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Prediction of air distribution in a ventilated room

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SYNOPSIS

This article examines a solution procedure which can determine the flow in an air-conditioned room. The method is based on the solution of a group of equations for the flow (four non-linear partial differential equations) by means of a numerical method.
Comparison with test results indicates that the method studied is suitable for prediction of air movement in an air-conditioned room when the flow is steady and two-dimensional.
The method can be extended to give the required information for the evaluation of thermal comfort in the room.

1. INTRODUCTION

The air-conditioning of a room serves several purposes: to remove or supply heat in order to maintain a convenient temperature level and to supply the room with a given amount of fresh air. However, it is not sufficient that the air is supplied in the correct amount with correct temperature; it is also necessary that it is distributed according to the heat sources in order to obtain a constant temperature level. It is also essential that this distribution is made by a good mixing with the room air and without causing high velocities.
Fig. 1 illustrates the problem by predicting these amounts. This shows a sectional elevation of a room. The air is injected from an outlet opening at the ceiling and develops into a wall jet running under the ceiling. In this jet it is easy to predict mixing, temperature and velocity profiles, and the outlet opening is often so dimensioned that the velocity has fallen to 200-250mm/s when the jet has reached three-quarters of the length of the room.
In the further progress of the jet the simple relations between, for example, velocity and distance from outlet opening will disappear — the jet is said to disperse — and other parameters such as the dimensions of the room will vitally influence the flow. If a calculation of mixing, temperature and velocity profiles in the occupation zone is wanted, all these parameters must be taken into consideration.
This paper suggests a calculation procedure which can predict the flow at any point in the room, including the occupation zone. The method is based on the solution of the equations for the flow (four non-linear partial differential equations) ie by means of a numerical method. In the following, the development of the equation system is mentioned, and subsequently the numerical method which necessitates the use of a computer for the solution. Finally, the predicted results for a averaged sized room and a model scale room are compared with the measured results.

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2. EQUATIONS

The equations applied here have been developed on the basis of Navier-Stokes' equations and the equation of continuity. We presuppose a steady two-dimensional air movement in the examined section of the room.
This demand does not exclude the use of air outlet openings which at first give a three-dimensional air movement, provided that this subsequently develops

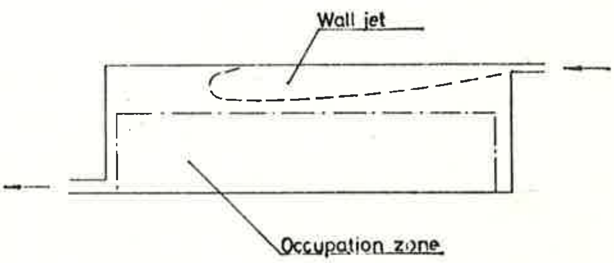

Fig 1. Sectional elevation in an air-conditioned room. It is usual that the air outlet jets are placed outside the occupation zone with the purpose of generating a constant velocity field in the occupation zone.

into a two-dimensional free jet or wall jet. This will often be the case when the outlet opening consists of sections sitting close along a line.

The examples with which comparison is made at the end of this paper give a constant two-dimensional air flow. In long, wide rooms an unsteady air movement may arise, so in case of doubt these conditions should be examined through, for example, small-scale modelling technique before the method is applied.

At the velocities and temperature differences which take place in air-conditioning, the air flow will always be turbulent. Navier-Stoke's equations and the continuity equation will provide in principle a complete equations system. It cannot however, be made the object of a numerical solution, since it expresses the turbulence at fluctuations which may be as minute as 0.1 mm, and will thus demand an inconceivably large number of difference equations at a numerical procedure.

Instead we choose to describe the turbulence by mean values of air movement and express it by an 'effective viscosity'. It is thereby possible to obtain an practical number of difference equations since the variation of the mean values over a given distance is relatively small. Such a description of the turbulence requires a turbulence model, ie an equation system which expresses the coherence between certain mean values of the turbulence. The turbulence model used here consists of a transport equation of turbulent kinetic energy k and an equation for the dissipation of turbulent kinetic energy ε. These are equations (3) and (4) respectively. Equation (5) shows how the turbulence is described by 'effective viscosity'. The quantities σ_k, σ_ε, C_1, C_2, and C_μ are various constants. The development of the equations is described more explicitly by Launder *et al*[1].

Navier-Stokes' equations and the equation of continuity have, in two directions at right angles to each other, the velocities v_1 and v_2, and the pressure p as variables. It was decided to leave these and change to the somewhat special variables: vorticity ω and stream function ψ, as it was then possible to reduce the number of differential equations by one. This would effect a saving in computing time and capacity of approximately 20 per cent.

The vorticity is the difference between the velocity gradients in two directions at right angles to each other. It is twice the angular velocity of the air at the examined point.

