ERRORS RESULTING FROM
THE USE OF SINGLE ZONE VENTILATION MODELS ON MULTI-ZONE BUILDINGS:
IMPLICATIONS FOR ENERGY CONSERVATION AND INDOOR AIR QUALITY STUDIES

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ABSTRACT

Errors resulting from treating a house as an enclosure surrounding a single, well-mixed volume of air are explored in detail for a ranch house with a basement. A fairly typical ventilation pattern is assumed and three quantities, the air exchange rate, the indoor pollutant concentration from a given emission, and the energy required to heat infiltrating air, are calculated and compared using both the one and two zone models for this house. In general, the errors were around 10-20% if the basement was included in the one zone models and 30-40% if the basement was neglected. Other factors that affect the magnitude of these errors include the length of a pollutant release, the outdoor temperature and the air exchange rate measurement protocol as well as the particular ventilation characteristics of the house.

Keywords: Air-Leakage, Convection, Heat-Transfer, Mass-Transfer, Modeling, Pollution, Ventilation.
INTRODUCTION

The energy price hikes of the 1970s caused many homeowners to search for methods that could be used to decrease their heating costs. Many found that reducing the amount of air entering their house by weatherstripping and crack sealing provided an easy first step. Builders found that plastic vapor barriers and tight sealing windows and doors were easy to incorporate into their existing designs and added to the attractiveness of their homes to buyers. The net result is that the average home in the colder climates is much tighter today than it was ten years ago.

Recent studies have shown, however, that the reduction in ventilation that has resulted from these tightening efforts has led to higher indoor pollutant levels. Indoor concentrations of pollutants such as radon, nitrogen dioxide, carbon monoxide and formaldehyde that are 10-100 times greater than outdoor levels are now commonly measured (Spengler and Sexton 1983). Combined with the results of time-budget surveys showing the average person in Europe or the U.S. spending 90% of their time indoors (National Research Council 1981), these elevated indoor pollutant levels can be seen to pose a health risk that far exceeds that for outdoor air pollution.

These concerns have led many countries to adopt or to consider adopting a minimum ventilation standard for newly constructed housing (Thompson 1984). The standards will rely heavily on the accuracy of air infiltration measurements and on mass and heat transfer models estimating the two basic quantities of interest, namely, the energy consumption due to heating infiltrating air and the concentration of an indoor pollutant given an indoor emission source.
In this paper we look at one basic assumption that is commonly made when making infiltration measurements in homes: that one can treat a home as an enclosure around a single, well-mixed volume of air. In order to illustrate the errors that may result from such an assumption, we use the simplest example, a ranch house with a basement, and compare the one and two zone models for this house using typical values for the infiltration, exfiltration and interzonal mixing rates for each zone.

Finally, we look at the effect of the single zone assumption on air exchange measurements using the tracer decay technique, on estimates of energy consumption due to air infiltration and on calculated indoor pollutant concentrations from emissions data.
RESULTS AND DISCUSSION

The equations which describe the mass balance within interconnected, well mixed enclosures can be found in many texts on transport theory. For convenience, these equations are listed in the appendix along with a few relevant examples of their use.

The mass transport equations allow us to calculate the errors that would be made if a one zone model was used on a two zone house. We will concentrate on three quantities of interest: (1) the air exchange rate, defined as the total volume flow rate of air entering a home divided by the total volume of the house, (2) the energy required to heat infiltrating air, and (3) the concentration of an indoor pollutant resulting from a constant emission source.

In order to get a feeling for the magnitude of the errors, we will use the house illustrated in figure 1. Results obtained using the dual tracer technique discussed in the appendix indicate that the air flow rates shown in figure 1 are probably typical of a ranch house with a basement when the outdoor temperature drops below 40°F.

When using the one zone model on this two zone house, we consider two likely scenarios. In Case A the basement of the house is totally ignored. The house volume and temperature are considered to be those of the living area alone, and the tracer is not mixed into the basement before decay measurements are made to determine the house air exchange rate.

