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### A Theory for the Effects of Convective Air Flow through Fibrous Thermal Insulation

Low density fibrous insulation usually possesses more than 95% void space which in ordinary application is occupied by air. The thermal conductivity of this material is a complex property because the three basic mechanisms of heat transfer, conduction, convection and radiation, play a part. The insulating value results from the fact that the gas contained in the pores is at rest, or that any convective motion present is small. Several papers have been written on the thermal properties of fibrous insulation,<sup>1,2</sup> however, the interaction between fluid conduction and convection has not been fully investigated.

Verschoor and Greebler<sup>1</sup> found that in guarded hot plate tests, where the insulation fills the space between plates, gas conduction is the mechanism which contributes the largest component of the heat transfer. A situation could arise in practice, however, where air spaces exist in contact with one or both sides of insulation. The insulation would then act as a permeable membrane, allowing air movement by natural convection between the spaces, as a result of the pressure difference created by different temperature conditions. This air movement would decrease the total heat transfer by fluid conduction and increase convective heat transfer. The resulting overall effect would be an increase in the apparent thermal conductance of the insulation.

Lotz<sup>3</sup> detected the presence of convection currents in several applications, including insulation with neither face, one face, or both faces permeable

to air flow. Negligible effects on the total heat transfer were observed for the first two cases; however, the last one showed a significant increase in heat transfer.

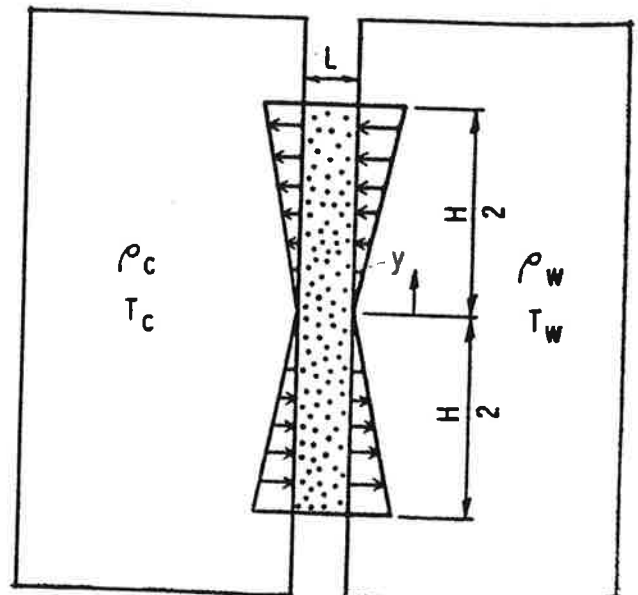
Two simplified models with both insulation faces permeable to air were analyzed to determine the heat transmission rate by combined fluid conduction and convection. Experimental results are included to substantiate the theoretical development.

#### THEORY

##### Model 1

The situation to be analyzed is schematically represented in Fig. 1. Two sealed cavities are sep-

Fig. 1 Natural convection through fibrous insulation



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arated by a vertical partition of fibrous insulation, having an air flow coefficient  $\lambda$ . The air densities and temperatures are assumed uniform over the insulation area at  $\rho_c$  and  $\rho_w$ , and  $T_c$  and  $T_w$ .

Since no net mass flow occurs across the insulation, a neutral axis must exist where the absolute pressure is everywhere equal. The pressure difference between the two air spaces at any level can be expressed as

$$P = P_w - P_c = (\rho_c - \rho_w) \frac{g}{g_c} y \quad (1)$$

The fluid velocity can be related to the pressure difference across the insulation. Assuming that the flow is laminar, the apparent velocity is given by the basic relationship of Darcy's Law as

$$V = \frac{AP}{\mu L} \quad (2)$$

According to the work of several investigators (e.g., Carman<sup>4</sup>), the value of A, based on both capillary and particle models, is a function of fiber diameter and porosity. For high porosities, as encountered in lightweight fibrous insulations, no reliable analytical prediction of A was available and an experimental method was used to determine an air infiltration coefficient, which in turn allowed determination of A. The infiltration tests showed that for the pressure differences in a normal application, the volume flow rate, and hence the velocity, is directly proportional to the pressure difference. This validates the assumption of laminar flow. The infiltration coefficient is defined as a proportionality constant between flow rate per unit area and pressure difference across unit thickness of the insulation. Hence, it follows that

$$A = \mu \lambda \quad (3)$$

Combining Eqs (1), (2) and (3), the velocity may be expressed as

$$V = \frac{\lambda}{L} (\rho_c - \rho_w) \frac{g}{g_c} y = B y \quad (4)$$

The convective air flow, Q, from the warm side to the cold side through the top half of the insulation of width W and height H, and the equal flow in the opposite direction through the bottom half can be found by integrating Eq (4) from 0 to H/2.

$$\begin{aligned} Q &= WB \int_0^{H/2} y dy \\ &= WB \frac{H^2}{8} \end{aligned} \quad (5)$$

