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Steady-State One-Dimensional Water Vapor Movement by Diffusion and Convection in a Multilayered Wall

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ABSTRACT

Current moisture analysis methods for walls ignore air leakage effects or are not directly applicable to multilayered walls. Mathematical equations were developed for water vapor flow, vapor pressures, and moisture accumulation under steady-state conditions with homogeneous one-dimensional airflow through a multilayered wall. A computer method was developed on the basis of these equations. With this method, a better qualitative understanding may be gained of the effects of air leakage on moisture accumulation.

Sample analyses show that significant moisture accumulation should not be expected in nonhygroscopic cavity insulation, with or without air leakage. The location of condensation may change with changing air leakage rates. Diffusion vapor flow and temperatures are affected by air leakage. Consequently, diffusion and convection may not be treated as independent and additive. Doing so may lead to significant overestimates of moisture accumulation rates. Although the practical use of this method is limited, it is a step toward the development of more comprehensive and realistic methods of moisture analysis.

The information presented should be useful to engineers, researchers, and others interested in building performance.

INTRODUCTION

Recent changes in building techniques have created conditions that could lead to increased moisture migration and condensation in walls. These changes include increased thickness of insulation, a sealed building envelope for reduced air infiltration, use of foil-backed and low-permeance sheathing, and increased use of panelized wood, aluminum, or plastic siding. Resulting condensation in the wall could result in lower R values, decay, paintpeeling, or the bleeding of extractives from the back of the siding onto the face of the siding. With this potential for damage to the structural and aesthetic integrity of wood frame and other buildings, it is important to know the conditions leading to moisture accumulation in walls. Measurements of moisture accumulation in high thermal performance walls have been made (Sherwood 1983), but the results are difficult to extrapolate to other conditions and wall constructions without an adequate method of moisture analysis.

Although it is generally recognized (ASHRAE 1977; Sherwood 1983; Sherwood and Tenwolde 1981) that water vapor transport through walls often takes place by a combination of air convection and vapor diffusion, most available methods to predict this flow and the potential for condensation are limited to vapor diffusion. The moisture profile method, as described in the ASHRAE Handbook-1977 Fundamentals (ASHRAE 1977), and the Kieper method (Kieper et al. 1976; Sherwood and TenWolde 1981; TenWolde 1982) do not include air convection effects. Others have proposed to calculate the aggregate effect by adding the diffusion and convection components (Burch et al. 1979; Stewart 1981). As will be shown in this paper, vapor diffusion rates are greatly affected by air leakage and a simple summation of the

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effects may lead to significant overestimates of moisture accumulation. Air leakage may also change the location at which moisture accumulates. The assumption that this location is unaffected further reduces the reliability of the summation approach.

Ogniewicz and Tien (1981) analyzed moisture accumulation in insulation materials, allowing for simultaneous heat conduction, vapor diffusion, and convection, but their analysis is not easily adapted to multilayered construction. Condensation in permeable cavity insulation requires conditions unlikely to occur in a multilayered wall. Instead, condensation is far more likely to occur on one or more of the interfaces between two different building materials.

The method of computer analysis presented here is based on the simplifying assumption that condensation only takes place on surfaces between prespecified material layers. A set of non-dimensional variables, x and y, is used to describe the steady-state heat and vapor flow in a multilayered wall, which greatly reduces the mathematical complexity of the flow equations. This approach was first introduced by Kieper et al. (1976) to describe vapor diffusion and is adapted here to include air convection effects. The analytical program, MOISTWALL-2, selects the most likely location(s) for condensation, includes the effect of latent heat release, and is suited for analysis of winter heating and summer cooling conditions.

This report contains the theoretical development of the equations that constitute the basis for this method, a short description of the algorithm of MOISTWALL-2, and two application examples.

THEORY

In this section, equations are developed for temperatures, vapor pressures, total vapor flow, and moisture accumulation in a multilayered wall with air movement through the wall. Equations for temperature can be derived from one-dimensional steady-state differential equations for conductive-convective heat flow. Similarly, vapor pressures may be determined from vapor transport differential equations. If condensation occurs, vapor pressures in the wall are lowered because vapor is removed from the air by condensation. Latent heat release also raises temperatures. However, this rise in temperature tends to lower the rate at which moisture accumulates.

