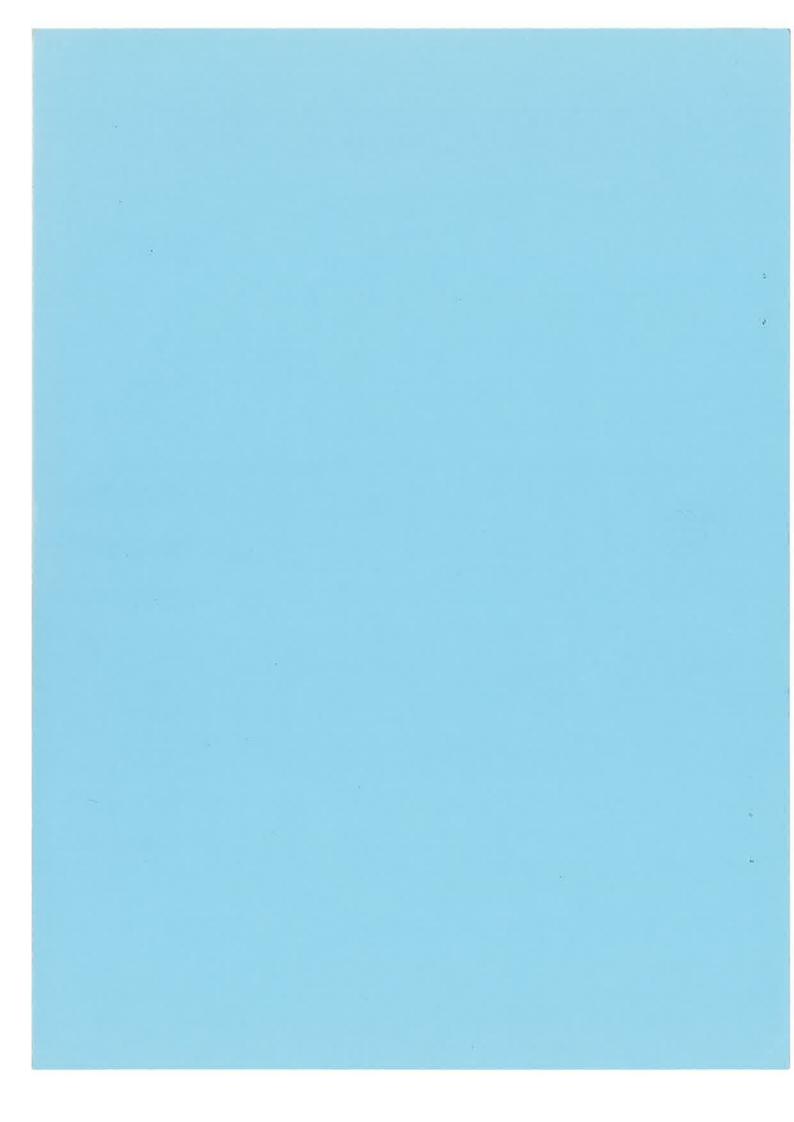
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# ENERGY AND BUILDINGS



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## A Simplified Model of Thermal Comfort

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#### SUMMARY

The purpose of conditioning the air in a building is to provide a safe and comfortable environment for its occupants. Satisfaction with the environment is composed of many components, the most important of which is thermal comfort. The principal environmental factors that affect human comfort are air temperature, mean radiant temperature, humidity, and air speed; virtually all heating, ventilating and air-conditioning (HVAC) systems, however, are usually controlled only by an air-temperature set-point. Significant efficiency improvements could be achieved if HVAC systems responded to comfort levels rather than air-temperature levels. The purpose of this report is to present a simplified model of thermal comfort based on the original work of Fanger, who related thermal comfort to total thermal stress on the body. The simplified solutions allow the calculation of predicted mean vote (PMV) and effective temperature which (in the comfort zone) are linear in the air temperature and mean radiant temperature, and quadratic in the dew point, and which can be calculated without any iteration. In addition to the mathematical expressions, graphical solutions are presented.

Key words: thermal comfort, predicted mean vote, effective temperature.

## INTRODUCTION

The primary purpose of conditioning buildings is to provide a comfortable environment in which to live and work, and a large amount of research [1] has already been compiled in this area. However, in an age in which energy cost and availability are key factors, using the least energy possible to accomplish that purpose becomes an important consideration. The designer or operator of a building who understands the effects of

environmental variables on human comfort and can manipulate them individually is capable of optimizing the building's heating, ventilating, and air-conditioning (HVAC) systems for maximum comfort at minimum cost. Many strategies are available for changing air temperature without sacrificing comfort conditions. For example, researchers have looked at the effect of night set-back/set-up for reducing heating/cooling loads [2, 3].

As discussed by Fanger and Valbjorn [1], there are many other aspects to acceptability of an indoor environment besides thermal comfort; in this report, however, we shall concern ourselves only with the thermal aspects of human comfort. Thermal comfort is that part of total human comfort which can be attributed to the thermal balance of the body. Specifically, it is the interaction of environmental variables (i.e., air temperature, mean radiant temperature, humidity, and air speed) with the occupant's personal variables (i.e., metabolic rate and clothing). The landmark work in the field of thermal comfort was the initial work of Fanger [4]; since that time there have been many good articles on thermal comfort [5 - 7] as well as large sections of books (e.g. ref. 1), whole journal issues [8], and ASHRAE standards [9] devoted to the topic.

Thermal comfort is a topic which is by nature multidisciplinary; it involves aspects of engineering and of human physiology. Because the human body has its own temperature-regulating responses (e.g., sweating, vaso-dilation/constriction, shivering, etc.), an occupant's response to (and hence sensation of) the environment will be a strong function of his/her physical condition; a young, healthy body recovers more quickly and therefore can respond to changes in thermal stress more quickly than can an older, ill-conditioned one.

In the building sciences, however, the usual goal is to predict the comfort needs for the

mean of the population who will occupy the structure (i.e., the average person). In general, we assume occupants represent a broad cross-section of the population, and knowing the mean response of the population is sufficient—that is, physiological variables can be omitted from the equations. (Of course, if the building is to be used primarily by a sub-set of the population that has significantly different physiological responses from the norm (e.g., housing for the elderly), predictions must be corrected accordingly.)

The purpose of this work is to derive simplified expressions for thermal comfort, expressions that can be used in engineering calculations and simplified thermal models to arrive at acceptable criteria for the thermal environment. As will be discussed later, we have used the basic equations of Fanger [4] but simplified them by making a few approximations. At the expense of complete generality, these simplifications make the form of the equations more compact. Many of the assumptions we have made are appropriate only when a person is near the comfort zone: we do not adequately model profuse sweating or shivering, or regimes of significant body heating or cooling. These simplifications should not significantly affect the precision of the predictions. (As Fanger reports, it is impossible to please more than about 95% of the people sampled; furthermore, even in the most carefully controlled experiments that use Fanger's original equation, there can be as much as a 25% variation in thermal sensation.)