Ignoring conduction effects, the predicted heat transfer associated with this air interchange would be

$$\begin{aligned} q_c &= \bar{\rho} c_p Q (T_w - T_c) \\ &= \bar{\rho} c_p WB \frac{H^2}{8} (T_w - T_c) \end{aligned} \quad (6)$$

This can also be expressed in terms of a conductance coefficient  $C_c$  such that

$$q_c = C_c WH (T_w - T_c)$$

then

$$C_c = \bar{\rho} c_p \frac{BH^2}{8} = \bar{\rho} c_p \frac{\lambda \Delta \rho Hg}{L 8g_c} \quad (7)$$

The heat transfer due to fluid flow as expressed above is modified, however, by conduction in the air in the insulation. Similarly, the heat transfer by conduction in the air in the insulation is modified by the air flow. This interaction is expressed by Brown<sup>5</sup> in the following heat conservation equation:

$$\frac{V}{dL} \frac{dT}{dL} = \alpha \frac{d^2 T}{dL^2}$$

This equation assumes conduction in the air only in the direction normal to the plane of the insulation and neglects conduction in other directions. Integration and substitution yields the following equation in differential form:

$$dq_{cc} = WV \bar{\rho} c_p \left( \frac{T_w e^{VL/\alpha} - T_c}{e^{VL/\alpha} - 1} \right) dy \quad (8)$$

Integrating from  $y = -\frac{H}{2}$  to  $+\frac{H}{2}$  gives:

$$q_{cc} = \bar{\rho} c_p W \frac{BH^2}{8} (T_w - T_c) \int_0^{H/2} \frac{e^{Bly/\alpha} + 1}{e^{Bly/\alpha} - 1} y dy$$

$$q_{cc} = \bar{\rho} c_p W \frac{BH^2}{8} (T_w - T_c) \frac{32 \alpha^2}{B^2 L^2 H^2} \int_0^{x=BLH/4\alpha} x \coth x dx \quad (9)$$

where

$$x = \frac{Bly}{2\alpha} = \frac{\lambda \Delta \rho yg}{2\alpha g_c}$$

if we write

$$\frac{32 \alpha^2}{B^2 L^2 H^2} \int_0^{x=BLH/4\alpha} x \coth x dx = K_{cc} \quad (10)$$

$$\text{then } q_{cc} = \bar{\rho} c_p W \frac{BH^2}{8} (T_w - T_c) K_{cc} \quad (11)$$

$$\text{or } q_{cc} = q_c K_{cc} \quad (12)$$

An apparent conductance  $C_{cc}$  can be used to express the combined conduction - convection heat flow:

$$C_{cc} = q_{cc} / WH (T_w - T_c) \quad (13)$$

$$= \bar{\rho} c_p \frac{BH}{8} K_{cc} \quad (14)$$

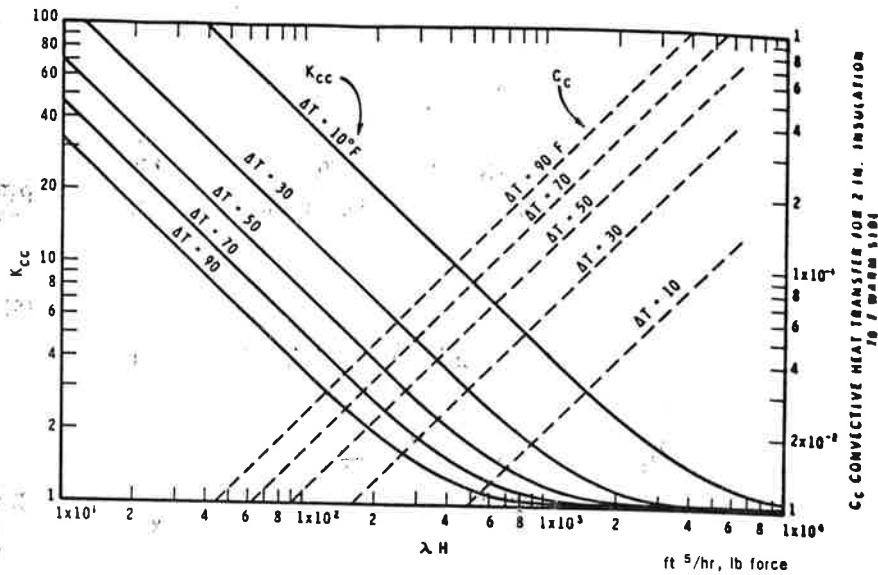


Fig. 2 Values of  $K_{cc}$  and  $C_{cc}$  vs  $\lambda H$

and

$$C_{cc} = C_c K_{cc}$$

The heat transfer by the conduction-convection mechanism is therefore the transfer that would be predicted on the basis of convective flow alone multiplied by  $K_{cc}$ .