Heat Transport

The effect of airflow on temperatures in a single layer of permeable insulation has been investigated by Berlad et al. (1980). They derived temperature equations for exfiltrative and infiltrative airflow from the steady-state energy conservation equation:

$$\rho c_p v \frac{dt}{d\ell} = k \frac{d^2 t}{d\ell^2} \tag{1}$$

where

ρ = density of air,
c_p = specific heat of air,
k = thermal conductivity of insulation,
ℓ = distance from the inside boundary of the insulation,
t = temperature,
v = velocity of air flowing from inside to outside.

With air flowing from the inside to the outside (exfiltration), v has a positive value, and with airflow from outside to inside (infiltration), v is negative. Equation 1 only applies to a single layer of homogeneous material. However, it is possible to use a similar differential equation for a multilayered wall by substituting a thermal parameter, x, for distance, ℓ. Kieper et al. (1976) introduced this parameter, which greatly simplifies energy balance equations for multilayered walls. Parameter x represents a fraction of the total thermal resistance of the wall and is most easily defined at the interface between layers:

$$x_n = \frac{1}{R_t} \sum_{i=1}^{n} R_{t,i} \tag{2}$$

where

x_n = value of x at interface between layer n and the next layer,
R_t = thermal resistance of wall,
$R_{t,i}$ = thermal resistance of layer i.

Within layer n, x can be defined as

$$x = x_{n-1} + \frac{D}{k_n R_t} \tag{3}$$

where

D = distance from interface between layer n-1 and layer n,
k_n = thermal conductivity of layer n.

Values of x are between 0 and 1, with 0 indicating the inside boundary and 1 the outside boundary of the system.

If no air movement takes place, temperatures in the wall are a linear function of x:

$$t = t_i - x(t_i - t_o) \tag{4}$$

where

t_i = indoor temperature,
t_o = outdoor temperature.

With a uniform airflow through all materials of the wall, the temperature in layer n is defined by Equation 1,

$$\rho c_p v \frac{dt}{dD} = k_n \frac{d^2t}{dD^2} \tag{5}$$

Equation 3 shows that

$$dD = k_n R_t dx \tag{6}$$

Substitution in Equation 5 yields

$$\rho c_p v \frac{dt}{dx} = \frac{1}{R_t} \frac{d^2t}{dx^2} \tag{7}$$

In Equation 7, the thermal conductivity specific to layer n is entirely contained in the definition of x. Equation 7 applies uniformly across the wall and can be solved analytically. Equation 7 is quite similar to Equation 1 and has a similar solution. The solution of differential Equation 7 is

$$t = t_i - (t_i - t_o) \frac{e^{Ax} - 1}{e^A - 1} \tag{8}$$

where

$A = \rho c_p v R_t$.

With exfiltrative airflow, v, and therefore A, is positive. With infiltrative airflow, A has a negative value.

Water Vapor Transport

If we assume that vapor pressures, p, are proportional to humidity ratios* W (W = cp), equations for steady-state vapor flow are very similar to those for heat flow. For a single layer of material with airflow, the governing steady-state equation is

$$\rho c v \frac{dp}{d\ell} = \mu \frac{d^2p}{d\ell^2} \quad (9)$$

where

ρ = density of air,
c = W/p = constant (W = humidity ratio),
μ = water vapor permeability,
p = water vapor pressure,
v = velocity of air flowing from inside to outside.

Similar to the thermal parameter, x, a vapor flow parameter, y, may be substituted for distance ℓ to describe vapor diffusion in multilayered walls. This substitution was first proposed by Kieper et al. (1976). Similar to Equation 3, y is defined as

$$y = y_{n-1} + \frac{D}{\mu_n R_v} \quad (10)$$

where

D = distance from interface between layer n-1 and layer n,
μ_n = permeability of layer n,
R_v = total vapor diffusion resistance of wall,
y_{n-1} = value of y at interface between layer n-1 and layer n.

