AIC 1594

2187

Building and Environment, Vol. 21, No. 1, pp. 35–39, 1986
Printed in Great Britain.

0360–1323/86 $3.00+0.00
Pergamon Journals Ltd.

Thermal Analysis of Naturally Ventilated Buildings

E. H. MATHEWS*

A simplified electric analogue method to analyse the thermal performance of naturally ventilated buildings is presented. One of the main features of the method is that empirical constants in some equations account for typical rates of natural ventilation in conventional buildings. Another feature is that a very high degree of lumping is attained by using a special calculation procedure to estimate effective capacity values of building elements. The method is therefore extremely easy to use. Predictions are compared with measurements. The comparison is acceptable for design purposes.

NOMENCLATURE

$\sum A$ total area of building's exposed shell [m^2]
$\sum C$ total active thermal capacity of building per unit exposed shell area [kJ °C^{-1} m^{-2}]
Q, q heat transfer rate [W]
R_s thermal resistance of shell [°C m^2 W^{-1}]
R all other resistances [°C W^{-1}]
T temperature [°C]
t time of day [hours].

Greek symbols
α phase shift between forcing function and response [rad]
β phase angle of heat generation [rad]
ϕ phase angle of outdoor forcing temperature [rad]
ω angular velocity [rad h^{-1}] = $2\pi/24$ for first frequency approximation.

Subscripts
E earth
ff forcing function
gen generated
i indoor air
NI non-ideal
o outdoor air
q response to heat generation
s shell
sa sol–air
T_{ff} response to outdoor forcing temperature
v ventilation.

Notation
$\tilde{}$ alternating part of alternating variable
$\bar{}$ mean part of alternating variable
| | amplitude of alternating variable
// parallel resistances.

1. INTRODUCTION

BUILDINGS can consume a large percentage of the total energy used in a country (e.g. approximately 20% for South Africa [1]). With increasing energy costs and diminishing energy sources, more emphasis has lately been placed on the design of energy efficient buildings. The ultimate in an energy efficient building is a completely passive building, where an acceptable indoor environment is achieved by natural ventilation, natural lighting and a good passive thermal performance. Although passive control can only be used where climatic conditions are favourable, there are many parts of the world where such conditions do exist.

Various procedures have been developed to aid in the thermal design of buildings [2]. As the more sophisticated procedures are not generally accessible to designers of buildings, simplified empirical methods are often used [3]. Empirical methods are easy to apply, but have some limitations. These methods are usually only applicable to buildings similar to the ones on which the methods are based, and can not be extended to analyse the thermal performance of other buildings.

Many of the thermal analysis methods do not account for the thermal effects of natural ventilation and, for those which do, measured or assumed values for expected natural ventilation rates must be provided [4]. Information on these rates of natural ventilation is not always available to building designers [5]. A useful empirical method to analyse the thermal performance of buildings with open windows was developed by Wentzel *et al.* [3]. This method however does not include the effects of outside surface colour and of heat generation inside a building, nor can phase lags between indoor and outdoor air temperatures be predicted.

A semi-empirical thermal analysis method, based on electric analogue principles as well as on measurements was developed to alleviate some of the limitations of existing methods. As the method is primarily based on theory (only some constants are derived from measurements), it can easily be extended. The empirical constants account for typical expected rates of natural ventilation in conventional South African buildings. The method is extremely easy to use and its predictions compare favourably with measurements.

2. DERIVATION OF EQUATIONS

2.1. *Electrical analogue*

Equations for the method are based on a very simple electric analogue for heat flow through the building as shown in Fig. 1.

The air temperature inside a building (T_i), is the result of the interaction of its thermal properties with the outdoor air temperature (T_o), the sol–air temperatures (T_{sa}) on the different exterior surfaces, as well as with indoor heat generation and direct sun penetration (q). In the simplified

* Department of Mechanical Engineering, University of Pretoria, Pretoria 0002, South Africa.

