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VENTILATION THEORY AND PRACTICE

Ventilation is one of the most overworked words in engineering technology with its meaning having undergone many changes with the passage of time. It even now carries conflicting connotations to different individuals. Much has been written about ventilation, including a recent history which called attention to the changing patterns of ventilation practices.[1] In this paper it is our intention to consider that ventilation is the circulation or passage of an air supply through an enclosure resulting in the displacement of some or all of the air contained in that enclosure by the supply air. Depending on the character and condition of the supply air it can dilute or change the quality of the air in an enclosure as well as alter its temperature and humidity. A mathematical analysis of the ventilation process appears later in this paper.

To indicate an earlier viewpoint, in 1931 the New York Commission in its final report on School Ventilation[2] stated:

> "The major objective of ventilation is, therefore, to remove excess of heat given off by the human body so as to maintain an atmosphere which will be comfortably cool but not too cold. Since heat loss from the body is determined by the temperature, the humidity and the movement of the atmosphere, the desired results may be attained by modifying any one or more of these factors.
>
> Finally, although the organic exhalations from the body are not poisonous, they may, in an unventilated room, become esthetically unpleasant and may even, as our earlier studies have shown, produce diminished appetite and hence exercise an indirect influence upon health. Ventilation should, therefore, produce sufficient air change to avoid an unpleasant concentration of such body odors."

This narrow viewpoint, it will be noticed, emphasized primarily the cooling aspect of outside air in winter.

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We must face the question of whether ventilation relates solely to *fresh air (outside air)* or whether it should also include mixtures of recirculated air with outside air as the ventilation air supply for an enclosed space. From a legalistic viewpoint, municipal and state codes largely consider ventilation air as outside air exclusively, although in certain circumstances consideration is given to limited amounts of recirculated air. More attention must be given to recirculation because in addition to the other problems which have taken place, even the character of the air used for ventilation has changed. The term fresh air implied that a supply of uncontaminated outside air could be brought into a space and used in dilution flow patterns to make desirable inside conditions. Now in the industrial districts of almost any major American city, the outside air may be so contaminated that the inside air is worsened by the invasion of this dirty air when ventilation takes place. In spite of this, most municipal codes are built around our former impressions of fresh air and these codes would lead you to believe that outside air ventilation solves many of the problems of building occupancy. The codes are often confusing and inconsistent, in that most of them try to relate the air flow both to the square footage of floor space and also to occupancy, two conditions which may or may not be compatible.

VENTILATION REQUIREMENTS

Carbon dioxide is not a satisfactory index of the quality and character of air; nevertheless it does provide a basis for analysis, and a high CO_2 level in a given occupied space would indicate a lower than normal quantity of air being supplied. The change in odor level in a space is a more suitable index of the ventilation requirement of an occupied space than is CO_2. However it is difficult, if not impossible, to set up adequate instrumentation to use an odor-level index as a parameter for showing the quality of the air. It is also true that in terms of human occupancy, variability in the odor produced by individuals is great, and the odor level is affected by numerous variables. Because of these difficulties there has been little research to add to our knowledge of this subject. The work of Lehmberg and his associates[3] provided data on odor production by individuals by

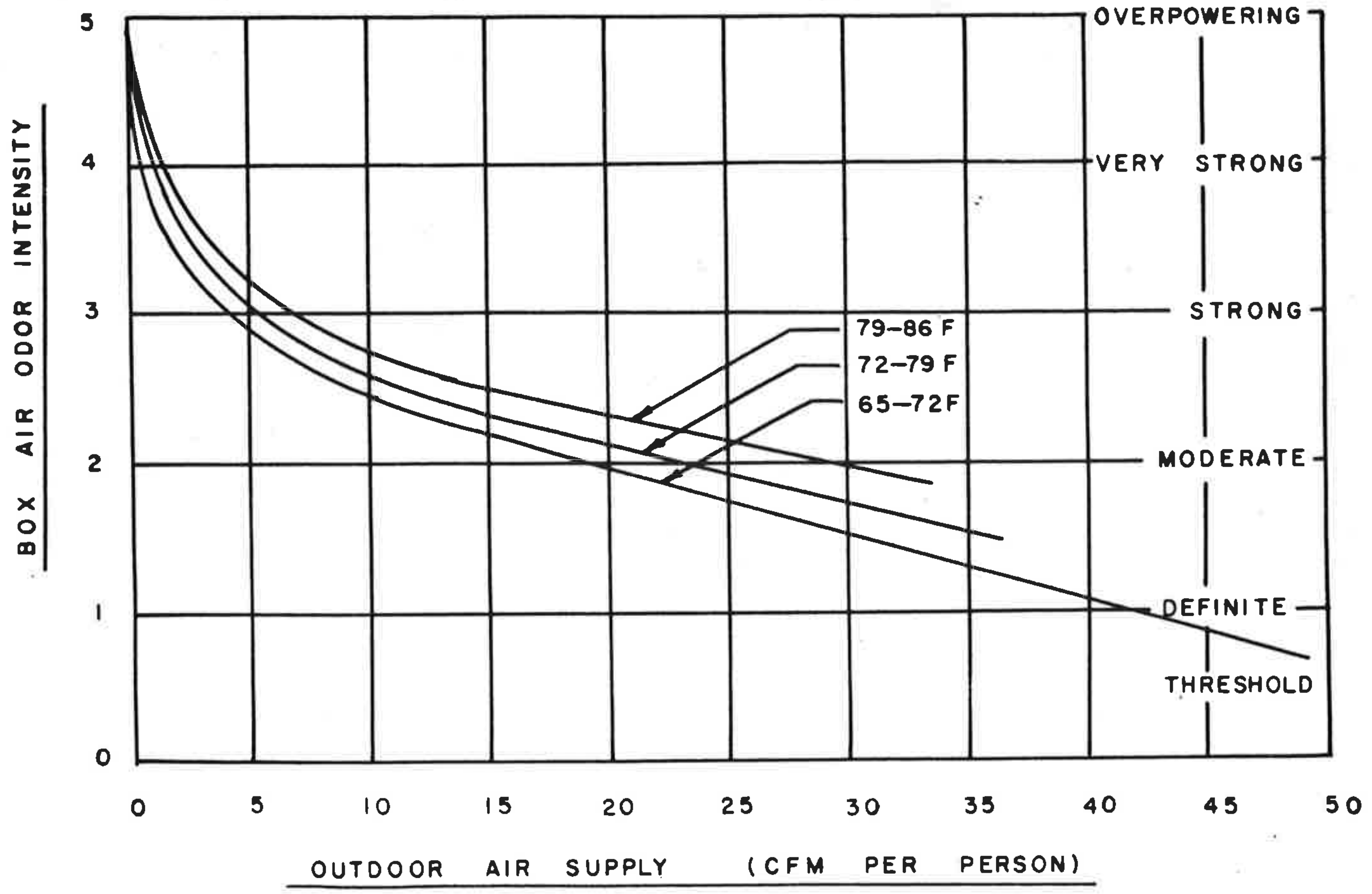


Fig. 1 Odor intensity levels in relation to ventilation rate and body surface area as found in box tests. After Lehmberg et al[3]

