studies include sensitivity and probability analysis. Sensitivity analysis tests the responsiveness of economic measurements to key factors such as discount rate, time horizons or fuel price escalation. Probability analysis determines benefits or costs based on their expected chance of occurrence. These techniques are not generally necessary for making home energy conservation investment decisions.



MODEL CONSERVATION STANDARDS (MCS)
CONSUMER ECONOMICS - INTERNAL RATE OF RETURN

PROBLEM #1 - SOLAR WATER HEATING

Assume that a homeowner will need to replace an aging conventional water heater soon and would like to compare a new efficient electric heater with an active solar water heating system. The homeowner's situation is as follows:

- present hot water use - 80 gal/day (heated from 50F to 130F)
- present cost of electricity - $0.045/kwh (forecasted to escalate at 2%/yr in real terms)
- solar availability - with roof mounted collectors, location is favorable (as determined by solar site survey)

Electric heating description:

- initial cost - $300 (installed)
- expected useful life - 12 years
- overall efficiency - 0.90 (including standby losses)
- first year operating cost (electricity) - $285/yr, calculated as follows:

$$\frac{(80\ \text{gal/day})(365\ \text{day/yr})(8.34\ \text{lb/gal})(130 - 50\text{F}^\circ)(1\text{Btu/lb} - \text{F}^\circ)(\$0.045/\text{kwh})}{(3413\ \text{Btu/kwh})(0.90)}$$

- replacement/repair costs - new heater in 13th year @ $300 (present value)
- maintenance costs - drain/flush tank once per year, 3 hr/yr @ $10/hr = $30/yr

Solar heating description:

- initial cost - $4500 (installed); federal tax credit - 40%($4500) = $1800; utility rebate - $300
- net initial cost - $2400
- expected useful life - 25 years (collectors, piping)
- overall efficiency - designed to meet 60% of hot water needs on an annual basis (with backup heating provided by single electric element); circulating pump consumes approximately 400 kwh/yr or 6% of total demand for net electric displacement of 54%
- first year operating cost (electricity) - with 54% savings, 0.46($285) = $131/yr

- replacement/repair costs - new storage tank in 13th year @ $500 (present value); new pump and controls in 8th and 16th years @ $200 each (present value)

- maintenance requirements - drain/flush tank once per year, 3 hr/yr; inspect, clean, maintain collectors once per year, 3 hr/yr @ $10/hr = $60/yr

To compare these two alternatives, the homeowner has decided to use the PW method to determine whether the incremental expense of the solar system is economically justified by its annual savings. An evaluation of incremental values is necessary since only the solar unit saves energy while the electric unit serves as a base case. Since the money spent on the solar system might also be invested in other long-term securities earning 10% interest (after tax), a 4% real discount rate is selected assuming a 6% long-term average inflation rate. The time period of evaluation selected is 25 years, the estimated useful life of the solar collectors. Both electric and solar storage tanks will require replacement halfway through this period (during the 13th year). After 25 years, it is assumed all components of both systems will require replacement and therefore no salvage values are included.

A summary of cash flows for both electric and solar heating systems, plus incremental values and present worth calculations are given in Table 2. Note that net present worth is determined by summing the individual equivalent present values of each cash flow under consideration. Since the net present worth that results is positive, the solar water heating system may be considered a good investment based on the economic parameters selected, earning better than a 4% real rate of return or 10% including inflation. In cases where the economic returns are marginally satisfactory, indirect benefits of each investment could be examined as well.

Although fairly typical values were selected, it is important to realize that the above problem represents a hypothetical situation and is only intended to demonstrate use of the analytical methods employed. Wide variety in the types of alternative water heating systems, in the actual conditions of use and in the selection of economic analysis factors plays a critical role in determining which investment will be most economic in any particular situation. For example, so little as removal of the utility rebate in the above case would result in a negative net present worth.

PROBLEM #2 - WOOD SPACE HEATING

Assume that an owner/builder is designing a new home and would like to compare electric space heating with a woodstove installation. The design conditions are as follows:

- o heating requirements of house - 8700 kwh/season (based on 1500 ft built to 1980 code standards)

- o present cost of electricity - $0.035/kwh (forecasted to

	Cash Flows				
	Electric Heating	Solar Heating	Incremental Value	Present Worth Factor*	Equivalent Present Value
Initial Cost	$ 300	$ 2400	$ -2100	1.00	$ -2100
<u>Annual</u> <u>Amounts</u> <u>(A)</u>					
Operating Costs (escalating)	285	131	154	19.61	3020
Maintenance Costs	30	60	-30	15.62	-469
<u>Future</u> <u>Amounts</u> <u>(F)</u>					
Replacement/repair costs (yr 8)	0	200	-200	0.731	-146
Replacement/repair costs (yr 13)	0	200	-200	0.601	-120
Replacement/repair costs (yr 16)	300	500	-200	0.534	-107
Net Present Worth					$ 78

* To locate the appropriate Present Worth Factors:
Converting from annual amounts, the P/A* factor for i=4%, n=25 years and e=2% is found in Table 5C and the P/A factor for i=4% and n=25 years is found in Table 4C. Converting from future amounts, the P/F factors for i=4% and n=8, 13 and 16 years are also found in Table 4C.

Table 2. Summary of Cash Flows and Present Worth Calculations - Solar Water Heating Problem #1.

escalate at 3%/yr in real terms)

- present cost of wood - $100/cord (forecasted to rise at the general inflation rate)

Electric heating description:

- initial cost of system - $1200 (installed)
- expected useful life - 15 years
- overall efficiency - 1.00
- first year operating cost (electricity) - $305/yr (=8700 kwh x $0.035/kwh)
- replacement/repair costs - entire system replaced in 15th year @ $1200 (present value)
- maintenance costs - none
- salvage value of replacement system (yr 20) - $800 (2/3 life remaining)

Wood heating description:

- initial cost of woodstove (including chimney and hearth) - $1100 (installed)
- expected useful life - 20 years
- overall efficiency - 0.50
- initial cost of baseboard backup - $200 (installed)
- expected useful life - 15 years
- overall efficiency - 1.00
- first year operating cost (wood) - $220/yr (to meet 80% total heating requirements = 2.2 cords)
- first year operating cost (electricity) - $61/yr (to meet 20% of total heating requirements)
- replacement/repair costs - woodstove, $20/yr (miscellaneous repair); electric units replaced in 15th year @ $200 (present value)
- maintenance costs - 2 woodstove cleanings per year @ $50 ea = $100/yr
- salvage value of replacement heaters (yr 20) - $133 (2/3 life remaining)

Assuming the owner/builder seeks an after tax return of 10% interest and predicts long-term inflation will average 5%, a real rate of return of 5% is desirable. The PW method is again useful to determine if the wood heating system satisfies this investment criteria. As in the last problem, Table 3 summarizes present worth calculations based on the incremental cash flows. Since the net present worth that results is negative, wood heating may be considered a poor investment based on the economic parameters selected, earning less than a 5% return. Investments in homes that incorporate conservation measures may be evaluated in another way, however. Since most of the cost is financed, the IRR of the investment may be based on the down payment. In this case, home appreciation is the main concern.

As previously mentioned, the intent of these problems is merely to demonstrate the use of life cycle costing techniques. The numerous individual and subjective factors that are inherent to the problem's solution make it impossible to generalize concerning the outcome of similar economic studies.

It is interesting to note how much information is provided by equivalent present value data given in Table 3. For instance the cost of wood is much less than electricity over the 20 year time period ($2741 vs. $4011) yet maintenance of the woodstove ($1246) almost exactly offsets those savings. Also, a slight economic advantage results from the increased useful life of the woodstove equal to the difference in replacement, less salvage, costs ($481 - $251 = $230).