## PREDICTED MEAN VOTE

Predicted mean vote (PMV) is a measure of the thermal sensation (not preference) that the mean of a population feels in a given environment. As defined by Fanger [4], predicted mean vote is based on a seven-point scale ranging from cold (-3), through neutral (0) to hot (+3). In deriving his equations Fanger correlated the PMV with the thermal stress on the body, relative to comfort conditions. Thus, using this correlation reduces the problem of calculating PMV to an engineering calculation of thermal load.

Conceptually, we can describe the thermal stress and, hence, the PMV as a function of all

the variables: personal (clothing and metabolic rate) and environmental (air temperature, radiant temperature, humidity, and air speed). This function can then be used to define comfort levels for different combinations of personal and environmental conditions. In order to derive an expression for PMV, one must construct a hypothetical heat balance for the body. Fanger did so by subtracting the heat load, as calculated from the comfort equation, from the heat generation; the thermal sensation is then empirically related to this difference. The complete derivation, including the individual heat loss terms, is contained in Appendix A.

Although the derivation in Appendix A follows Fanger's derivation quite closely, a following few differences have been introduced to simplify the results.

## (1) Linearized radiation

The radiation exchange terms have been linearized to remove the  $T^4$  dependence on temperature. This leads to a linear expression for the radiative heat transfer that is accurate to 5% for normal temperatures. If, however, the environment in question has sections with vastly different radiant temperatures (e.g., high-temperature radiant heaters), the error may be nonnegligible.

## (2) Simplified convection coefficient

In Fanger's original work the convection coefficient for low air movement was a function of the clothing temperature which was a function of the heat balance which depended on the convection coefficient. This process required an iterative solution and did not lend itself to easy interpretation. We have elected to use convection coefficients that can be evaluated directly. These two values will give the same results for all but a very few indoor environments.

## (3) Dew point for humidity

The humidity variable in Fanger's work was vapor pressure which can be calculated from the saturated vapor pressure and relative humidity. Because both these quantities are strong functions of air temperature, the effects of air temperature and humidity could not be easily separated. We therefore elected to use dew point, which is not a function of temperature, as our humidity variable.

For the vast majority of indoor environments, these three assumptions introduce very little additional uncertainty into the prediction of thermal comfort, and do allow the effects of air temperature, mean radiant temperature, and humidity to be separated.

Appendix A uses these assumptions to derive the thermal stress and then uses Fanger's correlation to calculate the PMV. The result is:

$$Y = Y_{o} + Y_{r} \frac{T_{r}}{T_{c}} + Y_{c} \frac{T_{a}}{T_{c}} + Y_{e} \frac{T_{d}^{2}}{T_{c}^{2}}$$
 (1)

The definitions and derivation of these quantities are supplied in the appendices. (Note that skin temperature,  $T_s$ , is determined only by the metabolic rate, m; we have used it throughout this report to simplify the units of the equations — it is not an independent variable.)

#### EFFECTIVE TEMPERATURES

Our expression for PMV allows us to calculate a comfort level for any given set of personal and environmental conditions. For many uses, however, it is desirable to have a temperature index that yields an equivalent comfort condition relative to a standard environment. Equivalently, the temperature index would be a corrected air temperature that took into account mean radiant temperature, dew point and air speed. Conceptually, we are comparing two environments: the first environment is the actual environment of interest and the second environment is one that has the same comfort level as the first but is described by a single temperature; we call this temperature the effective temperature. (Note that while our definition of effective temperature is similar to Gagge's, there are some differences. The differences are the combination of our comfort equations with our choice of standard conditions.)

In order to have an environment described by only one temperature we must constrain the other environmental variables in some way. We do this by defining a set of conditions for the standard environment; that is,

$$Y = Y_{o}' + Y_{r}' \frac{T_{r}'}{T_{s}} + Y_{c}' \frac{T_{a}'}{T_{s}} + Y_{e}' \left(\frac{T_{d}'}{T_{s}}\right)^{2}$$
 (2.1)

These standard conditions, given in detail in Appendix B, are as follows: the air temperature and mean radiant temperature equal to the effective temperature, dew point is standardized, and the wind speed is low. If we insert these conditions into the equation for PMV, we get an expression for effective temperature as a function of PMV and the personal variables:

$$Y = Y_{o}' + (Y_{r}' + Y_{c}') \frac{T_{e}}{T_{s}} + Y_{e}' \left( \frac{T_{e} - \frac{1}{2} T_{s}}{T_{s}} \right)$$
(2.2)

Solving for the effective temperature yields the following:

$$T_{\rm e} = \frac{T_{\rm s}}{Y_{\rm t}'} \left( Y - Y_{\rm o}' + \frac{1}{2} Y_{\rm e}' \right) \tag{3}$$

If we substitute the definition of the PMV into this expression, we get a simple expression for the effective temperature as a function of the environmental variables:

$$T_{\rm e} = A + BT_{\rm r} + CT_{\rm a} + DT_{\rm d}^{2} \tag{4}$$

Note that the dew-point term should be discarded for dew points less than zero.

The coefficients used above are defined as follows:

$$A = \frac{T_{s}}{Y_{t}'} \left( \frac{1}{2} Y_{e}' + Y_{o} - Y_{o}' \right) \qquad B = \frac{Y_{r}}{Y_{t}'}$$

$$C = \frac{Y_{c}}{Y_{t}'} \qquad \qquad D = \frac{1}{T_{s}} \frac{Y_{e}}{Y_{t}'}$$
(5)

## Comfort temperatures

Although the effective temperature gives a corrected temperature value for the existing conditions, it does not directly indicate the comfort level. However, since we have an expression that calculates the effective temperature as a function of PMV, we can use it to find the effective comfort temperatures. These comfort temperatures then become functions of the personal variables alone—independent of environmental conditions.

The optimal value of the effective temperature must occur for comfortable conditions. The optimal effective temperature, therefore, is calculated for PMV equal to zero:

$$T_{\rm o} = T_{\rm e}|_{Y=0}$$
 (6.1)

$$T_{o} = \frac{T_{s}}{Y_{t}'} \left( \frac{1}{2} Y_{e}' - Y_{o}' \right)$$
 (6.2)

Because both the personal and environmental parameters are variable, a comfort value alone is often insufficient; a range of acceptable temperatures is required. Fanger has found that while approximately 95% of the people polled will find the Y=0 (thermal neutrality) condition acceptable, 90% of people will find  $Y=\pm 1/2$  conditions acceptable. Accordingly, we shall define the comfort band to lie between those two limits:

$$\Delta T_{\rm o} = T_{\rm e}|_{Y=0.5} - T_{\rm e}|_{Y=-0.5} \tag{7.1}$$

$$\Delta T_{\rm o} = \frac{T_{\rm s}}{Y_{\rm t}'} \tag{7.2}$$

The size of the comfort band ranges from approximately 2 °C for conditions where occupants are lightly clothed and sedentary to over 10 °C for occupants who are heavily clothed and working hard. In Fig. 1 we have plotted the acceptable range of the comfort temperature as a function of the clothing level for three different activity levels.