In Case B the living area and basement are considered a single zone. The house volume is the sum of the living area and basement volumes and the house temperature is the average of those in the living area and basement.
To measure the air exchange rate, the door between the basement and living area is opened and a tracer is distributed uniformly throughout both zones. Then the basement door is closed and measurements of the concentration decay begin. The above assumptions for Case A and Case B are listed in Table I.

**Tracer Decay in a Two Zone House**

We can calculate the tracer concentration profiles in each zone by using the appendix equations A3a and A3b with \( s_{11} = s_{12} = 0 \). For Case A, with the boundary conditions \( C_{11} = 0 \) and \( C_{12} = C_{12}^o \) at \( t = 0 \)

\[
C_{11} = \frac{C_{12}^o \tau_{21}}{\lambda_1 - \lambda_2} [e^{-\lambda_1 t} - e^{-\lambda_2 t}] 
\]

(1a)

\[
C_{12} = \frac{C_{12}^o}{\lambda_1 - \lambda_2} [(\lambda_1 + r_1)e^{-\lambda_1 t} - (\lambda_2 + r_1)e^{-\lambda_2 t}] 
\]

(1b)

where

\[
\lambda_1 = \frac{r_1 + r_2}{2} - \sqrt{\frac{(r_1 - r_2)^2}{4} + r_1 r_2 \tau_{21}} 
\]

(2a)

\[
\lambda_2 = \frac{r_1 + r_2}{2} + \sqrt{\frac{(r_1 - r_2)^2}{4} + r_1 r_2 \tau_{21}} 
\]

(2b)

For Case B, \( C_{11} = C_{12} = C_{12}^o \) at \( t = 0 \). This results in

\[
C_{11} = \frac{C_{12}^o}{\lambda_1 - \lambda_2} [(\lambda_1 + r_2 + r_{21})e^{-\lambda_1 t} - (\lambda_2 + r_2 + r_{21})e^{-\lambda_2 t}] 
\]

(3a)

\[
C_{12} = \frac{C_{12}^o}{\lambda_1 - \lambda_2} [(\lambda_1 + r_1 + r_{12})e^{-\lambda_1 t} - (\lambda_2 + r_1 + r_{12})e^{-\lambda_2 t}] 
\]

(3b)
Substituting the values shown in figure 1 we find for Case A

\[
\frac{C_{11}}{C_{12}} = 0.313 \left( e^{-0.330t} - e^{-0.970t} \right)
\]  

(4a)

\[
\frac{C_{12}}{C_{12}^0} = 0.422e^{-0.330t} + 0.578e^{-0.970t}
\]

(4b)

and for Case B

\[
\frac{C_{11}}{C_{12}^0} = 0.891e^{-0.330t} + 0.109e^{-0.970t}
\]  

(5a)

\[
\frac{C_{12}}{C_{12}^0} = 1.203e^{-0.330t} - 0.203e^{-0.970t}
\]

(5b)

Equations 4a and 4b are plotted in figure 2. As noted by other authors (Hernandez and Ring 1982; Sinden 1978), this type of plot can be characterized by two distinct time segments: a transient period (~ hours 0-4 in our example house) in which the ratio \(C_{12}/C_{11}\) adjusts to its equilibrium value and a dominant period (>4 hours) where the concentration-profiles are dominated by a single exponential term yielding a constant value for \(C_{12}/C_{11}\). From equations 1 and 4 one can easily show that the value of \(U_{12}/C_{11}\), as \(t\) gets large, is \((\lambda_1 + x_1)/x_{21} = 1.35\).

The results from the living area \(C_{12}\) are replotted in semilogarithmic form in figure 3. As indicated in the appendix, the one zone model predicts that the house air exchange rate is the negative of the slope obtained from this type of plot. In the two zone case, the decay is the sum of two exponential terms. At long times, one of these terms becomes insignificant and the semilogarithmic plot becomes linear with a slope of \(-\lambda_1\).
In many cases $\lambda_1$ is close but not equal to the air exchange rate ($a_2$) for a two zone house where $a_2$ is defined as $a_2 = \frac{(R_{E1} + R_{E2})}{(V_1 + V_2)}$. The two values ($\lambda_1$ and $a_2$) will be equal only if the two zone volumes ($V_1$ and $V_2$) and the two zonal exfiltration rates ($R_{E1}$ and $R_{E2}$) are equal. Otherwise some disagreement will exist. For our model ranch house, $a_2 = 0.30 \text{ h}^{-1}$ while $\lambda_1 = 0.33 \text{ h}^{-1}$, a difference of 10%. Thus, one may be able to use the long term slope from a semilogarithmic tracer decay as a rough approximation of the air exchange rate in a multi-zone building.