means of box studies. In these investigations, each human test subject lay quietly on a cot in an open chamber for a resting period of 1/2 to 2 hours, depending upon his previous activities. Then just prior to starting a test, the subject was weighed and entered a box to lie on the bare wooden bottom with only a small pillow under his head. Air was supplied to the box through suitable openings to maintain desired temperature and humidity conditions. Other individuals serving as panelists then contrasted the odor level of the air leaving the box with that of the air entering the box and recorded their impressions of exit odor in terms of an arbitrary numerical scale with zero (0) representing no perceptible odor, (1) definite odor, (2) moderate odor, (3) strong odor, (4) very strong odor and (5) representing an overpowering nauseating odor. A graph (Fig. 1) representing some results of the Lehmberg studies shows how odor intensity varied in relation to ventilation rate and surface areas of the subjects. Fig. 2 records similar data as influenced by temperature of the air passing over the subjects. In summary, these tests show that a definite relationship exists between the intensity of body odor produced in the air of an occupied space and the quantity of fresh air supplied. The tests further show that with ventilation rates of 5 cfm per person or less, the odor level is disagreeable but that even with ventilation-rates as high as 50 cfm per person, complete elimination of odor does not take place. It is, of course, true that these experiments took place in a box of restricted volume so that the odor-masking effect of the large volume per individual which exists in most occupied rooms did not occur.

The next significant papers which gave experimental data relative to odor level produced by individuals were reported by Yaglou and his associates.[4,5] In the first of two papers work was done with children and adults occupying ordinary rooms and a partial summary part of his data is reproduced as a logarithmic-scale graph in Fig. 3. He employed the same odor scale, used by Lehmberg, on which level 1 represents a detectable odor which however, is not objectionable, 2 is a moderate odor with little or no objection and represents an allowable level for occupied spaces, 3 is at the point of being definitely objectionable. It will be noticed that the summary chart of Yaglou shows that the allowable level 2 was exceeded whenever the outdoor supply air was provided at less than 17 cfm per person. For a person entering the space the odor level appeared greater than for a person who, within the space, had become accustomed to the odor patterns in that space. Among other conclusions reached was the fact that there was wide individual variation in the amount of odor emitted by types of persons. This was often set by bathing habits, cleanliness of clothing and length of period between clothing changes. Even healthy, clean persons just after a bath produced sufficient odor to require from 15 to 18 cfm of outdoor air per person to dilute the odor to a concentration that was not objectionable to persons entering the space from an uncontaminated surrounding.

The use of untreated, recirculated air in Yaglou's test spaces did not appreciably reduce odor intensity. However, recirculated air which went through air conditioning processes of washing, humidifying, cooling or dehumidifying

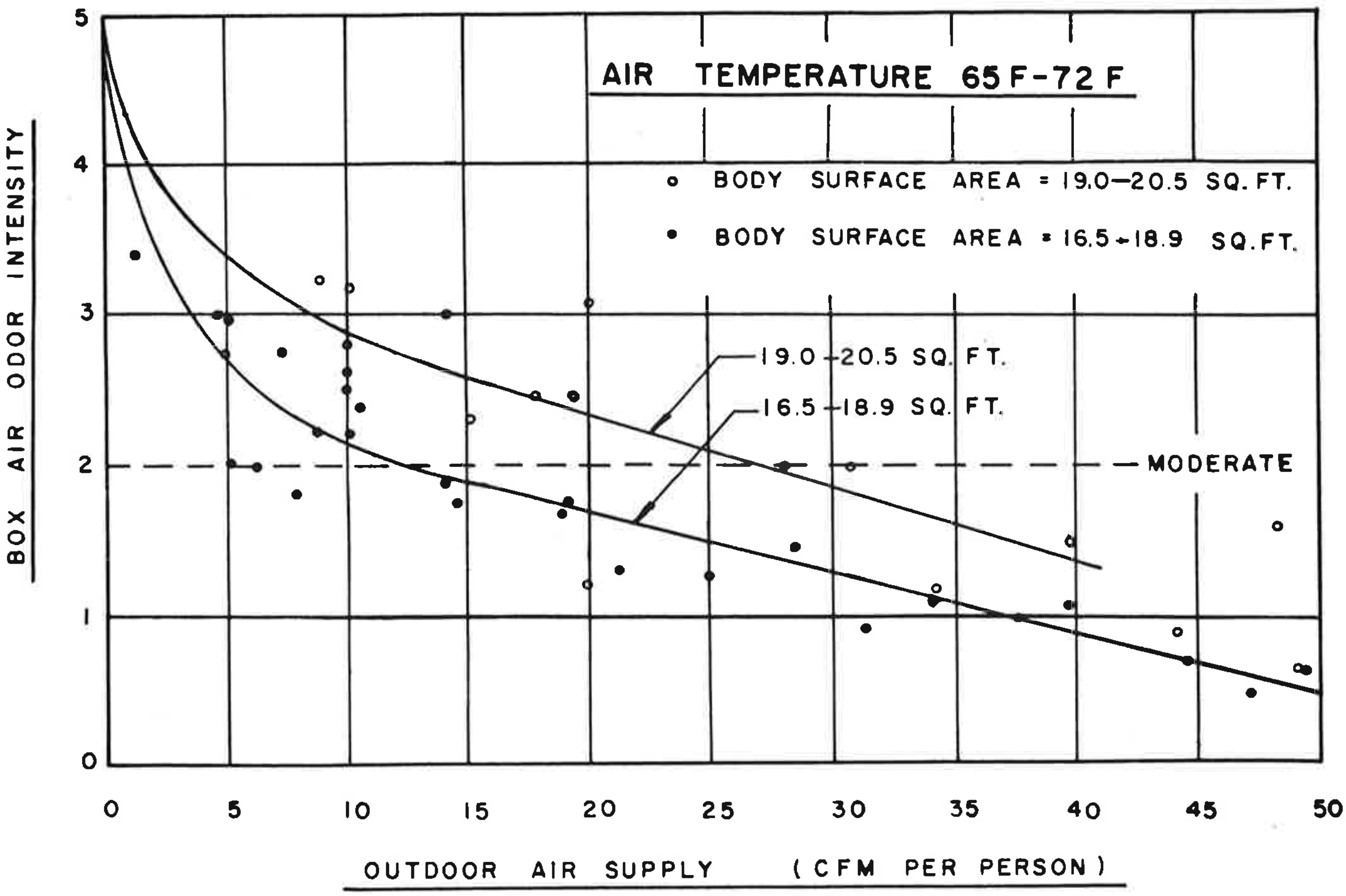


Fig. 2 Influence of air temperature on odor intensity of subjects as determined by box tests. After Lehmberg et al[3].

changed in character sufficiently to permit the substitution of some recirculated air for outside air. He also carried out tests relative to CO_2 in the air and found that it was an unreliable index of the character of ventilation from the standpoint of indicating the odor intensity that might exist in a space.