At the inside boundary of the wall system, y = 0, and at the outside, y = 1. Values for points within the wall fall between 0 and 1.

Without air movement, vapor moves by diffusion only, and vapor pressures in the wall are a linear function of y:

$$p_{dif.} = p_i - y(p_i - p_o) \quad (11)$$

where

p_i = indoor vapor pressure,
p_o = outdoor vapor pressure.

Equations 4 and 11 form the basis for the Kieper moisture analysis method for walls (Kieper et al. 1976, TenWolde 1982).

With simultaneous diffusion and air movement, conservation of mass gives

$$\rho c v \frac{dp}{dy} = \frac{1}{R_v} \frac{d^2p}{dy^2} \quad (12)$$

This differential equation has a solution similar to Equation 8 for temperature:

$$p = p_i - (p_i - p_o) \frac{e^{By} - 1}{e^B - 1} \quad (13)$$

*Humidity ratio is the ratio of the mass of the water vapor to the mass of dry air contained in a moist air sample.

where

$B = \rho c v R_v$.

With exfiltrative airflow, B has a positive value, and with infiltrative airflow, B is negative.

Total vapor flow toward the outside can be expressed as

$$w_H = -\frac{1}{R_v}\frac{dp}{dy} + \rho c v p \tag{14}$$

The first term on the right-hand side of Equation 14 represents the contribution of diffusion to the total vapor flow and the second term, the convective contribution. With continuity of flow (no condensation or evaporation inside the wall) the vapor flow from inside to outside is

$$w_{H,dif.} = \frac{p_i - p_o}{R_v} \tag{15}$$

for diffusion without airflow.

Application of Equation 14 to Equation 13 yields the vapor flow for combined diffusion and convection:

$$w_H = \frac{B}{R_v}\left(\frac{p_i - p_o}{e^B - 1} + p_i\right) \tag{16}$$

Condensation and Evaporation

Equations 11 and 13 through 16 are only valid for continuity of flow, i.e., moisture is not accumulating or evaporating anywhere in the wall. Condensation in a multilayered wall is most likely to occur at an interface between two different materials (Sherwood and TenWolde 1981; TenWolde 1982). If condensation takes place at a surface with y-coordinate y_s, vapor pressure at $y = y_s$ must be at saturation p_s. If condensation only occurs at $y = y_s$, the condensing surface divides the wall in two halves in which continuity of flow exists (Figure 1). Vapor flow to and from the condensing surface can be calculated from the vapor pressure profile in each half. The moisture accumulation rate is the difference between these two flows. The accumulation rate for diffusion is

$$w_{dif.} = \frac{1}{R_v}\,\frac{p_i - p_s - y_s(p_i - p_o)}{y_s(1 - y_s)} \tag{17}$$

With air movement the accumulation rate is

$$w = \frac{B}{R_v}\left(\frac{(p_i - p_s)e^{By_s}}{e^{By_s} - 1} - \frac{(p_s - p_o)}{e^{B(1 - y_s)} - 1}\right) \tag{18}$$

The derivation of these equations can be found in the appendix. If Equations 17 or 18 yield negative values for w and liquid moisture is present at that location, drying conditions exist and w represents the rate of evaporation at location (x_s, y_s).

Although condensation at one location often eliminates the potential for condensation at other locations, simultaneous condensation at two or more surfaces may occasionally occur. The vapor pressure at each of these surfaces is at saturation, and the condensing surfaces separate the sections of the wall in which continuous vapor flow may be assumed (Figure 2). Equations for moisture accumulation at each surface can be found in the appendix.

Latent Heat

So far, no assumptions have been made about the temperature and saturation pressure at the condensing surface. If the effect of the latent heat released by the condensing water vapor is ignored, the temperatures can be calculated with Equations 4 or 8. Although neglecting latent heat release is often permissible, the latent heat release sometimes has a significant effect on the temperature profile. The heat released is

$$q_L = c_L w \tag{19}$$

where

q_L = latent heat released,
w = moisture accumulation rate,
c_L = latent heat of vaporization or sublimation.

