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Determination of the Effective Leakage Areas of Houses by Multilinear Regression Analysis of the Energy Consumption Data

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ABSTRACT

The steady-state heat loss of a house can be expressed as the sum of the above-grade conduction loss, the below-grade conduction loss, and the infiltration loss, minus the solar gain. Each of these terms is the product of a weather related variable and a coefficient that describes a physical characteristic of the house.

If the infiltration driving force is properly defined, the infiltration coefficient is the equivalent leakage area. Thus a multilinear regression analysis of the total energy consumption of the house (including internal gains) against the four weather parameters will yield values of the four coefficients, including the equivalent leakage area.

This technique has been applied to two houses. The equivalent leakage areas determined correlate well enough with those measured by a blower door, indicating that the method has promise.

INTRODUCTION

There is a great potential for saving energy at reasonable cost by sealing air leaks in existing houses. There is also the potential for oversealing houses and lowering air quality below acceptable levels or creating problems with excessive humidity. These latter problems can be solved by mechanical ventilation, but few householders would be pleased to pay for ventilating equipment to increase airflow when they have just finished paying for sealing air leaks to reduce it.

An air-sealing contractor who uses a blower door to test before and after sealing can avoid this problem. The results of his preliminary airtightness test will allow him to estimate whether or not a house is leaky enough to be sealed without creating air quality problems and how extensive the sealing should be. However, there is a practical problem with this approach. By the time he has the results of the preliminary airtightness test, a contractor has invested time and money to make a sale and to carry out the test. Those who walk away from the houses that are tight enough already will lose money and, as a result, will have to charge more for sealing other houses. Others may be tempted to proceed anyway and seal houses that are already tight to the point where potential air quality problems exist.

In many energy audit programs, methods have been developed for estimating airtightness of houses. Some of these are based on the ASHRAE air change and crack length methods (ASHRAE, 1981). However, these methods were originally intended to estimate heating loads due to infiltration, for design purposes. They do not account for the wide differences in infiltration due to hidden differences in apparently identical houses. Other energy audit projects rely on moisture as a tracer gas to determine infiltration. However, wide differences in moisture production rates make this method less than reliable.

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If a cheap method could be developed to provide an estimate of airtightness of houses, it would be a boon to those administering energy audit programs and to air-sealing contractors. It would also allow the identification of unusually tight houses that may be a health hazard to their occupants. One possibility is multilinear regression analysis of fuel consumption against weather parameters, including an infiltration driving force. The coefficient determined from the regression analysis for the infiltration term will be the equivalent leakage area (ELA) of the house.

If this technique were successful it would allow utilities to analyze customers' bills to determine ELAs. They could then notify these customers of the airtightness of their houses and of the desirability of air sealing or ventilation. The cost of carrying out this analysis would be minimal for utilities with computerized billing systems.

To investigate the feasibility of using this technique, fuel consumption data from two houses in northern Canada were analyzed. These houses were chosen because the fuel data were available and because their ELAs had been measured with a blower door. The following chapters describe the analysis that was carried out on these houses and the results obtained.

THE STUDY

Description of Houses

The two houses analyzed in this study were two storey buildings without basements and with open crawl spaces. House No. 1 was built in 1978 and House No. 2 was built in 1980. The two houses had oil fired furnaces and domestic water heaters.

House 1 had 175 m^2 of floor area. The walls and floor were insulated to RSI = 4.7 m^2°C/W and the ceiling to RSI = 5.8 m^2°C/W. All windows were double glazed. House 2 had 183 m^2 of floor area. The walls were insulated to RSI = 7.1 m^2°C/W, the floor to RSI = 8.1 m^2°C/W, and the roof to RSI = 8.6 m^2°C/W. All windows were triple glazed. House 2 had an air-to-air heat exchanger installed in the ventilating system and a solar domestic water heater. Both houses were fitted with electricity, fuel, and water meters.