As an alternative we can use the last two expressions to rewrite the effective temperature equation in terms of the comfort temperature and the comfort band:

$$T_{\rm e} = T_{\rm o} + Y \Delta T_{\rm o} \tag{8}$$

These same two equations can be used to eliminate all primed terms (comfort coeffi-

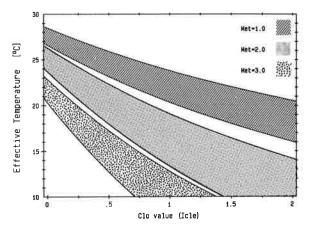


Fig. 1. Optimal value of simplified effective temperature (i.e., comfort temperature) as a function of personal parameters.

cients in the standard condition) from the definitions of the temperature coefficients:

$$A = T_{o} + Y_{o} \Delta T_{o} \qquad B = Y_{r} \frac{\Delta T_{o}}{T_{s}}$$

$$C = Y_{c} \frac{\Delta T_{o}}{T_{s}} \qquad D = Y_{e} \frac{\Delta T_{o}}{T_{s}^{2}}$$

$$(9)$$

This formulation has the advantage of not referring directly to standard conditions but, instead, to the value and width of the comfort temperature. Thus, if some other criterion for standard conditions is desired, these formulae can be used to calculate the effective temperature, as long as the value and width of the comfort temperature can be defined.

Comparison with standard effective temperature

The ASHRAE Handbook of Fundamentals uses as its effective temperature (ET\*) the Standard Effective Temperature (SET) of Gagge. As mentioned earlier, the assumptions we have used to define our effective temperature are somewhat different than Gagge's; we have used our simplified comfort equations with a set of standard conditions (see Appendix B) to define our effective temperature.

In Fig. 2, we compare the effective temperature of ASHRAE to our effective temperature. We have used the same criteria as that of ASHRAE: clo value of 0.6, low wind speed, met of 1.0. For cool and comfortable effective temperatures (i.e., below 30 °C) ET\* and our effective temperature agree quite well; however, for very warm temperatures (i.e., above 30 °C) there is significant divergence. The cause of this discrepancy is that our model does not, as Gagge does, correctly account for the thermal balance when sweating becomes the dominant heat loss mechanism (which, as Fanger points out, is well outside the comfort range). Because we are interested only in the behavior near comfort, this is not an important difference.

Figure 2 also compares the comfort zone of ASHRAE (Standard 55-74) with our predicted comfort zone. For this comparison we have truncated the comfort zone below the humidity ratio of 0.0043 and above the humidity ratio of 0.012 as is done in that standard. The ASHRAE comfort range extends 1 °C above the LBL zone, but this extra width is most likely due to the broader range of clothing

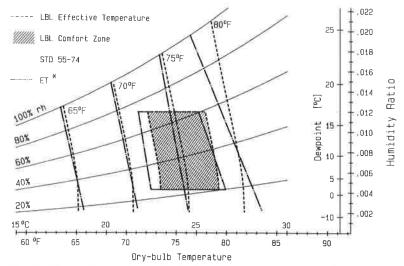


Fig. 2. Comparison of standard effective temperature (ET\*) with simplified effective temperature and comfort zones for the same conditions (see ASHRAE [17], Fig. 1b, p. 8.21.

and activity values used in the application of Standard 55-74.

#### TABULAR DATA

Although the equations for calculating the comfort and effective temperature coefficients (eqns. (1) - (5)) are straightforward, the procedure can be time-consuming. Furthermore, the clothing levels, metabolic rate, and air speed are rarely known to a high degree of accuracy. For these reasons, it may be practical to choose a single set of the comfort and effective temperature coefficients and use them to calculate comfort levels from the three environmental temperatures.

Table 1 displays all of the velocity-independent quantities  $(T_{\rm s},~T_{\rm o},~\Delta T_{\rm o})$  as well as the comfort coefficients  $(Y_{\rm r}',~Y_{\rm c}',~Y_{\rm e}',~Y_{\rm o}')$  and effective temperature coefficients (A', B', C',D') in their standard condition (i.e., zero air speed) as a function of the intrinsic parameters (clothing level and metabolic rate). Clothing level has been chosen to span the full range from no clothing whatsoever to heavy winter clothing and the metabolic rates cover sedentary to moderately active occupants. Table 2 displays the comfort coefficients  $(Y_r, Y_c, Y_e, Y_o)$  as a function of the intrinsic parameters and for three different air speeds. These wind speeds span the range normally considered to be acceptable for indoor work. (High wind speeds may cause local discomfort, especially to sedentary individuals.) Table 3 displays the effective temperature

coefficients (A, B, C, D) as a function of the same intrinsic parameters and air speeds.

#### **DESIGN APPLICATIONS**

A simplified comfort equation such as ours has many applications. It could, for example, be used as a control algorithm in a large HVAC system where a competent controller could adjust the environmental conditions to maintain acceptable comfort levels at a minimum cost. Other applications involve estimating the efficacy of radiant heating and the suitability of humidity control for comfort. One of the most important applications of a comfort model, and the one we treat below, is that of natural ventilation for cooling. We use natural ventilation here to mean intentional ventilation through conventional openings in the building shell (i.e., windows) where the driving pressures may either be natural (i.e., from the wind) or mechanical (e.g. from a whole-house fan).

During the heating season, free heat (generated within a structure by people, appliances, and solar radiation) assists the HVAC system in conditioning the air; during the cooling season, however, free heat is an added burden. Thus increased ventilation is rarely desirable from a thermal standpoint during the heating season, but may be quite desirable during the cooling season. Natural ventilation has two separable effects: the increased ventilation causes an increase in interior air speed which allows comfort at higher air temperatures through increased evaporative and convective

TABLE 1
Velocity-independent quantities and comfort and effective temperature coefficients