The value of  $K_{cc}$  is function of  $\frac{BLH}{4\alpha}$

$$\frac{\lambda \Delta \rho H g}{4\alpha} \text{ and } C_c \text{ is equal to } \frac{\rho c \lambda \Delta \rho H g}{8 L g_c}$$

In a given application, the heat transfer is therefore influenced primarily by  $\lambda$ ,  $H$  and  $\Delta \rho$ . The density difference is a function of temperature difference so that both  $K_{cc}$  and  $C_{cc}$  become a function of  $\lambda$ ,  $H$  and  $\Delta T$ . At low values of these three parameters,  $K_{cc}$  is very large and  $C_{cc}$  small. At high values,  $K_{cc}$  becomes asymptotic to 1.0, while  $C_{cc}$  increases in direct proportion to the product of the parameters.

Values of  $K_{cc}$  were determined by numerical integration of Eq (10) and are shown in Fig. 2. This figure shows  $K_{cc}$  vs  $\lambda H$  for various temperature differences and applies only to air at about 70 F on the warm side. Values of  $C_{cc}$  are also shown for the same conditions.

When  $\lambda H \Delta T$  is very small, the heat transfer is by pure conduction. This can best be shown by Eq (8) which, when  $V$  approaches zero, reduces to:

$$dq = W \frac{k}{L} (T_w - T_c) dy$$

$$\text{or } q = WH \frac{k}{L} (T_w - T_c)$$

The heat transfer coefficient  $C_{cc}$  can therefore range from a minimum of  $k/L$  to a maximum of  $C_c$  depending on the magnitude of  $\lambda H \Delta T$ .

By rearranging and adding terms in Eq (13)

the conductance factor  $C_{cc}$  can be expressed in terms of the following dimensionless groups:

$$\frac{C_{cc} H}{k} = \frac{\mu \lambda}{8g_c HL} \frac{c_p \mu}{k} \frac{g \bar{\rho} \Delta \rho H^3}{\mu^2} K_{cc} \quad (15)$$

$$Nu = G Pr Gr K_{cc} \quad (16)$$

where the term  $G$  can be regarded as a geometrical factor peculiar to the system.

$$G = \frac{\mu \lambda}{8g_c HL} = \frac{A}{8g_c HL} \quad (17)$$

### Model 2

In the second situation analyzed there is a uniform vertical temperature variation in the cavities on either side of the insulation. The neutral axis will be located at the horizontal centerline of the insulation, so that pressure difference, air velocity and flow rate will be the same as for model 1. The temperature in the cavities may be expressed in terms of the centerline temperature as follows:

$$T_w = T_{mw} + Dy \quad (18)$$

$$T_c = T_{mc} + Dy \quad (19)$$

$$\text{where } D = \frac{T_t - T_b}{H} \quad (20)$$

and  $\frac{H}{2} \leq y \leq \frac{H}{2}$

The total heat transfer rate due to fluid conduction and convection is obtained by substituting Eqs (18) and (19) into Eq (8), combining with Eq (4) and integrating from  $y = -H/2$  to  $y = +H/2$

$$q_{cc} = W \bar{\rho} c_p B \int_0^{H/2} [y (T_{mw} - T_{mc}) \coth \frac{BLy}{2a} + 2Dy]^2 dy$$

$$= W \bar{\rho} c_p (T_{mw} - T_{mc}) B \frac{H^2}{8} (K_{cc} + K_{tv}) \quad (21)$$

where

$$K_{tv} = \frac{2}{3} \left( \frac{T_t - T_b}{T_{mw} - T_{mc}} \right)$$

Comparing this result with that obtained for model 1, it is evident that the only modification is the addition of the temperature-variation variable,  $K_{tv}$ , to the  $K_{cc}$  factor. As before for the pure conduction case, when the factor  $BLH/4\alpha$  is very small, the heat-transfer rate defined in Eq (21) reduces to

$$q = WH \frac{k}{L} (T_w - T_c) \quad (22)$$

For the pure convection case, the total heat-transfer rate becomes:

$$q = WB \frac{H^2}{8} \bar{\rho} c_p (T_{mw} - T_{mc}) (1 + K_{tv}) \quad (23)$$

It is evident that  $K_{tv}$  acts to increase only the convective mode of heat transfer. The  $K_{tv}$  variable may be thought to occur because the significant temperature difference for convective heat transfer is between the temperature of the fluid entering and leaving through areas located an equal distance above and below the neutral axis.

The apparent heat-transfer coefficient due to fluid conduction and convection,  $C_{cc}$ , may now be defined so that

$$q_{cc} = C_{cc} WH (T_{mw} - T_{mc}) \quad (24)$$

The final correlation among dimensionless groups then becomes

$$Nu = G Pr Gr (K_{cc} + K_{tv}) \quad (25)$$

### EXPERIMENTAL PROGRAM

The experimental program was designed to demonstrate the increase in heat transmission of fibrous insulation due to convective air flow and, in addition, to verify the theoretical correlation among dimensionless groups.