In his investigations, Yaglou noted that the odor intensity in a space was strongly dependent upon the volume available for each individual in the space. With 400 cu ft per person available, the odor level rose at a much slower rate than was true with 200 cu ft per person. This conclusion would be expected with short-period tests since sufficient time had not elapsed in such tests for equilibrium to be attained. It is obvious and mathematically demonstrable that with a stable contaminant and with tests run for a sufficiently long period the concentration of the contaminant in a given space would ultimately be the same and independent of the volume per occupant. With odors, however, two other phenomena must be considered. The intensity level of body odors in a space tends to diminish with time, first as a result of possible oxidation along with breakdown of the complex molecules and second, because some surface absorption of odors occurs. The conclusions can readily be confirmed from tests run with high concentrations of valeric and butyric acid vapors in the air. Over a period of time even with no leakage in the spaces the odor concentration of these substances decreases by substantial amounts, a fact which is characteristic of many organic odors. Unfortunately tobacco smoke and fire-smoke odors diminish at an extremely slow rate. For example, if a room in which smoking has taken place is closed and sealed, the odor level in the room will be almost as high several hours later as it was during the height of smoke production. It is also true that tobacco-smoke odor is so strongly absorbed in clothing, drapes and carpet that this contributes to the slow decay of odor intensity.

Yaglou[6] made a further study of odor problems associated with cigarette smoke and showed that appreciably higher rates of air circulation were required for spaces in which smoking occurred. In fact, an outside air supply of 35-40 cfm per smoker was required to dilute objectionable odors in a space when judged by observers entering such a space from uncontaminated air. For people within a space in which smoking was taking place, clean air at a rate of 25 cfm per smoker produced a level which was acceptable both for smokers and non-smokers in the space. The previously mentioned figure of 17 cfm per person is inadequate from an odor-level viewpoint for spaces in which smoking is taking place.

Particulates in air are also contaminants and this is certainly true for tobacco smoke. Leopold[7] made tobacco smoke studies in a sports arena by making optical density (transparency) studies by passing light beams through a smoke-filled space. His study showed that in a room in which there was very heavy smoking, 31 cfm of supply air per person was required to maintain acceptable conditions in the space.

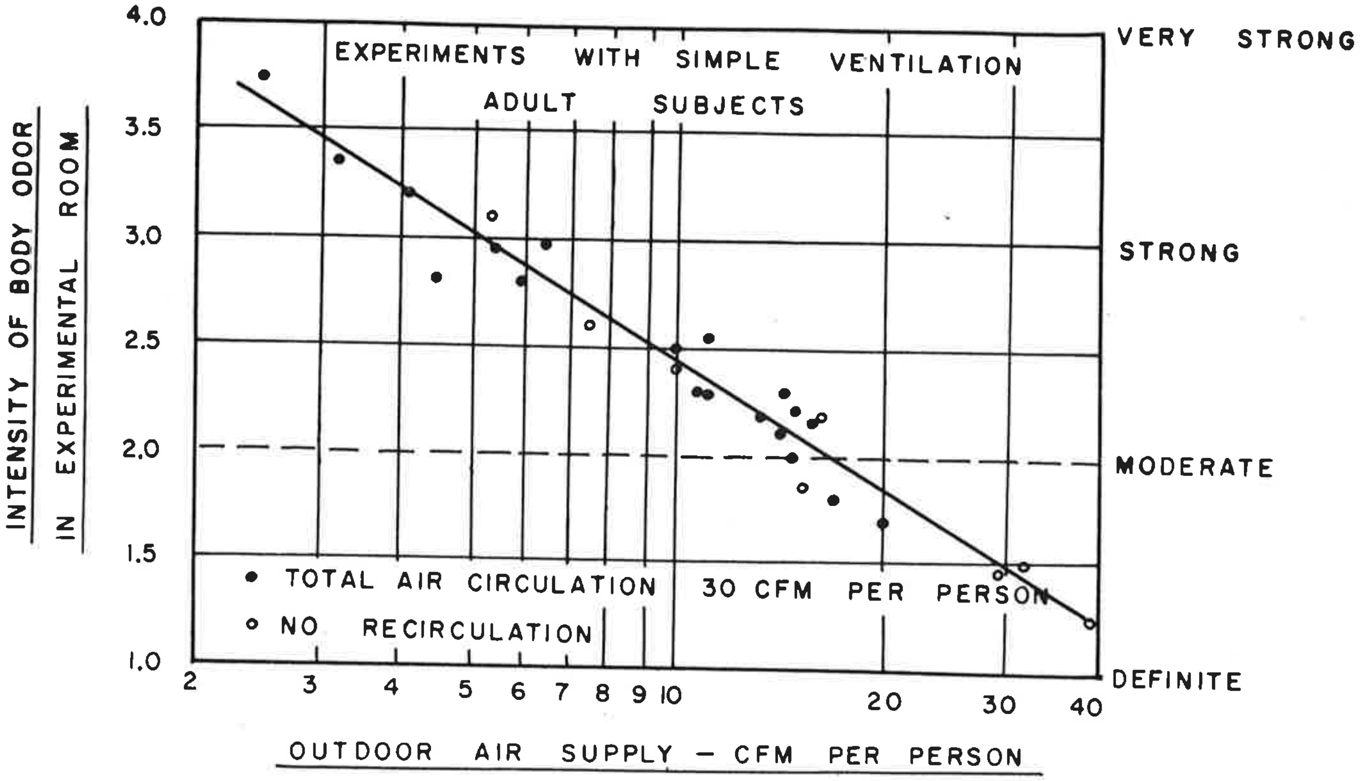


Fig. 3 Outdoor air supply rate in relation to odor intensity (net air space per person 200 cu ft). After Yaglou et al[4].