Both houses were relatively airtight. The ELA of House 1 was measured to be .0290 m^2 using the Canadian General Standards Board method (CGSB, 1985). That of House 2 was .0330 m^2. Both houses had airlock entrances with vestibules large enough that one door would be closed before the other was opened.

The Regression Model

If transient effects are neglected, the net heat requirements of a house during a time period τ can be calculated as:

$$Q_L = (\Sigma U_j A_j)\ \Delta T_a \tau + (\ U_k A_k)\ \Delta T_g \tau + L \rho\ C_p\ \Delta T_a \tau \sqrt{P_s\ \Delta T_a + P_w V^2} - f_i Q_i - F_w f_s \sum A_{wi} S_{wi} \qquad (1)$$

where:

- U_j = the conductance of the jth above ground surface of the house envelope (W/m^2 °C)
- A_j = the area of the jth above ground surface of the house envelope (m^2)
- T_a = temperature difference, interior to ambient (°C)
- A_k = the area of the kth below ground surface (m^2)

ΔT_g = temperature difference, interior to ground (°C)

L = effective leakage area (m^2)

ρ = density of air (kg/m^3)

C_p = specific heat of air (kJ/kg *°C)

P_s = stack parameter (a coefficient for buoyancy induced infiltration) ($m^2 \times s^2/°C$)

P_w = wind parameter (a coefficient for wind induced infiltration)

V = wind velocity (m/s)

f_i = interior gain utilization factor

Q_i = internal heat gains during the time period (kJ)

F_w = fraction of solar radiation penetrating window

f_s = solar utilization factor

A_{wi} = area of windows facing in the ith direction (m^2)

S_{wi} = solar flux per unit area incident on the windows facing the ith direction during the time period (W/m^2)

In the present case, the below ground term does not apply. If f_i is estimated to be 1.0, the internal gains term can be added to Q_L to leave three terms with unknown variables in them.

It would be possible to determine P_s and P_w by regression analysis, but to reduce the number of unknown coefficients and to keep the equation linear, a simpler approach has been taken. Typical values of P_s = .015 m_2/s_2 x°C and P_w = .015 have been used so that the equation is linear in three weather related terms.

The three coefficients are:

$$C_1 = \Sigma U_j A_j \tag{2}$$

$$C_2 = L = X(ELA) \tag{3}$$

<u>where</u>:

ELA = the equivalent leakage area of the house (m^2), as defined by CGSB Method (ASHRAE, 1981).

X = the ratio of effective leakage area to equivalent leakage area = 0.53

$$C_3 = F_w f_s \tag{4}$$

These three coefficients may be determined by multilinear regression analysis of the energy consumption data against three weather parameters. These parameters are:

x_1 = the degree-hours below the indoor temperature during the week

$$x_1 = \Sigma \Delta T_a \tag{5}$$

where:

Σ = the sum over all the hours in the week

x_2 = the infiltration heat loss driving force

$$x_2 = \sum(\rho C_p \Delta T_a \sqrt{P_s \Delta T_a + P_w V^2}) \qquad (6)$$

x_3 = the radiation incident on the windows of the house during the week

$$x_3 = \Sigma A_{wi} S_{wi} \qquad (7)$$

where:

Σ = the sum over

A_{wi} = the window area on the ith wall (m^2)

S_{wi} = the solar radiation falling on a unit area of vertical surface facing in the ith direction during the week (W/m^2)

Statistical Analysis of Meter Readings

Both houses were equipped with meters on the oil lines to their furnaces and hot water tanks as well as on their electricity supplies. These meters were read weekly for a full year. Also available were hourly values of the required weather parameters (temperature, wind speed, and solar radiation) for the periods of interest.

For house 2, 18 weeks of data in 1983 were chosen for statistical analysis. These weeks were selected from a set of 28 weeks with no apparent errors in the meter readings. (The other 10 weeks were saved for an independent check of the results.) The statistical analysis yielded the results shown in Table 1.