In practice, how close one comes to the long term slope will be determined by the time period over which the slope is averaged. Table 2 lists average slopes obtained from various segments of figure 3. The computed slopes are the results of least square fits over the given time period with a sampling frequency of one point per hour. The tracer injection is considered time zero and instantaneous mixing is assumed.

The results in table 2 indicate that the one zone assumptions of Case A can lead to large errors in the computed air exchange rate. Even if one assumes -0.33 to be the desired value, errors greater than 50% can be obtained by sampling for times as long as four hours directly after a tracer injection. To get to within 10% of the desired value, one would have to wait about four hours under any practical scenario.

Notice that computed values of $R^2$, the square of the correlation coefficient, are all greater than 0.98 confirming what many researchers have noted in the past: that in this type of plot, the correlation coefficient is a deceptive indicator of linearity. In practice, the degree of non-linearity in the underlying processes is overshadowed by scatter in the data due to non-ideal mixing and analytical error.
The magnitude of the errors can be reduced by using the scenario of Case B. Figure 4 shows the tracer decay in both the basement and living area after both are adjusted to the same initial concentration.

The transient period is now much shorter because the ratio $C_{12}/C_{11}$ starts out much closer to its equilibrium value of 1.35. After 1-2 hours the semilogarithmic decay plot for both zones 1 and 2 become quite linear as can be seen in Figure 5. Repeating the least squares fitting procedure used for Case A on the results obtained for Case B (Table 3) shows that a waiting period of only one hour is necessary to get an air exchange rate that is within 10% of the long term value. Thus the waiting period can be reduced from 4 hours to 1 hour by using the scenario of Case B rather than that of Case A.

Indoor Pollution Concentrations

Indoor pollution levels can result from short term emissions, continuous emissions or some combination of both. Examples of short term indoor pollution sources are cigarettes, gas stoves and shower water laced with radon gas while kerosene heaters and formaldehyde based pressed wood products would be considered continuously emitting sources.

The concentration profile from a short term source can be characterized by a buildup curve and by a decay curve. In the one zone approximation the buildup curve is generated by integrating equation A1 using the boundary condition $C = 0$ at $t = 0$. This results in the solution

$$C = \frac{S}{K_L} \left(1 - e^{-\lambda t}\right)$$

The decay curve is given by equation A2.
In the two zone case we use equations A3a and A3b and assume that the pollutant is released only in the living area (zone 2) so that \( s_{11} = 0 \). For the buildup curve the boundary conditions are \( C_{11} = C_{12} = 0 \) at \( t = 0 \) and the results are

\[
C_{11} = s_{12} \left[ \frac{1}{\lambda_1 \lambda_2} + \frac{e^{-\lambda_1 t}}{\lambda_1 (\lambda_1 - \lambda_2)} - \frac{e^{-\lambda_2 t}}{\lambda_2 (\lambda_1 - \lambda_2)} \right].
\]  
(7a)

\[
C_{12} = s_{12} \left[ \frac{\lambda_1}{\lambda_1 \lambda_2} + \frac{(\lambda_1 - \lambda_2)e^{-\lambda_1 t}}{\lambda_1 (\lambda_1 - \lambda_2)} - \frac{(\lambda_1 - \lambda_2)e^{-\lambda_2 t}}{\lambda_2 (\lambda_1 - \lambda_2)} \right].
\]  
(7b)

