Summary A large proportion of heated rooms depend primarily on natural convection for the distribution of heat within the occupied zone. A method of predicting air temperature variations with height is presented. Using the heat and mass flow rates of the driving convection plume, along with the corresponding parameters of downward flowing air streams at cool surfaces, an estimation of the maximum (upper) temperature and the minimum (lower) temperature can be made. Points of entry of incoming air can be taken into account. It is shown that there are clear advantages of employing over-sized well controlled radiators.

The prediction of air temperature variations in naturally ventilated rooms with convective heating

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List of Symbols

A	See Equation 12
A_w	area of window
C_p	specific heat capacity of air
Gr_w	Grashof number
H	height of warm/cold convecting surface
H_H	height of radiator
K_1, K_2, K_3	constants
L_c	length of cold wall
L_H	length of radiator
m	exponent (Eq. 10)
$\dot{m}$	mass flow rate in convection boundary layer
$\dot{m}_H$	mass flow rate in plume leaving radiator
$\dot{m}_p$	mass flow rate in plume developing above radiator
n	exponent (Eq. 6)
Pr	Prandtl number
Q	heat transfer rates
Q_c	heat transferred by convection to the ceiling
Q_F	heat transferred by convection to the floor
Q_H	convective heat output of radiator
Q_v	ventilation heat loss
$Q_{v,w}$	ventilation heat loss at the window by infiltration
Q_w	heat transferred by convection to window
Ra	Rayleigh number
T	mean temperature of air in convection boundary layer
$\triangle T$	temperature difference $T_R - T_C$
T_C	temperature of cool vertical surface
T_H	mean temperature of radiator surface
T_L	temperature of air in the core of the room above the neutral plane
T_{Pl}	mean temperature of air in plume leaving radiator
T_R	mean temperature of room air
T_U	temperature of air in the core of the room below the neutral plane
T_W	temperature of window surface
X	distance above the floor
X^1	height of neutral plane
X_o	imaginary origin of radiator (Eq. 9)
θ	See Eq. 17
	(basic SI units throughout, i.e., kg, m, s, J etc)

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1 Introduction

Temperature distributions in rooms have long been known to produce relatively cool conditions at low levels in the occupied zone. The sensitivity of the ankles to low temperatures (Wyon[1], Gonzales[2] and Homma[3]) often results in discomfort due to these inherent variations of temperature, whilst the heating of the unoccupied zone at high levels represents a degree of inefficiency of heat distribution. Indeed, the two effects may conspire to worsen the situation as occupants demand increased input of heat to keep the ankles warm whilst temperatures at high levels exceed the values necessary for comfort. At the same time, temperatures around neck and head level may well be elevated and a feeling of stuffiness becomes increasingly likely, especially in an environment with low levels of air movement. In well sealed, well insulated buildings, the volume flow rates of air are reduced because of the relative warmth of cool convecting surfaces of building fabric and because of the reduced ventilating air flows. The general lack of air movement places more importance on the natural convection around the body for dissipation of metabolic heat so that it remains important to maintain *uniform* temperatures in the occupied rooms as well as maintaining a minimum *mean* air temperature.

This paper describes a method of assessing the magnitude of-room temperature variation with height in rooms heated by convective sources, exemplified by the use of single panel radiators. The method enables reasoned decisions of sizing and siting to be made.

2 Temperatures and flows in enclosures

A wide range of problems is to be found in the field of heat transfer in enclosures, with heat transfer mechanisms ranging from a conduction régime applicable to very narrow tall cavities through a transition régime in which rising and falling boundary layers can be identified at the hot and cold sides of the cavity (but interfere with each other in the centre) to the present problem of very wide cavities in which the boundary layer thickness is small compared with the width of the enclosure. Batchelor[4], in an early appraisal of the situation in wide cavities, was unable to obtain a solution consistent with a core which was assumed to be isothermal. Eckert and Carlson[5] concluded that it was doubtful whether a core of uniform temperature would ever exist unless the height to width ratio was extremely small.

Parameters of importance are cavity width and temperature difference (hot side to cold side, so a convenient method of grouping the terms is to define a Rayleigh Number,

$$\mathrm{Ra} = \mathrm{Gr_w\,Pr} \tag{1}$$

where $\mathrm{Gr_w}$ is Grashof number based on the width of enclosure and temperature difference from one side to the other. The case of rooms with very large width in relation to boundary layer thickness involves $\mathrm{Ra} > 10^{10}$.

Wilkes[6] produced results from a numerical solution of the equations of fluid flow and heat transfer for Rayleigh numbers up to 10^5. His results gave clear indications of a stratified core in the centre of the enclosure.