The standard deviation of the predicted results from the measured results was 528.4 W or 15.2% and the mean deviation was 47 W or 1.35%. This latter number would be expected to be small, since the correlation being tested is based on the numbers against which it is being tested. The size of the standard deviation might be due to faults in the model, to random variations not accounted for in the model, or to errors in the data.

There are faults in the model. One is that the seasonal variation in furnace efficiency is ignored. Also, the effect of solar radiation on opaque walls is ignored, the sensitivity of infiltration to wind direction is ignored, the variation of F_w with the sun's altitude angle is ignored, the variation of f_s with solar load ratio is ignored, and the gain/losses from the domestic water system are ignored. However, these terms are all of lesser magnitude than the three main terms that are considered.

There are also bound to be random variations not accounted for in the model, due to short-term lifestyle changes. Finally, there are errors in the data. The most common is the misreading of a single digit in a meter reading. Many of these were detected where the error was in a leading number that should not have changed or in the first significant figure. Errors of lower magnitude could not be detected or corrected and, undoubtedly, some of them have contributed to the scatter of this data.

The value of the overall heat loss coefficient of the house determined from this analysis was:

$$UA = 57.4\ W/°C$$

This comparison with 55.08 W/°C predicted from the nominal RSI values of the various wall surfaces and their areas. This 4% deviation is considerably less than could be expected from this kind of analysis, and must be regarded as fortuitous.

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The equivalent leakage area determined from this analysis was .0388 m^2. This compares with .033 m^2 measured with a blower door. The deviation is 17.5% of the value measured with the blower door. This agreement between the results of two very different methods is close enough to indicate that they are both reasonably accurate.

The product of the shading coefficient and the solar utilization factor determined by this method was $C_3 = F_w f_s = 1.33$. This value is clearly impossible, since neither of these numbers can be greater than 1.0. An expected value for a house with triple glazed windows would be between 0.5 and 0.7. This result may be due to one of several factors. The first is the effect of the solar hot water heating system. No reasonable estimate could be made of the amount of energy from the solar hot water heating system that ultimately ended up in the house, so this factor was ignored. If there was an appreciable loss of heat from the solar energy system to the house, it could account for this apparent extra solar gain. A second factor is that the solar gain by the opaque walls of the house is not accounted for in our simple model. This will contribute to an increase in the value of $F_w f_s$.

The third factor causing errors in any of the coefficients determined by multilinear regression analysis is correlation between the independent variables. For example, if the weekly solar radiation were perfectly correlated with the weekly average temperature, it would be impossible to determine the heat loss and radiation gain coefficients. There is obviously a relationship between solar radiation and air temperature, although not a perfect correlation, and this allows variations in the two coefficients from their true values.

In spite of the deviation of the solar gains coefficient from the expected value, the overall agreement between the coefficients determined from the energy consumption data and those determined from the house description and blower door measurements is very good.

The final test of the coefficients obtained by the analysis of the first 18 weeks of energy use data was to use them to predict the remaining ten weeks of data. Table 2 presents the results.

The standard error of the estimates is 792 W and the mean error is 265 W, or 8% of the mean value.

A similar study was carried out for House 1 using 34 weeks of data to determine the coefficients in the house model, and 19 weeks of independent data to test the coefficients. The statistical analysis yielded the results shown in Table 3.

The standard deviation of the predicted energy consumptions from the measured energy consumptions was 349 W, or 8.7% and the mean deviation was 23 W, or 0.6%. As in the analysis of House 2, this variance is undoubtedly caused in part by the approximations made in developing a simplified model, and in part by errors in the recorded data.

The value of the overall heat loss coefficient determined by regression analysis was 93.5 W/°C, compared to a value calculated from the house description of 111.5 W/°C. It is unlikely that the actual heat loss will be less than the computed value, so the difference between these two numbers must be due to the uncertainty in the coefficient determined by regression analysis.

The equivalent leakage area determined by the regression analysis was .0103 m^2, compared to .0290 m^2 determined by using a blower door, and the value of the product $F_w f_s$ determined by regression analysis was 0.19, compared to an expected value of around 0.6. These values are too far from the expected values to confirm them in any way. It is likely that this result is due to the fact that the independent variables are not truly independent, but are correlated with one another.