where \( \lambda_1 \) and \( \lambda_2 \) are given by equations 2a and 2b. The decay curve can be generated by setting \( s_{12} = 0 \) and using the boundary conditions \( C_{11} = C_{11}^0 \) and \( C_{12} = C_{12}^0 \) at \( t = 0 \). This yields

\[
C_{11} = \frac{1}{\lambda_1 - \lambda_2} \left[ (C_{11}^0 (\lambda_1 + \lambda_2) + C_{12}^0 \tau_{21})e^{-\lambda_1 t}ight. \\
\left. - (C_{11}^0 (\lambda_2 + \lambda_2) + C_{12}^0 \tau_{21})e^{-\lambda_2 t} \right].
\]  
(8a)

\[
C_{12} = \frac{1}{\lambda_1 - \lambda_2} \left[ (C_{12}^0 (\lambda_1 + \lambda_1) + C_{11}^0 \tau_{12})e^{-\lambda_1 t}ight. \\
\left. - (C_{12}^0 (\lambda_2 + \lambda_1) + C_{11}^0 \tau_{12})e^{-\lambda_2 t} \right].
\]  
(8b)

In order to compare the one and two zone models and their effect on calculated pollutant concentrations within a home we will now look at the effect of turning on and off a constant emission source in the home such as a gas stove. Emissions of NO or NO\(_2\) from a well tuned gas stove with two burners on high average approximately \( 1.5 \times 10^5 \) \( \mu L/h \).
We assume that the outdoor and initial indoor NO\textsubscript{x} concentrations are all zero \(C_{11}^0 = C_{12}^0 = 0\), that there are no losses due to surface reactions within the house, and that there is a constant emission \(s_{12} = 1.5 \times 10^5 \text{ µL/h/200 m}^3 = 750 \text{ µL/h/m}^3\). Using the flow specifications of the model ranch house in figure 1, we can now solve equations 7a and 7b to yield

\[
\begin{align*}
C_{11} &= 469 - 710e^{-0.330t} - 241e^{-0.970t} \quad (9a) \\
C_{12} &= 1406 - 959e^{-0.330t} - 447e^{-0.970t} \quad (9b)
\end{align*}
\]

Taking a typical scenario, let's assume that the burners stay on for two hours and then are shut off completely. We can then calculate the concentration decay by using equations 8a and 8b using initial concentrations calculated by substituting \(t = 2\text{ h}\) into equations 9a and 9b. The results are

\[
\begin{align*}
C_{11} &= 344e^{-0.330t} - 207e^{-0.970t} \quad (10a) \\
C_{12} &= 464e^{-0.330t} + 382e^{-0.970t} \quad (10b)
\end{align*}
\]

For the one zone approximation, we can calculate the concentration buildup by using equation 6 with \(R_T = a_1V = 0.33V\) to yield

\[
\begin{align*}
\text{Case A: } C_{12} &= 2273(1 - e^{-0.330t}) \quad (11a) \\
\text{Case B: } C_{12} &= 1136(1 - e^{-0.330t}) \quad (11b)
\end{align*}
\]

With the emissions shut off after 2 hours, the concentration decay given by equation A2 is
Case A: $C_{12} = 1098e^{-330t}$ (12a)
Case B: $C_{12} = 549e^{-330t}$ (12b)

The results of this simulated two hour release are plotted in figure 6 for both the one and two zone models. Neither of the one zone models come very close to predicting the maximum concentration generated by the two zone model. Case A overestimates and Case B underestimates the peak concentration by about 35%. Notice also that the rate of rise and rate of decay of the two zone concentration profiles are substantially faster than for the one zone cases.

If the emission were continuous, as may occur when a kerosene heater is used, then the steady state concentration that results can be predicted by evaluating the buildup equations at $t \to \infty$. This yields: two zone, $C_{12} = 1406$ PPB; one zone, $C_{12} = 2273$ PPB (Case A), $C_{12} = 1136$ PPB (Case B). Thus Case B, which includes the basement volume, more accurately predicts the concentrations resulting from continuous pollutant emissions.

**Energy Consumption Due to Infiltrating Air**

The energy implications of outdoor air entering buildings continues to be an area of concern to both scientists, engineers and consumers. Given the appropriate flow information, the energy required to heat infiltrating air can be calculated in the following manner.