The evidence, therefore, points to an inevitability of the existence of a region in the core of rooms in which low vertical velocity components exist and with stratified fluid layers at temperatures increasing with height.

Fig. 1 shows the development of the circulation pattern in a full scale room when heated by a single vertical surface as observed by Howarth[8] using a flow visualisation technique. The rising convction boundary layer (A) flows upwards into a 'buffer zone' (B and C) under the ceiling. The air then disperses downwards into the core (D) of the room to be subsequently entrained into the rising boundary layer, so completing its circulation.

Two different scales of the same problem were studied by Baines and Turner[9], who considered the repercussions of a buoyant plume generating recirculatory flows in the atmosphere, and by Mouton and De Roëck[10], who studied the free convection in liquids enclosed in cylindrical vessels with insulated ends.

Baines and Turner consider the effects of continuous convection from small sources of buoyancy on the properties of the environment when the region of interest is bounded. The main assumption, apart from the assumed form of the plume itself, is that the buoyant stream of fluid spreads out at the top of the region and becomes part of the environment at that level. The solutions for the various cases considered (point and line sources, periodically released thermals) reveal a stably stratified environment with the temperature profile changing with time at all levels and everywhere descending. Experimental verification was successfully carried out.

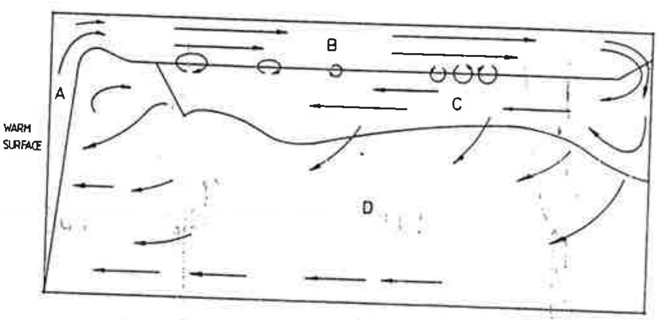

Fig. 1. The circulation of air by convection in a room with a single heated wall.

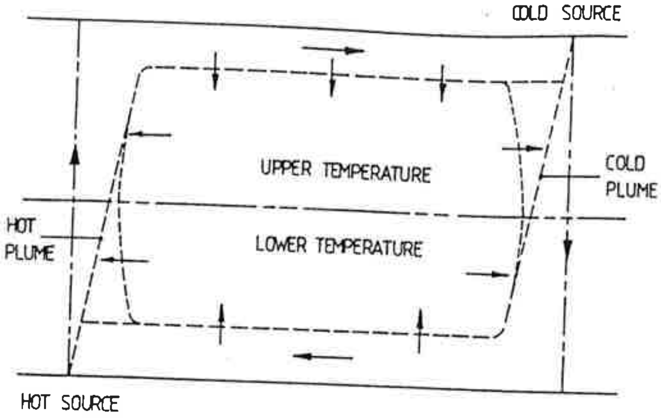

Fig. 2. The circulation of air produced by opposing warm and cool plumes (Baines and Turner).

An application of particular interest which Baines and Turner investigated, is the case of two equal sources, one discharging upwards with positive buoyancy and the other discharging downwards with negative buoyancy. Fig. 2 illustrates the environment produced by the pair of sources. When equilibrium is reached, the rate of change of temperature in the core with time is zero. With equal sources, a line of zero velocity half way up the environment exists so that the entrainment into both plumes in either layer is fed entirely by the plume which ends in the particular layer. Self entrainment by this plume ultimately results in constant temperature throughout each of the two regions. This asymptotic state is analogous to that existing in an enclosure with equally matched opposing convective flows.

Mouton and De Roëck[10] concentrated on the development of temperature profiles in a cylinder of liquid heated through the curved vertical surface. They considered the movements of 'fronts' of fluid at increasing temperature down the core after being fed into a buffer region under the surface by the annular perimeter convection boundary layer.

Mouton and De Roëck assumed Eckert and Jackson's[11] turbulent boundary layer velocity and temperature profiles at the perimeter and then integrated the core velocity with respect to time to give the distance covered by the first front of warm fluid. The nature of the flow which was assumed bears close resemblance to that observed by Howarth described above. Mouton and De Roëck investigated only the unsteady heating up period and the relationship between the position of the first front and the attained mean temperature. No allowance is made for the influence of vertical temperature gradient upon the convective flows at the boundaries.