CARBON DIOXIDE, OXYGEN AND THE RESPIRATORY PROCESS

One of the most important aspects of ventilation relates to the manner in which the human body makes use of the air available to it. Thus an understanding of human physiology in relation to air is essential. The breathing system in human beings provides air (oxygen) for the lungs and removes waste products from the respiratory system of the body in the form of waste air containing higher amounts of carbon dioxide and humidity (water vapor) than it had on entry. Tests[8] run on a group of young men (age 22-23 years) showed, for the breathing process, that the tidal volume of gas expired or inspired in a respiratory cycle, ranged from 0.45 liters minimum to 3.8 liters maximum. Of this tidal air only about two thirds of that inspired during breathing reaches the lungs. The other third remains in the mouth, trachea, windpipe and bronchi and is, of course, exhaled as the body delivers air. Even under maximum exhalation the volume of residual gas in the lungs was 1.2 liters while the volume of gas remaining at a resting expiratory level was 2.2 liters. The maximum inspiratory capacity from resting level was measured at 3.8 liters. The vital capacity or maximum volume that could be expired after maximum inspiration was 4.8 liters. The total volume of gas in the respiratory system at the end of maximum inspiration was 5.97 liters.

Breathing is an automatic process taking place in an individual under resting conditions about 10 to 30 times/min. However, under stress conditions or when purposely controlled, the breathing rate rises. Increasing the breathing rate provides hyperventilation which means that the relative concentration of oxygen in the lungs is increased. Breathing is continuous but under body control can be stopped for short periods of time. The breath, under normal breathing conditions, can only be held for some 30 sec without discomfort before its resumption is necessary. However, when hyperventilation is practiced for several minutes beforehand, so that the oxygen content of the blood stream is increased, the breath may be held for 2 to 4 min or even longer. Native pearl divers make use of this phenomenon to enable them to stay submerged for long periods.

The lungs are incapable of storing oxygen and, in fact, the only oxygen that is stored in the body is that which is in transport by the blood stream. The muscles need oxygen to function although they can act for very short periods without an oxygen supply. With oxygen lack, the muscles rapidly build up waste products and muscular activity becomes inhibited. The central nervous system is most critical in its oxygen requirements and if deprived of oxygen for as few as 5 min, as in the case of a person submerged in water, the system may be so irreparably damaged as to lead to death for the individual. In particular, the brain and eyes are most susceptible to inadequate oxygen in the blood supply.

For proper functioning of the body it is necessary that the bloodstream have the same saturation even at higher altitudes. At reduced atmospheric pressure this can be brought about by hyperventilation (more rapid breathing) as in the case of people engaged in moderate mountain climbing, while for aircraft travellers it becomes necessary either to increase cabin pressure to approach standard atmospheric pressure or to provide direct oxygen to the individual. For equivalent oxygen concentration in the bloodstream the following oxygen concentrations are needed at different elevations.

Oxygen Concentrations

Elevation in ft	Atmospheric Pressure in Hg	mm Hg	Oxygen Required in % by Volume
0	29.921	760.0	21
2500	27.321	694.0	23
5000	24.897	632.4	26
10,000	20.581	522.8	32
15,000	16.893	429.1	38
20,000	13.761	349.5	48
25,000	11.118	282.4	60
30,000	8.903	226.1	78
33,000	7.756	197.0	90

At an altitude of 23,000 ft the atmospheric pressure is so low that it can barely force sufficient air into the lungs to provide adequate oxygen for breathing. Thus pressurization of atmospheric air is a necessity for life at altitudes above 23,000. With almost pure oxygen provided for breathing it is possible to keep the lungs functioning to about 33,500 ft but this altitude represents a top limit for breathing with non-pressurized oxygen. At 50,000 ft, tests have shown that no oxygen exists in the lungs from a non-pressurized supply and only CO_2 and H_2O along with nitrogen are present.

Just as too little oxygen is undesirable in the lungs, it is also true that too much oxygen can cause difficulty. In fact when the partial pressure of oxygen in the lungs exceeds 175 mm Hg, unpleasant reactions appear and when oxygen is present in the lungs at full atmospheric pressure, 760 mm Hg, it is dangerous. The symptoms of oxygen toxicity are nausea, dizziness, coughing and respiratory unpleasantness. If the oxygen supply is carried to yet higher pressures, at about 2000 mm Hg fainting, dizziness and convulsions will occur.

In the alveoli of the lungs, carbon dioxide is transferred out of the blood stream and oxygen is added to it. In order for this process to take place effectively, the vapor pressures (concentrations) of the oxygen and of the carbon dioxide in the lung must desirably be kept in specified ranges. The partial pressure of the carbon dioxide in the lungs is closely in the range of 40 mm of mercury which corresponds to a concentration of closely 5.25% of CO_2 by volume under standard barometric conditions (760 mm of mercury). For the same conditions the corresponding oxygen pressure is about 103 mm of mercury. The mouth, trachea and lungs even when provided with dry air rapidly humidify the air so that water vapor always exerts a vapor pressure of its own in the respiratory system. At 98.6 F (37 C) this corresponds to 47 mm Hg. A tabulation[9,10] of the changes in the variables with dry air entering the lungs is tabulated below.

Breathing Dry Air

Altitude ft	Pressure mm Hg	Volumetric percentages N_2	CO_2	H_2O	O_2	Pressure O_2 mm Hg
0	760.0	79.02	0.03	0.0	20.95	159.2
5000	632.3	79.02	0.03	0.0	20.95	132.5
Tracheal gas with dry air intake						
0	760.0	74.13	0.03	6.18	19.66	149.4
5000	632.3	73.14	0.03	7.43	19.39	122.6
Alveolar gas with dry air intake						
0	760.0	75.00	5.26	6.18	13.55	103.0
5000	632.3	74.06	6.01	7.43	12.49	79.0

Atmospheric air at standard sea level conditions (760 mm Hg) contains closely 21% of oxygen by volume and at this concentration permits the lungs to produce about 97% oxygen saturation in the bloodstream.

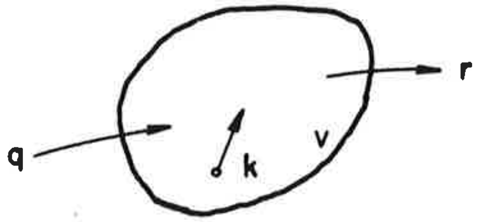
Thus the character of the ventilation air must be kept within controlled limits to cause proper interaction with the human body. The contaminants which exist in an air supply may or may not be undesirable or toxic. For example, carbon dioxide in small concentrations causes no difficulty while in larger amounts, say over 6% by volume, it could be undesirable to the point of having to be considered a toxic element. Fortunately, in most ventilation applications supply air is provided in sufficiently great amounts that the CO_2 increase in a given space is not critical and a reduction in supply air is noticed more by an increase in odor level than by the rise in CO_2 concentration. However, in restricted air-flow spaces such as in shelters or in vehicular tunnels, CO_2 often does increase along with odor level rise and increase in other contaminants.