One factor not accounted for in this economic analysis is the time consuming and often laborious act of building and tending fires in the woodstove. Including this indirect cost in the evaluation will make it more difficult to justify wood heating. On the other hand, the overall benefits of wood heat could be improved if the wood is self-harvested or domestic water heating were included. In the final analysis, the economics of wood heating, as with many other alternative energy systems, highly depends on how much one values his or her time.

SUMMARY

More useful than simple payback analysis, life cycle costing methods offer the opportunity to evaluate energy conservation investments based on overall economic efficiency. PW and IRR calculations account for both total ownership benefits and costs as well as the time value of money. Since economic evaluations rely on numerous subjective assumptions and the prediction of future benefits and costs, results are highly individualized. Non-quantifiable, or indirect, economic factors may also highly influence the attractiveness of a particular investment. For these reasons, differences in returns of 1-2% among most energy conservation investments, competing against each other or a fixed minimum rate of return, are probably not significant.

	Cash Flows				
	Electric Heating	Wood Heating	Incremental Value	Present Worth Factor*	Equivalent Present Value
Initial Cost	$ 1200	$ 1300	$ -100	1.00	$ -100
Annual Amounts (A)					
Operating Costs - electric (escalating)	305	61	244	16.44	4011
Operating Costs - wood	0	220	-220	12.46	-2741
Replacement/repair costs	0	20	-20	12.46	-249
Maintenance costs	0	100	-100	12.46	-1246
Future Amounts (F)					
Replacement/repair costs (yr 15)	1200	200	1000	0.481	481
Salvage value (yr 20)	-800	-133	-667	0.377	-251
Net Present Worth					$ -95

* To locate the appropriate Present Worth Factors:
Converting from annual amounts, the P/A* factor for i=5%, n=20 years and e=3% is found in Table 5D and the P/A factor for i=5% and n=20 years is found in Table 4D. Converting from future amounts, the P/F factors for i=5% and n=15 and 20 years are also found in Table 4D.

Table 3. Summary of Cash Flows and Present Worth Calculations - Wood Heating Problem #2.

Both the PW and IRR methods are similar in that they involve converting future cash flows to an equivalent time basis, typically present values. They differ in that the PW method compares the profitability of investments to a fixed rate of return, the assumed discount rate, while the IRR method solves for the actual rate of return by finding the discount rate that corresponds to a net present worth of zero. Due to this difference, each method offers distinct benefits.

Present Worth:

- determines the optimum level of related (non-mutually exclusive) conservation investments
- determines if minimum acceptable rate of return is met
- simpler to compare investments of equal useful lives

Internal Rate of Return:

- able to rank mutually exclusive investments competing for funds
- determines actual rate of return
- easier to compare investments of variable useful lives

While each method has its advocates, it is best to understand both and how they are interrelated. Either method will serve the investor with a practical approach to making sound economic decisions. Their reliability depends entirely on the accuracy of one's estimates and assumptions. For further reading, a list of reference texts and publications is included.

This factsheet was written by Jack Brautigam. Artwork was provided by Rudi Fyles and Mike Nelson.

SUGGESTED READING

- Marshall, Harold E. and Ruegg, Rosalie T. Simplified Energy Design Economics National Bureau of Standards Special Publication 544, U.S. Government Printing Office. Washington, D.C. 1980.

  Informative overview of various life cycle costing methods, with sample problems relating specifically to conservation and solar investments in buildings.

- Marshall, Harold E. and Ruegg, Rosalie T. Energy Conservation in Buildings: An Economics Guidebook for Investment Decisions. National Bureau of Standards Handbook 132, U.S. Government Printing Office. Washington, D.C. 1980.

  A more comprehensive and complex treatment of economics than the above publication; detailed illustration and numerous sample problems are used throughout.

- Grant, Eugene L. and Ireson, W. Grant. Principles of Engineering Economy, 5th Edition. The Ronald Press Co., New York, NY 1970.

  An in-depth textbook review of the principles and techniques needed for making decisions concerning the acquisition and retirement of capital goods.

- "How to Cash In on Energy Buys". Solar Age. Solar Vision Inc. Harrisonville, NH. December 1984 and February 1985 with corrections (p. 6).

  Terse but detailed summary of economic methods and formulas appropriate to investments in conservation and renewable energy - including worksheets.

- "Life Cycle Cost Analysis versus Payback for Evaluating Project Alternatives". Heating/Piping/Air Conditioning. Reinhold Publishing Division of Penton/IPC. Stamford, CT. September 1984.

  Review of advantages of life cycle costing over payback for choosing among energy conservation investment alternatives.

- "Rate of Return Analysis in the Evaluation of Project Alternatives". Heating/Piping/Air Conditioning. Reinhold Publishing Division of Penton/IPC. Stamford, CT. September 1983.

  Comparison of Rate of Return and Present Worth life cycle costing methods.

TABLE 4A

2% Discount Factors

n	Compound Amount Factor F/P	Present Worth Factor P/F	Sinking Fund Factor A/F	Capital Recovery Factor A/P	Compound Amount Factor F/A	Present Worth Factor P/A	n
1	1.0200	0.9804	1.000 00	1.020 00	1.000	0.980	1
2	1.0404	0.9612	0.495 05	0.515 05	2.020	1.942	2
3	1.0612	0.9423	0.326 75	0.346 75	3.060	2.884	3
4	1.0824	0.9238	0.242 62	0.262 62	4.122	3.808	4
5	1.1041	0.9057	0.192 16	0.212 16	5.204	4.713	5
6	1.1262	0.8880	0.158 53	0.178 53	6.308	5.601	6
7	1.1487	0.8706	0.134 51	0.154 51	7.434	6.472	7
8	1.1717	0.8535	0.116 51	0.136 51	8.583	7.325	8
9	1.1951	0.8368	0.102 52	0.122 52	9.755	8.162	9
10	1.2190	0.8203	0.091 33	0.111 33	10.950	8.983	10
11	1.2434	0.8043	0.082 18	0.102 18	12.169	9.787	11
12	1.2682	.7885	0.074 56	0.094 56	13.412	10.575	12
13	1.2936	0.7730	0.068 12	0.088 12	14.680	11.348	13
14	1.3195	0.7579	0.062 60	0.082 60	15.974	12.106	14
15	1.3459	0.7430	0.057 83	0.077 83	17.293	12.849	15
16	1.3728	0.7284	0.053 65	0.073 65	18.639	13.578	16
17	1.4002	0.7142	0.049 97	0.069 97	20.012	14.292	17
18	1.4282	0.7002	0.046 70	0.066 70	21.412	14.992	18
19	1.4568	0.6864	0.043 78	0.063 78	22.841	15.678	19
20	1.4859	0.6730	0.041 16	0.061 16	24.297	16.351	20
21	1.5157	0.6598	0.038 78	0.058 78	25.783	17.011	21
22	1.5460	0.6468	0.036 63	0.056 63	27.299	17.658	22
23	1.5769	0.6342	0.034 67	0.054 67	28.845	18.292	23
24	1.6084	0.6217	0.032 87	0.052 87	30.422	18.914	24
25	1.6406	0.6095	0.031 22	0.051 22	32.030	19.523	25
26	1.6734	0.5976	0.029 70	0.049 70	33.671	20.121	26
27	1.7069	0.5859	0.028 29	0.048 29	35.344	20.707	27
28	1.7410	0.5744	0.026 99	0.046 99	37.051	21.281	28
29	1.7758	0.5631	0.025 78	0.045 78	38.792	21.844	29
30	1.8114	0.5521	0.024 65	0.044 65	40.568	22.396	30
31	1.8476	0.5412	0.023 60	0.043 60	42.379	22.938	31
32	1.8845	0.5306	0.022 61	0.042 61	44.227	23.468	32
33	1.9222	0.5202	0.021 69	0.041 69	46.112	23.989	33
34	1.9607	0.5100	0.020 82	0.040 82	48.034	24.499	34
35	1.9999	0.5000	0.020 00	0.040 00	49.994	24.999	35
40	2.2080	0.4529	0.016 56	0.036 56	60.402	27.355	40
45	2.4379	0.4102	0.013 91	0.033 91	71.893	29.490	45
50	2.6916	0.3715	0.011 82	0.031 82	84.579	31.424	50
55	2.9717	0.3365	0.010 14	0.030 14	98.587	33.175	55
60	3.2810	0.3048	0.008 77	0.028 77	114.052	34.761	60
65	3.6225	0.2761	0.007 63	0.027 63	131.126	36.197	65
70	3.9996	0.2500	0.006 67	0.026 67	149.978	37.499	70
75	4.4158	0.2265	0.005 86	0.025 86	170.792	38.677	75
80	4.8754	0.2051	0.005 16	0.025 16	193.772	39.745	80
85	5.3829	0.1858	0.004 56	0.024 56	219.144	40.711	85
90	5.9431	0.1683	0.004 05	0.024 05	247.157	41.587	90
95	6.5617	0.1524	0.003 60	0.023 60	278.085	42.380	95
100	7.2446	0.1380	0.003 20	0.023 20	312.232	43.098	100