3 Temperatures and air flows in heated rooms

The method used by Baines and Turner[9] and by Mouton and De Roëck[10] is potentially suitable for investigating the development of temperature profiles in rooms with convective heating by analysing the gradual displacement of strata in the core under the influence of heater plumes and convection boundary layers near the vertical surfaces (Fig. 3). The buoyant driving flows can be represented by empirical formulae in the same way as Baines and Turner represented the plume which formed the subject of their investigation. Similarly convection boundary layers can be represented by empirical equations. Then the formation of strata and their gradual vertical displacements in the core can be predicted by regard to the continuity of flow in horizontal planes (Fig. 4).

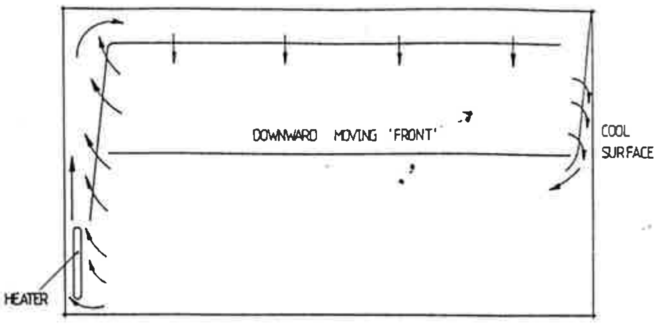

Fig. 3. The heating of room air by convection from a heater.

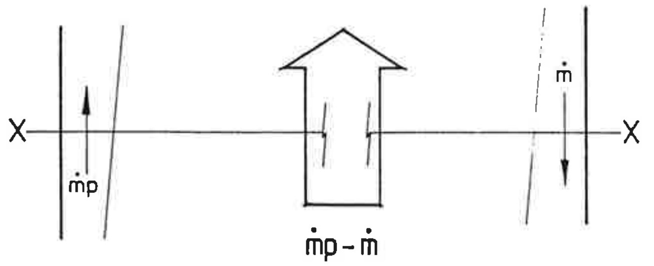

Fig. 4. Mass balance at a horizontal plane in a room with convective heating.

3.1 The generating convective flows

The technique requires information on the convective flows and heat transfer rates in the room. These processes occur at a range of surfaces shown in Fig. 5 classified as follows:

(1) cool elements of building fabric; natural convection downwards

(2) surfaces, affected by the natural convection of the heater, which are subject to forced convection due to the generation of flow elsewhere

(3) the floor where heat is lost downwards from the stably stratified relatively warm room air, moving in a manner dependent on the strength of convective flows from (1) above and on low level infiltration heater surface; natural convection upwards

Although values of convective heat transfer coefficient are widely used in heat loss calculations, their particular values in these widely varying conditions are not well established. The CIBSE Guide[12] quotes a range of values based on traditional heat transfer research. Improved correlations are proposed by Alamdari and Hammond[13] based on more recent and diverse investigations for isothermal surfaces of type 1 in isothermal surroundings. When the ambient temperature varies with height, the velocity and temperature profiles in these convection boundary layers are distorted as shown by Eichhorn[14], Cheesewright[15] and Howarth *et al*[16].

Since the temperature variation with height is dependent on the convective flows, a relationship between the surface flows and the room temperature may exist, as suggested by the recirculatory nature of the confined flows. A limited range of results by Howarth[16] at the wall of a room with a vertical temperature gradient is consistent with the following expressions for air mass flow rate and mean temperature:

$$\dot{m} = 0.0033 \quad \triangle T^{0.25} H^{0.75} L_c \tag{2}$$

$$\text{and } T = 0.5\, T_C + 0.5 T_R \tag{3}$$

It follows from these equations that the heat transferred from the air to the surface is given by

$$Q = 1.66 \triangle T^{1.25} H^{0.75} L_c \tag{4}$$

in which the volumetric specific heat capacity of air is assumed to be 1006 kJ/kg°C.

The flow and entertainment characteristics of the developing heater plume need to to be found by experiment. Surfaces of type 2 (Fig. 6) remove heat from this warm air current before it enters the main body of the room. A general expression for the heat transfer from a radiator plume to a window, based on forced convection correlation, has been developed by Howarth[17] as

$$Q_W = 4\,(Tp_1 - Tw)\,Aw \tag{5}$$

There is scope for further investigation of this aspect of surface heat transfer in relation to window design features as well as the interacting plume characteristics. It should be noted that the heat transfer is based on plume temperature rather than the mean room air temperature, producing higher heat transfer rates which differ from those estimated using CIBSE Guide[12] U-values.

Similar estimations of surface 3 (floor) heat transfer coefficients reveal a value of approximately 1W/m² °C, a value which is consistent with the correlation presented by Alamdari and Hammond[13].