Although the coefficients determined by the regression analysis are not individually accurate, the prediction based on them can still be accurate if the errors compensate. This will occur if the period for which the coefficients are used for prediction shows the same correlation between the weather parameters as the period used to determine the coefficients.

The chance of this correlation between weather parameters is best if a long period of time is used in each case and if the two periods of time cover the same part of the year.

The independent test of the house model did not meet this later criterion. The coefficients were fitted using data for the winter period from September 18, 1982, through May 15, 1983. They were tested against an independent set of data for the summer period from May 15 to September 24, 1983. The prediction of summer heating loads is the most challenging task for a house thermal model, because the solar heat gain is of the same magnitude as the conduction and convection losses. In the present case the predictive program performs well in spite of this difficulty. Table 4 shows the comparison of predicted and measured energy consumptions.

The standard deviation of the predictions from the measured values is 285 W, and the mean deviation is only 17 W or 1.8% of the mean value. This comparison indicates that the house model is a good one, even though the individual values of the wind and solar coefficients are not accurate. It should be noted that the large percentage errors in weeks 54, 57, and 60 are not particularly significant, since they are errors in particularly small energy consumption.

DISCUSSION

Because of the limited budget for the present study, it was not possible to carry out an experiment to test the proposed method of determining ELA. It was necessary to make use of data collected by others at a remote location the author has not visited. If the results had been conclusively positive this would not have been a problem. It could have been considered unlikely there were errors in the metered data and in the house descriptions, which would result in ELAs that agreed with the values measured with the blower door results in two cases.

In the present situation, this uncertainty about the metered data and the experimental setup creates a problem, because the results are not conclusive. The results of the analysis of House 2 were good. The ELA determined by regression analysis was 17.5% higher than that determined by the blower door. This deviation is in the direction that would be expected, since the former measurement will include the averaged effect of door openings. However, the results of the analysis of House 1 were not good enough to indicate that regression analysis will be useful in determining ELA. The value determined in this way was only 35% of that determined by a blower door test.

It is possible that this result is an indication that regression analysis cannot be relied on to determine ELA. Another possibility is that the present model is not good enough. It may be that House 1 has an unusual spatial distribution of leaks, and that the typical values of P_s and P_w used are incorrect. It may be necessary to develop a new model with P_s and P_w determined by regression analysis instead of ELA. A third possibility is that there are problems with the data, the house description, or the ELA test. Because of the second-hand nature of the data, the probability that this may be the case is hard to assess.

CONCLUSIONS

Although the results of this limited test are not conclusive, good agreement between the two methods of determining Equivalent Leakage Area was obtained in one of the two cases studied. This is enough to encourage further work. We are presently carrying out a much larger and better controlled study of the multilinear regression analysis technique. It is hoped that other researchers will be encouraged by the results reported here to carry out their own investigations of the possibility that multilinear regression analysis of fuel bills against weather data can be an economical method of obtaining information about house airtightness.

REFERENCES

1. ASHRAE Fundamentals Handbook, 1981, Chapter 22.

2. "Determination of the Airtightness of Buildings by the Fan Depressurization Method," CGSB Standard CAN2-149.10-M84, 9th Draft, January 1985.