TABLE 4B

3% Discount Factors

n	Compound Amount Factor F/P	Present Worth Factor P/F	Sinking Fund Factor A/F	Capital Recovery Factor A/P	Compound Amount Factor F/A	Present Worth Factor P/A	n
1	1.0300	0.9709	1.000 00	1.030 00	1.000	0.971	1
2	1.0609	0.9426	0.492 61	0.522 61	2.030	1.913	2
3	1.0927	0.9151	0.323 53	0.353 53	3.091	2.829	3
4	1.1255	0.8885	0.239 03	0.269 03	4.184	3.717	4
5	1.1593	0.8626	0.188 35	0.218 35	5.309	4.580	5
6	1.1941	0.8375	0.154 60	0.184 60	6.468	5.417	6
7	1.2299	0.8131	0.130 51	0.160 51	7.662	6.230	7
8	1.2668	0.7894	0.112 46	0.142 46	8.892	7.020	8
9	1.3048	0.7664	0.098 43	0.128 43	10.159	7.786	9
10	1.3439	0.7441	0.087 23	0.117 23	11.464	8.530	10
11	1.3842	0.7224	0.078 08	0.108 08	12.808	9.253	11
12	1.4258	0.7014	0.070 46	0.100 46	14.192	9.954	12
13	1.4685	0.6810	0.064 03	0.094 03	15.618	10.635	13
14	1.5126	0.6611	0.058 53	0.088 53	17.086	11.296	14
15	1.5580	0.6419	0.053 77	0.083 77	18.599	11.938	15
16	1.6047	0.6232	0.049 61	0.079 61	20.157	12.561	16
17	1.6528	0.6050	0.045 95	0.075 95	21.762	13.166	17
18	1.7024	0.5874	0.042 71	0.072 71	23.414	13.754	18
19	1.7535	0.5703	0.039 81	0.069 81	25.117	14.324	19
20	1.8061	0.5537	0.037 22	0.067 22	26.870	14.877	20
21	1.8603	0.5375	0.034 87	0.064 87	28.676	15.415	21
22	1.9161	0.5219	0.032 75	0.062 75	30.537	15.937	22
23	1.9736	0.5067	0.030 81	0.060 81	32.453	16.444	23
24	2.0328	0.4919	0.029 05	0.059 05	34.426	16.936	24
25	2.0938	0.4776	0.027 43	0.057 43	36.459	17.413	25
26	2.1566	0.4637	0.025 94	0.055 94	38.553	17.877	26
27	2.2213	0.4502	0.024 56	0.054 56	40.710	18.327	27
28	2.2879	0.4371	0.023 29	0.053 29	42.931	18.764	28
29	2.3566	0.4243	0.022 11	0.052 11	45.219	19.188	29
30	2.4273	0.4120	0.021 02	0.051 02	47.575	19.600	30
31	2.5001	0.4000	0.020 00	0.050 00	50.003	20.000	31
32	2.5751	0.3883	0.019 05	0.049 05	52.503	20.389	32
33	2.6523	0.3770	0.018 16	0.048 16	55.078	20.766	33
34	2.7319	0.3660	0.017 32	0.047 32	57.730	21.132	34
35	2.8139	0.3554	0.016 54	0.046 54	60.462	21.487	35
40	3.2620	0.3066	0.013 26	0.043 26	75.401	23.115	40
45	3.7816	0.2644	0.010 79	0.040 79	92.720	24.519	45
50	4.3839	0.2281	0.008 87	0.038 87	112.797	25.730	50
55	5.0821	0.1968	0.007 35	0.037 35	136.072	26.774	55
60	5.8916	0.1697	0.006 13	0.036 13	163.053	27.676	60
65	6.8300	0.1464	0.005 15	0.035 15	194.333	28.453	65
70	7.9178	0.1263	0.004 34	0.034 34	230.594	29.123	70
75	9.1789	0.1089	0.003 67	0.033 67	272.631	29.702	75
80	10.6409	0.0940	0.003 11	0.033 11	321.363	30.201	80
85	12.3357	0.0811	0.002 65	0.032 65	377.857	30.631	85
90	14.3005	0.0699	0.002 26	0.032 26	443.349	31.002	90
95	16.5782	0.0603	0.001 93	0.031 93	519.272	31.323	95
100	19.2186	0.0520	0.001 65	0.031 65	607.288	31.599	100

TABLE 4C

4% Discount Factors

n	Compound Amount Factor F/P	Present Worth Factor P/F	Sinking Fund Factor A/F	Capital Recovery Factor A/P	Compound Amount Factor F/A	Present Worth Factor P/A	n
1	1.0400	0.9615	1.000 00	1.040 00	1.000	0.962	1
2	1.0816	0.9246	0.490 20	0.530 20	2.040	1.886	2
3	1.1249	0.8890	0.320 35	0.360 35	3.122	2.775	3
4	1.1699	0.8548	0.235 49	0.275 49	4.246	3.630	4
5	1.2167	0.8219	0.184 63	0.224 63	5.416	4.452	5
6	1.2653	0.7903	0.150 76	0.190 76	6.633	5.242	6
7	1.3159	0.7599	0.126 61	0.166 61	7.898	6.002	7
8	1.3686	0.7307	0.108 53	0.148 53	9.214	6.733	8
9	1.4233	0.7026	0.094 49	0.134 49	10.583	7.435	9
10	1.4802	0.6756	0.083 29	0.123 29	12.006	8.111	10
11	1.5395	0.6496	0.074 15	0.114 15	13.486	8.760	11
12	1.6010	0.6246	0.066 55	0.106 55	15.026	9.385	12
13	1.6651	0.6006	0.060 14	0.100 14	16.627	9.986	13
14	1.7317	0.5775	0.054 67	0.094 67	18.292	10.563	14
15	1.8009	0.5553	0.049 94	0.089 94	20.024	11.118	15
16	1.8730	0.5339	0.045 82	0.085 82	21.825	11.652	16
17	1.9479	0.5134	0.042 20	0.082 20	23.698	12.166	17
18	2.0258	0.4936	0.038 99	0.078 99	25.645	12.659	18
19	2.1068	0.4746	0.036 14	0.076 14	27.671	13.134	19
20	2.1911	0.4564	0.033 58	0.073 58	29.778	13.590	20
21	2.2788	0.4388	0.031 28	0.071 28	31.969	14.029	21
22	2.3699	0.4220	0.029 20	0.069 20	34.248	14.451	22
23	2.4647	0.4057	0.027 31	0.067 31	36.618	14.857	23
24	2.5633	0.3901	0.025 59	0.065 59	39.083	15.247	24
25	2.6658	0.3751	0.024 01	0.064 01	41.646	15.622	25
26	2.7725	0.3607	0.022 57	0.062 57	44.312	15.983	26
27	2.8834	0.3468	0.021 24	0.061 24	47.084	16.330	27
28	2.9987	0.3335	0.020 01	0.060 01	49.968	16.663	28
29	3.1187	0.3207	0.018 88	0.058 88	52.966	16.984	29
30	3.2434	0.3083	0.017 83	0.057 83	56.085	17.292	30
31	3.3731	0.2965	0.016 86	0.056 86	59.328	17.588	31
32	3.5081	0.2851	0.015 95	0.055 95	62.701	17.874	32
33	3.6484	0.2741	0.015 10	0.055 10	66.210	18.148	33
34	3.7943	0.2636	0.014 31	0.054 31	69.858	18.411	34
35	3.9461	0.2534	0.013 58	0.053 58	73.652	18.665	35
40	4.8010	0.2083	0.010 52	0.050 52	95.026	19.793	40
45	5.8412	0.1712	0.008 26	0.048 26	121.029	20.720	45
50	7.1067	0.1407	0.006 55	0.046 55	152.667	21.482	50
55	8.6464	0.1157	0.005 23	0.045 23	191.159	22.109	55
60	10.5196	0.0951	0.004 20	0.044 20	237.991	22.623	60
65	12.7987	0.0781	0.003 39	0.043 39	294.968	23.047	65
70	15.5716	0.0642	0.002 75	0.042 75	364.290	23.395	70
75	18.9453	0.0528	0.002 23	0.042 23	448.631	23.680	75
80	23.0500	0.0434	0.001 81	0.041 81	551.245	23.915	80
85	28.0436	0.0357	0.001 48	0.041 48	676.090	24.109	85
90	34.1193	0.0293	0.001 21	0.041 21	827.983	24.267	90
95	41.5114	0.0241	0.000 99	0.040 99	1 012.785	24.398	95
100	50.5049	0.0198	0.000 81	0.040 81	1 237.624	24.505	100