In shelters under enforced occupancy, an increase in carbon dioxide concentration to 3% by volume along with an oxygen decrease to 17% is considered the practical limit for safe occupancy.[11] Dilution of contaminant concentration by means of supply air still represents the basic approach to space control. In this paper, any constituent of the supply air can be considered a contaminant and can be present in varying concentrations. Let us recognize that under certain circumstances even oxygen, available in excess amount, is a contaminant.

Before developing a general dilution equation the work of several authors who formulated mathematical approaches to dilution ventilation should be cited. In a paper on air distribution, Houghton and Blackshaw[12] used an exponential decay equation developed for studying the dilution of carbon monoxide in automobile garages. Equations considering the dilution of contaminants in traffic tunnels were developed by Bijdendyk,[13] Burgers,[14] and Broer.[15] A paper by Jones[16] used a similar decay equation to illustrate the way thermal conditions in a room approach equilibrium values after an air conditioning start-up. Turk[17] developed a dilution equation to measure odorous vapors in test chambers. His analysis included the concept of odor sources and sinks in a control volume and his paper was the first to emphasize that the ventilation rate is independent of the control volume size.

DEVELOPMENT OF THE DILUTION EQUATION

In this paper an approach has been taken which gives a broad mathematical analysis of dilution ventilation.



Consider a control volume v to contain a source or sink which emits or absorbs a contaminant K at a volumetric flow rate k. Supply air with a contaminant concentration β_K is introduced into v at a volumetric rate q and exhaust air at a contaminant concentration α_K is removed from v at a volumetric rate r.

Consider the rates q, r, and k and the concentrations β_K and α_K to be functions of time. The control volume v and the densities $\rho_s, \rho_v, \rho_K, \rho_{Ks}$, and ρ_{Kv} are assumed to be constant. The temperatures of the supply air, the control volume air and the effluent of the source are also assumed to be constant, although not necessarily equal. Assume the mixing of the supply air with the control volume air to be complete and instantaneous.

The principle of conservation of mass states that the rate of accumulation of mass within the control volume is equal to the excess of the incoming rate of flow over the outgoing rate of flow, or, more simply:

$$\text{Increase} = \text{Inflow} - \text{Outflow} \tag{1}$$

Applying the mass conservation principle to the contaminant K in time dt, the incoming rate of contaminant flow is:

$$\text{Inflow} = \rho_{Ks}\, q\beta_K\, dt + \rho_K\, k\, dt$$

where ρ_{Ks} = Density of the contaminant K evaluated at the supply air conditions;

ρ_K = Density of the contaminant K evaluated at the source conditions.

The first term on the right hand side considers the concentration of the contaminant in the supply air. The second term considers a contaminant source in the control volume. If a sink existed in the control volume instead of a source, this term would be negative and ρ_K would be replaced by ρ_{Kv} which is the density of the contaminant K evaluated at the control volume air conditions.

Since a certain mass of air is supplied to v, an equivalent mass must be displaced from it. In time dt the outgoing rate of contaminant flow is:

$$\begin{aligned}\text{Outflow} &= \alpha_K\, \rho_{Kv}\, r\, dt \\ &= \alpha_K\, \rho_{Kv} \frac{\rho_s}{\rho_v} q\, dt \\ &= \alpha_K\, \rho_{Kv}\, \Omega\, q\, dt\end{aligned}$$

where $r = \dfrac{\rho_s}{\rho_v} q$

and $\dfrac{\rho_s}{\rho_v} \equiv \Omega$

The total contaminant at time t in v is $\rho_{Kv}\, v\, \alpha_K$. Hence the increase in time dt is:

$$\text{Increase} = d(\rho_{Kv}\, v\, \alpha_K)$$

or

$$\text{Increase} = \rho_{Kv}\, v\, d\alpha_K$$

Since v and ρ_{Kv} are constant.

Eq (1) becomes:

$$\rho_{Kv}\, v\, d\alpha_K = \rho_{Ks}\, q\, \beta_K\, dt + \rho_K\, k\, dt - \rho_{Kv}\, \Omega\, q\, \alpha_K\, dt$$

Dividing through by $\rho_{Kv} v$

$$d\,\alpha_K = \frac{\rho_{Ks}}{\rho_{Kv}} \frac{q}{v} \beta_K\, dt + \frac{\rho_K}{\rho_{Kv}} \frac{k}{v} dt - \frac{\rho_{Kv}}{\rho_{Kv}} \Omega \frac{q}{v} \alpha_K\, dt$$

Define $\dfrac{\rho_{Ks}}{\rho_{Kv}} \equiv \Psi$

and $\dfrac{\rho_K}{\rho_{Kv}} \equiv \theta$

So $d\,\alpha_K = \Psi \dfrac{q}{v} \beta_K\, dt + \theta \dfrac{k}{v} dt - \Omega \dfrac{q}{v} \alpha_K\, dt$

Note that if a sink is present in the control volume, $\theta = 1$ since $\rho_K = \rho_{Kv}$. Taking the time derivative of the above equation

$$\frac{d\,\alpha_K}{dt} = \Psi \frac{q}{v} \beta_K + \theta \frac{k}{v} - \Omega \frac{q}{v} \alpha_K$$

or

$$\frac{d\,\alpha_K}{dt} + \Omega \frac{q}{v} \alpha_K = \Psi \frac{q}{v} \beta_K + \theta \frac{k}{v}$$

Let $F(t) = \Omega \dfrac{q}{v}$

and $\phi(t) = \Psi \dfrac{q}{v} \beta_K + \theta \dfrac{k}{v}$

Then

$$\frac{d\alpha_K}{dt} + F(t)\alpha_K = \phi(t) \tag{2}$$

Eq (2) is a linear differential equation of order one. The solution is obtained by multiplying the equation through by the integrating factor $e^{\int F(t)dt}$

$$e^{\int F(t)dt} \frac{d\alpha_K}{dt} + e^{\int F(t)dt} F(t)\alpha_K = e^{\int F(t)dt} \phi(t)$$

or

$$\frac{d}{dt}\left[e^{\int F(t)dt} \alpha_K\right] = e^{\int F(t)dt} \phi(t)$$

Integrating

$$\alpha_K\, e^{\int F(t)dt} = \int \phi(t)\, e^{\int F(t)dt} dt + c$$

$$\alpha_K = e^{-\int F(t)dt} \int \phi(t)\, e^{\int F(t)dt} dt + c e^{-\int F(t)dt}$$

Substituting for F(t) and $\phi(t)$, the general solution is:

$$\alpha_K = e^{-\int \Omega \frac{q}{v} dt} \int \left[\Psi \frac{q}{v} \beta_K + \theta \frac{k}{v} \right] e^{\int \Omega \frac{q}{v} dt} dt + c e^{-\int \Omega \frac{q}{v} dt} \qquad (3)$$

The number of air changes per unit time is:

$$n = \frac{q}{v}$$

Substituting n into Eq (3), the general solution in terms of air changes per unit time is:

$$\alpha_K = e^{-\int \Omega \, n dt} \int \left[\Psi \, n\beta_K + \theta \frac{k}{v} \right] e^{\int \Omega \, n dt} dt + c e^{-\int \Omega \, n dt} \qquad (4)$$

The constant of integration c is evaluated by considering the initial concentration of the contaminant in v at time $t = 0$.