### TEST APPARATUS

Heat transmission rates were measured with the wall panel test unit which has previously been described.<sup>6</sup> Basically, the apparatus consists of a cold-box/hot-box combination; each box is 4 ft deep with an 8-ft sq opening. Panels lining the inside walls are heated in the warm box and cooled in the cold box by circulating liquid. Additional surface area for cooling is provided in the cold box by finned tubing. The warm-box liquid can be controlled between 65 and 75 F to within  $\pm 0.01$  F. The cold-box temperature can be varied down to about -25 F.

Heat transfer to the test wall on the warm side is by natural convection. The surface conductance at the warm side of the test panel simulates that obtained in practice and agrees closely with values published in the ASHRAE Guide And Data Book. Facilities for forced circulation are available in the cold box but for this test series natural convection conditions were provided.

### TEST WALL AND TEST PROCEDURE

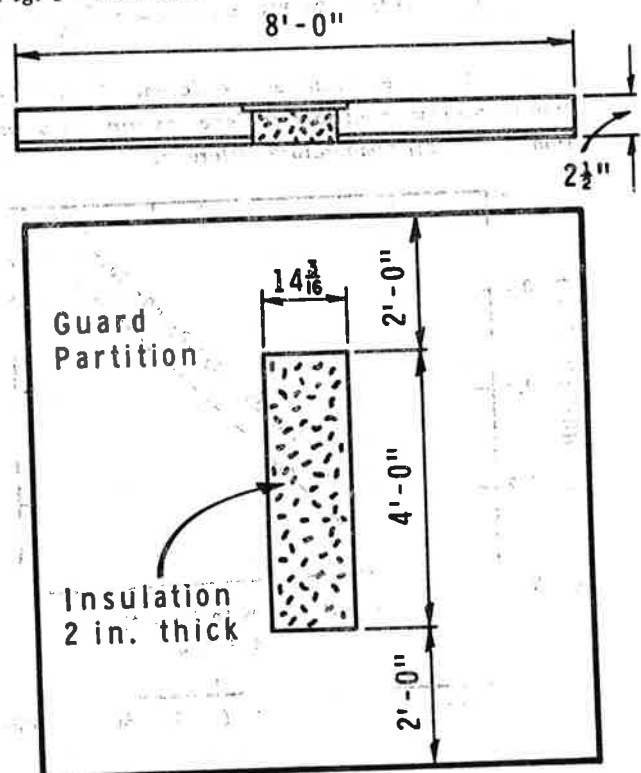
Glass fiber insulation manufactured without paper backing, of 2-in. nominal thickness, having a density between 0.8 and 0.9 lb/cu ft and an average fiber diameter of 0.4 mil was used in the experimental program. A standard 4-ft batt was installed in a 14 3/16- by 48-in. opening with the long dimension vertical; this was centrally located in an 8-ft sq partition fabricated from 2-in. thick foamed polystyrene glued to 1/2-in. thick plywood backing (Fig. 2). The insulation was maintained at the desired 2-in. thickness by 1-in. hexagonal-netted wire mesh stretched across both sides.

To isolate the effect of convective air flow on heat transfer, tests were performed with and without polyethylene sheeting over the insulation surface. Tests were conducted with the warm box maintained at about 72 F, while the cold box was varied in steps down to about -20 F. Temperatures of inside and outside air and insulation surfaces were measured by copper-constantan thermocouples. Temperature readings under steady-state conditions are considered accurate to within  $\pm 0.1$  F, and heat transfer measurements to  $\pm 1\%$ .

The heat transmitted through the insulation was obtained by subtracting the heat transmitted through the partition from the total measured. The heat transmitted through the partition was calculated from partition-conductance values determined in previous tests.

To compare the test results with the theory just presented, the heat transfer by radiation must be known. An apparent radiation coefficient was determined from measurements with an 18-in.

Fig. 3 Test wall



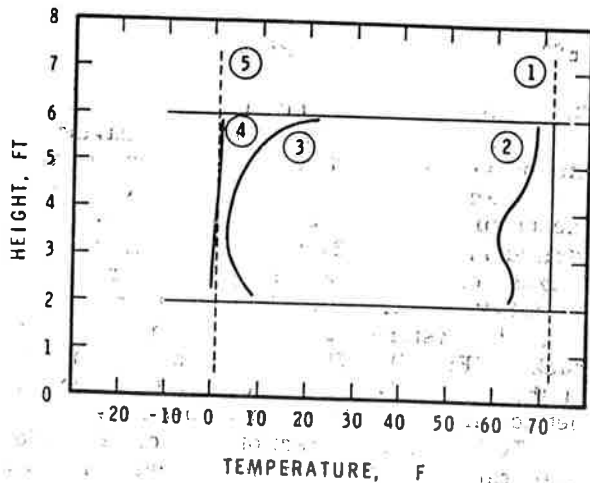


Fig. 4. Vertical temperature variations with polyethylene covering: (1) warm-side air; (2) warm surface; (3) cold surface; (4) cold-side air; (5) cold-box air

guarded hot-plate conductivity apparatus by subtracting the calculated heat transfer by fluid conduction and solid fiber conduction from the total measured. Measurements on a 2-in. thick sample of 0.90 lb/cu ft density at a mean temperature of 55 F and a temperature difference of 40 F resulted in a total conductivity of 0.255 and a radiation conductivity of 0.079 Btu/hr, sq ft, F/in. The radiation conductivity may be expressed as:

$$h_r = 4 \epsilon \frac{L_f}{\epsilon^2} T_m^3 \quad (26)$$

The factor  $1/\epsilon^2$  is an opacity factor dependent upon temperature. For the limited temperature range encountered in the experiment,  $30 \text{ F} < T_m < 55 \text{ F}$ ,