TABLE 4D

5% Discount Factors

n	Compound Amount Factor F/P	Present Worth Factor P/F	Sinking Fund Factor A/F	Capital Recovery Factor A/P	Compound Amount Factor F/A	Present Worth Factor P/A	n
1	1.0500	0.9524	1.000 00	1.050 00	1.000	0.952	1
2	1.1025	0.9070	0.487 80	0.537 80	2.050	1.859	2
3	1.1576	0.8638	0.317 21	0.367 21	3.153	2.723	3
4	1.2155	0.8227	0.232 01	0.282 01	4.310	3.546	4
5	1.2763	0.7835	0.180 97	0.230 97	5.526	4.329	5
6	1.3401	0.7462	0.147 02	0.197 02	6.802	5.076	6
7	1.4071	0.7107	0.122 82	0.172 82	8.142	5.786	7
8	1.4775	0.6768	0.104 72	0.154 72	9.549	6.463	8
9	1.5513	0.6446	0.090 69	0.140 69	11.027	7.108	9
10	1.6289	0.6139	0.079 50	0.129 50	12.578	7.722	10
11	1.7103	0.5847	0.070 39	0.120 39	14.207	8.306	11
12	1.7959	0.5568	0.062 83	0.112 83	15.917	8.863	12
13	1.8856	0.5303	0.056 46	0.106 46	17.713	9.394	13
14	1.9800	0.5051	0.051 02	0.101 02	19.599	9.899	14
15	2.0789	0.4810	0.046 34	0.096 34	21.579	10.380	15
16	2.1829	0.4581	0.042 27	0.092 27	23.657	10.838	16
17	2.2920	0.4363	0.038 70	0.088 70	25.840	11.274	17
18	2.4066	0.4155	0.035 55	0.085 55	28.132	11.690	18
19	2.5270	0.3957	0.032 75	0.082 75	30.539	12.085	19
20	2.6533	0.3769	0.030 24	0.080 24	33.066	12.462	20
21	2.7860	0.3589	0.028 00	0.078 00	35.719	12.821	21
22	2.9253	0.3418	0.025 97	0.075 97	38.505	13.163	22
23	3.0715	0.3256	0.024 14	0.074 14	41.430	13.489	23
24	3.2251	0.3101	0.022 47	0.072 47	44.502	13.799	24
25	3.3864	0.2953	0.020 95	0.070 95	47.727	14.094	25
26	3.5557	0.2812	0.019 56	0.069 56	51.113	14.375	26
27	3.7335	0.2678	0.018 29	0.068 29	54.669	14.643	27
28	3.9201	0.2551	0.017 12	0.067 12	58.403	14.898	28
29	4.1161	0.2429	0.016 05	0.066 05	62.323	15.141	29
30	4.3219	0.2314	0.015 05	0.065 05	66.439	15.372	30
31	4.5380	0.2204	0.014 13	0.064 13	70.761	15.593	31
32	4.7649	0.2099	0.013 28	0.063 28	75.299	15.803	32
33	5.0032	0.1999	0.012 49	0.062 49	80.064	16.003	33
34	5.2533	0.1904	0.011 76	0.061 76	85.067	16.193	34
35	5.5160	0.1813	0.011 07	0.061 07	90.320	16.374	35
40	7.0400	0.1420	0.008 28	0.058 28	120.800	17.159	40
45	8.9850	0.1113	0.006 26	0.056 26	159.700	17.774	45
50	11.4674	0.0872	0.004 78	0.054 78	209.348	18.256	50
55	14.6356	0.0683	0.003 67	0.053 67	272.713	18.633	55
60	18.6792	0.0535	0.002 83	0.052 83	353.584	18.929	60
65	23.8399	0.0419	0.002 19	0.052 19	456.798	19.161	65
70	30.4264	0.0329	0.001 70	0.051 70	588.529	19.343	70
75	38.8327	0.0258	0.001 32	0.051 32	756.654	19.485	75
80	49.5614	0.0202	0.001 03	0.051 03	971.229	19.596	80
85	63.2544	0.0158	0.000 80	0.050 80	1 245.087	19.684	85
90	80.7304	0.0124	0.000 63	0.050 63	1 594.607	19.752	90
95	103.0357	0.0097	0.000 49	0.050 49	2 040.694	19.806	95
100	131.5013	0.0076	0.000 38	0.050 38	2 610.025	19.848	100