For a two zone house, conservation of energy around each zone at steady-state yields the following two equations.

$$R_{11}HT_0 + R_{21}HT_2 + Q_1 = R_{E1}HT_1 + R_{12}HT_1$$ (13a)

$$R_{12}HT_0 + R_{12}HT_1 + Q_2 = R_{E2}HT_2 + R_{21}HT_2$$ (13b)
where

\[ H_T = \text{enthalpy per unit volume of air at temperature } T \]
\[ T_0 = \text{outdoor temperature} \]
\[ T_1 = \text{zone 1 temperature} \]
\[ T_2 = \text{zone 2 temperature} \]
\[ Q_1 = \text{heat generated in zone 1} \]
\[ Q_2 = \text{heat generated in zone 2} \]

and the flows \( R_{11}, R_{12}, R_{E1}, R_{E2}, R_{12}, \text{ and } R_{21} \) are the same as described in the appendix and shown in figure A1. Here, since we are concerned only with the energy necessary to heat infiltrating air, we have assumed that the zonal boundaries are perfect insulators to conductive and radiational heat transfer.

If we measure the enthalpies in equation 13a relative to \( H_T \) and those in equation 13b relative to \( H_{T2} \) and assume \( \Delta H = C_p \Delta T \), then the above equations can be rewritten as:

\[
Q_1 = C_p [R_{11}(T_1 - T_0) + R_{21}(T_1 - T_2)] \tag{14a}
\]
\[
Q_2 = C_p [R_{12}(T_2 - T_0) + R_{12}(T_2 - T_1)] \tag{14b}
\]

A similar analysis for the one zone case shows that the energy required to heat infiltrating air can be calculated by

\[
Q = C_p R_1(T - T_0) \tag{15}
\]

where for case A (basement ignored), \( T = T_2 \) and \( R_1 \) is based on the volume of the living area only and for case B, \( T = (T_1 + T_2)/2 = T_{AVG} \) and \( R_1 \) is based on the total volume of the house.

The error induced by the one zone assumptions can be seen by taking the ratio of the energy required to heat infiltrating air in the one zone case
to that in the two zone case, i.e. \( Q/(Q_1+Q_2) \). This ratio is plotted as a function of outdoor temperature, \( T_0 \), in figure 7 assuming our example ranch house with an "unheated" basement and zonal temperatures \( T_2 = 68^\circ F \) and \( T_1 = 54^\circ F \).

As can be seen in figure 7, the one zone model and the case B scenario do a very good job at approximating the convective energy losses of this example house, especially when the outdoor temperature is low. The error using the case A scenario is much larger for any outdoor temperature, mainly due to an underestimate of \( R_1 \) resulting from ignoring the basement in the house volume.

Note that since we have assumed no conductive heat transfer through zonal boundaries, this implies that the "unheated" basement is, in fact, being internally heated, presumably from waste heat emitted from the heating system. In this case, any one and two zone heating cost comparison match the comparison of energy input since all the energy input is from the heating system.

There are other circumstances, however, where this comparison would not be the same. For example, what if the basement was constructed so that it was well insulated from the living area, but not from the surrounding soil and that the heating system was insulated to make the waste heat input to the basement negligible. In this case, \( Q_1 \) would be approximately the same, but the heat would come not from the heating system, but from conductive transfer from the surrounding soil. The heat input ratio would still be \( Q/(Q_1+Q_2) \) but the heating cost ratio would be \( Q/Q_2 \), possibly leading to significant errors in estimating heating costs using a one zone model. For our example house insulated in the above manner, the one zone model using the
Case B scenario would overestimate the heating costs by more than 40% at outdoor temperatures below 30°F.
SUMMARY

In this paper we have looked at the practical results of approximating a typical two zone ranch house as one, well mixed zone. The two zone model was compared with two scenarios in which the one zone model might be used. In Case A, the basement was totally ignored in the infiltration measurement and in the calculations while in Case B the basement was included.