ACKNOWLEDGMENTS

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TABLE 1

Statistical Analysis of House 2

TEST NO.	WEEK ENDING ON	TEMP. DIFFER. [C]	INFILTRATION DRIVING FORCE [W/m²]	SOLAR RADIATION [WATTS]	MEASURED ENERGY INPUT [WATTS]	PREDICTED ENERGY INPUT [WATTS]	PERCENTAGE DEVIATION
1	1983 1 8	54.7	95635.0	156.7	5470.8	5197.5	-5.0
2	1983 1 15	55.2	109397.0	178.1	5737.0	5524.9	-3.7
3	1983 1 22	58.4	110317.0	289.2	5929.9	5583.2	-5.8
4	1983 1 29	54.2	115571.0	357.4	5752.8	5377.2	-6.5
5	1983 2 5	52.7	82155.0	368.4	5011.7	4480.7	-10.6
6	1983 2 12	53.9	120206.0	583.9	5527.2	5169.4	-6.5
7	1983 2 26	52.1	97147.0	887.3	4366.5	4115.2	-5.8
8	1983 3 12	43.1	85357.0	1153.2	2514.3	2961.3	17.8
9	1983 3 26	50.9	109054.0	1861.3	2755.7	3025.7	9.8
10	1983 4 9	40.4	63433.0	2137.6	1402.1	975.1	-30.5
11	1983 10 8	23.7	31059.0	889.8	1063.7	908.6	-14.6
12	1983 10 15	24.1	38568.0	692.4	1273.5	1373.8	7.9
13	1983 10 22	26.9	49839.0	523.4	1616.4	2027.4	25.4
14	1983 10 30	28.3	49800.0	595.8	1616.4	2012.9	24.5
15	1983 11 12	28.6	43989.0	324.5	1659.3	2252.8	35.8
16	1983 11 26	42.4	101595.0	494.0	4210.8	4184.4	-0.6
17	1983 12 10	35.6	64555.0	164.6	3158.4	3352.5	6.1
18	1983 12 31	50.1	92896.0	178.7	3481.0	4840.5	39.1

TABLE 2

Comparison of Model Predictions with an Independent Set of Data for House 2

TEST NO.	WEEK ENDING ON	TEMP. DIFFER. [C]	INFILTRATION DRIVING FORCE [W/m²]	SOLAR RADIATION [WATTS]	MEASURED ENERGY INPUT [WATTS]	PREDICTED ENERGY INPUT [WATTS]	PERCENTAGE DEVIATION
7	1983 2 19	54.8	87475.0	684.5	4786.1	4308.0	-10.0
9	1983 3 5	49.8	100003.0	970.9	3109.9	3934.0	26.5
11	1983 3 19	48.9	101449.0	1517.4	3061.4	3190.7	4.2
13	1983 4 2	45.2	73852.0	2249.8	1563.4	1349.3	-13.7
15	1983 4 17	43.0	79585.0	2538.7	2242.5	970.8	-56.7
44	1983 11 5	34.8	68274.0	474.7	2162.4	2986.5	38.1
46	1983 11 20	35.7	67815.0	373.7	2058.6	3157.1	53.4
48	1983 12 3	40.7	73653.0	268.3	3384.0	3728.7	10.2
50	1983 12 17	46.9	87678.0	163.5	4060.8	4555.4	12.2
51	1983 12 24	53.6	105429.0	164.1	4463.5	5360.4	20.1
					30892.6	33549.9	8.6