TABLE 4E

6% Discount Factors

n	Compound Amount Factor *F/P*	Present Worth Factor *P/F*	Sinking Fund Factor *A/F*	Capital Recovery Factor *A/P*	Compound Amount Factor *F/A*	Present Worth Factor *P/A*	*n*
1	1.0600	0.9434	1.000 00	1.060 00	1.000	0.943	1
2	1.1236	0.8900	0.485 44	0.545 44	2.060	1.833	2
3	1.1910	0.8396	0.314 11	0.374 11	3.184	2.673	3
4	1.2625	0.7921	0.228 59	0.288 59	4.375	3.465	4
5	1.3382	0.7473	0.177 40	0.237 40	5.637	4.212	5
6	1.4185	0.7050	0.143 36	0.203 36	6.975	4.917	6
7	1.5036	0.6651	0.119 14	0.179 14	8.394	5.582	7
8	1.5938	0.6274	0.101 04	0.161 04	9.897	6.210	8
9	1.6895	0.5919	0.087 02	0.147 02	11.491	6.802	9
10	1.7908	0.5584	0.075 87	0.135 87	13.181	7.360	10
11	1.8983	0.5268	0.066 79	0.126 79	14.972	7.887	11
12	2.0122	0.4970	0.059 28	0.119 28	16.870	8.384	12
13	2.1329	0.4688	0.052 96	0.112 96	18.882	8.853	13
14	2.2609	0.4423	0.047 58	0.107 58	21.015	9.295	14
15	2.3966	0.4173	0.042 96	0.102 96	23.276	9.712	15
16	2.5404	0.3936	0.038 95	0.098 95	25.673	10.106	16
17	2.6928	0.3714	0.035 44	0.095 44	28.213	10.477	17
18	2.8543	0.3503	0.032 36	0.092 36	30.906	10.828	18
19	3.0256	0.3305	0.029 62	0.089 62	33.760	11.158	19
20	3.2071	0.3118	0.027 18	0.087 18	36.786	11.470	20
21	3.3996	0.2942	0.025 00	0.085 00	39.993	11.764	21
22	3.6035	0.2775	0.023 05	0.083 05	43.392	12.042	22
23	3.8197	0.2618	0.021 28	0.081 28	46.996	12.303	23
24	4.0489	0.2470	0.019 68	0.079 68	50.816	12.550	24
25	4.2919	0.2330	0.018 23	0.078 23	54.865	12.783	25
26	4.5494	0.2198	0.016 90	0.076 90	59.156	13.003	26
27	4.8223	0.2074	0.015 70	0.075 70	63.706	13.211	27
28	5.1117	0.1956	0.014 59	0.074 59	68.528	13.406	28
29	5.4184	0.1846	0.013 58	0.073 58	73.640	13.591	29
30	5.7435	0.1741	0.012 65	0.072 65	79.058	13.765	30
31	6.0881	0.1643	0.011 79	0.071 79	84.802	13.929	31
32	6.4534	0.1550	0.011 00	0.071 00	90.890	14.084	32
33	6.8406	0.1462	0.010 27	0.070 27	97.343	14.230	33
34	7.2510	0.1379	0.009 60	0.069 60	104.184	14.368	34
35	7.6861	0.1301	0.008 97	0.068 97	111.435	14.498	35
40	10.2857	0.0972	0.006 46	0.066 46	154.762	15.046	40
45	13.7646	0.0727	0.004 70	0.064 70	212.744	15.456	45
50	18.4202	0.0543	0.003 44	0.063 44	290.336	15.762	50
55	24.6503	0.0406	0.002 54	0.062 54	394.172	15.991	55
60	32.9877	0.0303	0.001 88	0.061 88	533.128	16.161	60
65	44.1450	0.0227	0.001 39	0.061 39	719.083	16.289	65
70	59.0759	0.0169	0.001 03	0.061 03	967.932	16.385	70
75	79.0569	0.0126	0.000 77	0.060 77	1 300.949	16.456	75
80	105.7960	0.0095	0.000 57	0.060 57	1 746.600	16.509	80
85	141.5789	0.0071	0.000 43	0.060 43	2 342.982	16.549	85
90	189.4645	0.0053	0.000 32	0.060 32	3 141.075	16.579	90
95	253.5463	0.0039	0.000 24	0.060 24	4 209.104	16.601	95
100	339.3021	0.0029	0.000 18	0.060 18	5 638.368	16.618	100

TABLE 4F

8% Discount Factors

n	Compound Amount Factor *F/P*	Present Worth Factor *P/F*	Sinking Fund Factor *A/F*	Capital Recovery Factor *A/P*	Compound Amount Factor *F/A*	Present Worth Factor *P/A*	*n*
1	1.0800	0.9259	1.000 00	1.080 00	1.000	0.926	1
2	1.1664	0.8573	0.480 77	0.560 77	2.080	1.783	2
3	1.2597	0.7938	0.308 03	0.388 03	3.246	2.577	3
4	1.3605	0.7350	0.221 92	0.301 92	4.506	3.312	4
5	1.4693	0.6806	0.170 46	0.250 46	5.867	3.993	5
6	1.5869	0.6302	0.136 32	0.216 32	7.336	4.623	6
7	1.7138	0.5835	0.112 07	0.192 07	8.923	5.206	7
8	1.8509	0.5403	0.094 01	0.174 01	10.637	5.747	8
9	1.9990	0.5002	0.080 08	0.160 08	12.488	6.247	9
10	2.1589	0.4632	0.069 03	0.149 03	14.487	6.710	10
11	2.3316	0.4289	0.060 08	0.140 08	16.645	7.139	11
12	2.5182	0.3971	0.052 70	0.132 70	18.977	7.536	12
13	2.7196	0.3677	0.046 52	0.126 52	21.495	7.904	13
14	2.9372	0.3405	0.041 30	0.121 30	24.215	8.244	14
15	3.1722	0.3152	0.036 83	0.116 83	27.152	8.559	15
16	3.4259	0.2919	0.032 98	0.112 98	30.324	8.851	16
17	3.7000	0.2703	0.029 63	0.109 63	33.750	9.122	17
18	3.9960	0.2502	0.026 70	0.106 70	37.450	9.372	18
19	4.3157	0.2317	0.024 13	0.104 13	41.446	9.604	19
20	4.6610	0.2145	0.021 85	0.101 85	45.762	9.818	20
21	5.0338	0.1987	0.019 83	0.099 83	50.423	10.017	21
22	5.4365	0.1839	0.018 03	0.098 03	55.457	10.201	22
23	5.8715	0.1703	0.016 42	0.096 42	60.893	10.371	23
24	6.3412	0.1577	0.014 98	0.094 98	66.765	10.529	24
25	6.8485	0.1460	0.013 68	0.093 68	73.106	10.675	25
26	7.3964	0.1352	0.012 51	0.092 51	79.954	10.810	26
27	7.9881	0.1252	0.011 45	0.091 45	87.351	10.935	27
28	8.6271	0.1159	0.010 49	0.090 49	95.339	11.051	28
29	9.3173	0.1073	0.009 62	0.089 62	103.966	11.158	29
30	10.0627	0.0994	0.008 83	0.088 83	113.283	11.258	30
31	10.8677	0.0920	0.008 11	0.088 11	123.346	11.350	31
32	11.7371	0.0852	0.007 45	0.087 45	134.214	11.435	32
33	12.6760	0.0789	0.006 85	0.086 85	145.951	11.514	33
34	13.6901	0.0730	0.006 30	0.086 30	158.627	11.587	34
35	14.7853	0.0676	0.005 80	0.085 80	172.317	11.655	35
40	21.7245	0.0460	0.003 86	0.083 86	259.057	11.925	40
45	31.9204	0.0313	0.002 59	0.082 59	386.506	12.108	45
50	46.9016	0.0213	0.001 74	0.081 74	573.770	12.233	50
55	68.9139	0.0145	0.001 18	0.081 18	848.923	12.319	55
60	101.2571	0.0099	0.000 80	0.080 80	1 253.213	12.377	60
65	148.7798	0.0067	0.000 54	0.080 54	1 847.248	12.416	65
70	218.6064	0.0046	0.000 37	0.080 37	2 720.080	12.443	70
75	321.2045	0.0031	0.000 25	0.080 25	4 002.557	12.461	75
80	471.9548	0.0021	0.000 17	0.080 17	5 886.935	12.474	80
85	693.4565	0.0014	0.000 12	0.080 12	8 655.706	12.482	85
90	1 018.9151	0.0010	0.000 08	0.080 08	12 723.939	12.488	90
95	1 497.1205	0.0007	0.000 05	0.080 05	18 701.507	12.492	95
100	2 199.7613	0.0005	0.000 04	0.080 04	27 484.516	12.494	100