The characteristics of the heater plume itself may be expressed in the form

$$\dot{m}_H = K_1 (T_H - T_R)^n \tag{6}$$

$$\text{and } T_{p1} - T_R = K_2 (T_H - T_R) \tag{7}$$

$$\text{with } Q_H = C_P K_1 K_2 (T_H - T_R)^{n+1} \tag{8}$$

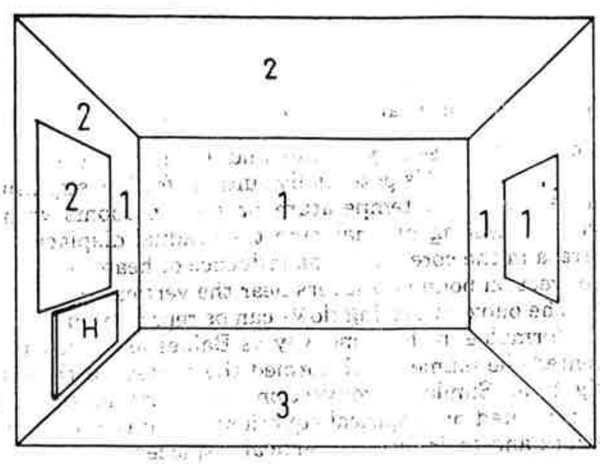

Fig. 5. Identification of heat-transferring surfaces within a room.

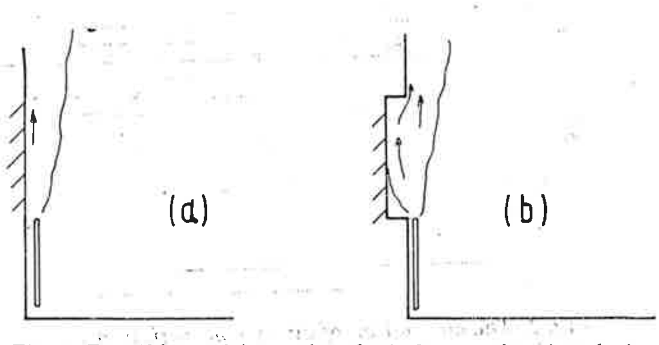

Fig. 6. Possible positions of cool window surface in relation to heater beneath.

As the plume develops, its mass flow rate increases with height in a manner which may be expressed in general terms as:

$$\dot{m}_P = \dot{m}_H \left[\frac{x - x_o}{H_H - x_o}\right]^m \tag{9}$$

The values of these parameters have been found for a single panel radiator to be

$$K_1 = 0.015\ H_H L_H$$

$$K_2 = 0.26;\ K_3 = 3.92 H_H L_H$$

$$n = 0.4$$

$$m = 0.8$$

$$x_o = \tfrac{1}{3} H_H \tag{10}$$

3.2 The estimations of room temperature distribution

The procedure is carried out in three stages:

(1) The radiator temperature required to produce a convective output which offsets the convective losses at a set room temperature is calculated.

(2) The position of the 'neutral plane' at which the upward flow in the radiator plume is balanced by the downward flows in the convective boundary layers is located.

(3) The temperatures above and below the neutral plane are calculated.

3.2.1. The radiator temperature

The convective heat output of the radiator has been established in Eq. 8 above.

Before the air enters the core of the room, heat is lost to the surfaces past which the plume flows. Thus the residual convective heat input to the core of the room is

$$K_3 (T_H - T_R)^{n+1} - Q_W - Q_C$$

When thermal equilibrium exists, this input is balanced by losses at the vertical surfaces, the floor and in the ventilation air, i.e.,

$$\Sigma 1.66 \Delta T^{1.25} H^{0.75} + Q_F + Q_V$$

Hence, the heat balance is given by

$$K_3 (T_H - T_R)^{n+1} - Q_W - Q_C = \Sigma 1.66 \Delta T^{1.25} H^{0.75} + Q_F + Q_V$$

from which $T_H = T_R +$

$$\left\{\frac{\Sigma 1.66 \Delta T^{1.25} H^{0.75} + Q_W + Q_C + Q_F + Q_V}{K_3}\right\}^{\frac{1}{n+1}} \tag{11}$$

3.2.2 The position of the neutral plane

There exists in the core of the room a horizontal surface across which the net vertical volume flow rate is zero. This coincides with the height at which the upward flow in the heating convective plume(s) is equal to the downward flow in the boundary layers adjacent to the cold walls. Thus, from Eqs. 2, 6 and 9 the height (x') of this horizontal surface (or neutral plane) is given by

$$0.0033\ \Delta T^{0.25} H^{0.75} L_C = K_1 (T_H - T_R)^n \left[\frac{X' - x_o}{H_H - x_o}\right]^{0.8}$$