Fig. 6 Total heat-transfer coefficient ( $C_t$ ) of insulation with and without polyethylene covering as a function of overall temperature difference

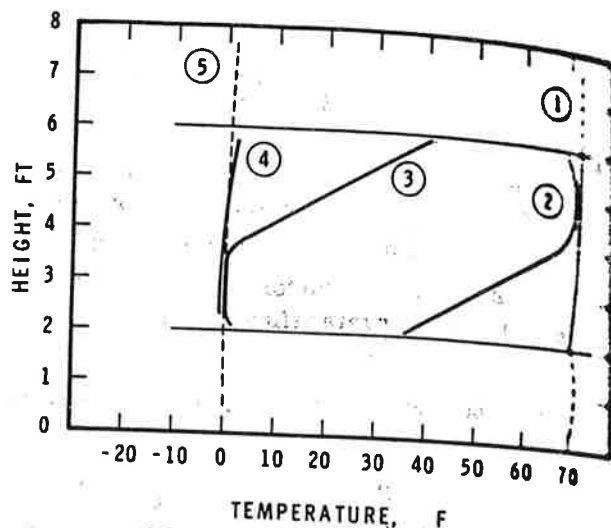
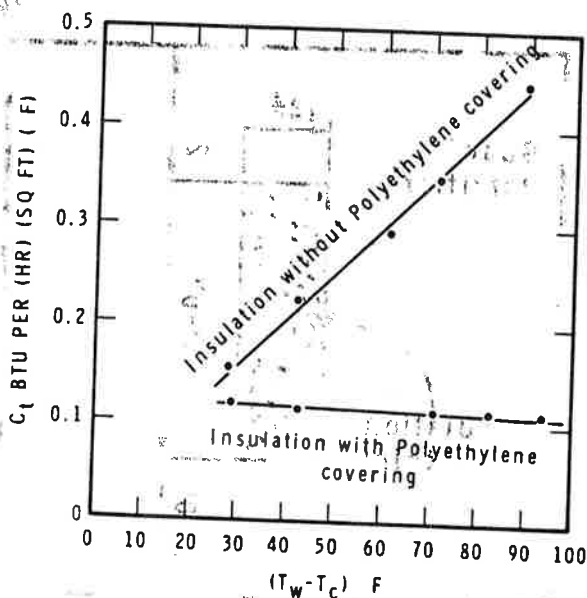


Fig. 5 Vertical temperature variations - without polyethylene covering: (1) warm-side air; (2) warm surface; (3) cold surface; (4) cold-side air; (5) cold box air

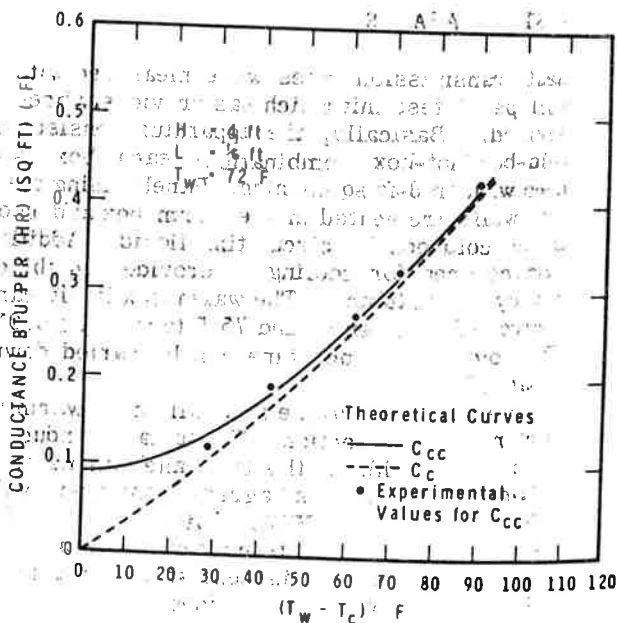
the radiation conductivity (applicable at a bulk insulation density of 0.85 pcf) was determined as

$$h_r = 6.0 \times 10^{-10} T_m^3 \quad (27)$$

### TEST RESULTS AND DISCUSSION

Figs. 4 and 5 show vertical variations in temperature at the centerline of the insulation section, with and without a polyethylene cover over the insulation surface. In Fig. 5, the occurrence of convective air flow is indicated by the decreased temperature over the bottom half of the warm surface of the insulation and the increased temperature

Fig. 7 Apparent thermal conductance of insulation due to gas conduction and convection vs air temperature difference



over the top half of the cold surface. These variations can be explained with reference to the air movement. Cold air moves from the cold side to the warm side below the neutral axis, thereby lowering the warm side surface temperature; an opposite air movement and temperature effect occurs above the axis.