TABLE 4G

10% Discount Factors

n	Compound Amount Factor F/P	Present Worth Factor P/F	Sinking Fund Factor A/F	Capital Recovery Factor A/P	Compound Amount Factor F/A	Present Worth Factor P/A	n
1	1.1000	0.9091	1.000 00	1.100 00	1.000	0.909	1
2	1.2100	0.8264	0.476 19	0.576 19	2.100	1.736	2
3	1.3310	0.7513	0.302 11	0.402 11	3.310	2.487	3
4	1.4641	0.6830	0.215 47	0.315 47	4.641	3.170	4
5	1.6105	0.6209	0.163 80	0.263 80	6.105	3.791	5
6	1.7716	0.5645	0.129 61	0.229 61	7.716	4.355	6
7	1.9487	0.5132	0.105 41	0.205 41	9.487	4.868	7
8	2.1436	0.4665	0.087 44	0.187 44	11.436	5.335	8
9	2.3579	0.4241	0.073 64	0.173 64	13.579	5.759	9
10	2.5937	0.3855	0.062 75	0.162 75	15.937	6.144	10
11	2.8531	0.3505	0.053 96	0.153 96	18.531	6.495	11
12	3.1384	0.3186	0.046 76	0.146 76	21.384	6.814	12
13	3.4523	0.2897	0.040 78	0.140 78	24.523	7.103	13
14	3.7975	0.2633	0.035 75	0.135 75	27.975	7.367	14
15	4.1772	0.2394	0.031 47	0.131 47	31.772	7.606	15
16	4.5950	0.2176	0.027 82	0.127 82	35.950	7.824	16
17	5.0545	0.1978	0.024 66	0.124 66	40.545	8.022	17
18	5.5599	0.1799	0.021 93	0.121 93	45.599	8.201	18
19	6.1159	0.1635	0.019 55	0.119 55	51.159	8.365	19
20	6.7275	0.1486	0.017 46	0.117 46	57.275	8.514	20
21	7.4002	0.1351	0.015 62	0.115 62	64.002	8.649	21
22	8.1403	0.1228	0.014 01	0.114 01	71.403	8.772	22
23	8.9543	0.1117	0.012 57	0.112 57	79.543	8.883	23
24	9.8497	0.1015	0.011 30	0.111 30	88.497	8.985	24
25	10.8347	0.0923	0.010 17	0.110 17	98.347	9.077	25
26	11.9182	0.0839	0.009 16	0.109 16	109.182	9.161	26
27	13.1100	0.0763	0.008 26	0.108 26	121.100	9.237	27
28	14.4210	0.0693	0.007 45	0.107 45	134.210	9.307	28
29	15.8631	0.0630	0.006 73	0.106 73	148.631	9.370	29
30	17.4494	0.0573	0.006 08	0.106 08	164.494	9.427	30
31	19.1943	0.0521	0.005 50	0.105 50	181.943	9.479	31
32	21.1138	0.0474	0.004 97	0.104 97	201.138	9.526	32
33	23.2252	0.0431	0.004 50	0.104 50	222.252	9.569	33
34	25.5477	0.0391	0.004 07	0.104 07	245.477	9.609	34
35	28.1024	0.0356	0.003 69	0.103 69	271.024	9.644	35
40	45.2593	0.0221	0.002 26	0.102 26	442.593	9.779	40
45	72.8905	0.0137	0.001 39	0.101 39	718.905	9.863	45
50	117.3909	0.0085	0.000 86	0.100 86	1 163.909	9.915	50
55	189.0591	0.0053	0.000 53	0.100 53	1 880.591	9.947	55
60	304.4816	0.0033	0.000 33	0.100 33	3 034.816	9.967	60
65	490.3707	0.0020	0.000 20	0.100 20	4 893.707	9.980	65
70	789.7470	0.0013	0.000 13	0.100 13	7 887.470	9.987	70
75	1 271.8952	0.0008	0.000 08	0.100 08	12 708.954	9.992	75
80	2 048.4002	0.0005	0.000 05	0.100 05	20 474.002	9.995	80
85	3 298.9690	0.0003	0.000 03	0.100 03	32 979.690	9.997	85
90	5 313.0226	0.0002	0.000 02	0.100 02	53 120.226	9.998	90
95	8 556.6760	0.0001	0.000 01	0.100 01	85 556.760	9.999	95
100	13 780.6123	0.0001	0.000 01	0.100 01	137 796.123	9.999	100

TABLE 4H

12% Discount Factors

n	Compound Amount Factor F/P	Present Worth Factor P/F	Sinking Fund Factor A/F	Capital Recovery Factor A/P	Compound Amount Factor F/A	Present Worth Factor P/A	n
1	1.1200	0.8929	1.000 00	1.120 00	1.000	0.893	1
2	1.2544	0.7972	0.471 70	0.591 70	2.120	1.690	2
3	1.4049	0.7118	0.296 35	0.416 35	3.374	2.402	3
4	1.5735	0.6355	0.209 23	0.329 23	4.779	3.037	4
5	1.7623	0.5674	0.157 41	0.277 41	6.353	3.605	5
6	1.9738	0.5066	0.123 23	0.243 23	8.115	4.111	6
7	2.2107	0.4523	0.099 12	0.219 12	10.089	4.564	7
8	2.4760	0.4039	0.081 30	0.201 30	12.300	4.968	8
9	2.7731	0.3606	0.067 68	0.187 68	14.776	5.328	9
10	3.1058	0.3220	0.056 98	0.176 98	17.549	5.650	10
11	3.4785	0.2875	0.048 42	0.168 42	20.655	5.938	11
12	3.8960	0.2567	0.041 44	0.161 44	24.133	6.194	12
13	4.3635	0.2292	0.035 68	0.155 68	28.029	6.424	13
14	4.8871	0.2046	0.030 87	0.150 87	32.393	6.628	14
15	5.4736	0.1827	0.026 82	0.146 82	37.280	6.811	15
16	6.1304	0.1631	0.023 39	0.143 39	42.753	6.974	16
17	6.8660	0.1456	0.020 46	0.140 46	48.884	7.120	17
18	7.6900	0.1300	0.017 94	0.137 94	55.750	7.250	18
19	8.6128	0.1161	0.015 76	0.135 76	63.440	7.366	19
20	9.6463	0.1037	0.013 88	0.133 88	72.052	7.469	20
21	10.8038	0.0926	0.012 24	0.132 24	81.699	7.562	21
22	12.1003	0.0826	0.010 81	0.130 81	92.503	7.645	22
23	13.5523	0.0738	0.009 56	0.129 56	104.603	7.718	23
24	15.1786	0.0659	0.008 46	0.128 46	118.155	7.784	24
25	17.0001	0.0588	0.007 50	0.127 50	133.334	7.843	25
26	19.0401	0.0525	0.006 65	0.126 65	150.334	7.896	26
27	21.3249	0.0469	0.005 90	0.125 90	169.374	7.943	27
28	23.8839	0.0419	0.005 24	0.125 24	190.699	7.984	28
29	26.7499	0.0374	0.004 66	0.124 66	214.583	8.022	29
30	29.9599	0.0334	0.004 14	0.124 14	241.333	8.055	30
31	33.5551	0.0298	0.003 69	0.123 69	271.292	8.085	31
32	37.5817	0.0266	0.003 28	0.123 28	304.847	8.112	32
33	42.0915	0.0238	0.002 92	0.122 92	342.429	8.135	33
34	47.1425	0.0212	0.002 60	0.122 60	384.520	8.157	34
35	52.7996	0.0189	0.002 32	0.122 32	431.663	8.176	35
40	93.0510	0.0107	0.001 30	0.121 30	767.091	8.244	40
45	163.9876	0.0061	0.000 74	0.120 74	1 358.230	8.283	45
50	289.0022	0.0035	0.000 42	0.120 42	2 400.018	8.305	50
∞				0.120 00		8.333	∞