TABLE 3

Statistical Analysis of House 1

TEST NO.	WEEK ENDING ON	TEMP. DIFFER. [°C]	INFILTRATION DRIVING FORCE [W/m²]	SOLAR RADIATION [WATTS]	MEASURED ENERGY INPUT [WATTS]	PREDICTED ENERGY INPUT [WATTS]	PERCENTAGE DEVIATION
1	1982 9 25	23.5	43856.0	1806.4	3125.7	2137.3	-31.6
2	1982 10 3	22.1	43313.0	1495.1	1847.7	2056.9	11.3
3	1982 10 9	21.1	39761.0	1516.4	1880.4	1938.2	3.1
4	1982 10 16	20.0	30234.0	942.7	1530.7	1883.2	23.0
5	1982 10 23	28.8	50705.0	1093.7	2594.4	2804.1	8.1
6	1982 10 30	34.9	79990.0	1105.0	3835.2	3560.6	-7.2
7	1982 11 6	26.6	41738.0	690.4	2900.1	2625.1	-9.5
8	1982 11 13	34.0	66030.0	689.4	3835.2	3462.9	-9.7
9	1982 11 20	36.8	66251.0	488.2	4479.3	3767.2	-15.9
10	1982 11 27	35.2	83950.0	326.6	4576.3	3765.3	-17.7
11	1982 12 4	40.0	74101.0	341.2	4415.0	4140.3	-6.2
12	1982 12 11	47.8	140976.0	237.1	4883.1	5318.6	8.9
13	1982 12 18	52.6	86746.0	327.1	4995.9	5406.2	8.2
14	1982 12 24	44.1	88266.0	221.3	4737.6	4638.7	-2.1
15	1982 12 31	55.1	102152.0	237.0	5752.8	5757.3	0.1
16	1983 1 8	55.6	97944.0	239.1	5654.7	5773.5	2.1
17	1983 1 15	55.2	109540.0	303.5	5962.6	5798.6	-2.8
18	1983 1 22	57.4	107854.0	472.6	5817.1	5963.4	2.5
19	1983 1 29	54.2	115414.0	578.5	5414.4	5684.6	5.0
20	1983 2 5	51.7	80025.0	629.3	5011.7	5222.6	4.2
21	1983 2 12	54.8	122857.0	966.3	5398.6	5721.8	6.0
22	1983 2 19	53.9	85401.0	1165.4	5494.5	5361.1	-2.4
23	1983 2 26	52.1	97011.0	1504.0	5091.8	5202.0	2.2
24	2983 3 5	49.8	100003.0	1663.4	4834.6	4972.3	2.8
25	1983 3 12	42.2	82968.0	2051.8	4157.8	4081.0	-1.8
26	1983 3 19	49.8	103846.0	2680.4	4786.1	4810.0	0.5
27	1983 3 26	50.9	109221.0	3324.8	4866.2	4829.1	-0.8
28	1983 4 2	44.3	71726.0	3957.4	3642.3	3853.8	5.8
29	1983 4 9	40.4	63326.0	3923.5	3255.4	3439.9	5.7
30	1983 4 17	42.0	77357.0	4501.3	3863.4	3576.4	-7.4
31	1983 4 24	27.0	46297.0	3067.3	2336.1	2239.2	-4.1
32	1983 4 30	22.2	29636.0	3467.3	1410.0	1617.1	14.7
33	1983 5 7	24.5	41202.0	4780.5	1627.7	1651.2	1.4
34	1983 5 15	29.9	53647.0	5274.8	2030.4	2142.1	5.5

TABLE 4

Comparison of Model Predictions with an Independent Set of Data for House 1

TEST NO.	WEEK ENDING ON	TEMP. DIFFER. [°C]	INFILTRATION DRIVING FORCE [W/m²]	SOLAR RADIATION [WATTS]	MEASURED ENERGY INPUT [WATTS]	PREDICTED ENERGY INPUT [WATTS]	PERCENTAGE DEVIATION
52	1983 5 22	26.3	41679.0	5781.8	1208.1	1640.8	35.8
53	1983 5 28	21.9	47696.0	5245.0	1560.0	1369.6	-12.2
54	1983 6 4	18.8	22164.0	5406.4	338.4	882.3	-160.7
55	1983 6 11	18.3	24876.0	4702.6	935.1	1004.9	7.5
56	1983 6 18	17.0	24876.0	3441.2	1176.5	1105.9	-6.0
57	1983 6 25	14.1	16851.0	3747.0	289.9	717.8	147.6
58	1983 7 2	14.8	22460.0	2754.8	789.6	1012.9	28.3
59	1983 7 9	12.0	13498.0	3478.0	966.7	552.7	-42.8
60	1983 7 16	10.7	10784.0	3163.3	322.6	474.5	47.1
61	1983 7 23	8.1	8640.0	3756.4	128.6	104.7	-18.6
62	1983 7 30	10.4	13314.0	3144.5	418.5	470.9	12.5
63	1983 8 6	11.6	20858.0	2697.1	902.4	715.1	-20.8
64	1983 8 13	12.8	12344.0	1862.3	1015.2	928.4	-8.6
65	1983 8 20	12.2	14485.0	1864.4	982.5	878.1	-10.6
66	1983 8 27	14.3	17180.0	1795.8	1337.8	1105.6	-17.4
67	1983 9 3	16.6	17765.0	2051.9	1225.0	1276.0	4.2
68	1983 9 10	16.0	17854.0	3013.8	1499.1	1039.9	-30.6
69	1983 9 17	16.2	23174.0	2519.3	1595.0	1184.9	-25.7
70	1983 9 24	16.7	28694.0	1658.3	1530.7	1434.7	-6.3
					18221.7	17899.0	1.77