met	$T_{\mathbf{s}}$	clo	$T_{\mathbf{o}}$	$\Delta T_{\mathbf{o}}$	A'	B'	C'	D'	Yo'	$Y_{\mathbf{r}}$	$Y_{\mathbf{c}}$	Ye'
1.0	33.5	0.0	27.5	1.9	1.07	0.57	0.363	0.00190	-14.03	10.19	6.46	1.13
		0.5	24.5	2.8	1.37	0.56	0.359	0.00243	-8.37	6.78	4.36	0.99
		1.0	22.1	3.5	1.54	0.55	0.358	0.00274	-5.92	5.33	3.46	0.89
		1.5	20.0	4.0	1.64	0.54	0.357	0.00292	-4.55	4.52	2.97	0.81
		2.0	18.3	4.5	1.71	0.54	0.357	0.00304	-3.67	4.01	2.65	0.76
		3.0	15.5	5.3	1.77	0.54	0.358	0.00315	-2.60	3.40	2.28	0.67
1.5	32.5	0.0	26.5	2.4	1.31	0.45	0.470	0.00248	-10.68	6.18	6.45	1.11
		0.5	22.6	3.8	1.69	0.43	0.461	0.00320	-5.54	3.74	3.97	0.89
		1.0	19.4	4.9	1.88	0.43	0.458	0.00357	-3.58	2.83	3.04	0.77
		1.5	16.7	5.8	1.99	0.42	0.457	0.00377	-2.54	2.35	2.56	0.69
		2.0	14.4	6.6	2.05	0.42	0.457	0.00390	-1.89	2.06	2.26	0.63
		3.0	10.8	7.8	2.12	0.41	0.457	0.00403	-1.12	1.72	1.91	0.55
2.0	31.4	0.0	25.0	2.6	1.41	0.39	0.516	0.00286	-9.01	4.73	6.18	1.08
		0.5	20.1	4.4	1.84	0.38	0.504	0.00373	-4.16	2.71	3.60	0.84
		1.0	16.2	5.8	2.05	0.37	0.500	0.00416	-2.45	2.01	2.72	0.71
		1.5	13.0	6.9	2.17	0.36	0.498	0.00441	-1.56	1.65	2.27	0.63
		2.0	10.3	7.8	2.25	0.36	0.498	0.00456	-1.02	1.44	1.99	0.57
		3.0	6.0	9.3	2.34	0.35	0.498	0.00475	-0.40	1.19	1.68	0.50
2.5	30.3	0.0	23.3	2.6	1.46	0.36	0.543	0.00318	-8.29	4.15	6.25	1.11
		0.5	17.5	4.6	1.93	0.34	0.529	0.00420	-3.42	2.28	3.51	0.84
		1.0	13.0	6.1	2.16	0.33	0.524	0.00470	-1.78	1.67	2.62	0.71
		1.5	9.2	7.3	2.29	0.33	0.522	0.00500	-0.95	1.36	2.18	0.63
		2.0	6.1	8.3	2.38	0.32	0.520	0.00520	-0.45	1.18	1.91	0.58
		3.0	1.3	9.8	2.50	0.32	0.520	0.00544	0.12	0.98	1.61	0.51
3.0	29.2	0.0	21.6	2.5	1.49	0.34	0.562	0.00348	-7.90	3.86	6.46	1.17
		0.5	15.0	4.5	1.99	0.32	0.546	0.00465	-2.87	2.05	3.53	0.88
		1.0	9.8	6.0	2.23	0.31	0.540	0.00523	-1.25	1.49	2.61	0.74
		1.5	5.6	7.3	2.38	0.30	0.537	0.00558	-0.44	1.21	2.16	0.66
		2.0	2.1	8.2	2.49	0.29	0.535	0.00582	0.05	1.05	1.90	0.60
		3.0	-3.2	9.8	2.62	0.29	0.533	0.00614	0.60	0.86	1.60	0.54
4.0	27.1	0.0	18.1	2.3	1.50	0.30	0.588	0.00409	-7.17	3.51	6.86	1.29
		0.5	9.8	4.3	2.04	0.28	0.568	0.00557	-1.82	1.77	3.59	0.95
		1.0	3.5	5.8	2.31	0.27	0.560	0.00631	-0.20	1.27	2.63	0.80
		1.5	-1.5	6.9	2.48	0.26	0.555	0.00679	0.58	1.02	2.17	0.72
		2.0	-5.6	7.9	2.61	0.26	0.552	0.00712	1.05	0.88	1.90	0.66
		3.0	-11.8	9.3	2.77	0.25	0.548	0.00758	1.57	0.72	1.60	0.60

cooling\*, and the increased ventilation removes internally generated heat and humidity, thus lowering the effective temperature. In other words, for many cooling climates it may be possible to eliminate cooling plants or reduce cooling loads by using natural ventilation.

The effect of increased air speed on the comfort zone can be calculated directly from our comfort equations. In Fig. 3 we display the comfort zones for different internal air

speeds using the conditions of clo = 0.5 and met = 1. This Figure could be used, for example, to estimate the internal air speed that would need to be created by a fan in order to extend upward the acceptable air temperature; by allowing air-conditioning thermostats to be set higher, energy savings would be realized. Although useful, this type of information tells us only the *inside* temperature and humidity conditions that would be comfortable for different internal air speeds; for natural ventilation considerations, we wish to know the *outside* conditions that would be appropriate for different internal air speeds.

<sup>\*</sup>Technically this may not be true for air temperatures that are higher than skin temperatures. Such a situation, however, is very unlikely in the comfort range.