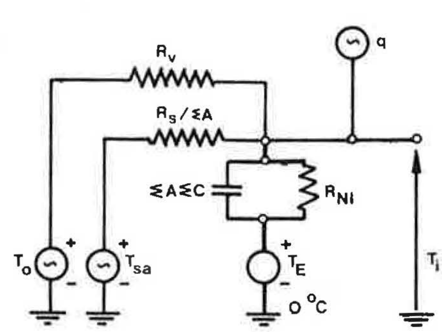

Fig. 1. Simplified electric analogue for building thermal analysis.

electric analogue the highest degree of lumping is used, as the thermal properties of the building are described by single values for the ventilation resistance (R_v), the shell resistance per unit shell area ($R_s/\sum A$) and for the total active capacity ($\sum C$).

The ventilation resistance is dependent on the flow rate of outdoor air entering the building, while the shell resistance is dependent on the thermal properties of the building's envelope. The total active capacity of the building is that portion of its thermal capacity that is effective in storing heat. This value therefore accounts for the relative position of mass and insulation. It also includes the contributions of interior walls, floors and other high mass objects inside the building. The calculation of active capacity is based on theory as well as on empirical data and is described in detail by Wentzel *et al.* [3]. The active capacity is calculated down to a depth of 300 mm below ground level, where the temperature has a negligible diurnal swing [3]. The one leg of the active capacity in Fig. 1 is therefore referred to a 'constant' earth temperature (T_E) which is measured at 300 mm below ground level. Similar to an electric capacitor, the thermal capacitor ($\sum C$) will not be ideal. Tiny leakage flow will occur 'around' its two junction points, through a very large resistance, the non-ideal capacitor resistance (R_{NI}). As the analogue model is very simplified, i.e. the entire building's active capacity is lumped into *one* value, the physical meaning of R_{NI} cannot be accurately described. The non-ideal resistance may however in part be seen as the thermal resistance of the walls to heat flowing down towards the ground. The value of R_{NI} will be much larger than the values of the other resistances in the proposed analogue [4].

By using a single forcing temperature (T_{ff}) instead of two, namely T_o and T_{sa}, as shown in Fig. 2, the analogue in Fig. 1 may be further simplified. The equation for the single forcing temperature can be derived from Fig. 2 as

$$T_{ff} = \frac{(R_v)T_{sa} + (R_s/\sum A)T_o}{R_v + R_s/\sum A} \tag{1}$$

where the sol–air temperature (T_{sa}) is approximated as the average of the sol–air temperatures on the different outside surfaces. It is assumed that the phase shift between the outdoor and averaged sol–air temperature is negligible, which seems a fair assumption for the purpose of this study [4]. Equation (1) shows that for *very* high ventilation rates T_{ff} can be approximated by the outdoor air temperature, while for no ventilation T_{ff} is given by the sol–air temperature. The value of T_{ff} for most naturally ventilated buildings, however, will be between these extremes. As a first simplifying approximation the magnitude of the single forcing function (T_{ff}) is taken as the average of the sol–air and outdoor air temperatures and is given by

$$T_{ff} = 0.5(T_{sa} + T_o). \tag{2}$$

From equation (1) it is seen that the approximate equation (2) will be exactly satisfied when the ventilation resistance is equal to the shell resistance per unit shell area.

As the analogue system in Fig. 1 is linear, the indoor responses to the different thermal excitation forces are additive. The value of the indoor air temperature is therefore given by

$$T_i = (T_i)_{Tff} + (T_i)_q \tag{3}$$

where $(T_i)_{Tff}$ denotes the response to T_o as a result of natural ventilation as well as the response to the sol–air temperatures (T_{sa}) on the outside surfaces. The indoor response to heat generation (q) and direct penetration (q) is denoted by $(T_i)_q$.

2.2. *Response to periodic excitation*

The equation for the amplitude of the indoor air temperature $|(\tilde{T}_i)_{Tff}|$, as a result of the single external forcing temperature (T_{ff}) is derived from the electric analogues in Figs. 1 and 2 as [4]

$$|(\tilde{T}_i)_{Tff}| = |\tilde{T}_{ff}| \left[\left(\frac{R_v R_s}{R_s/\sum A + R_v} \right)^2 (\omega \sum C)^2 + 1^2 \right]^{-1/2} \tag{4}$$

where ω is the angular velocity of the forcing temperature variation.