The variation in surface temperature of the insulation section with polyethylene covering is due to convection currents within the insulation. Such internal convection currents were also noticed by Lotz who found, however, that the average heat-transfer coefficient was practically the same as that determined by guarded hot-plate tests.

The total heat-transfer coefficients for the insulation  $C_t$ , with and without the polyethylene covering, were obtained by dividing the total heat transfer through the insulation by its area and the overall air temperature difference. These coefficients, plotted as a function of overall temperature difference in Fig. 6, demonstrate the effect of convection in increasing the total heat transfer. At an overall temperature difference of 90 F. the apparent thermal conductance of the insulation has increased by a factor of about four.

The insulation heat-transfer coefficients due to fluid conduction and convection only,  $C_{cc}$ , were determined by subtracting the calculated heat transfer due to radiation and solid fiber conduction from the measured total. These values are plotted in Fig. 7, and are compared with the theoretical curves calculated from Eqs (7) and (13) applicable to model 1, which simulates the actual test conditions more closely than model 2. Values obtained from Eq (7) are for heat transfer by convective air flow only, while those from Eq (13) are for com-

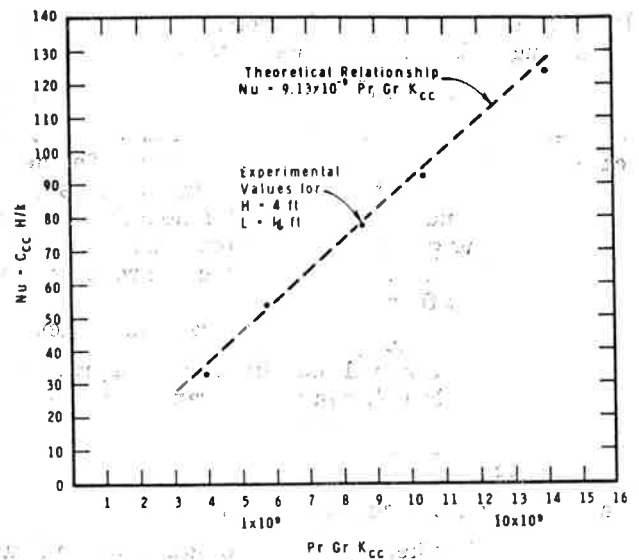


Fig. 8 Convective air flow through fibrous insulation in a vertical partition

bined conduction and convection. It is seen that gas conduction predominates at low-temperature differences, and gas convection at high-temperature differences. In Fig. 8, the Nu numbers based on test values of  $C_{cc}$  are plotted against the corresponding values of  $Pr Gr K_{cc}$ , based on average fluid properties and a measured value of the air flow coefficient,  $\lambda$ , of 460 cu ft/hr, sq ft, lb/sq ft pressure difference/ft of thickness; definitions are in accordance with Eq (15). Also shown is a theoretical Nu curve based on values of  $G$  calcu-

Table I. Test Results

Test No.	11	12	13	14	15	21	22	23	24	25
q Btuh	222	339	503	599	787	217	321	518	598	672
$t_{ac}$ F	44.4	30.9	11.0	0.8	-17.4	44.3	30.0	112	-10.2	-21.6
$t_{le}$ F	73.2	72.9	72.8	72.6	72.5	73.1	73.0	72.8	72.8	72.6
$q_i$ Btuh	21	45	86	119	190	16	23	39	44	50
$q_r$ Btuh	4.5	6.1	7.9	8.6	9.4					
$q_s$ Btuh	0.1	0.2	0.2	0.2	0.3					
$C_t$ Btu/hr, sq ft, F	0.155	0.224	0.295	0.350	0.447	0.12	0.11	0.11	0.11	0.11
$C_{cc}$ Btu/hr, sq ft, F	0.120	0.193	0.275	0.325	0.426					
$K_{cc}$	1.22	1.10	1.04	1.03	1.02					
$Gr Pr \times 10^9$	3.26	5.20	8.26	10.1	13.7					
Nu	32.8	53.5	77.6	92.5	124.0					

lated from Eq (17). The theoretical correlation according to Eq (15) then becomes

$$Nu = 9.13 \times 10^{-9} Pr Gr K_{cc} \quad (28)$$

Agreement between the experimental and theoretical correlation is within a few per cent. The heat flow attributable to the insulation in the tests was a relatively small part of the total measured. Even small percentage errors in the total therefore result in relatively large percentage errors in the insulation heat flow.