TABLE 2
Comfort coefficients for different intrinsic parameters and air speeds

met	clo	v = 0.20				v = 0.50				v = 1.00			
		$Y_{o}$	$Y_{\mathbf{r}}$	$Y_{\mathbf{c}}$	$Y_{\rm e}$	$\overline{Y_{\mathbf{o}}}$	$Y_{\mathbf{r}}$	$Y_{\mathbf{c}}$	$Y_{\mathbf{e}}$	$\overline{Y_{\mathbf{o}}}$	$Y_{\mathbf{r}}$	$Y_{\mathbf{c}}$	$Y_{\mathbf{e}}$
1.0	0.0	-15.59	10.2	7.8	1.31	-21.02	10.2	18.2	2.68	-27.28	10.2	18.2	2.68
	0.5	-9.02	6.6	5.1	1.11	-10.98	5.9	7.4	1.47	-12.78	5.3	9.5	1.79
	1.0	-6.30	5.1	4.0	0.97	-7.37	4.4	5.5	1.21	-8.27	3.8	6.9	1.39
	1.5	-4.81	4.3	3.4	0.88	-5.50	3.6	4.6	1.04	-6.06	3.1	5.6	1.16
	2.0	-3.86	3.8	3.0	0.81	-4.35	3.2	4.0	0.92	-4.74	2.6	4.8	1.01
	3.0	-2.72	3.2	2.6	0.70	-3.02	2.6	3.4	0.78	-3.24	2.2	4.0	0.82
1.5	0.0	-10.68	6.2	6.5	1.11	-12.12	6.2	11.1	1.72	-15.91	6.2	11.1	1.72
	0.5	-5.54	3.7	4.0	0.89	-6.02	3.6	4.5	0.98	-7.12	3.2	5.8	1.18
	1.0	-3.58	2.8	3.0	0.77	-3.83	2.7	3.4	0.82	-4.38	2.3	4.2	0.93
	1.5	-2.54	2.4	$^{2.6}$	0.69	-2.70	2.2	2.8	0.72	-3.04	1.9	3.4	0.79
	2.0	-1.89	2.1	2.3	0.63	-2.00	1.9	2.5	0.65	-2.24	1.6	3.0	0.70
	3.0	-1.12	1.7	1.9	0.55	-1.19	1.6	2.1	0.56	-1.33	1.3	2.5	0.59
2.0	0.0	-9.01	4.7	6.2	1.08	-9.01	4.7	8.5	1.38	-11.65	4.7	8.5	1.38
	0.5	-4.16	2.7	3.6	0.84	-4.16	2.7	3.6	0.84	-4.92	2.4	4.5	0.97
	1.0	-2.45	2.0	2.7	0.71	-2.45	2.0	2.7	0.71	-2.82	1.8	3.3	0.79
	1.5	-1.56	1.7	$^{2.3}$	0.63	-1.56	1.7	2.3	0.63	-1.80	1.4	2.7	0.68
	2.0	-1.02	1.4	2.0	0.57	-1.02	1.4	2.0	0.57	-1.18	1.2	2.3	0.61
	3.0	-0.40	1.2	1.7	0.50	-0.40	1.2	1.7	0.50	-0.49	1.0	1.9	0.52
2.5	0.0	-8.29	4.1	6.3	1.11	-8.29	4.1	7.5	1.27	-9.71	4.1	7.5	1.27
	0.5	-3.42	2.3	3.5	0.84	-3.42	2.3	3.5	0.84	-3.81	2.1	4.0	0.91
	1.0	-1.78	1.7	2.6	0.71	-1.78	1.7	2.6	0.71	-1.97	1.5	2.9	0.75
	1.5	-0.95	1.4	2.2	0.63	-0.95	1.4	2.2	0.63	-1.07	1.2	2.4	0.66
	2.0	-0.45	1.2	1.9	0.58	-0.45	1.2	1.9	0.58	-0.53	1.1	2.1	0.59
	3.0	0.12	1.0	1.6	0.51	0.12	1.0	1.6	0.51	0.07	0.9	1.7	0.52
3.0	0.0	-7.90	3.9	6.5	1.17	-7.90	3.9	7.0	1.24	-8.52	3.9	7.0	1.24
	0.5	-2.87	2.1	3.5	0.88	-2.87	2.1	3.5	0.88	-3.03	2.0	3.7	0.91
	1.0	-1.25	1.5	2.6	0.74	-1.25	1.5	2.6	0.74	-1.32	1.4	2.7	0.75
	1.5	-0.44	1.2	2.2	0.66	-0.44	1.2	2.2	0.66	-0.49	1.2	2.3	0.67
	2.0	0.05	1.0	1.9	0.60	0.05	1.0	1.9	0.60	0.01	1.0	2.0	0.61
	3.0	0.60	0.9	1.6	0.54	0.60	0.9	1.6	0.54	0.58	0.8	1.7	0.54
4.0	0.0	-7.17	3.5	6.9	1.29	-7.17	3.5	6.9	1.29	-7.17	3.5	6.9	1.29
	0.5	-1.82	1.8	3.6	0.95	-1.82	1.8	3.6	0.95	-1.82	1.8	3.6	0.95
	1.0	-0.20	1.3	2.6	0.80	-0.20	1.3	2.6	0.80	-0.20	1.3	2.6	0.80
	1.5	0.58	1.0	2.2	0.72	0.58	1.0	2.2	0.72	0.58	1.0	2.2	0.72
	2.0	1.05	0.9	1.9	0.66	1.05	0.9	1.9	0.66	1.05	0.9	1.9	0.66
	3.0	1.57	0.7	1.6	0.60	1.57	0.7	1.6	0.60	1.57	0.7	1.6	0.60

Accurate calculation of the internal and outside conditions for a given house normally requires a complex computer program. On a mainframe computer, hour-by-hour simulation programs [10, 11] calculate energy use by doing a detailed thermal balance for each component, and user-friendly, microprocessor-based programs [12] use correlation techniques to calculate monthly energy usage. For the high ventilation rates typically associated with natural ventilation, very simple steady-state calculations can be used because the energy flows are dominated by the ventilation. In addition, the free heat and moisture

generation and the thermal resistance of the building envelope have a relatively small effect on internal conditions; high-accuracy calculations are not needed.

## Example\*

As an example, we will calculate the daytime comfort zones, using different internal air speeds, for a naturally ventilated

<sup>\*</sup>The specific assumptions used in this example are necessarily crude. The effect of these air speed and internal generation assumptions will only be significant when the ventilation rate (and, hence, the equilibrium outside temperature) is low.

TABLE 3
Effective temperature coefficients for different intrinsic parameters and air speeds

met	clo	v = 0.20	)			v = 0.50				v = 1.00			
		$\overline{A}$	В	C	D	$\overline{A}$	В	C	D	Ā	В	C	D
1.0	0.0	-1.88	0.57	0.44	0.00220	-12.11	0.57	0.71	0.00326	-23.92	0.57	1.02	0.00449
	0.5	-0.43	0.54	0.42	0.00273	-5.84	0.49	0.61	0.00361	-10.84	0.43	0.78	0.00441
	1.0	0.23	0.53	0.41	0.00300	-3.47		0.57	0.00371	-6.59	0.39	0.71	0.00428
	1.5	0.61	0.52	0.41	0.00315	-2.19		0.55	0.00373	-4.43	0.37	0.67	0.00415
	2.0	0.86	0.51	0.41	0.00323	-1.38		0.54	0.00371	-3.13	0.36	0.65	0.00404
	3.0	1.15	0.50	0.41	0.00331	-0.43	0.41	0.53	0.00365	-1.62	0.34	0.63	0.00386
1.5	0.0	1.31	0.45	0.47	0.00248	-2.08		0.56	0.00285	-11.06	0.45	0.81	0.00385
	0.5	1.69	0.43	0.46	0.00320	-0.14		0.53	0.00352	-4.28	0.37	0.68	0.00423
	1.0	1.88	0.43	0.46	0.00357	0.62		0.51	0.00382	-2.05	0.35	0.63	0.00434
	1.5	1.99	0.42	0.46	0.00377	1.03		0.51	0.00398	-0.93	0.33	0.62	0.00437
	2.0	2.05	0.42	0.46	0.00390	1.29		0.51	0.00406	-0.25	0.32	0.60	0.00437
	3.0	2.12	0.41	0.46	0.00403	1.58	0.38	0.50	0.00415	0.52	0.31	0.59	0.00435
2.0	0.0	1.41	0.39	0.52	0.00286	1.41	0.39	0.52	0.00286	-5.52	0.39	0.71	0.00368
	0.5	1.84	0.38	0.50	0.00373	1.84	0.38	0.50	0.00373	-1.48	0.34	0.63	0.00434
	1.0	2.05	0.37	0.50	0.00416	2.05	0.37	0.50	0.00416	-0.12	0.32	0.60	0.00461
	1.5	2.17	0.36	0.50	0.00441	2.17		0.50	0.00441	0.57	0.31	0.59	0.00475
	2.0	2.25	0.36	0.50	0.00456	2.25		0.50	0.00456	0.99	0.31	0.58	0.00483
	3.0	2.34	0.35	0.50	0.00475	2.34	0.35	0.50	0.00475	1.47	0.30	0.57	0.00493
2.5	0.0	1.46	0.36	0.54	0.00318	1.46		0.54	0.00318	-2.27	0.36	0.65	0.00365
	0.5	1.93	0.34	0.53	0.00420	1.93		0.53	0.00420	0.16	0.32	0.60	0.00455
	1.0	2.16	0.33	0.52	0.00470	2.16		0.52	0.00470	1.01	0.31	0.58	0.00495
	1.5	2.29	0.33	0.52	0.00500	2.29		0.52	0.00500	1.45	0.30	0.57	0.00519
	2.0	2.38	0.32	0.52	0.00520	2.38		0.52	0.00520	1.72	0.29	0.57	0.00534
	3.0	2.50	0.32	0.52	0.00544	2.50		0.52	0.00544	2.05	0.29	0.56	0.00554
3.0	0.0	1.49	0.34	0.56	0.00348	1.49		0.56	0.00348	-0.10	0.34	0.61	0.00370
	0.5	1.99	0.32	0.55	0.00465	1.99		0.55	0.00465	1.24	0.31	0.58	0.00481
	1.0	2.23	0.31	0.54	0.00523	2.23		0.54	0.00523	1.75	0.30	0.56	0.00534
	1.5	2.38	0.30	0.54	0.00558	2.38		0.54	0.00558	2.03	0.29	0.56	0.00566
	2.0	2.49	0.29	0.54	0.00582	2.49		0.54	0.00582	2.21	0.28	0.56	0.00589
	3.0	2.62	0.29	0.53	0.00614	2.62	0.29	0.53	0.00614	2.43	0.27	0.55	0.00618
4.0	0.0	1.50	0.30	0.59	0.00409	1.50		0.59	0.00409	1.50	0.30	0.59	0.00409
	0.5	2.04	0.28	0.57	0.00557	2.04		0.57	0.00557	2.04	0.28	0.57	0.00557
	1.0	2.31	0.27	0.56	0.00631	2.31		0.56	0.00631	2.31	0.27	0.56	0.00631
	1.5	2.48	0.26	0.55	0.00679	2.48		0.55	0.00679	2.48	0.26	0.55	0.00679
	2.0	2.61	0.26	0.55	0.00712	2.61		0.55	0.00712	2.61	0.26	0.55	0.00712
	3.0	2.77	0.25	0.55	0.00758	2.77	0.25	0.55	0.00758	2.77	0.25	0.55	0.00758