It was found [4] that the following equation is valid for

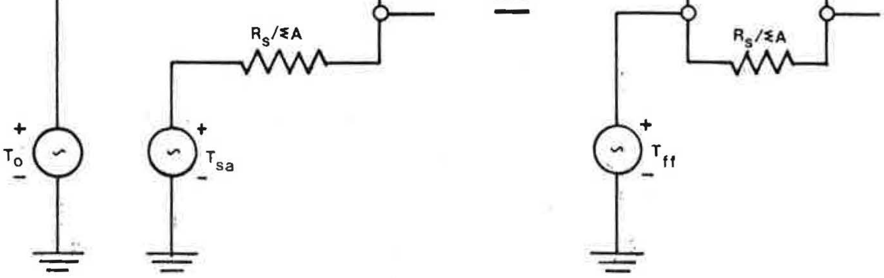

Fig. 2. Two forcing temperatures combined into one value.

most conventional buildings

$$\left(\frac{R_v R_s}{R_s/\sum A + R_v}\right)^2 (\omega \sum C)^2 \gg 1^2. \tag{5}$$

Equation (4) can therefore be simplified to

$$|(\tilde{T}_i)_{T_{ff}}| = |\tilde{T}_{ff}| \frac{1}{\omega \sum C} \left[\frac{R_s/\sum A + R_v}{R_v R_s}\right]. \tag{6}$$

It was further found in practice [4] that for conventional, naturally ventilated buildings, the term in square brackets in equation (6) can be substituted by an empirical constant, as a certain relationship between R_s, R_v and $\sum A$ usually exists. If the outdoor forcing temperature is approximated by a sine wave ($\omega = 2\pi/24$ rad h^{-1}), equation (6) for naturally ventilated buildings can be approximated as [4]

$$\frac{|(\tilde{T}_i)_{T_{ff}}|}{|\tilde{T}_{ff}|} = \frac{150.41}{\sum C}. \tag{7}$$

The amplitude of the indoor air temperature that results from the outdoor forcing temperature, is therefore dependent on an empirical constant, the amplitude of the forcing temperature and on the active capacity. The empirical constant accounts for the relationship between shell resistance, exposed shell area and ventilation rates, while the active capacity is dependent on the mass and resistances of building elements, their position relative to each other, as well as on the shell resistance.

Similarly the equation for the amplitude of the indoor air temperature $|(\tilde{T}_i)_q|$, as a result of heat generation or direct sun penetration can be derived as [4]

$$\frac{|(\tilde{T}_i)_q|}{|\tilde{q}|} = \frac{3.57}{\omega \sum C \sum A}. \tag{8}$$

The equation to calculate phase shifts (α) between the thermal forcing functions (T_{ff} or q) and its appropriate indoor responses can also be derived from Figs. 1 and 2 and is given by [4]

$$\alpha = \arctan\left[\omega \sum C \Big/ \left(\frac{R_s/\sum A + R_v}{R_s R_v}\right)\right]. \tag{9}$$

By using the empirical constant that accounts for the relationship between R_s, R_v and $\sum A$, equation (9) can be simplified to

$$\alpha = \arctan\left(\frac{\sum C}{150.41}\right). \tag{10}$$

2.3. *Response to constant excitation*

The equation relating the mean indoor response $[(\bar{T}_i)_{T_{ff}}]$ to the mean value of the outdoor forcing temperature ($\bar{T}_{ff}$) is derived from Figs. 1 and 2 and is given by [4]

$$(\bar{T}_i)_{T_{ff}} = \bar{T}_{ff} \Big/ \left(1 + \frac{R_v // R_s/\sum A}{\bar{T}_{ff}/Q + R_{NI}}\right) \tag{11}$$

where Q is the mean heat flow through the combination of parallel resistances R_v and $R_s/\sum A$. The term $R_v//R_s/\sum A$ denotes that resistances R_v and $R_s/\sum A$ are in parallel. As the non-ideal resistance (R_{NI}) is much larger than the other terms in equation (11), the ratio between $(\bar{T}_i)_{T_{ff}}$ and $\bar{T}_{ff}$ reduces to