The stream function describes the vector field (v_1, v_2) by a single scalar quantity ψ. The stream function ψ is a practical variable when describing an air-conditioning problem, because lines through constant ψ values are stream lines, ie lines parallel to the velocity vector.

The two differential equations replacing Navier-Stokes' equations and the continuity equation under the aforementioned change in variables, are called the vorticity transport equation and the transport equation for the stream function. These are equations (1) and (2).

The four differential equations (1), (2), (3) and (4) have the four unknown quantities ω, ψ, k, and ε, and therefore represent a complete description of the air flow, and they form the basis of the numerical solution method.

This paper concerns determination of the velocity distribution at isothermal flow. The results also apply to small temperature differences where the forced convection is predominant compared to the natural convection. If a prediction of the temperature field is required, the four differential equations are increased by one equation, the energy equation.

If the natural convection is considerable, one cannot ignore the generation of vorticity by the density gradients; a term for this is introduced into the vorticity transport equation.

$$\frac{\partial}{\partial x_1}\left(\omega\frac{\partial \psi}{\partial x_2}\right)-\frac{\partial}{\partial x_2}\left(\omega\frac{\partial \psi}{\partial x_1}\right)= \frac{\partial}{\partial x_1}\left(\frac{\partial \mu_{\text{eff}}\omega}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{\partial \mu_{\text{eff}}\omega}{\partial x_2}\right) \quad \ldots. (1)$$

$$\frac{\partial}{\partial x_1}\left(\frac{1}{\rho}\frac{\partial \psi}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{1}{\rho}\frac{\partial \psi}{\partial x_2}\right)=-\omega \quad \ldots. (2)$$

$$\frac{\partial}{\partial x_1}\left(k\frac{\partial \psi}{\partial x_2}\right)-\frac{\partial}{\partial x_2}\left(k\frac{\partial \psi}{\partial x_1}\right)= \frac{\partial}{\partial x_1}\left(\frac{\mu_{\text{eff}}}{\sigma_k}\frac{\partial k}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{\mu_{\text{eff}}}{\sigma_k}\frac{\partial k}{\partial x_2}\right) + \mu_{\text{eff}}\left[2\left(\left(\frac{\partial v_1}{\partial x_1}\right)^2+\left(\frac{\partial v_2}{\partial x_2}\right)^2\right)+ \left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right)^2\right]-\rho\varepsilon \quad \ldots. (3)$$

$$\frac{\partial}{\partial x_1}\left(\varepsilon\frac{\partial \psi}{\partial x_2}\right)-\frac{\partial}{\partial x_2}\left(\varepsilon\frac{\partial \psi}{\partial x_1}\right)= \frac{\partial}{\partial x_1}\left(\frac{\mu_{\text{eff}}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{\mu_{\text{eff}}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_2}\right) + C_1\mu_{\text{eff}}\left[2\left(\left(\frac{\partial v_1}{\partial x_1}\right)^2+\left(\frac{\partial v_2}{\partial x_2}\right)^2\right)+ \left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right)^2\right]\frac{\varepsilon}{k}-C_2\rho\frac{\varepsilon^2}{k} \quad \ldots. (4)$$

$$\text{where } \mu_{\text{eff}} = C_\mu \rho\frac{k^2}{\varepsilon} \quad \ldots. (5)$$

3. NUMERICAL METHOD

The principal of the numerical method is to replace the differential equations by a number of difference equations which can be solved by a systematic procedure.

The examined section is divided up into a number of points. These points are distributed so that it is permissible to consider ω, ψ, k, and ε as constant within the quadratic range around the single points. In ranges where the gradients of the individual quantities are large, the points are close, and in ranges where the gradients are small, the distance between the points is greater.

The four differential equations are replaced by four difference equations in each point. If a number of points 21×21 is used, it results in an equation system consisting of 21 × 21 × 4 = 1764 equations with 1764 unknown quantities. This equation system can now be solved in a computer by an iterative procedure. Because of the high number of equations, a computer in the size range 100 K bytes is required. The examples examined in this paper have been computed on an IBM system 370/145.

The development of the difference equations and the structure of the iterative procedure are described in detail by Gosman *et al*[2]. Part of the computer programme used has been developed at Imperial College in London, and this is also described by Gosman.

4. RESULTS

In the following, comparisions are made with test results obtained in a test room in the Technical University of Norway at Trondheim[3]. The room is 8.3 m long, 3.4 m wide and 2.8 m high. The air outlet opening is placed immediately under the ceiling at one end-wall. It runs along the width of the room and has an opening of 15 mm. Two return openings are in the bottom corners of the opposite wall. The velocity is measured by spherical anemometers and the air flow directions obtained by injecting smoke into the room.