If the functional relationships with respect to time for β_K, k, q and therefore n are known, the above equations can be solved for the concentration of contaminant K in the control volume for any time.

AIR MIXING AND STRATIFICATION

A basic assumption in the derivation of the dilution equation is that of complete and instantaneous mixing. In actual practice, however, it is only possible to approach this ideal condition. In order to correct for this assumption, a mixing factor γ may be included as follows:

$$\alpha_K = e^{-\int \Omega\gamma \frac{q}{v} dt} \int \left[\Psi\gamma \frac{q}{v} \beta_K + \theta \frac{k}{v} \right] e^{\int \Omega\gamma \frac{q}{v} dt} dt + c e^{-\int \Omega\gamma \frac{q}{v} dt} \qquad (5)$$

or, in terms of the number of air changes:

$$\alpha_K = e^{-\int \Omega\gamma n \, dt} \int \left[\Psi\gamma n \, \beta_K + \theta \frac{k}{v} \right] e^{\int \Omega\gamma n \, dt} dt + c e^{-\int \Omega\gamma n \, dt} \qquad (6)$$

The mixing factor is a function of the supply-air distribution, the circulation of air in the volume, the distribution of the contaminant sources and the consequent uniformity of the contaminant concentration within the volume, and finally the size of the control volume. Small test chambers that are carefully designed to insure complete mixing can reasonably approach ideal conditions, thus having a mixing factor of 1. In general ventilation, air mixing is necessarily incomplete, resulting in stratification of contaminants or inhomogeneity. Brief[18] suggests that a mixing factor varying from 1/3 to 1/10 be employed for general ventilation systems. Thus, if a particular room has a mixing factor of 1/3 and the room has 9 actual air changes, then the room would have only 3 effective air changes.

The choice of a mixing factor in a particular application is reached through engineering experience and judgment. In order to change this method of choosing γ from a subjective to an objective engineering basis, experimental studies should be conducted on ventilation efficiency. In this regard, a direct correlation can be drawn between the mixing of air in ventilation applications and the mixing of two miscible liquids in continuous-flow mixing vessels. A paper by Cholette and Cloutier[19] discusses several models for various mixing conditions in continuous-flow systems. Differential equations are derived for mixing vessels which take into account an effective volume of mixing, possible short-circuiting, hold-up time of the system, and partial displacement or piston flow. The values of the different factors contained in the integrated equations are determined experimentally by the particular response of a given system to a sudden change in composition of the feed. The equations developed in this paper by Cholette and Cloutier are appliable to ventilation test chambers where the mixing conditions are the same as the mixing tanks, that is, partial mixing, short circuiting, piston flow, and combinations of these. It may also be possible in a ventilation study to determine the effect of contaminant sources distributed throughout the test enclosure by the use of a tracer gas, such as helium, emanating from calibrated sources. Concentrations at various points of the enclosure could then be measured and compared with theory.

SOLUTIONS FOR THE VENTILATION EQUATION

A solution from Eq (5) of specific interest in actual ventilation studies may be determined for the following conditions that:

1. The contaminant concentration in the supply air β_K is constant;
2. The contaminant flow rate of the source or sink k is constant;
3. The flow rate of the supply air q is constant;
4. Therefore, the number of air changes n is constant;
5. At the initial time $t = 0$, the contaminant concentration in the control volume α_K equals a constant α_o.

Eq (5) becomes:

$$\alpha_K = e^{-\Omega\gamma \frac{q}{v} t} \int \left[\Psi\gamma \frac{q}{v} \beta_K + \theta \frac{k}{v} \right] e^{\Omega\gamma \frac{q}{v} t} dt + c e^{-\Omega\gamma \frac{q}{v} t}$$

$$\alpha_K = e^{-\Omega\gamma \frac{q}{v} t} \left[\Psi\gamma \frac{q}{v} \beta_K + \theta \frac{k}{v} \right] \frac{v}{\Omega\gamma q} e^{\Omega\gamma \frac{q}{v} t} + c e^{-\Omega\gamma \frac{q}{v} t}$$

$$\alpha_K = \frac{\Psi}{\Omega} \beta_K + \frac{\theta}{\Omega} \frac{k}{\gamma q} + c e^{-\Omega\gamma \frac{q}{v} t}$$

At $t = 0$ $\quad \alpha_o = \frac{\Psi}{\Omega} \beta_K + \frac{\theta}{\Omega} \frac{k}{\gamma q} + c$

$$\therefore c = \alpha_o - \frac{\Psi}{\Omega}\beta_K - \frac{\theta}{\Omega}\frac{k}{\gamma q}$$

$$\alpha_K = \left[\frac{\Psi}{\Omega}\beta_K + \frac{\theta}{\Omega}\frac{k}{\gamma q}\right]\left[1 - e^{-\Omega\gamma\frac{q}{v}t}\right] + \alpha_o e^{-\Omega\gamma\frac{q}{v}t} \qquad (7)$$

or, in terms of n:

$$\alpha_K = \left[\frac{\Psi}{\Omega}\beta_K + \frac{\theta}{\Omega}\frac{k}{\gamma nv}\right]\left[1 - e^{-\Omega\gamma nt}\right] + \alpha_o e^{-\Omega\gamma nt} \qquad (8)$$

The equilibrium condition in the control volume is reached as $t \to \infty$. Eq (7) becomes:

$$\alpha_K = \frac{\Psi}{\Omega}\beta_K + \frac{\theta}{\Omega}\frac{k}{\gamma q} \qquad (9)$$

This shows that the contaminant concentration at equilibrium is dependent solely on the rate of contaminant generation k and on the ventilation rate q and is not a function of the control volume. If the mixing factor, γ, is equal to 1 and the air temperatures of the supply, the control volume and the source are equal, then the density ratios, Ψ, Ω, and θ, are all equal to unity and Eq (9) becomes equivalent to the dilution equation given in the 1968 ASHRAE GUIDE and DATA BOOK:[20]

$$Q_a = \frac{Q_c \times 10^6}{(MAC) - (SAC)}$$

where $Q_a = q$
$Q_c = k$

MAC = maximum allowable concentration of the contaminant in ppm and α_K of Eq (9)
SAC = supply air concentration of the contaminant in ppm and β_K of Eq (9)

For the case of no ventilation, i.e., n = 0, the expression for α_K is obtained by taking the limit of Eq (6) as $n \to 0$:

$$\lim_{n\to o}\alpha_K = \lim_{n\to o}\left\{e^{-\int\Omega\gamma\, ndt}\int\left[\Psi\gamma n\,\beta_K + \theta\frac{k}{v}\right] e^{\int\Omega\gamma\, ndt} dt + ce^{-\int\Omega\gamma\, ndt}\right\}$$

$$\alpha_K = \int\theta\frac{k}{v}dt + c$$

$$\alpha_K = \theta\frac{k}{v}t + c$$

At t = 0, $\qquad \alpha_o = c$

$$\therefore \alpha_K = \theta\frac{k}{v}t + \alpha_o$$

Thus, Eq (6) reduces to the equation of a straight line.