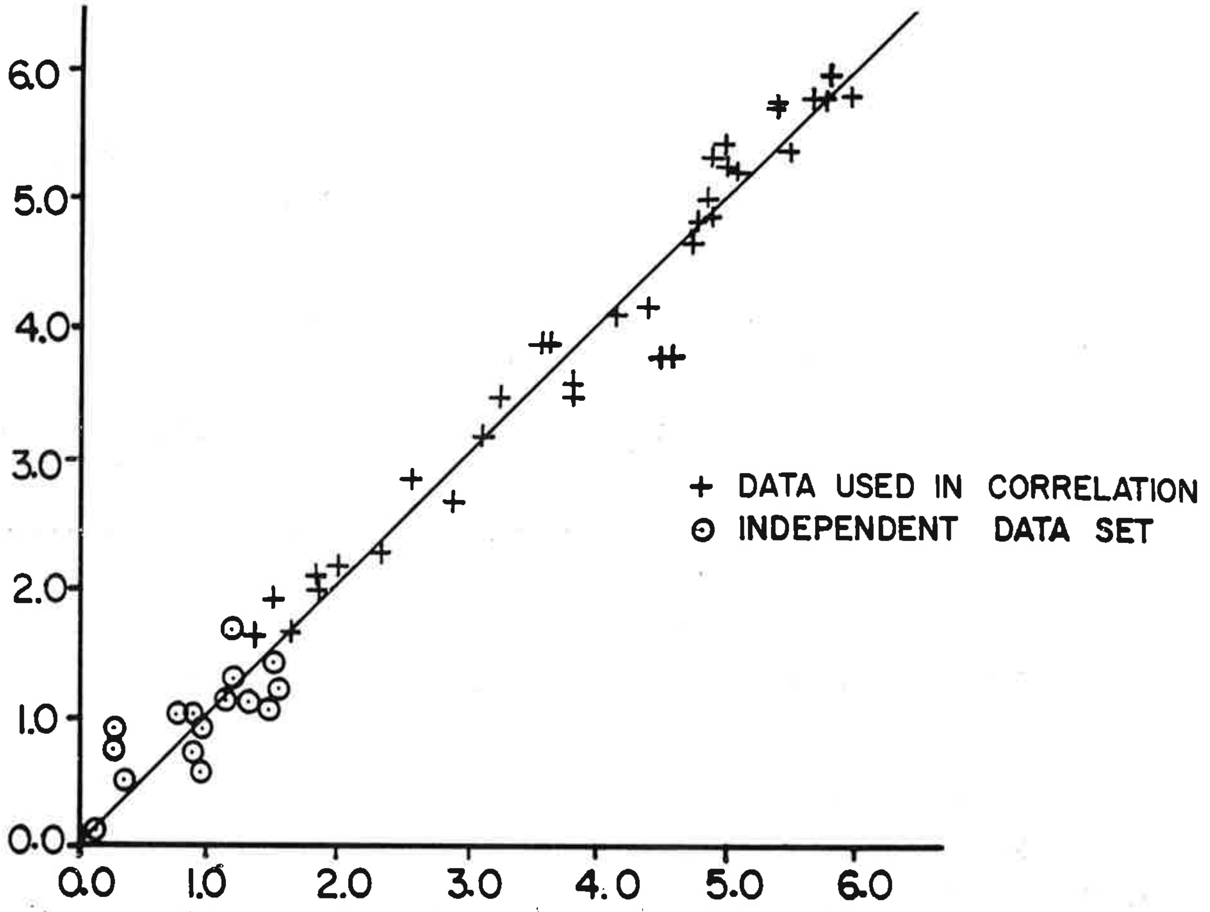

Figure 1. Measured and predicted energy input for house one