Putting $x_o = 0.33\ H_H$, and using values of constants from Eq. (10)

$$\frac{x' - 0.33 H_H}{0.67 H_H} = \left[\frac{0.215\ (T_R - T_C)^{0.25} L_C}{L_H\ H_H (T_H - T_R)^{0.4}}\right]^{1.25} (H - x')^{0.9375}$$

within 10% accuracy for all practical purposes

$$(H - x')^{.9375} \cong H - x'$$

and putting $A = \left[\dfrac{0.215\ (T_R - T_C)^{0.25} L_C}{L_H\ H_H\ (T_H - T_R)^{0.4}}\right]^{1.25}$ (12)

$$\text{then } x' = \frac{2H\ H_H A + H_H}{2 H_H A + 3} \tag{13}$$

3.2.3 The upper and lower temperatures

Fig. 7 shows the flow directions in the plume and in the convection boundary layer. The neutral plane may be expected to occur near to the top of the radiator, so the radiator plume is discharged almost immediately into the upper region which theoretically tends towards a uniform temperature. Thus the upper region of the room is in equilibrium at the departing temperature of the radiator plume, modified by convective losses to adjacent surfaces such as window and ceiling. Mixing of ventilation air entering through crackage adjacent to the radiator plume also reduces the temperature of the upper part (by an amount related to the ventilation heat loss of the room). The temperature in the upper region of the room is now given by

$$T_U = 0.26\ T_H + 0.74\ T_L - \frac{Q_W + Q_C + Q_{V,W}}{\dot{m}_P C_P} \tag{14}$$

The temperature in the lower region is then estimated by regard to the energy content of the room air, based on the original assumed mean temperature, T_R hence

$$x' T_L + (H - x') T_U = H\ T_R \tag{15}$$

Simultaneous solution of Eqs. (13) and (14) gives

$$T_U = \frac{(H\ T_R - x' T_L)}{(H - x')} \tag{16}$$

$$\text{and, with } \theta = \frac{Q_W + Q_C + Q_{V,W}}{\dot{m}_P C_P} \tag{17}$$

$$T_L = \frac{H T_R - (0.26 T_H - \theta)(H - x')}{0.74\ (H - x') + x'} \tag{18}$$

Using Eqs. (15) and (16), the temperatures in the upper and lower regions of the room can be assessed. The form of the vertical temperature profile implied by this analysis is

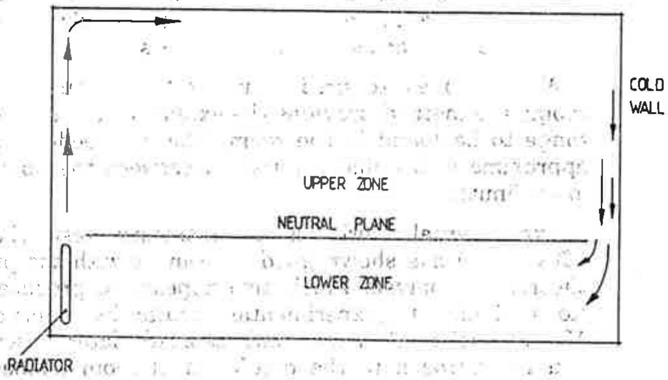

Fig. 7: Two-zone model of room air in thermal equilibrium with convecting surfaces.