APPLICATION OF THE DILUTION EQUATION

The following example illustrates the use of the dilution equation.

EXAMPLE

Consider a classroom of volume v to have initially a CO_2 concentration of 0.03% by volume. This room is to be occupied by 50 people. Consider an average person at rest in a seated position to emit CO_2 at a rate of 0.0136 cfm at room temperature. This is equivalent to approximately 18 breaths/min, exhaling each time 500 milli-liters of air containing 4.2% CO_2 by volume. The supply air to the room contains 0.03% CO_2 by volume. Assume the supply air temperature to be the same as the air in the room. Consider the flow rate of the supply air, q, to be constant with respect to time. Assuming a mixing factor of 1, evaluate the system for different values of v and q.

For the above conditions, the general dilution equation reduces to Eq (7) of the previous section:

$$\alpha_{CO_2} = \left[\frac{\Psi}{\Omega}\beta_{CO_2} + \frac{\theta k_{CO_2}}{\Omega\gamma q}\right]\left[1 - e^{-\Omega\gamma\frac{q}{v}t}\right] + \alpha_o e^{-\Omega\gamma\frac{q}{v}t} \qquad (7)$$

Since $\quad T_s = T_v, \quad \Psi = \Omega = \Theta = 1$

$\beta_{CO_2} = 0.0003$

$k_{CO_2} = (0.0136 \text{ cfm/person})(50 \text{ people})$

$\alpha_{CO_2}(0) = 0.0003$

$\gamma = 1$

Eq (7) becomes

$$\alpha_{CO_2} = \left[0.0003 + \frac{0.68}{q}\right]\left[1 - e^{\frac{-qt}{v}}\right] + 0.0003\, e^{\frac{-qt}{v}}$$

The above equation can be readily evaluated for any values of q, v and t. The CO_2 concentrations from 15 min to 24 hours were calculated for volumes ranging from 2500 cu ft to 15,000 cu ft and supply rates varying from 25 cfm to 250 cfm. Fig. 4 shows the situation for a classroom of 10,000 cu ft. After a given period of time, the CO_2 concentration reaches a certain equilibrium value for each supply rate. For the case of no ventilation, i.e., q = 0, the CO_2 concentration is seen to increase linearly. For example, in 3 hours the CO_2 concentration is seen to be 1.27% by volume for no ventilation. The threshold limit value for CO_2 is currently given as 5000 ppm or 0.5% by volume. For this particular classroom, Fig. 4 shows that a supply rate of 150 cfm is necessary to limit the equilibrium CO_2 concentration to less than 0.5%. This is 3 cfm of supply air per person. It is seen from Fig. 4 that the concentration reaches its terminal value of 0.48% in 6 hours for a supply rate of 150 cfm.

Fig. 5 shows the relationship between various room volumes and a constant supply rate of 150 cfm or 3 cfm per person. It is seen that the larger the volume, the longer the time period required to reach the same equilibrium condition. Again it can be seen that the equilibrium concentration level of the contaminant, in this case CO_2, is not a function of the volume of the room, or in particular, the room air volume per person. Ventilation requirements for human occupancy were earlier thought to be a function of the space volume per occupant where the larger the volume, the smaller the ventilation rate needed.[22,23] This is true only during the transient condition preceding the equilibrium condition.

It should be emphasized that the supply rate of 3 cfm/person is concerned only in maintaining the CO_2 concentration in the classroom at a level below the threshold limit value. To maintain the odor level in this classroom at a desired threshold limit, it is usually necessary to deliver more supply air per person than required to satisfy respiratory needs. The acceptable odor level would then become the controlling parameter for determining the ventilation requirements. Tables of ventilation standards which provide generally acceptable environments have been established.[21]

Eq (5) or more specifically Eq (7) is applicable to the ventilation procedures which should be followed in relation to industrial contaminants. Dilution ventilation must always be adequate in amount to keep the concentration of the contaminant below a maximum (safe) allowable concentration.

The authors wish very much to express their appreciation to ASHRAE for providing the original grant which made this study possible. A number of graduate students at Northwestern have worked on the project and contributed to the results reported here. Earlier studies, particularly in relation to the background literature were carried out by John A. Welch. The co-author James A. Armstrong, Jr. made a most significant contribution to the study by refining the ventilation equation into its final form and developing a computer program to produce the data for the graphs.

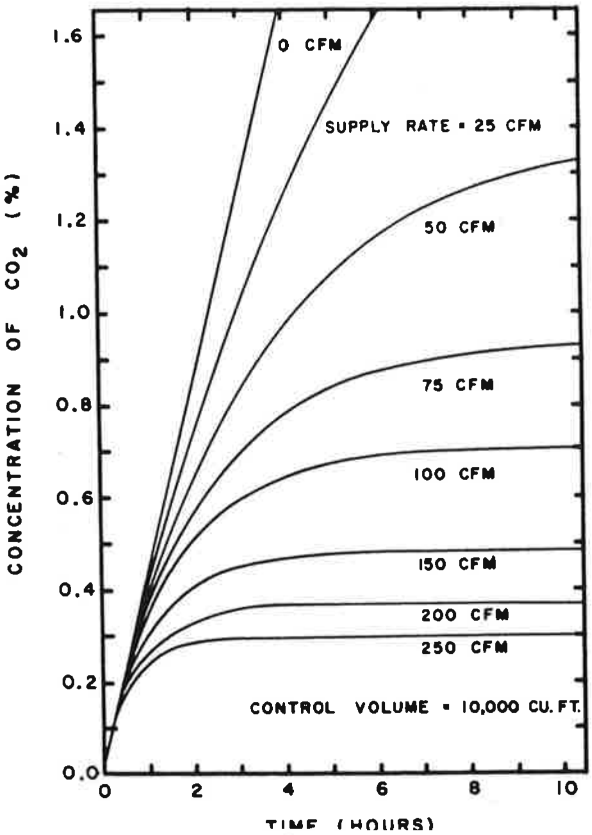


Fig. 4 Concentrations of CO_2 in relation to time for various supply rates in a 10,000 cu ft control volume.