$$\frac{(\bar{T}_i)_{T_{ff}}}{\bar{T}_{ff}} = 1. \tag{12}$$

The indoor response $[(\bar{T}_i)_q]$ to the mean value of q can also be derived from Fig. 1 and is given by

$$(\bar{T}_i)_q = (R_v // R_s/\sum A)\bar{q}. \tag{13}$$

By utilising the empirical constant mentioned in the previous section, equation (13) can be simplified to

$$\frac{(\bar{T}_i)_q}{\bar{q}} = \frac{0.091}{\sum A}. \tag{14}$$

2.4. *Total indoor response*

By using equations (3), (7), (8), (10), (12) and (14), the following equation for a first frequency approximation of the indoor air temperature can be derived:

$$T_i = \frac{0.091\,\bar{q}}{\sum A} + \bar{T}_{ff} + \frac{1}{2}\frac{150.41|\tilde{T}_{ff}|}{\sum C}\sin\left(\frac{2\pi}{24}t - \phi - \arctan\frac{\sum C}{150.41}\right) + \frac{1}{2}\frac{13.64|\tilde{q}|}{\sum C \sum A}\sin\left(\frac{2\pi}{24}t - \beta - \arctan\frac{\sum C}{150.41}\right) \tag{15}$$

where ϕ is the phase angle of T_{ff}, β is the phase angle of q, and t is the time of day in hours. All the unknowns on the RHS of equation (15) can be estimated from design weather data and from building plans. Equation (15) can therefore be used to predict the thermal performance of a building, while it is still at the design stage.

3. MEASUREMENTS AND DISCUSSION

Five buildings that differed in values of active capacity, shell resistance, exterior colour and exposed shell area were used to verify the proposed method [4]. Three of them were empty, low-cost, experimental houses with different thermal properties, the other a furnished, middle-income house and the fifth was a furnished office in a high-rise building. The effect of heat generation was investigated in one of the low-cost houses. All these buildings were naturally ventilated.

Measured and predicted indoor air temperature variations, as well as the measured outdoor air temperature variations for completely passive buildings, where direct sun penetration was negligible, are shown in Figs. 3–5.

The effect of heat generation on indoor air temperature was investigated in one of the experimental, low-cost houses. Heaters were used to generate a constant heat value (q_{gen}) of 4600 W during typical working hours, i.e. from 0800 to 1600 h. The heat pulse, as well as a one-frequency and a five-frequency approximation of the pulse is shown in Fig. 6. A one-frequency approximation for the heat pulse was used for prediction purposes. The equation for such an approximation is given by [4]

$$q = 1/3 q_{gen} + 0.55 q_{gen} \sin(2\pi/24t - 1.57). \tag{16}$$

Measured and predicted indoor air temperature variations for two identical experimental houses, where heat was generated in one of them, are shown in Figs. 7(a) and (b).

Figures 3–5 and Fig. 7 show that the simplified method can to a fair extent predict the influence of different thermal effects on the indoor air temperature in naturally

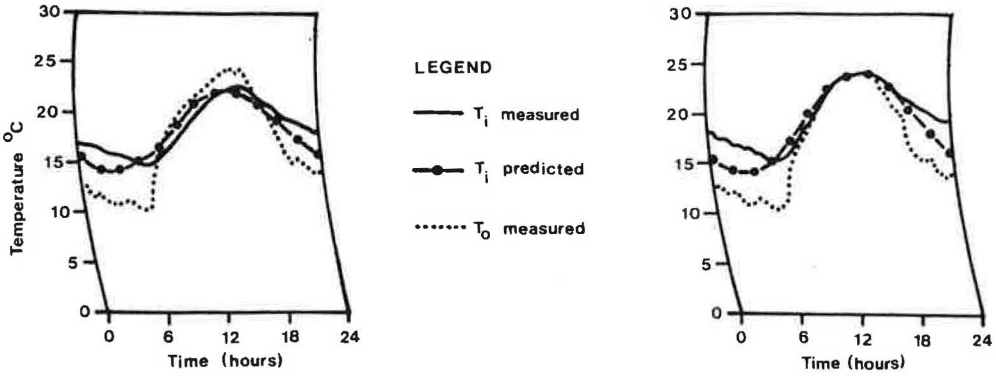

Fig. 3. Comparison between measured and calculated indoor air temperatures for two low-income houses ($\sum C = 395$ kJ °C^{-1} m^{-2}) where only the exterior colour differed. Building shells completely exposed to outdoor environment. (a) House 1: white exterior walls and weathered roof (date: 26 April 1984). (b) House 2: dark brown exterior walls and black roof (date: 26 April 1984).