In terms of the accuracy of an air exchange rate measurement using the tracer decay technique, we found that Case B does a much better job at approaching the two zone results, especially if the concentration measurements are begun a few hours after the tracer injection. A comparison of the one and two zone theories shows, however, that even under the most ideal measurement protocol, the air exchange rate computed using the one zone technique differs from the true value except when the house has certain flow symmetries. For our model ranch house, the one zone model overpredicts the air exchange rate by 10%.

A comparison of the one and two zone models and their predictions of indoor pollutant concentrations from both continuous and non-continuous emission sources was also presented. It was found that the one zone model overpredicts the pollutant concentrations for Case A while it underpredicts for Case B. Errors ranged from 20-60% depending on the house volume used in the calculation and on the length of the emission. Again, Case B was more accurate in predicting the results of the two zone model.

Calculations, using the one and two zone models, of the energy required to heat infiltrating air in our example ranch house shows that errors resulting from the use of the one zone model and the Case B scenario were less than 15% over a range of outdoor temperatures. Errors using the Case A
scenario were much higher (35-45%). Depending on how the house is insulated, these errors may or may not be the same as the errors in calculating actual heating costs.

It is interesting to note that the heat load and pollutant concentration equations which describe the one zone model errors do not depend linearly on errors in the air exchange rate. In fact, significant errors would result even if there was no error in the air exchange rate. This indicates that an accurate air exchange rate measurement combined with a one zone model can only give a reasonable first approximation to energy conservation and indoor air quality calculations involving multi-zone dwellings.
APPENDIX

One Zone Model

Conservation of mass for a single zone system can be expressed by:
\[ V \frac{dC}{dt} = S + R_I C_A - R_E C \]

where

- \( C \) = concentration of the component of interest within the zone (vol./vol. air)
- \( C_A \) = concentration of the component outside the zone (vol./vol. air)
- \( S \) = emission rate into the zone (vol./time)
- \( R_E \) = air exfiltration rate from the zone (vol. air/time)
- \( R_I \) = air infiltration rate from the zone (vol. air/time)
- \( V \) = volume of the zone
- \( t \) = time

Here we have assumed that all volumes are measured at some standard temperature and pressure and that a uniform concentration exists throughout volume \( V \). For most applications where this model is used, \( C_A \ll C \) and since \( R_E = R_I \),
\[ V \frac{dC}{dt} = S - R_I C \]  \hspace{1cm} (A1)

Equation A1, together with the appropriate boundary conditions, describes how the concentration of a given substance within the zone changes with time.

The measurement of air infiltration by the tracer decay method is an example of how this model is used in practice (Hunt 1980). Here a quantity
of a tracer gas, often SF$_6$, is injected and mixed thoroughly in a home and its concentration decay is monitored over several hours. Referring to equation A1 with $S = 0$ and the boundary condition $C = C_0$ at $t = 0$,

$$C = C_0 e^{-a_1 t}$$  

(A2)

where $a_1 = R_I/V$ is called the air exchange rate. Plotting ln $C$ vs. $t$ should yield a straight line with the slope equal to the negative of the air exchange rate.

The air exchange rate is the value which is most often used as a measure of air infiltration. Usually reported in units of h$^{-1}$, it indicates how many house volumes of air pass through the house in one hour. A typical value for a home in the northern U.S. is 0.3 - 0.5 h$^{-1}$.

**Two Zone Model**

A two zone model for a house is illustrated in figure A1. Here, for a or b equal to 1 or 2,

- $C_{ab}$ = concentration of component a in zone b (vol./vol. air)
- $C_{ab}$ = ambient concentration of component a (vol./vol. air)
- $R_{ab}$ = air transfer rate from zone a to zone b (vol. air/time)
- $R_{Ib}$ = air infiltration rate into zone b (vol. air/time)
- $R_{Eb}$ = air exfiltration rate from zone b (vol. air/time)
- $S_{ab}$ = emission rate of component a into zone b (vol./time)

$V_a = \text{volume of zone a}$

For component 1, a mass balance on each zone yields,

**Zone 1:**

$$V_1 \frac{dC_{11}}{dt} = S_{11} + R_{21}C_{12} + R_{11}C_{1A} - (R_{12} + R_{E1})C_{11}$$