This causes the temperature at the condensing surface to rise, which increases the heat flow from the condensing surface to the cold side and reduces the heat flow from the warm side into the wall. Under steady-state conditions, the latent heat release is the difference between these two heat flows. The relationship between latent heat release and temperature t_s at the condensing surface $x = x_s$ for a single point of condensation is

$$c_L w_{dif.} = \frac{1}{R_t} \frac{t_s - t_i + x_s(t_i - t_o)}{x_s(1 - x_s)} \tag{20}$$

for diffusion, and

$$c_L w = \frac{A}{R_t}\left(\frac{t_s - t_o}{e^{A(1 - x_s)} - 1} - \frac{(t_i - t_s)e^{Ax_s}}{e^{Ax_s} - 1} \right) \tag{21}$$

for combined diffusion and convection. The derivations can be found in the appendix. With Equations 4 and 8, the increase in temperature Δt_L attributable to latent heat can be expressed. For diffusion this increase is

$$\Delta t_{L,dif.} = c_L w_{dif.} R_t x_s (1 - x_s) \tag{22}$$

The increase for combined diffusion and convection is

$$\Delta t_L = \frac{R_t c_L w}{A} \frac{\left(e^{x_s A} - 1\right)\left(e^{(1 - x_s)A} - 1\right)}{(e^A - 1)} \tag{23}$$

Equations 22 and 23 cannot be solved analytically because w depends on the saturation pressure p_s at $x = x_s$, which is a nonlinear function of t_s. An increase in temperature (Δt_L) leads to an increase in saturation pressure, causing a decrease in the moisture accumulation rate. This in turn decreases the magnitude of Δt_L. Thus, latent heat release tends to moderate the rate of moisture accumulation, especially when condensation takes place near the "thermal center" of the wall (i.e., x_s is close to 1/2). In the more usual case, the condensing surface is closer to the cold side of the wall (i.e., x_s is close to 1 during the heating season or close to 0 when the house is air-conditioned), and the influence of latent heat release is limited or even negligible.

Equations 22 and 23 equally apply to evaporation in case moisture is present and drying conditions exist. Of course heat is being absorbed during evaporation, and the value of Δt_L would be negative.

Equations 22 and 23 are for a single condensing or evaporating surface. The more general formulas for multiple locations can be found in the appendix.

THE MOISTWALL-2 COMPUTER PROGRAM

This program calculates temperatures, vapor pressures, relative humidities, and moisture accumulation rates in a multilayered wall, assuming uniform one-dimensional heat and vapor flow by diffusion and convection. The algorithm is based on the equations described previously. Figure 3 is a flow diagram of the program. Calculations are only made for the interfaces between the specified layers, and continuous flow of heat and vapor is assumed to exist through each layer. The user may subdivide any layer into several thinner layers of the same material. This can be especially useful for thick layers such as cavity insulation. The program allows up to 20 separate layers.

The program is written in FORTRAN and currently runs interactively on the central mainframe computer at the University of Wisconsin in Madison. After specifying indoor and outdoor temperatures and relative humidities, the user is asked to define the wall design by entering the thermal and vapor flow resistance of each layer. Finally the airflow velocity and direction are specified.

The program first calculates the x and y parameter values for each interface. Then it proceeds by identifying the interface with the highest moisture accumulation rate. If no positive accumulation is found at any interface, the rest of the program is skipped and output values printed if so desired. If condensation takes place, the accumulation rate and temperature are adjusted for latent heat release. Because condensation affects vapor pressures and, to a lesser extent, temperatures elsewhere in the wall, recalculation is necessary. The program then searches for additional locations where condensation may be occurring simultaneously. The program is able to handle one additional location on each side of the initially identified surface, enabling it to accurately analyze up to three simultaneous locations with moisture accumulation. If a second or third location is found, adjustment for latent heat is made, and moisture accumulation at other previously identified points is recalculated until the convergence criterion is satisfied.

Specific values of physical parameters used in the program are: specific heat of air, c_p = 0.24 Btu/lb· F (1.6 kJ/kg·°C); constant, c = 145 grains/lb·in Hg (6.14 Pa^{-1}); latent heat of vaporization, c_L = 0.15 Btu/grain (2,400 kJ/kg); latent heat of sublimation, c_L = 0.17 Btu/grain (2,800 kJ/kg).