The factor  $K_{cc}$  is useful in predicting whether convective air flow will be a problem in a practical installation. Eq (14) can be rearranged to read:

$$1/K_{cc} = \frac{C_c}{C_{cc}} \quad (29)$$

In this form,  $1/K_{cc}$  represents the ratio of the convective flow heat-transfer potential to the total heat-transfer coefficient (exclusive of radiation transfer and fiber conduction). As  $1/K_{cc}$  approaches zero,  $C_c$  also approaches zero and  $C_{cc}$  approaches the fluid conduction coefficient  $k/L$ . At low values of  $1/K_{cc}$ , Eq (29) can be expressed approximately as:

$$1/K_{cc} \approx \frac{C_c}{k/L + C_c} \quad (30)$$

The value of  $K_{cc}$  required to limit the increase in heat flow due to convective air flow to any specified ratio of the total conduction-convection through the insulation can be calculated from Eq (30). For example, if the increase in heat flow due to convective air flow at a specific temperature difference is to be limited to 10%, then the value of  $K_{cc}$  must be 10 or greater. If the insulation properties will not provide a sufficiently high  $K_{cc}$  value at the design condition, then an air barrier must be incorporated into the system.

The value of  $\lambda H$  for the insulation used in the test was  $460 \times 4 = 1840$  which gives  $K_{cc}$  values of about 2.5 at 10 F temperature difference and close to 1.0 for the 90 F temperature difference. The air leakage coefficient required for a  $K_{cc}$  greater than 10.0 is very low;  $\lambda H$  must be lower than about 35 for 90 F temperature difference and, if  $H$  were 4 ft,  $\lambda$  must be less than 9.

### CONCLUSION

Theoretical relationships have been developed to describe the heat transfer by combined fluid conduction-convection through air-permeable insulation with vertical air spaces adjacent to both surfaces. The fluid conduction-convection is shown to be a function of fluid properties, air flow coefficient of the insulation, insulation height and thickness, and temperature difference; a correlation in terms of dimensionless groups has been derived. Results of measurements on a 4-ft high insulation specimen over a temperature difference range

from 30 to 90 F were in agreement with the theory.

The relationships suggest that very low air flow coefficients are required to minimize convection effects and, in practice, an air barrier in contact with the insulation is required if air spaces occur on both sides.

### ACKNOWLEDGMENT

The author is grateful to Dr. W.G. Brown for advice in connection with the theoretical development and to Messrs. A.G. Wilson and K.R. Solvason for their assistance with the experimental program and the preparation of the paper. This paper is a contribution from the Div of Building Research, National Research Council, Canada, and is published with the approval of the Director of the Division.

### NOTATION

#### English Letter Symbols

A, B, C, D	=	constants defined when introduced
$c_p$	=	specific heat of fluid
$g$	=	acceleration of gravity
$g_c$	=	gravitational constant
H	=	height of insulation
$C_t$	=	total heat-transfer coefficient for insulation
$C_c$	=	apparent heat-transfer coefficient due to convective air flow
$C_{cc}$	=	apparent heat-transfer coefficient due to fluid conduction and convection
$k_a$	=	conductivity of air
$k_r$	=	apparent radiation conductivity through insulation
$K_{cc}$	=	conduction-convection variable
$K_{tv}$	=	temperature-variation variable
L	=	thickness of insulation
$L_f$	=	mean free path at very low pressures for molecule-fiber collisions
P	=	pressure
q	=	total heat-transfer rate
T	=	temperature
$T_{ie}$	=	equivalent warm side temperature
$T_{mc}$	=	bulk mean air temperature on cold side
$T_{mw}$	=	bulk mean air temperature on warm side
V	=	fluid velocity through insulation
W	=	width of insulation

- y = vertical co-ordinate
- x = variable defined in Eq (9)

**Greek Letter Symbols**

- $\alpha$  = thermal diffusivity
- $\Delta$  = property difference
- $\epsilon$  = fraction of incident radiant energy absorbed by a fiber
- $\rho$  = fluid density
- $\bar{\rho}$  = mean fluid density
- $\lambda$  = air infiltration coefficient/unit thickness
- $\mu$  = fluid viscosity
- $\sigma$  = Stefan-Boltzmann radiation constant

**Dimensionless Groups**

- G = Geometric factor  $\frac{\lambda \mu}{8g_c HL}$
- Gr = Grashof number  $g \rho \Delta \rho H^3 / \mu^2$
- Nu = Nusselt number  $C_{cc} H/k$
- Pr = Prandtl number  $c_p \mu / k$

**Subscripts**

- b = air on one side at bottom
- c = cold-side air
- i = insulation
- m = mean of surfaces
- p = partitions
- r = radiation
- s = solid fiber
- t = air on one side at top
- w = warm-side air