**Zone 2:**

$$V_2 \frac{dC_{12}}{dt} = S_{12} + R_{12}C_{11} + R_{E2}C_{1A} - (R_{21} + R_{E2})C_{12}$$
For component 2,

Zone 1: \( V_1 \frac{dC_{21}}{dt} = S_{21} + R_{21}C_{22} + R_{I1}C_{2A} - (R_{12} + R_{E1})C_{21} \)

Zone 2: \( V_2 \frac{dC_{22}}{dt} = S_{22} + R_{12}C_{21} + R_{I2}C_{2A} - (R_{21} + R_{E2})C_{22} \)

Making the substitutions \( r_{ab} = R_{ab}/V_b, \) \( s_{ab} = S_{ab}/V_b \) and \( r_a = (R_{ab} + R_{Ea})/V_a \) and assuming all \( C_{aA} \) small we find for component 1,

Zone 1: \( \frac{dC_{11}}{dt} = s_{11} + r_{21}C_{12} - r_1C_{11} \) \( \quad \text{(A3a)} \)

Zone 2: \( \frac{dC_{12}}{dt} = s_{12} + r_{12}C_{11} - r_2C_{12} \) \( \quad \text{(A3b)} \)

and for component 2,

Zone 1: \( \frac{dC_{21}}{dt} = s_{21} + r_{21}C_{22} - r_1C_{21} \) \( \quad \text{(A4a)} \)

Zone 2: \( \frac{dC_{22}}{dt} = s_{22} + r_{12}C_{21} - r_2C_{22} \) \( \quad \text{(A4b)} \)

As an example of how this two zone model is used, let's look at how the six air flows, \( R_{E1}, R_{E2}, R_{I1}, R_{I2}, R_{12} \) and \( R_{21} \) can be obtained using a constant emission release of a different tracer gas in each zone (I'Anson et al., 1982; Dietz et al., 1983). Referring to equations A3 and A4, setting \( s_{12} = s_{21} = 0 \) and allowing the system enough time to come to steady state we obtain,
\[ s_{11} + r_{21}C_{12} - r_{1}C_{11} = 0 \]
\[ r_{12}C_{11} - r_{2}C_{12} = 0 \]
\[ r_{21}C_{22} - r_{1}C_{21} = 0 \]
\[ s_{22} + r_{12}C_{21} - r_{2}C_{22} = 0 \]

If the emission rates \( s_{11} \) and \( s_{22} \) are known and if the concentrations \( C_{11}, C_{12}, C_{21}, \) and \( C_{22} \) are measured, then we have a system with four equations and four unknowns \( (r_{1}, r_{2}, r_{12}, \) and \( r_{21}) \). Once these unknowns are obtained,

\[ R_{12} = V_{2}r_{12} \]
\[ R_{21} = V_{1}r_{21} \]
and \[ R_{E1} = V_{1}r_{1} - R_{12} \]
\[ R_{E2} = V_{2}r_{2} - R_{21} \]

Finally a mass balance for each zone yields

\[ R_{11} = R_{E1} + R_{12} - R_{21} \]
\[ R_{12} = R_{E2} + R_{21} - R_{12} \]
REFERENCES


Fig. 1
Fig. 3

The graph shows the concentration ratio $\frac{C_{12}}{C_{12}^0}$ plotted against time (in hours) from 0 to 7 hours. The concentration ratio decreases linearly with time.
Fig. 4
Fig. 5

Graph showing the concentration ratio over time. The graph plots the ratio $\frac{C_{11}}{C_{12}}$ and $\frac{C_{12}}{C_{12}^\circ}$ against time (hr). The concentration ratio decreases over time, indicating a decrease in the concentration of the substances.
Fig. 6

- ONE ZONE, Case A
- TWO ZONE
- ONE ZONE, Case B

NO or NO₂ conc. (ppb)

TIME (hr)
FIGURE CAPTIONS

1. Ventilation characteristics of the ranch house with a basement that is used as an example in the text. All flows are in m³/h.