house. To estimate the ventilation rate, we will assume that the internal air speed is proportional to the ventilation rate (specifically, that the number of air changes per hour is 100 times the internal air speed (m/s) with a minimum of one air change per hour). We will calculate the increase in humidity from outside to inside from the total internal moisture generation, 454 g/h, and the total ventilation; the total increase in air temperature from outside to inside is calculated from the total free heat generation, 3000 W, the conductance of the envelope 300 W/°C, and ventilation.

Figure 4 displays the comfort zones as a function of outside temperature and humidity for different internal air speeds. For the higher wind speeds the comfort zones in Figs. 3 and 4 are quite similar, indicating that the inside and outside conditions are comparable; for low wind speed, however, there is a significant shift between the two situations because of the presence of internal gains. (Note that once the internal air speed is below approximately 0.1 m/s, its direct effect on comfort vanishes, but, since the ventilation rate and air speed are linked, it affects the thermal balance of the building.) The range of

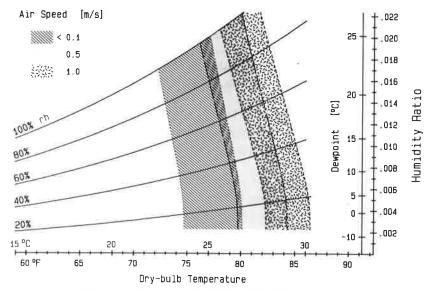


Fig. 3. Comfort zones of a typical building with different internal air speeds for a clo value of 0.5 and a met value of unity as a function of *inside* temperature and humidity.

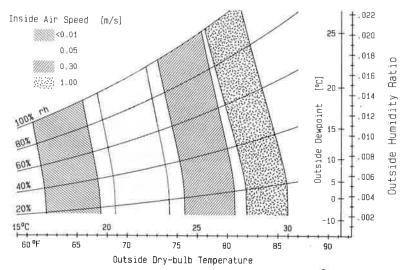


Fig. 4. Comfort zones of a typical building with different internal air speeds (ventilation rates) for a clo value of 0.5 and a met value of unity as a function of *outside* temperature and humidity.

comfort zones in Fig. 4 indicates that this building could be naturally ventilated in the outdoor temperature range of approximately 15 °C to 30 °C, if the internal air speed (via ventilation) could be controlled.

Although such design charts indicate the optimal amount of internal air speed consonant with human comfort, they do not indicate how the air speed is to be provided. If the air flow occurs as a result of mechanical ventilation, the problem is a straightforward one of equipment sizing; if the air flow is associated with naturally induced ventilation, using architectural design is more difficult.

A discussion of appropriate passive-design features is outside the scope of this report, but many authors have devoted themselves to this classic issue [13 - 15]. More modern work has been done in the areas of wind channeling and stack-induced ventilation [16].

#### CONCLUSIONS

In this report we have used the original work of Fanger to derive a simplified PMV expression for predicting thermal sensation. In doing so, we have made some simplifying assumptions to allow a closed-form expression of the predicted mean vote that is accurate near the comfort zone. The results of this simplified calculation have compared favorably with exact expressions developed and used by Fanger and Gagge. Concise tabular data that allow quick computation of comfort levels for different clothing, metabolic rates, and air speeds as a function of environmental temperatures have also been presented.

These simplifying assumptions allow the definition of a simplified effective temperature scale that converts the actual environmental conditions into an equivalent temperature. The simplified effective temperature compares well with the effective temperature (ET\*) in current usage. The use of the PMV scale creates a unique definition of the optimal value and acceptable range of the simplified effective temperature. Concise tabular data have also been presented that allow a quick computation of the simplified effective temperature for different clothing, metabolic rate, and air speed as a function of the other environmental temperatures.

Finally, we have included sample plots of the comfort zones for different air speeds and conditions. These plots allow the designer to estimate the air speed necessary to keep a particular space comfortable under specific conditions. The amount of natural ventilation required for a prototypical house for arbitrary outdoor conditions can be estimated from such plots.