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**APPENDIX A**

The conduction-convection variable in Eq (10) may be rewritten as

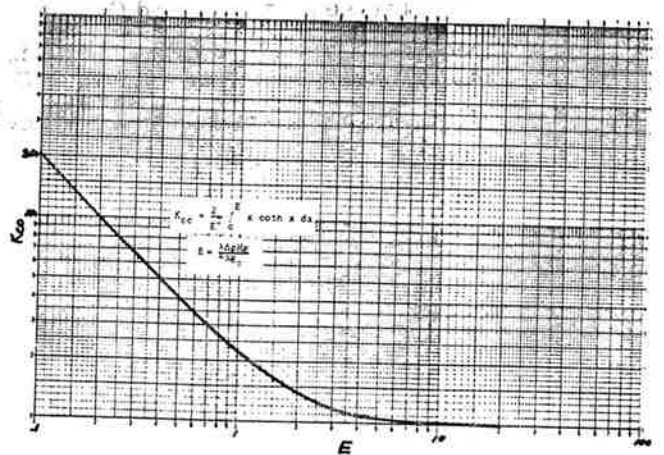
$$K_{cc} = \frac{2}{E^2} \int_0^E x \coth x \, dx$$

where E is defined as

$$E = \frac{BLy}{4\alpha} = \frac{\lambda \Delta \rho Hg}{4\alpha g_c}$$

Values of  $K_{cc}$  as a function of the single dimensionless group E were determined by numerical integration and are shown in Fig. A-1. The application of this figure is not restricted as that in Fig. 2; but rather applies to any fluid at any temperature.

**Fig. A-1  $K_{cc}$  as a function of E**





## DISCUSSION

B. E. ENO, Brookings, S. D. (Written): I would like first to question the results indicated in Fig. 7. The  $C_c$  curve comes from Eq. (7) where  $C_c$  is shown as linearly proportionate to  $\Delta\rho$ , which in turn is linear to  $\Delta T$  across the insulation.  $C_c$  in Fig. 7 is not a straight line. Also the gap between the  $C_c$  and  $C_{cc}$  curves should represent conduction.

If the  $\Delta T$  across the insulation is increasing it does seem reasonable that the ratio  $\frac{C_{cc}}{C_c} = K_{cc}$  should decrease but it does not seem reasonable that the conduction effect reduces to zero. The latter should still be an effect superimposed upon the convection effect.

Secondly, Table I seems to indicate that radiation heat transfer is considerably greater than conduction. Since the technique for calculating radiation heat transfer is not a very exacting one, I would be suspicious of the accuracy of predictions on conduction.

Finally, it appears that only one type and density of insulation was chosen. This may not be sufficient to prove the theory although it readily shows the convection effect being predominant. I think the grand conclusion is simply to keep polyethylene covers on.

AUTHOR WOLF, (Written): The apparent heat-transfer coefficient due to convective air flow,  $C_c$ , is by Eq. (7) linearly proportional to  $\rho\Delta\rho$  which is in turn, by assuming the application of the perfect gas law, proportional to  $(1/T_c^2 - 1/T_w^2)$ . Hence,  $C_c$  is not a linear function of  $(T_w - T_c)$  and the  $C_c$  curve in Fig. 7 should not be a straight line.

The gap between the  $C_c$  and  $C_{cc}$  curves in Fig. 7 decreases as  $\Delta T$  increases, and in the limit, as  $\Delta T$  approaches infinity, the ratio  $C_{cc}/C_c = K_{cc}$  approaches one, as is also shown in

Fig. A-1. This trend is as expected, since it will be recognized that the heat-transfer, conduction-convection equation, Eq. (8) or (9), reduces to the pure convection form for high velocity or low  $\Delta T$ . Similarly, for low velocity or low  $\Delta T$  the equation reduces to the pure conduction form so that as  $\Delta T$  approaches zero,  $C_{cc}$  approaches  $k/L$ .

In Table I, Tests 11 to 15 were performed without polyethylene sheeting over the insulation. The rate of heat transfer for the insulation due to combined fluid conduction and convection,  $(q_i - q_s)$ , varied from 78 to 95% of the total insulation heat-transfer rate,  $q_i$ . Regarding the effect of an error in  $q_r$  upon the accuracy of the fluid conduction-convection heat-transfer rate,  $q_{cc}$ , it will be recognized from Table I, considering for example test 15, that a 2% error in  $q_r$  will result in an error in  $q_{cc}$  of only about one-tenth of 1%.

The last point of the discussion is well taken. Only one type of insulation was tested to demonstrate the usefulness of the theory. However, the theory was further proven by application to a different type of insulation, as reported in the companion paper by Wolf, Solvason and Wilson (1966).

G. D. DAVIS, Providence, R. I.: What steps did you take to ensure that there was no condensation or freezing of the moisture or the air, if one happens to have sub-zero temperatures?

K. R. SOLVASON, Ottawa, Ont., Canada: This did not become a problem because the dew point in the entire apparatus very quickly came to a dew point in the cold box. Both boxes, in effect, came down to the dew point temperature in the cold box. The samples were considerably above the cold box or the dew point temperature so that there was no possibility of any condensation in the system. These boxes were sealed well enough so that there was no infiltration into the boxes.