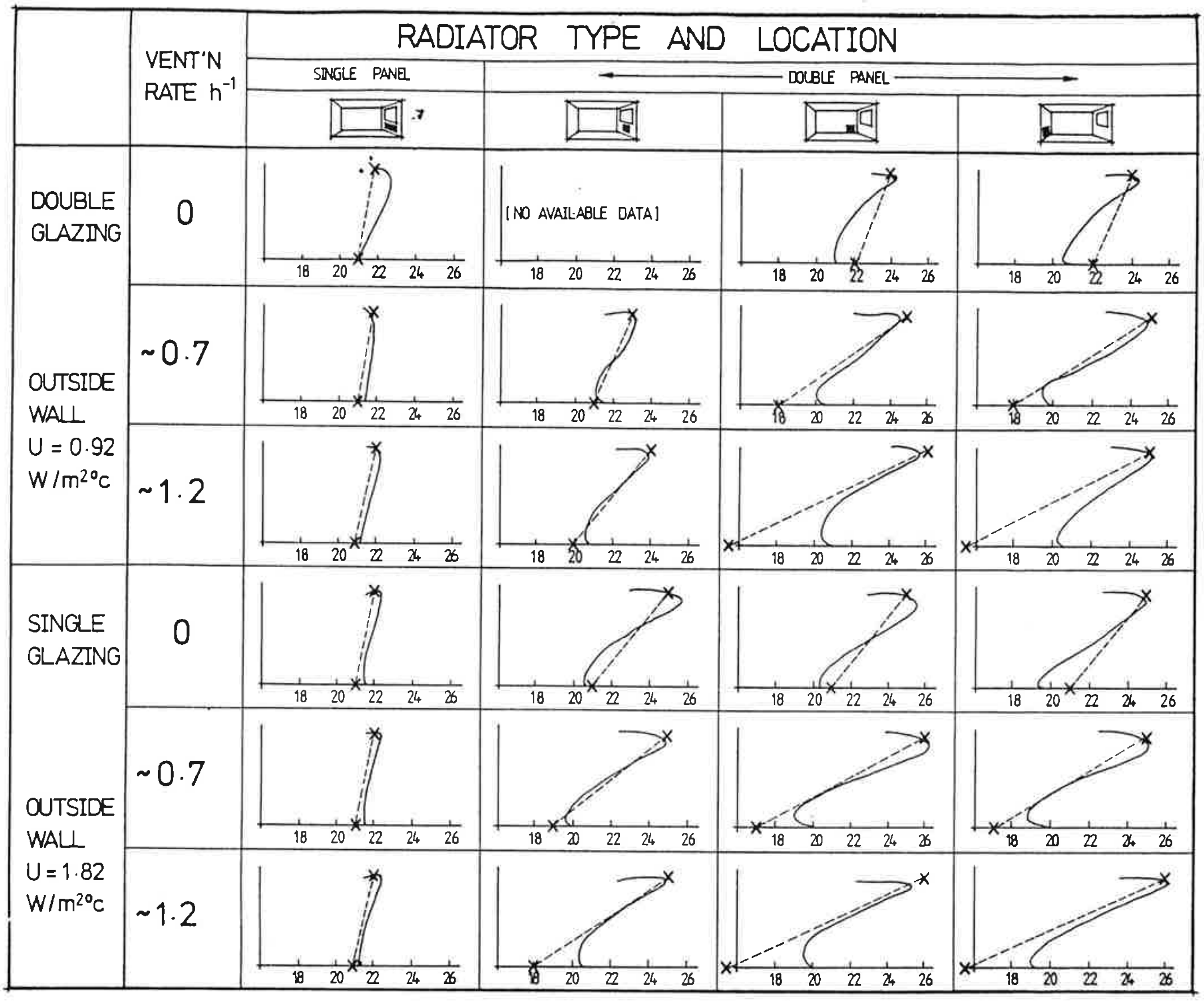

Fig. 8. Comparisons with experimental measurements (Lebrun and Marret).

Z-shaped with a sharp discontinuity at the neutral plane and uniform temperatures above and below. In practice, the discontinuity is smoothed out by internal turbulent momentum transfer probably initiated by disturbances at the edges of the boundary layers. The discontinuity would be more noticeable in the analogous heated water tank since water has a much larger Prandtl number implying a greater degree of viscous damping of convective flows arising from small local temperature variations. Furthermore, the ventilation flows are assumed, in the above theory, to mix thoroughly with either the air in the heater plume or with the air below the neutral plane without creating significant local air movements.

When applied to predictions of temperatures in real rooms the method provides the extent of the temperature range to be found in the room. The intermediate values approximate to a linear variation between the upper and lower limits.

Experimental work in a laboratory test chamber (Howarth[17]) has shown good agreement with the present theoretical analysis which also appears to produce solutions similar to experimental results by Lebrun and Marret[18] (Fig. 8). Laret and Lebrun[19] later describe a similar approach to the prediction of room temperature variations for use in building thermal performance modelling.

4 Specimen calculations

A room 5 m × 3.5 m × 2.7 m high with a 1.1 m × 2.2 m window in an end wall is considered with heating by either a 2.2 m radiator or a 1.1 m radiator either under the window or remote from the window. One of the 5 m × 2.7 m walls is also external. The U-values are as follows: roof 0.3W/m² °C, walls 0.6W/m² °C, floor 0.75W/m² °C, window 2.8W/m² °C. Three ventilation rates of zero, 0.5 and 1.0 air changes per hour are considered with fresh air entering through crackage uniformily distributed over the window area. When part (or all) of the window is not directly affected by the radiator plume, the infiltration of air is assumed to enter the lower levels of the room without mixing en route with air at higher levels. Thus in some circumstances there is an exaggerated temperature gradient.