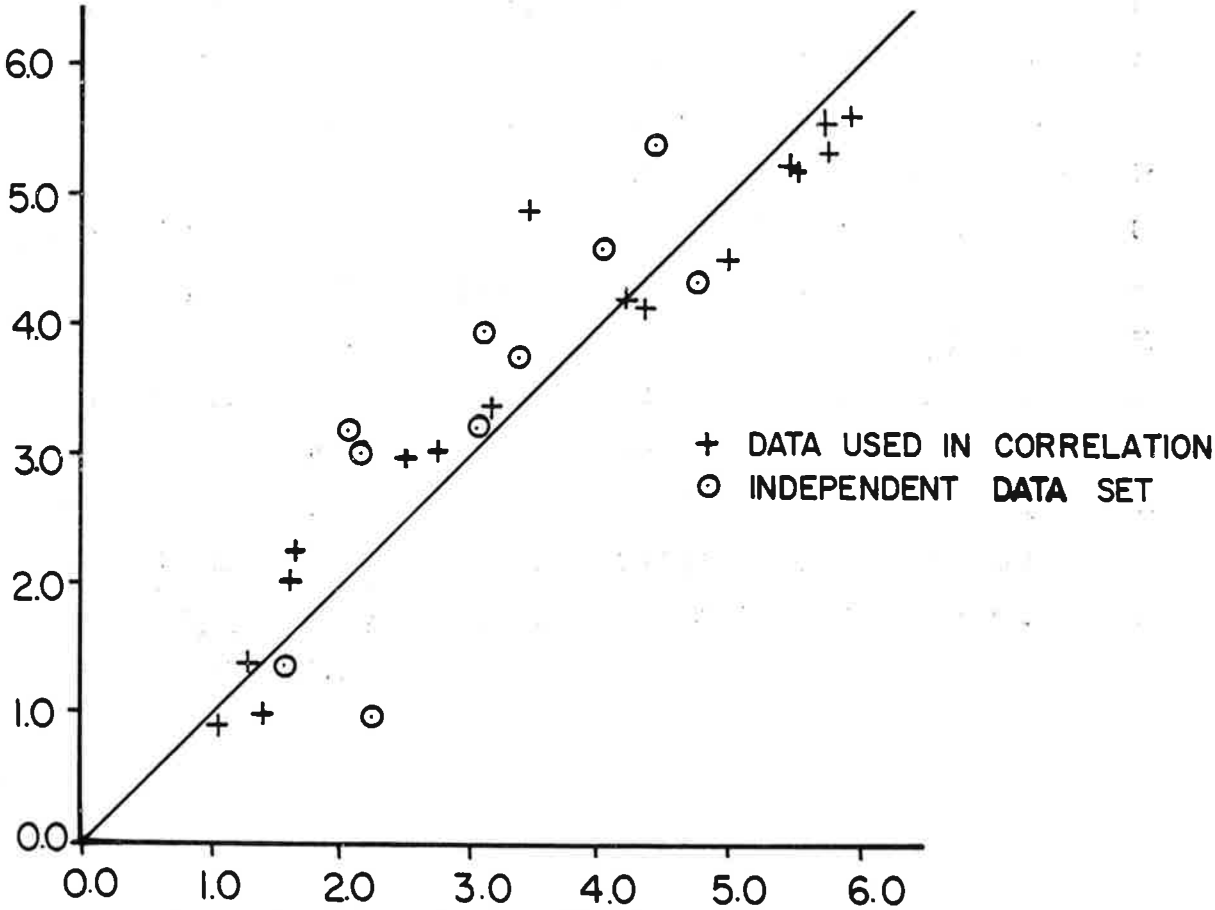

Figure 2. Measured and predicted energy input for house two