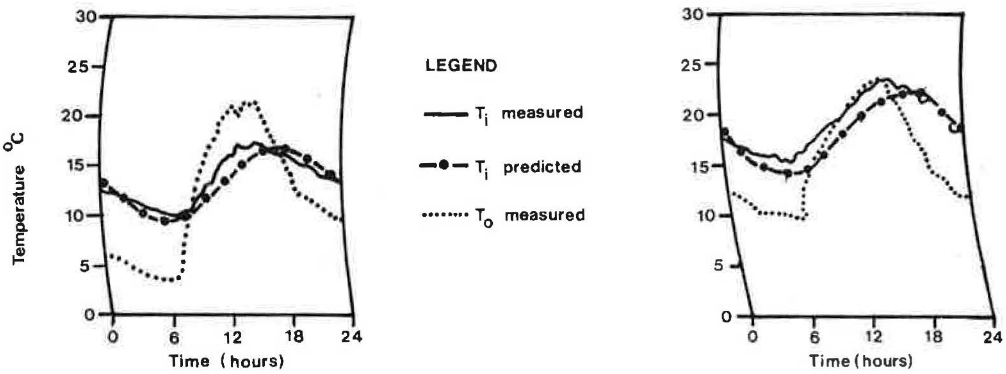

Fig. 4. Comparison between measured and calculated indoor air temperatures for low-income house 3 ($\sum C = 464$ kJ °C^{-1} m^{-2}). Building shell completely exposed to outdoor environment. (a) White exterior walls and white roof (date: 16 June 1985). (b) Dark brown exterior walls and black roof (date: 22 April 1984).

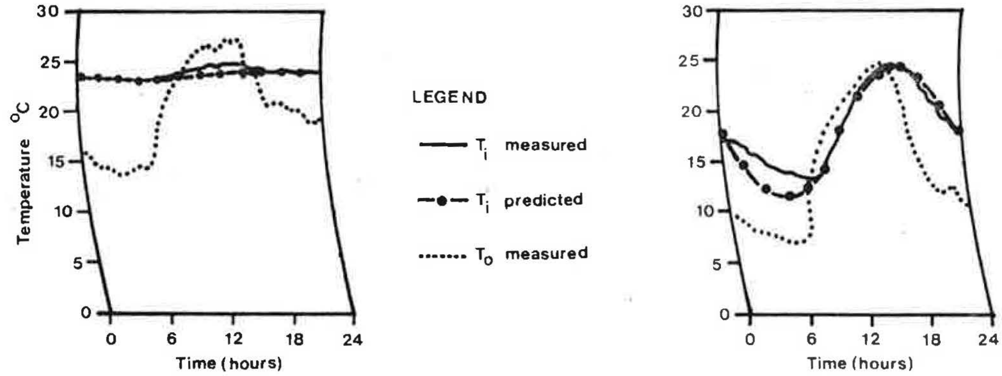

Fig. 5. Comparison between measured and calculated indoor air temperatures for an office ($\sum C =$ 1761 kJ °C^{-1} m^{-2}) and for a room in a middle income house ($\sum C = 340$ kJ °C^{-1} m^{-2}). (a) Office in high-rise building. Only northern wall exposed to outdoor environment (date: 7 April 1984). (b) Room in middle-income house. Only northern wall and roof exposed to outdoor environment (date: 19 April 1985).