Fig. 9 shows some results of the analysis of the room air temperature variations with height. It can be seen that positioning of the largest radiator under the window provides the most uniform temperature distribution whilst the positioning of the radiator elsewhere provides the least favourable temperature gradient. Comparatively large radiators are to be preferred because they initiate movements of larger volumes of air at more modest temperatures.

All the results are based on a *mean* room air temperature of 23 °C with an outside temperature of −1 °C. The tem-

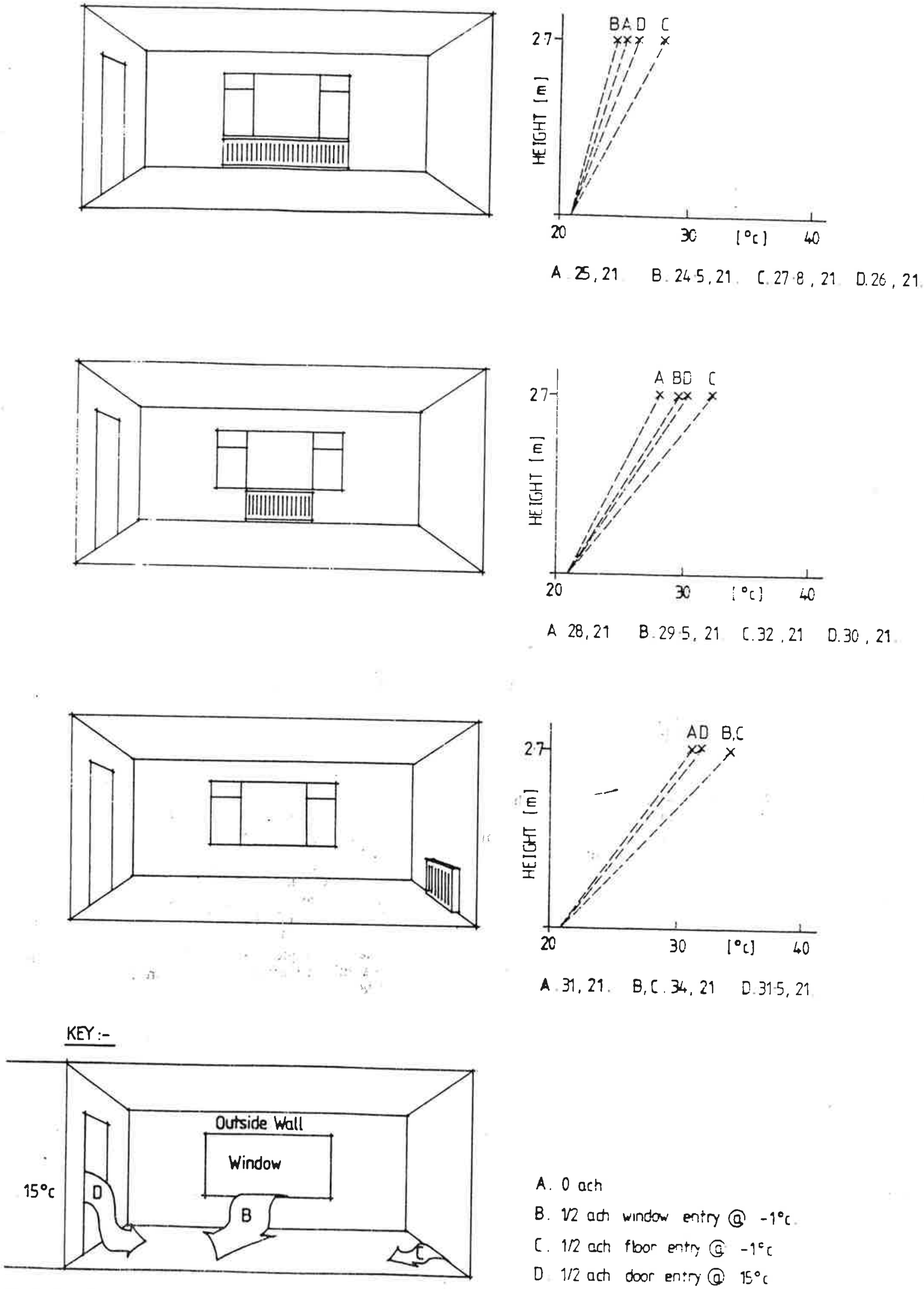

Fig 9. Specimen calculations of room temperature distribution.

perature of the air in the lower region of the room varied between 19.2 °C (large radiator, under window, zero air changes per hour) and 10.3 °C (small radiator, not under window, one air change per hour).