#### LIST OF SYMBOLS

- $\boldsymbol{A}$ basic effective temperature coefficient
- $\boldsymbol{B}$ radiative effective temperature coefficient (unitless)
- $\boldsymbol{C}$ convective effective temperature coefficient (unitless)
- evaporative effective temperature coef-Dficient  $(1/^{\circ}C)$
- convective heat loss (W/m<sup>2</sup>)  $E_{\rm conv}$
- $E_{
  m in}$ internal heat generation (W/m<sup>2</sup>)
- maximum evaporative heat loss (W/  $E_{\rm max}$  $m^2$ )
- radiative heat loss (W/m<sup>2</sup>)  $E_{\rm rad}$
- respired heat loss (W/m<sup>2</sup>)  $E_{\mathrm{res}}$
- metabolic evaporative heat loss (sweat- $E_{\rm sw}$ ing)  $(W/m^2)$

- effective thermal efficiency of cloth- $F_{
  m cle}$
- permeation efficiency  $F_{\rm pcl}$
- convective heat transfer coefficient  $(W/m^2 °C)$
- $h_r$ radiative heat transfer coefficient  $(4.8 \text{ W/m}^2 \, ^{\circ}\text{C})$
- basic clo value (clo)  $I_{
  m cl}$
- effective clo value (clo)  $I_{
  m cle}$
- thermal body load (thermal stress)  $(W/m^2)$
- metabolic rate (58.1 W/m<sup>2</sup>)  $M_{\rm o}$
- met rate (met) m
- saturated vapor pressure of water at  $P_{\mathbf{d}}$ dew point (torr)
- $P_{\rm s}$ saturated vapor pressure of water at skin temperature (torr)
- air temperature (°C)  $T_{\mathbf{a}}$
- optimal effective temperature, i.e., comfort temperature (°C)
- dew point temperature (°C)  $T_{\rm d}$
- $T_{
  m e} \ T_{
  m r}$ effective temperature (°C)
- mean radiant temperature (°C)
- $T_{
  m s}$ skin temperature (°C) mean air speed (m/s)
- convective comfort coefficient
- $Y_{
  m c}$   $Y_{
  m e}$   $Y_{
  m o}$   $Y_{
  m r}$ evaporative comfort coefficient
- basic comfort coefficient
- radiative comfort coefficient
- $Y_{\rm t}$ total comfort coefficient
- $\boldsymbol{Y}$ predicted mean vote
- parameter x in the standard condition,

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## Appendix A

#### DERIVATION OF THE PREDICTED MEAN VOTE

The predicted mean vote (PMV) is an expression for representing the thermal sensation of occupants exposed to the environment. It is a seven-point scale centered on zero where positive values represent warm sensations and negative values represent cold sensations. Fanger [4] has found an empirical

relation between the PMV and the physiological load on the body.

$$Y = [1.6 + 17.6 \exp(-2.1m)] \frac{L}{M_{\odot}}$$
 (A1)

The load on the body is defined as the difference between the internal heat generated and the heat loss that would occur in the actual conditions if the body were in comfort. The total load can then be written as follows:

$$L = E_{\rm in} - E_{\rm res} - E_{\rm diff} - E_{\rm sw} - E_{\rm conv} - E_{\rm rad}$$
(A2)

Each of these terms represents a particular energy generation or loss and will be discussed below. The derivations for these terms as well as tables of clo  $(I_{\rm cle})$  and met (m) values can be found in ASHRAE's Handbook of Fundamentals [17].

## Internal heat generation

A human body generates a certain amount of heat for any given activity level. The activity level is usually specified by the met value, m, which is in units of the metabolic rate of a resting sedentary person,  $M_o$  (58.1 W/m<sup>2</sup>).

$$E_{\rm in} = mM_{\rm o} \tag{A3}$$

 $M_{\rm o}$  is the metabolic rate of a resting sedentary person.

#### Convective heat loss

Both free and forced convection cause heat exchange between the surface of the body and the air. Thus, the heat transfer will be proportional to the temperature difference between air and skin:

$$E_{\rm conv} = F_{\rm cle} h_{\rm c} (T_{\rm s} - T_{\rm a}) \tag{A4}$$

Definitions of the factors are given in Appendix B.

## Radiative heat loss

In any indoor environment the surface of the body is exchanging energy through radiation with other surfaces. The linearized heat exchange will be proportional to the difference between mean radiant temperature and skin temperature:

$$E_{\rm rad} = F_{\rm cle} h_{\rm r} (T_{\rm s} - T_{\rm r}) \tag{A5}$$

Definitions of the factors are given in Appendix B below.

Evaporative heat loss

Evaporative heat loss comes from three sources: diffusion through the skin, sweating due to (comfortable) metabolic activity, and sweating for temperature regulation. (Because here we are concerned only with the thermal load relative to comfort conditions, sweating as a means of regulating temperature away from comfort conditions does not enter into the calculation of PMV.)

The total evaporative heat loss from skin cannot be more than what would occur if the body were completely covered by a film of water. This maximum heat loss,  $E_{\text{max}}$ , depends only on the evaporative power of the environment and is given by the *Lewis Relation* (for air):

$$E_{\text{max}} = 2.2h_{\text{c}}F_{\text{pel}}(P_{\text{s}} - P_{\text{d}}) \tag{A6}$$

The three mechanisms are explained in the following paragraphs.

#### Sweating

Fanger has found that sedentary individuals do not sweat when comfortable, but that individuals who engage in activity will sweat to remain comfortable. He uses the following expression to denote the amount of sweating caused by activity:

$$E_{\rm sw} = 0.42M_{\rm o}(m-1) \tag{A7}$$

(This term does not, of course, contribute for m < 1.)

#### Diffusion

The heat loss due to diffusion is equal to 6% of  $E_{\rm max}$  times the fraction of the skin that is not covered by water. (See, for example, ASHRAE [17], Chap. 8, pp. 8.3 - 8.4.) Since the fraction of skin that is covered by water can be approximated by the ratio of  $E_{\rm sw}$  to  $E_{\rm max}$ , we can write the diffusion term as follows:

$$E_{\rm diff} = 0.06(E_{\rm max} - E_{\rm sw}) \tag{A8}$$

(This term does not contribute for  $E_{\rm sw} > E_{\rm max}$ .)

#### Respired heat loss

Respiration causes two forms of heat exchange with the environment: dry heat loss and latent heat loss. This heat-loss mechanism is generally not important unless the temperature is quite low and the person is heavily

clothed. We use the following expression to approximate the total respired heat loss:

$$E_{\text{res}} = mM_{\text{o}}[0.0014(T_{\text{s}} - T_{\text{a}}) + 0.0024(P_{\text{s}} - P_{\text{d}})]$$
 (A9)

The first term represents the dry heat loss and the second the latent loss.

Substitution of dew point for vapor pressure

Before totaling the load, we will make one more simplification: we will replace the terms that depend on vapor pressure with ones that depend on dew point. Some authors have used a linearized expression for dew point as a function of vapor pressure:

$$P_{\rm s} - P_{\rm d} = 1.92(T_{\rm s} - T_{\rm d})$$
 (A10.1)

This expression is accurate in the 25 °C to 35 °C range, but begins to deviate sharply for dew points below 20 °C. Because dew points below 20 °C will be important in most instances, we have decided to use a more accurate quadratic expression to relate dew point to vapor pressure; we have used an exact calculation of vapor pressure and dew point over the normal range of skin temperatures to generate an empirical relationship. The expression we use for  $P_{\rm s}$  is as follows:

$$P_{\rm s} - P_{\rm d} = \frac{T_{\rm s}^2 - T_{\rm d}^2}{T_{\rm c}} \tag{A10.2}$$

This expression has a maximum error of less than 1 torr in the range 0 - 40 °C, which corresponds to a mean scatter of about 0.5 °C in dew point. Below a dew point of 0 °C the dew point has little effect on the vaporpressure difference and, hence, on comfort, and we shall ignore its effect in this range. Thus, for any dew point below 0 °C the term containing the dew point should be discarded.