Fig 2 shows the predicted and measured velocity distribution in the room. The velocity is indicated as 'isovels', ie lines through points of the same velocity, and both the predicted and the measured velocity field are, in Fig 2, indicated as dimensionless compared to the outlet velocity.

The outlet air stream forms a wall jet which flows under the ceiling. In its further progress down the left wall the velocity is reduced considerably in the corners and remains lower than in a wall jet of the corresponding length. In the occupation zone the velocity increases to 8 per cent of the outlet velocity, and then falls back to below 2 per cent in the upper and the right section of the occupation zone.

The prediction and the test have been made at an outlet velocity of 1.8 m/s; the velocity in the occupation zone, therefore, lies between 140 and 40 mm/s. It appears that in practice the dimensionless velocity is rather independent of larger variations in outlet velocity, because the structure of the turbulence in the recirculating flow will be similar and thereby independent of Reynolds numbers. If the outlet velocity increases to 3.6 m/s, it results in a velocity in the occupation zone which lies between 280 and 80 mm/s. It will be seen that a doubling of the outlet velocity means both a doubling of the maximum velocity and a doubling of the velocity difference in the occupation zone. Thus it is more difficult to obtain a uniform state of thermal comfort in the whole occupation zone at high outlet velocities than at low.

Fig 3 shows the distribution of the stream-lines in the room. The amount of air which is transported between two stream-lines is constant, and therefore the velocity is high where the stream-lines are close, and low where the distance is great. The stream function is dimensionless compared with the amount of outlet air. Between stream-lines 4 and 6, for example, twice the outlet ammount is transported, and in all, an amount of air approximately seven times the injected amount is set into motion.

It must be pointed out that the stream-lines are only an expression of mean values. In an examined point, the stream-line is parallel with the mean velocity, but because of the various directions of the instant velocities, a motion of mass and energy takes places across the stream-lines.

The test results indicated in Fig 4 are derived from Urbach[4]. They are tests made in a model 2 m long, 1 m wide, and 1 m high. The outlet opening is situated under the ceiling at one end-wall and runs along all the width of the model. The amount of the opening can be set at various values. The return opening is situated at the bottom of the opposite wall.

It can be seen from the results that the maximum value of the stream function is an expression of the amount of air set into motion compared with the injected amount. Fig 4 shows how this quantity increases with the decreasing outlet opening. The injected amount is always the same, ie the outlet velocity increases with decreasing outlet opening. The outlet opening h/H is indicated as dimensionless compared with the height of the model.

At a given amount of outlet air — the quantity of which may be set according to a thermal or hygenic criterion — the mixing and the amount of air set into motion in the room are limited. The reason is

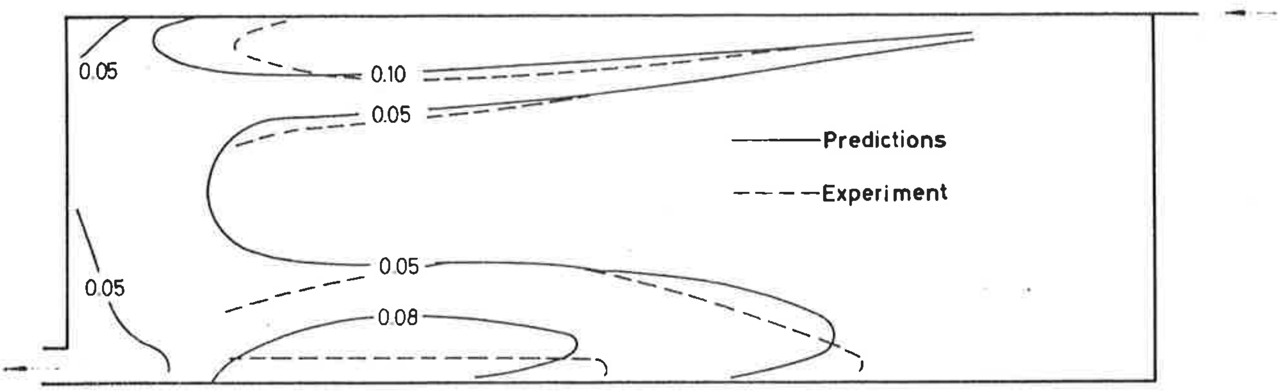

Fig 2. Predicted and measured velocity distribution in a room.

that the outlet velocity increases with decreasing outlet opening and causes an increasing velocity in the occupation zone. So this velocity and noise generating from the outlet sets a limit to the minimum size of the outlet opening.

5. CONCLUSION

Comparison with test results indicates the suggested prediction method herein is suitable for investigation of the flow in an air-conditioned room when the flow is steady and two-dimensional in the examined section.