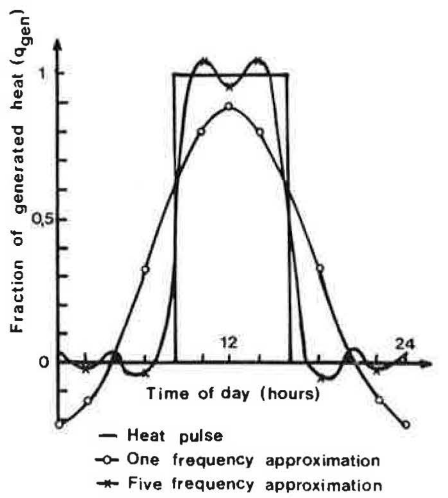

Fig. 6. Heat pulse and two possible approximations.

ventilated buildings. Figures 3(a), (b) and 4(a), (b) show the effect of exterior surface colour, while the important influence of different active capacity values is demonstrated by comparing Figs. 3, 4, 5(a) and 5(b). The effect of indoor heat generation is illustrated by Figs. 7(a) and (b).

One of the most important reasons for the difference between measurements and predictions is that a single frequency approximation was used for all the alternating variables. If however a more accurate analysis is necessary, more than one frequency component can be used by extending equation (15). Amplitude and phase shift values for these components can be calculated from equations (6), (8) and (9).

The method should produce more accurate predictions if the degree of lumping is decreased by lumping the major building elements separately, i.e. walls, roof, floor, etc. More than one forcing temperature will then be used. By numerically solving the resulting electric analogue, the forcing temperatures need not be approximated as harmonic. It should, however, still be possible to implement the extended method on a micro-computer.

4. CONCLUSIONS

It was shown that a very simplified method can be used to predict the thermal performance of naturally ventilated buildings. The method can be extended to analyse the thermal performance of air-conditioned buildings [4] and is ideally suited for use by designers of energy efficient buildings.

Acknowledgements—The research described in this paper was done while the author was employed by the South African National Building Research Institute.

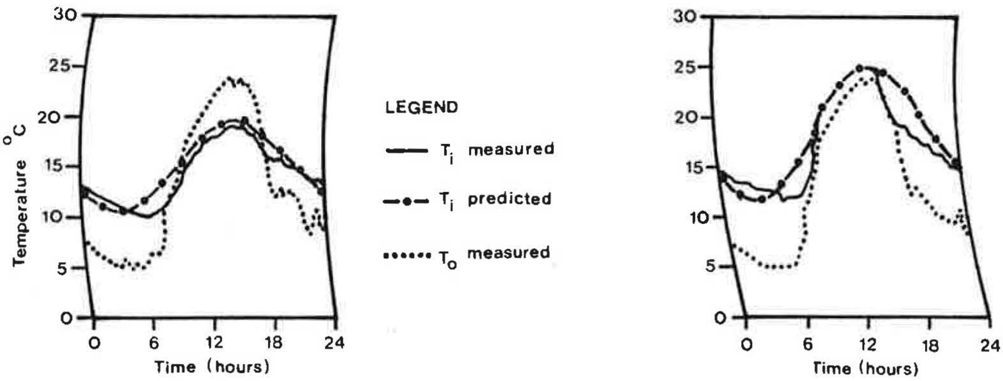

Fig. 7. Comparison between measured and calculated indoor air temperatures for two identical, low-income houses, except that heat was generated inside house 2. (a) House 1: no heat generated inside building (date: 22 June 1985). (b) House 2: 4600 W heat generated between 0800 and 1600 h (date: 22 June 1985).

REFERENCES

1. R. Mathews, Energy effective design. In *Heating, Airconditioning and Refrigeration*, Vol. 16, p. 3. Pithead Press, Randburg, RSA (1984).
2. K. Källbald, Calculation methods to predict energy savings in residential buildings, Research Report D4, Swedish Council for Building Research, Stockholm, Sweden (1983).
3. J. D. Wentzel, R. J. Page-Shipp and J. A. Venter, The prediction of the thermal performance of buildings by the CR-method. NBRI research report BRR 396, CSIR, Pretoria (1981).
4. E. H. Mathews, The prediction of natural ventilation in buildings, D.Eng. Thesis, University of Potchefstroom, South Africa (1985).
5. A. G. Loudon, Summertime temperatures in buildings without air-conditioning. *J. Inst. Heat. Vent. Engrs* 37, 288 (1970).