#### SUMMARY

We can now rewrite the equation for the load in terms of the environmental parameters  $(T_a, T_r, T_d, \text{ and } v)$  and the intrinsic parameters  $(m \text{ and } I_{cle})$ .

$$L = mM_0$$
 generation 
$$-0.42M_0(m-1)$$
 sweating

$$\begin{split} -mM_{\rm o} \bigg[ & 0.0014(T_{\rm s}-T_{\rm a}) + \\ & + 0.0024 \, \frac{{T_{\rm s}}^2 - {T_{\rm d}}^2}{T_{\rm s}} \bigg] \qquad \text{respiration} \\ -0.132 F_{\rm pcl} h_{\rm c} ({T_{\rm s}}^2 - {T_{\rm d}}^2) - \\ & - 0.02 M_{\rm o} (m-1) \qquad \text{diffusion} \\ -F_{\rm cle} h_{\rm c} (T_{\rm s} - T_{\rm a}) \qquad \text{convection} \\ -F_{\rm cle} h_{\rm r} (T_{\rm s} - T_{\rm r}) \qquad \qquad \text{radiation} \end{split}$$

Equivalently, the predicted mean vote can be calculated from its definition. The equation below combines several of the terms above into a set of comfort coefficients:

$$Y = Y_{\rm o} + Y_{\rm r} \frac{T_{\rm r}}{T_{\rm s}} + Y_{\rm c} \frac{T_{\rm a}}{T_{\rm s}} + Y_{\rm e} \frac{T_{\rm d}^2}{T_{\rm s}^2}$$
 (A12)

The definitions of these quantities, including the comfort coefficients  $(Y_o, Y_r, Y_c, Y_e)$ , are given in Appendix B.

## Appendix B

#### LIST OF DEFINITIONS

## Basic definitions

Skin temperature

Fanger has suggested that in the comfort range the skin temperature,  $T_s$ , is only a function of the activity level, m

$$T_{\rm s} = 35.7 - 2.16m \tag{B1}$$

## Convective heat transfer coefficient

The convective heat-transfer coefficient,  $h_{\rm c}$ , can be dominated either by air speed or by thermal buoyancy. The larger of the two equations, as defined by Gagge [18], should be used

$$h_{c} = 8.3v^{0.53} \tag{B2.1}$$

$$h_c = 5.66(m - 0.85)^{0.39}$$
 (B2.2)

subject to a minimum value.

$$h_{\rm c} = 2.9$$
 (B2.3)

Note that very similar forms for the wind-dominated convection coefficient have been found by others [19, 20].

## Radiative heat-transfer coefficient

We have used a linearized form of the radiation equations, thus implying that the heat-transfer coefficient will depend on the surface temperature of the body. However, for the normal range of environmental conditions we can assume a constant value of  $4.7 \, (\text{W/m}^2\,^{\circ}\text{C})$  for the coefficient:

$$h_{\rm r} = 4.7 \tag{B3}$$

## Effective thermal efficiency of clothing

The effective thermal efficiency of clothing is a measure of the effectiveness of clothing in insulating the skin surface from heat exchange:

$$F_{\rm cle} = \frac{1 + 0.23I_{\rm cle}}{1 + 0.178I_{\rm cle}(h_{\rm c} + h_{\rm r})}$$
 (B4)

## Permeation efficiency of clothing

The permeation efficiency of clothing is a measure of its ability to allow the transfer of moisture from the skin surface:

$$F_{\rm pcl} = \frac{1}{1 + 0.143 I_{\rm cle} h_{\rm c}}$$
 (B5)

## Clo value

The insulation value of clothing is given in units of clo; one clo is equal to  $0.155\,\mathrm{m}^2\,^\circ\mathrm{C/W}$ . Clo values are usually quoted as either a basic clo value,  $I_{\rm cl}$ , or an effective clo value,  $I_{\rm cle}$ . The average relationship between these two values is as follows:

$$I_{\rm cl} = 1.16I_{\rm cle} \tag{B6}$$

## Metabolic rate

The activity level of the body is given in units of met, m; one met is equal to the basic metabolic rate,  $M_o$ , and has a value of 58.1 W/m<sup>2</sup>.

$$M_{\rm o} = 58.1$$
 (B7)

This value incorporates a mechanical efficiency of work (i.e., the amount of body energy that is converted into useful work) which, for the activities considered here is a very small effect.

## Comfort coefficients

The comfort coefficients as used in this paper are defined as follows:

$$Y_{\rm r} = \left[ F_{\rm cle} \frac{h_{\rm r} T_{\rm s}}{M_{\rm o}} \right] \times [1.6 + 17.6 \exp(-2.1 \, m)]$$
(B8.1)

Convective comfort coefficient:

$$Y_{\rm c} = \left[0.0014T_{\rm s}m + F_{\rm cle}\frac{h_{\rm c}T_{\rm s}}{M_{\rm o}}\right] \times \\ \times \left[1.6 + 17.6 \exp(-2.1 m)\right]$$
 (B8.2)

Evaporative comfort coefficient:

$$Y_{e} = \left[0.0024T_{s}m + 0.132F_{pcl} \frac{h_{c}T_{s}}{M_{o}}\right] \times$$

$$\times [1.6 + 17.6 \exp(-2.1 m)]$$
 (B8.3)

Basic comfort coefficient:

$$Y_o = [0.4 + 0.6m] \times \times [1.6 + 17.6 \exp(-2.1m) - -Y_r - Y_c - Y_c]$$
 (B8.4)

For convenience in calculating effective temperature, we have defined the total comfort coefficient as follows:

$$Y_{t} = Y_{r} + Y_{c} + Y_{e} \tag{B8.5}$$

#### Standard conditions\*

In order to define an effective temperature or a comfort temperature we must define a set of standard conditions to which the actual conditions must be corrected. In our nomenclature, a prime indicates that the quantity is in the standard condition, which is defined as follows:

zero air speed,

$$v' = 0 \tag{B9.1}$$

air temperature equal to the effective temperature,

$$T_a' = T_e \tag{B9.2}$$

mean radiant temperature equal to the effective temperature,

$$T_{\rm r}' = T_{\rm e} \tag{B9.3}$$

standardized dew point,

$$(T_{\rm d}')^2 = T_{\rm s} \left( T_{\rm e} - \frac{1}{2} T_{\rm s} \right)$$
 (B9.4)

We empirically developed this equation to approximate a 50% relative humidity over the range of interest. For effective temperatures between 15 °C and 30 °C, this assumption causes no more than a 10% difference in the evaporative heat transfer when compared to an exact calculation assuming 50% relative humidity.

The standard comfort coefficients  $(Y_s)$  are calculated using the same formulae as the non-primed versions except that the low airspeed value of the convection coefficient is used. Thus these comfort coefficients represent the comfort coefficients that would exist under the standard conditions described above.

<sup>\*</sup>We have not assumed a standard value for the clo and met values; therefore, our comfort coefficients will depend on the actual values of the personal variables.