TABLE 5A

2% P/A* Discount Factors

n	e=1%	e=2%	e=3%	e=4%	e=5%	e=6%
1	0.990	1.000	1.010	1.020	1.029	1.039
2	1.971	2.000	2.029	2.059	2.089	2.119
3	2.942	3.000	3.059	3.119	3.180	3.242
4	3.903	4.000	4.099	4.200	4.303	4.408
5	4.855	5.000	5.149	5.302	5.459	5.620
6	5.797	6.000	6.209	6.425	6.649	6.880
7	6.731	7.000	7.280	7.571	7.874	8.189
8	7.655	8.000	8.361	8.739	9.135	9.549
9	8.570	9.000	9.453	9.930	10.433	10.963
10	9.476	10.000	10.555	11.144	11.769	12.432
11	10.374	11.000	11.669	12.383	13.145	13.958
12	11.262	12.000	12.793	13.645	14.561	15.545
13	12.142	13.000	13.928	14.932	16.018	17.194
14	13.013	14.000	15.074	16.244	17.519	18.907
15	13.876	15.000	16.232	17.583	19.064	20.688
16	14.730	16.000	17.401	18.947	20.654	22.538
17	15.576	17.000	18.581	20.338	22.291	24.462
18	16.413	18.000	19.773	21.756	23.976	26.460
19	17.242	19.000	20.977	23.203	25.710	28.537
20	18.064	20.000	22.192	24.677	27.496	30.695
21	18.877	21.000	23.420	26.181	29.334	32.938
22	19.682	22.000	24.659	27.714	31.226	35.269
23	20.479	23.000	25.911	29.277	33.174	37.691
24	21.268	24.000	27.175	30.870	35.179	40.209
25	22.050	25.000	28.451	32.495	37.243	42.825
26	22.824	26.000	29.739	34.152	39.368	45.543
27	23.591	27.000	31.041	35.841	41.555	48.369
28	24.349	28.000	32.355	37.564	43.807	51.305
29	25.101	29.000	33.682	39.320	46.125	54.356
30	25.845	30.000	35.022	41.110	48.511	57.527

TABLE 5B

3% P/A* Discount Factors

n	e=1%	e=2%	e=3%	e=4%	e=5%	e=6%
1	0.981	0.990	1.000	1.010	1.019	1.029
2	1.942	1.971	2.000	2.029	2.059	2.088
3	2.885	2.942	3.000	3.059	3.118	3.178
4	3.810	3.904	4.000	4.098	4.198	4.300
5	4.716	4.856	5.000	5.148	5.299	5.454
6	5.605	5.799	6.000	6.207	6.421	6.642
7	6.477	6.733	7.000	7.277	7.565	7.865
8	7.332	7.658	8.000	8.358	8.732	9.123
9	8.170	8.574	9.000	9.448	9.921	10.418
10	8.992	9.481	10.000	10.550	11.133	11.750
11	9.798	10.380	11.000	11.662	12.368	13.122
12	10.588	11.269	12.000	12.785	13.628	14.533
13	11.363	12.150	13.000	13.919	14.912	15.985
14	12.123	13.022	14.000	15.064	16.221	17.480
15	12.868	13.886	15.000	16.220	17.555	19.018
16	13.599	14.742	16.000	17.387	18.915	20.602
17	14.316	15.589	17.000	18.565	20.302	22.231
18	15.018	16.428	18.000	19.755	21.716	23.907
19	15.707	17.259	19.000	20.957	23.157	25.633
20	16.383	18.081	20.000	22.170	24.626	27.408
21	17.045	18.896	21.000	23.395	26.123	29.236
22	17.695	19.703	22.000	24.632	27.650	31.117
23	18.332	20.502	23.000	25.881	29.206	33.052
24	18.956	21.293	24.000	27.142	30.793	35.044
25	19.569	22.077	25.000	28.415	32.410	37.094
26	20.170	22.853	26.000	29.700	34.059	39.203
27	20.758	23.621	27.000	30.999	35.740	41.374
28	21.336	24.382	28.000	32.309	37.453	43.608
29	21.902	25.136	29.000	33.633	39.200	45.907
30	22.458	25.882	30.000	34.969	40.980	48.274

TABLE 5C

4% P/A* Discount Factors

n	e=1%	e=2%	e=3%	e=4%	e=5%	e=6%
1	0.971	0.981	0.990	1.000	1.010	1.019
2	1.914	1.943	1.971	2.000	2.029	2.058
3	2.830	2.886	2.943	3.000	3.058	3.117
4	3.720	3.811	3.905	4.000	4.097	4.196
5	4.584	4.719	4.858	5.000	5.146	5.296
6	5.423	5.609	5.801	6.000	6.205	6.417
7	6.237	6.482	6.736	7.000	7.275	7.560
8	7.028	7.338	7.661	8.000	8.354	8.724
9	7.797	8.178	8.578	9.000	9.444	9.911
10	8.543	9.001	9.486	10.000	10.544	11.121
11	9.268	9.809	10.385	11.000	11.655	12.354
12	9.972	10.601	11.276	12.000	12.777	13.611
13	10.655	11.378	12.158	13.000	13.910	14.892
14	11.319	12.140	13.031	14.000	15.053	16.198
15	11.964	12.887	13.896	15.000	16.207	17.528
16	12.590	13.620	14.753	16.000	17.373	18.885
17	13.198	14.339	15.602	17.000	18.550	20.267
18	13.788	15.044	16.442	18.000	19.737	21.676
19	14.362	15.735	17.274	19.000	20.937	23.112
20	14.918	16.413	18.099	20.000	22.148	24.576
21	15.459	17.079	18.915	21.000	23.370	26.068
22	15.984	17.731	19.723	22.000	24.605	27.588
23	16.495	18.371	20.524	23.000	25.851	29.138
24	16.990	18.998	21.317	24.000	27.109	30.717
25	17.471	19.614	22.103	25.000	28.379	32.327
26	17.938	20.217	22.880	26.000	29.662	33.968
27	18.392	20.809	23.651	27.000	30.957	35.641
28	18.832	21.390	24.414	28.000	32.264	37.345
29	19.260	21.959	25.169	29.000	33.584	39.083
30	19.676	22.518	25.918	30.000	34.916	40.854

TABLE 5D

5% P/A* Discount Factors

n	e=1%	e=2%	e=3%	e=4%	e=5%	e=6%
1	0.962	0.971	0.981	0.990	1.000	1.010
2	1.887	1.915	1.943	1.972	2.000	2.029
3	2.777	2.832	2.887	2.943	3.000	3.057
4	3.633	3.722	3.813	3.906	4.000	4.096
5	4.457	4.587	4.721	4.859	5.000	5.145
6	5.249	5.428	5.612	5.803	6.000	6.203
7	6.011	6.244	6.487	6.738	7.000	7.272
8	6.744	7.037	7.344	7.665	8.000	8.351
9	7.449	7.808	8.185	8.582	9.000	9.440
10	8.127	8.556	9.010	9.491	10.000	10.539
11	8.779	9.283	9.819	10.391	11.000	11.649
12	9.407	9.989	10.613	11.282	12.000	12.769
13	10.010	10.675	11.392	12.165	13.000	13.900
14	10.591	11.341	12.156	13.040	14.000	15.042
15	11.149	11.989	12.905	13.906	15.000	16.195
16	11.686	12.618	13.641	14.764	16.000	17.359
17	12.203	13.229	14.362	15.614	17.000	18.534
18	12.700	13.822	15.069	16.456	18.000	19.720
19	13.178	14.399	15.763	17.290	19.000	20.917
20	13.638	14.959	16.444	18.116	20.000	22.126
21	14.080	15.503	17.111	18.934	21.000	23.346
22	14.506	16.031	17.766	19.744	22.000	24.578
23	14.915	16.545	18.409	20.546	23.000	25.821
24	15.309	17.043	19.039	21.341	24.000	27.077
25	15.688	17.528	19.658	22.128	25.000	28.344
26	16.052	17.998	20.264	22.908	26.000	29.624
27	16.402	18.456	20.859	23.680	27.000	30.915
28	16.739	18.900	21.443	24.445	28.000	32.219
29	17.064	19.331	22.015	25.203	29.000	33.536
30	17.375	19.750	22.577	25.953	30.000	34.865