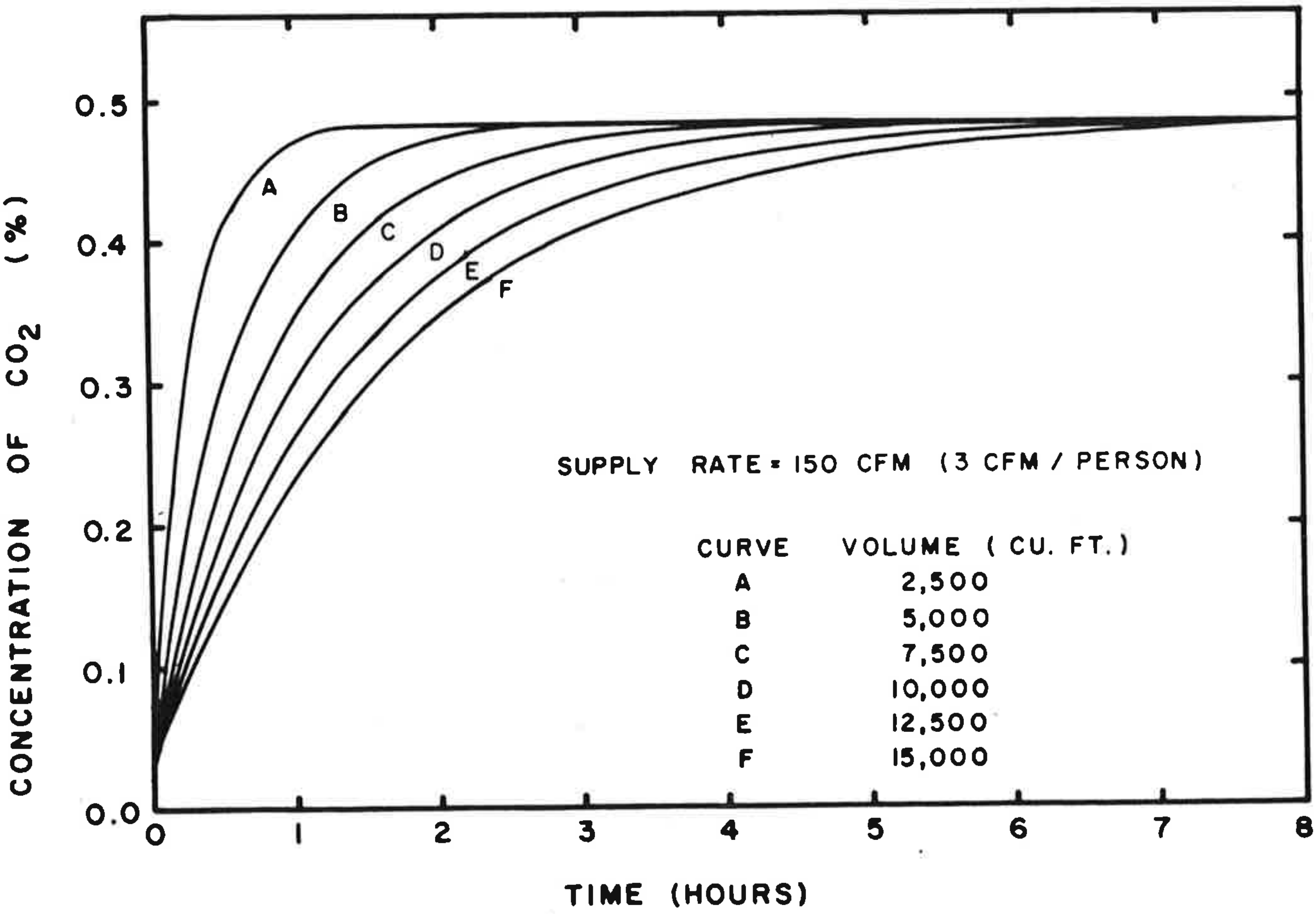


Fig. 5 Concentrations of CO_2 in relation to time for various control volumes supplied with 150 cfm air.

CONCLUSIONS

(1) This paper presents a study in depth of our current knowledge of ventilation as it relates to the air conditioning industry while more importantly it develops a usable equation to provide a rigid mathematical basis on which to analyze dilution aspects of the ventilation processes.

(2) The dilution method of controlling the contaminant level of an enclosure is best applied to diffusible gases and vapors. Where gaseous or vaporous contaminants arising from industrial operations cover large areas of a plant, dilution ventilation is often the only satisfactory method to economically control the concentration level. Care should be used, however, in applying the dilution method to control dusts, fumes, and mists. The reason is that the allowable air motion in an enclosure is usually lower than the capture velocity of the particles resulting in the enclosure becoming a large settling chamber for the particles. Though the particulate concentration in the atmosphere may be always within allowable health and safety limits, the buildup of particulate matter over long periods of time on inaccessible surfaces can constitute a nuisance or hazard.

(3) A detailed study of municipal codes was not made beyond ascertaining that they run an extensive gamut in their manner of application and in range values. Strong inconsistencies develop when a ventilation requirement in terms of occupancy arbitrarily clashes with a ventilation requirement based on floor areas.

(4) The volume associated with individual occupants is an important parameter in relation to ventilation, only in so far as it provides air volume in which to dilute a contaminant and the larger the volume the slower will be responses, in relation to ventilation to appear. The mathematical analysis shows that when sufficient time elapses either a large or a small volume per individual will ultimately reach a fixed level of contaminant concentration.

The value given in the ASHRAE GUIDE AND DATA BOOK[21] for minimum air requirement of 15 cfm per person (non-smoking) is adequate for physiological need but will not eliminate a detectable odor level unless this air flow is supplemented by washed (or otherwise-cleaned) recirculated air. For smoking conditions the value of 40 cfm per person should dilute the odor to levels which are not objectionable in most instances.

(5) Insufficient recognition has been given to the need of making more effective use of recirculated air in ventilation systems in which the air can be washed, deodorized or cleaned. Increasing emphasis will have to be given to air recirculation because the character of the air which exists in many cities appears to be becoming increasingly less satisfactory.

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DISCUSSION

CHAIRMAN P.E. McNALL, JR., (Kansas State University, Manhattan, Kan.): The Society is wrestling with terms, as many of you know, and we shouldn't speak of "fresh air," we should speak of "outdoor air." The Society is currently revising its ventilation standard and we hope to make some of these things more clear in the future so that anyone who reads it and uses it can be more precise.