5 Applications

Skaret and Mathisen[20] approach the problem of contaminant disposal in a similar manner and illustrate the principle by use of a low level air supply to an office in which the contaminants are essentially displaced upwards in the convection currents around the sources of production whilst the ventilation air moves in beneath. Supply air change rates of 1 to 3 air changes per hour at temperatures close to room temperature were employed.

The current work, conducted simultaneously and independently, concentrates on the problem of maintaining uniform vertical temperature distributions in heated spaces with natural ventilation. Here, the incoming air possesss the *undesirable* quality of low temperature and therefore *requires* mixing before coming in contact with the occupants. In modern domestic accommodation the ventilation rate tends to be lower than 1 air change per hour and represents a fraction of the total room heat requirement. The results encourage the continued practice of installing radiators beneath windows, even double-glazed windows, especially if the ventilation occurs in that region. It is also clear that oversized radiators are of advantage because of the movement of larger air volumes at lower temperature. Thus, if properly controlled, radiators which run the full length of windows are ideal.

With regard to ventilation, although the removal of contaminants is most effectively achieved by an upward displacement flow of air, this does not provide a satisfactory air temperature distribution in the naturally ventilated room. In any event there is little control over the direction of air flow through the room. Thus, crackage at floor level encourages a depression of temperature in the lower region whilst convective heating may achieve a 'satisfactory' mean temperature only by overheating the upper zone. Suspended floors therefore need to be well sealed.

Other applications of the technique may include assessments of temperature gradient in larger spaces, strategy for selection and location of de-stratifying fans and the design and siting of ventilation openings.

References

1 Wyon, D. P., *et al*, Thermal comfort during surgical operation, JIHVE (October 1969).

2 Gonzalez, R. R., and Nishi, Y., Effect of cooling environments on local thermal sensation, discomfort and clothing selection, ASHRAE Trans, No. 2384, 1976.

3 Homma, H., The effects of free convection around a human body on indoor environment, CIB W67 Symposium on Energy Conservation in the Built Environment, Copenhagen (1979).

4 Batchelor, G. K., Heat transfer by free convection across a closed cavity between vertical boundaries at different temperatures, Quart. Appl. Math. V 12 (1954).

5 Eckert, E. R. G., and Carlson, W. O., Natural convection in air layer enclosed between two vertical plates with different temperatures, Int. J. Heat Mass Transfer, **2**, 106 (1961).

6 Wilkes, J. O., Finite difference computation of natural convection in an enclosed rectangular cavity, PhD Thesis, University of Michigan (1963).

7 Elder, J. W., Laminar free convection in a vertical slot, J. Fluid Mech., **23**, 1 (1965).

8 Howarth, A. T., An investigation of factors affecting air movement in mechanically ventilated rooms, MPhil. Thesis, University of Nottingham (1970).

9 Baines, W. D., and Turner, J. S., Turbulent buoyant convection from a source in a confined region, J. Fluid Mech., **37**, 1 (1969).

10 Mouton, H., and De Roëck, M., Convection naturelle au sein d'un liquide, Int. J. Heat Mass Transfer, **22** (1977).

11 Eckert, E. R. G., and Jackson, T. W., Analysis of turbulent free convection boundary layer at flat plate, NACA Rep. 1015 (1951).

12 CIBSE Guide, Thermal properties of building structures, Section A3 (1980).

13 Alamdari, F., and Hammond, G. P., Improved data correlations for buoyancy-driven convection in rooms, BSER&T, 4, 3 (1983).

14 Eichhorn, R. I., Natural convection in a thermally stratified fluid, Progress in Heat and Mass Transfer, 2 (1969).

15 Cheesewright, R., Natural convection from a plane vertical surface in non-isothermal surroundings, Int. J. Heat Mass Transfer, **10** (1967).

16 Howarth, A. T., Morton, A. S., Sherratt, A. F. C., Air movement in an enclosure with a single heated wall, JIHVE, **40** (1972).

17 Howarth, A. T., Temperature distributions and air movements in rooms with convective heat sources, PhD. Thesis, University of Manchester (1983).

18 Lebrun, J., and Marret, D., Convection exchanges inside a dwelling room in winter, International Seminar of ICHMT, Dubrovnik (September 1976).

19 Laret, L., and Lebrun, J., Recherche d'un meilleur modele convectif pour la simulation thermique des bâtiments, l'Institut International du Froid, Paris (August 1983).

20 Skaret, E., and Mathisen, H. M., Ventilation efficiency —a guide to efficient ventilation, ASHRAE Trans. (2B, 1983).