TABLE 5E

6% P/A* Discount Factors

n	e=1%	e=2%	e=3%	e=4%	e=5%	e=6%
1	0.953	0.962	0.972	0.981	0.991	1.000
2	1.861	1.888	1.916	1.944	1.972	2.000
3	2.726	2.779	2.833	2.888	2.944	3.000
4	3.550	3.637	3.725	3.815	3.907	4.000
5	4.335	4.462	4.591	4.724	4.860	5.000
6	5.084	5.256	5.433	5.616	5.805	6.000
7	5.797	6.019	6.251	6.491	6.741	7.000
8	6.476	6.755	7.046	7.350	7.668	8.000
9	7.124	7.462	7.818	8.192	8.586	9.000
10	7.740	8.143	8.568	9.019	9.496	10.000
11	8.328	8.798	9.298	9.830	10.397	11.000
12	8.888	9.428	10.006	10.625	11.289	12.000
13	9.422	10.034	10.695	11.406	12.173	13.000
14	9.930	10.618	11.364	12.172	13.049	14.000
15	10.414	11.180	12.014	12.924	13.916	15.000
16	10.876	11.720	12.645	13.661	14.776	16.000
17	11.316	12.240	13.259	14.384	15.627	17.000
18	11.735	12.740	13.856	15.094	16.470	18.000
19	12.134	13.222	14.435	15.790	17.305	19.000
20	12.515	13.685	14.998	16.473	18.132	20.000
21	12.877	14.131	15.546	17.144	18.952	21.000
22	13.223	14.560	16.077	17.801	19.764	22.000
23	13.552	14.973	16.594	18.447	20.568	23.000
24	13.865	15.370	17.096	19.080	21.364	24.000
25	14.164	15.752	17.584	19.701	22.153	25.000
26	14.449	16.120	18.058	20.310	22.935	26.000
27	14.720	16.474	18.519	20.908	23.709	27.000
28	14.979	16.815	18.966	21.495	24.476	28.000
29	15.225	17.143	19.401	22.070	25.236	29.000
30	15.460	17.458	19.824	22.635	25.988	30.000

TABLE 5F

8% P/A* Discount Factors

n	e=1%	e=2%	e=3%	e=4%	e=5%	e=6%
1	0.935	0.944	0.954	0.963	0.972	0.981
2	1.810	1.836	1.863	1.890	1.917	1.945
3	2.628	2.679	2.731	2.783	2.836	2.890
4	3.393	3.474	3.558	3.643	3.730	3.818
5	4.108	4.226	4.347	4.471	4.598	4.729
6	4.777	4.936	5.099	5.269	5.443	5.623
7	5.402	5.606	5.817	6.036	6.264	6.500
8	5.987	6.239	6.501	6.776	7.062	7.361
9	6.534	6.837	7.154	7.488	7.838	8.207
10	7.046	7.401	7.777	8.173	8.593	9.036
11	7.525	7.935	8.370	8.834	9.326	9.850
12	7.972	8.438	8.937	9.469	10.039	10.649
13	8.391	8.914	9.476	10.082	10.733	11.434
14	8.782	9.363	9.991	10.671	11.407	12.203
15	9.148	9.787	10.483	11.239	12.062	12.959
16	9.490	10.188	10.951	11.786	12.699	13.700
17	9.810	10.566	11.398	12.312	13.319	14.428
18	10.110	10.924	11.824	12.819	13.921	15.142
19	10.390	11.261	12.230	13.307	14.507	15.843
20	10.651	11.580	12.618	13.777	15.076	16.532
21	10.896	11.881	12.987	14.230	15.629	17.207
22	11.125	12.166	13.340	14.666	16.167	17.870
23	11.339	12.434	13.676	15.086	16.691	18.520
24	11.539	12.688	13.996	15.490	17.199	19.159
25	11.727	12.928	14.302	15.879	17.694	19.786
26	11.902	13.154	14.594	16.254	18.174	20.401
27	12.066	13.367	14.872	16.615	18.642	21.004
28	12.219	13.569	15.137	16.963	19.096	21.597
29	12.362	13.760	15.390	17.297	19.538	22.178
30	12.496	13.940	15.631	17.620	19.967	22.749

TABLE 5G

10% P/A* Discount Factors

n	e=1%	e=2%	e=3%	e=4%	e=5%	e=6%
1	0.918	0.927	0.936	0.945	0.955	0.964
2	1.761	1.787	1.813	1.839	1.866	1.892
3	2.535	2.584	2.634	2.684	2.735	2.787
4	3.246	3.324	3.403	3.483	3.566	3.649
5	3.899	4.009	4.123	4.239	4.358	4.480
6	4.498	4.645	4.797	4.953	5.115	5.281
7	5.048	5.234	5.428	5.628	5.837	6.053
8	5.553	5.781	6.019	6.267	6.526	6.796
9	6.017	6.288	6.572	6.871	7.184	7.513
10	6.443	6.758	7.090	7.441	7.812	8.203
11	6.834	7.194	7.575	7.981	8.411	8.868
12	7.193	7.598	8.030	8.491	8.983	9.510
13	7.523	7.972	8.455	8.973	9.530	10.127
14	7.825	8.320	8.853	9.429	10.051	10.723
15	8.103	8.642	9.226	9.860	10.549	11.297
16	8.358	8.941	9.576	10.268	11.024	11.849
17	8.593	9.218	9.903	10.653	11.477	12.382
18	8.808	9.475	10.209	11.018	11.910	12.896
19	9.005	9.713	10.496	11.362	12.323	13.390
20	9.187	9.934	10.764	11.688	12.718	13.867
21	9.353	10.139	11.015	11.996	13.094	14.326
22	9.506	10.329	11.251	12.287	13.454	14.769
23	9.647	10.505	11.471	12.562	13.797	15.196
24	9.776	10.668	11.678	12.822	14.124	15.607
25	9.894	10.819	11.871	13.069	14.437	16.003
26	10.003	10.960	12.052	13.301	14.735	16.384
27	10.102	11.090	12.221	13.521	15.020	16.752
28	10.194	11.211	12.380	13.729	15.291	17.107
29	10.278	11.323	12.528	13.926	15.551	17.448
30	10.355	11.426	12.667	14.112	15.799	17.778

TABLE 5H

12% P/A* Discount Factors

n	e=1%	e=2%	e=3%	e=4%	e=5%	e=6%
1	0.902	0.911	0.920	0.929	0.938	0.946
2	1.715	1.740	1.765	1.791	1.816	1.842
3	2.448	2.495	2.543	2.591	2.640	2.690
4	3.110	3.183	3.258	3.335	3.413	3.492
5	3.706	3.810	3.916	4.025	4.137	4.252
6	4.244	4.380	4.521	4.666	4.816	4.970
7	4.729	4.900	5.078	5.262	5.452	5.650
8	5.166	5.373	5.589	5.814	6.049	6.294
9	5.561	5.804	6.060	6.328	6.609	6.903
10	5.916	6.197	6.492	6.804	7.133	7.480
11	6.237	6.554	6.890	7.247	7.625	8.026
12	6.526	6.880	7.256	7.658	8.086	8.542
13	6.787	7.176	7.593	8.039	8.518	9.031
14	7.022	7.446	7.902	8.394	8.923	9.494
15	7.234	7.692	8.187	8.723	9.303	9.931
16	7.426	7.916	8.449	9.028	9.659	10.346
17	7.598	8.120	8.689	9.312	9.993	10.738
18	7.754	8.306	8.911	9.575	10.306	11.109
19	7.894	8.475	9.114	9.820	10.599	11.461
20	8.020	8.629	9.302	10.047	10.874	11.793
21	8.134	8.769	9.474	10.258	11.132	12.108
22	8.237	8.897	9.632	10.454	11.374	12.405
23	8.330	9.013	9.778	10.636	11.600	12.687
24	8.414	9.119	9.912	10.805	11.813	12.954
25	8.489	9.216	10.035	10.961	12.012	13.207
26	8.557	9.304	10.148	11.107	12.199	13.445
27	8.619	9.384	10.252	11.242	12.374	13.672
28	8.674	9.456	10.348	11.368	12.538	13.886
29	8.724	9.523	10.436	11.484	12.692	14.088
30	8.769	9.583	10.517	11.593	12.836	14.280