2. Decay of a tracer in the model ranch house using the Case A measurement protocol (basement neglected). C₁₁ = basement concentration, C₁₂ = living area concentration.

3. Semilogarithmic plot of the living area concentration profile in figure 4.

4. Decay of a tracer in the model ranch house using the Case B measurement protocol (basement included). C₁₁ = basement concentration, C₁₂ = living area concentration.

5. Semilogarithmic plot of the concentration profiles in figure 6.

6. NO or NO₂ concentration profiles resulting from a gas stove left on for two hours in the example ranch house. The emission rate, S = 1.5 \times 10^5 μL/h, is equivalent to that resulting from two burners on high.

7. Q/(Q₁+Q₂) is the ratio of the energy required to heat infiltrating air in the example ranch house using a one zone model to the energy computed using a two zone model.

A1. Schematic for the two zone mass balance model. See text for details.
<table>
<thead>
<tr>
<th>Case</th>
<th>House volume</th>
<th>House temperature</th>
<th>Initial tracer concentration for tracer decay method</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$V_2$</td>
<td>$T_2$</td>
<td>$C_{11} = 0$, $C_{12} = C_{12}^0$</td>
</tr>
<tr>
<td>B</td>
<td>$V_1 + V_2$</td>
<td>$(T_1 + T_2)/2$</td>
<td>$C_{11} = C_{12}^0$, $C_{12} = C_{12}^0$</td>
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</tbody>
</table>
TABLE 2

Slope of the decay curve of figure 3 computed over various time segments. $R^2$ is the square of the correlation coefficient.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Slope</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4</td>
<td>-.518</td>
<td>.993</td>
</tr>
<tr>
<td>0-6</td>
<td>-.462</td>
<td>.988</td>
</tr>
<tr>
<td>0-8</td>
<td>-.425</td>
<td>.987</td>
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<tr>
<td>1-4</td>
<td>-.476</td>
<td>.996</td>
</tr>
<tr>
<td>1-6</td>
<td>-.428</td>
<td>.993</td>
</tr>
<tr>
<td>1-8</td>
<td>-.399</td>
<td>.993</td>
</tr>
<tr>
<td>2-4</td>
<td>-.439</td>
<td>.998</td>
</tr>
<tr>
<td>2-6</td>
<td>-.401</td>
<td>.997</td>
</tr>
<tr>
<td>2-8</td>
<td>-.378</td>
<td>.997</td>
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<tr>
<td>3-6</td>
<td>-.380</td>
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<tr>
<td>3-8</td>
<td>-.363</td>
<td>.999</td>
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<td>3-10</td>
<td>-.353</td>
<td>.999</td>
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<tr>
<td>4-6</td>
<td>-.367</td>
<td>1.000</td>
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<tr>
<td>4-8</td>
<td>-.353</td>
<td>.999</td>
</tr>
<tr>
<td>4-10</td>
<td>-.345</td>
<td>1.000</td>
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</table>
TABLE 8

Slopes of the decay curve of figure 5 (Zone 2) computed over various time segments. $R^2$ is the square of the correlation coefficient.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Slope</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4</td>
<td>-0.389</td>
<td>.997</td>
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<tr>
<td>0-6</td>
<td>-0.304</td>
<td>.998</td>
</tr>
<tr>
<td>0-8</td>
<td>-0.312</td>
<td>.998</td>
</tr>
<tr>
<td>1-4</td>
<td>-0.304</td>
<td>.999</td>
</tr>
<tr>
<td>1-6</td>
<td>-0.314</td>
<td>.999</td>
</tr>
<tr>
<td>1-8</td>
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<td>1.000</td>
</tr>
<tr>
<td>2-4</td>
<td>-0.313</td>
<td>1.000</td>
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<tr>
<td>2-6</td>
<td>-0.320</td>
<td>1.000</td>
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<tr>
<td>2-8</td>
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<tr>
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<tr>
<td>5-8</td>
<td>-0.321</td>
<td>1.000</td>
</tr>
</tbody>
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