This prediction method can be extended to give the required information for the evaluation of thermal comfort, ie air velocity, air temperature, surface temperature, velocity and temperature gradients, and turbulence intensity.

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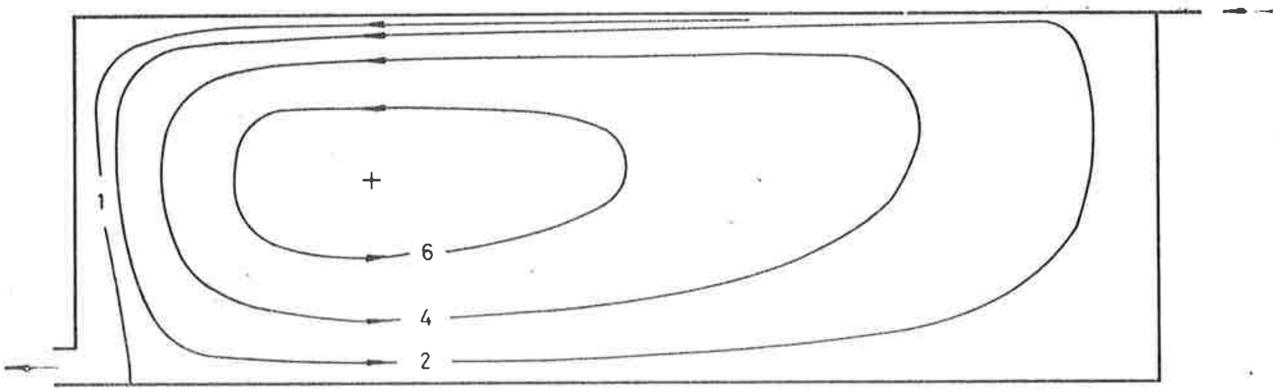

Fig 3. Predicted stream-line distribution in a room.

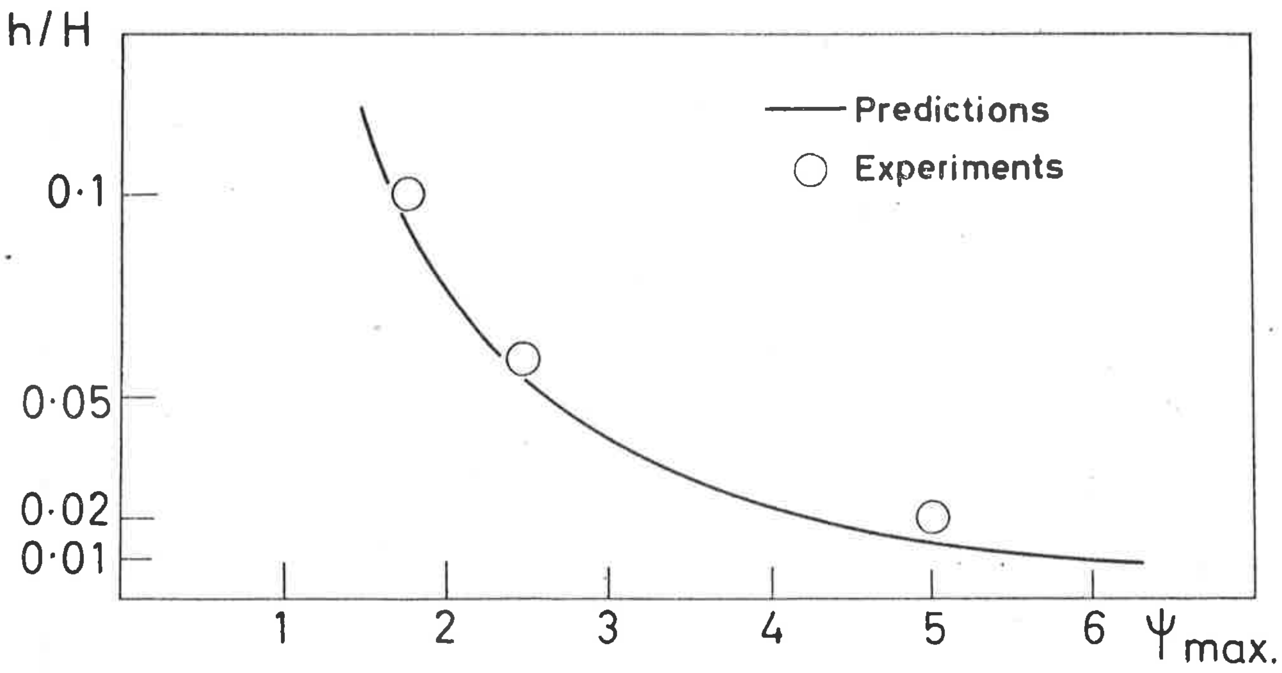

Fig 4. Effect of outlet geometry on amount of recirculation air.