Air density ρ is sensitive to temperature. In all equations ρ appears in the product ρv, the air mass flow, which is constant throughout the wall. Both ρ and v are therefore defined at the inside temperature, i.e., at the inside surface of the wall, and ρ is calculated with

$$\rho = 353.05/T_i \text{ kg/m3} \qquad (24)$$

where

T_i is the inside air temperature expressed in K.

APPLICATION

The following examples demonstrate the MOISTWALL-2 program capabilities and show the interaction between vapor diffusion, air leakage, material properties, and wall design. The first example demonstrates how vapor pressures, temperatures, and the location and rate of moisture accumulation vary with air leakage in a single layer of insulation. The second example, with a complete wall, shows the difference in accumulation rates as calculated with MOISTWALL and rates calculated with the additive method assuming that diffusion and convection flows are independent.

For both examples, the indoor conditions are 68 F (20°C), 40% relative humidity (RH), and the outdoor conditions, 0 F (-17.8°C), 50% RH.

Example 1, Single Layer of Insulation

The layer is 4 in (100 mm) thick with a thermal resistance of R-13.6 (2.40 m^2·K/W) and a vapor diffusion resistance of 0.03 Rep* (5.24 x 10^8 m/s**). The following results are for three

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*1 Rep = 1 $Perm^{-1}$ = 1 in Hg·hr·ft^2/grain.

**1 m/s = 1 Pa·s·m^2/kg.

situations: no air leakage, infiltrative, and exfiltrative air movement at a rate of 10 ft/hr (8.47×10^{-4} m/s). The temperature profiles are shown in Figure 4 for infiltration and exfiltration, and they are identical to the results published by Berlad et al. (1980). The vapor pressures are shown in Figure 5. The results confirm that vapor pressures and temperatures within the wall are greatly influenced by air leakage.

The results also show that an infiltrative airflow increases the vapor pressure gradient across the inside section of the insulation, resulting in more vapor entering the insulation by diffusion. Toward the outside, the vapor pressure gradient gradually decreases, causing a decrease in diffusional flow. As Figure 6 shows, this variation in diffusional flow is offset by a variation in convective flow in the opposite direction, resulting in a constant total vapor flow throughout the insulation.

Diffusion with an exfiltrative airflow follows a different pattern. Here the convective and diffusional flows are in the same direction. The convective component dominates toward the inside, and the diffusional component near the outside surface (Figure 7). Figures 6 and 7 assume continuity of vapor flow.

Figure 8 shows how relative humidities in the insulation are affected by airflow. Without airflow, relative humidity reaches a maximum of 94%, approximately 20 mm in from the outside surface. Infiltrative airflow tends to lower this maximum and move it toward the inside surface of the insulation. Exfiltrative airflow tends to increase the maximum relative humidity and move it toward the outside surface. With an exfiltration rate of 10 ft/hr (8.47×10^{-4} m/s), moisture is condensing approximately 10 mm from the outside surface at a rate of 2.4 grain/ft^2·hr (0.46×10^{-6} kg/m^2·s). Both the accumulation rate and location depend on the airflow rate. Additional analysis showed that condensation begins to appear with an exfiltration rate of 4 ft/hr (3.4×10^{-4} m/s) at approximately 15 mm from the outside surface.

Example 2, Insulated Light-Frame Wall

The same insulation of example 1 is placed in a light-frame wall with gypsum board, plywood sheathing, and wood siding. Table 1 lists the thermal and vapor diffusion resistances of each material. The addition of the gypsum board, sheathing, and siding greatly changes the humidity conditions in the insulation (Figure 9). All condensation has moved out of the insulation to the siding and sheathing, and relative humidities in the insulation stay below 100%.

TABLE 1
Thermal and Vapor Diffusion Resistance of Light-Frame Wall Materials (Example 2)

	Thermal Resistance		Vapor Diffusion Resistance	
	hr·ft^2·F/Btu	(m^2·K/W)	Rep	(10^8 m/s)
Air film (still)	0.68	(0.12)	0.0026	(0.45)
Gypsum board	0.45	(.08)	0.11	(19.1)
Insulation	13.64	(2.40)	0.03	(5.2)
Plywood sheathing	0.62	(.11)	0.5	(87.0)
Wood siding	1.05	(.18)	0.029	(5.0)
Air film (6.7 m/s wind)	0.17	(.03)	0.00029	(.05)
Total	16.61	(2.92)	0.672	(116.8)

Figure 10 shows the accumulation rate as a function of air leakage rate. Without air leakage 1.5 grain/hr·ft^2 (0.29 x 10^{-6} kg/m^2·s) is condensing on the sheathing. Exfiltration causes a rapid increase in the accumulation rate to 10 grain/hr·ft^2 (2.0 x 10^{-6} kg/m^2·s), with an exfiltration rate of 4.72 ft/hr (400 x 10^{-6} m/s). Drying conditions prevail at sheathing surface for infiltration rates above 1.3 ft/hr (110 x 10^{-6} m/s).

DISCUSSION

Comparison of Figures 8 and 9 shows that humidity conditions in the insulation greatly depend on the presence and properties of other materials in the wall. Although some moisture may condense in a single layer of insulation, condensation is much less likely in the cavity insulation in an actual light-frame wall because of the influence of the other materials.

Moisture accumulation rates as shown in Figure 10 were compared with results calculated with the additive approach (Burch et al. 1979; Stewart 1981). The additive method is based on the assumption that temperatures, condensation location, and diffusional flow are unchanged by air leakage. There is no agreement in the literature about the equations that should be used for this calculation, especially for exfiltrative airflow. For infiltration we used

$$w_{inf.} = w_{dif.} - \rho cv(p_s - p_o) \tag{25}$$

and for exfiltration,

$$w_{exf.} = w_{dif.} + \rho cv(p_i - p_s) \tag{26}$$

Equation 26 for exfiltration yields lower moisture accumulation rates than the equations proposed in the literature. Nevertheless, Equations 25 and 26 yielded substantially higher accumulation rates than MOISTWALL-2 for infiltration and exfiltration (Figure 11).

In this example, the additive approach gives results that are as much as 30% above the rates calculated with MOISTWALL. This is due primarily to the interaction between convection and diffusion and the effect of air leakage on temperatures. Latent heat effects, which are included in MOISTWALL, may in some cases add to the discrepancy in results from the two methods.

The MOISTWALL-2 program has several advantages over other methods. It is able to determine the location for condensation and calculate the moisture accumulation rate with or without air leakage into the wall. The method is not as complex as finite differencing methods and does not require as many input data. However, this simplicity and the underlying assumptions limit the applicability of MOISTWALL results. The equations are limited to steady-state conditions, and the air is assumed to move uniformly from inside to outside or in the opposite direction. Air convection loops within the cavity, between the cavity and the indoors or cavity ventilation with outdoor air, are not specifically handled in MOISTWALL, but the effect may be approximated by entering appropriate equivalent values for thermal and vapor diffusion resistances.

The method is one-dimensional; corners, tears, cracks, or holes in materials cannot be adequately modeled. The flow of liquid water in hygroscopic materials, such as wood-based panels and cellulose insulation, is ignored. Reliable input values for vapor diffusion resistance are not always available, but this problem is generic to any method of moisture analysis. Research is planned to experimentally evaluate the model.

SUMMARY

Mathematical equations were developed for water vapor flow, vapor pressures, and moisture accumulation in a composite wall with uniform air leakage. The MOISTWALL-2 program is based on these equations and is especially designed for moisture analysis of multilayered walls and is capable of including the effects of air leakage. On the basis of a series of sample analyses, it may be concluded that significant moisture accumulation is unlikely in nonhygroscopic cavity insulation, with or without air leakage into the wall. The location of condensation may move as a result of air leakage. Vapor flow by diffusion is affected by air leakage, as are the temperatures in the wall. It is, therefore, not correct to treat diffusional and convective flows as independent and sum the separate effects. This may lead to significant overestimates of moisture accumulation rates.

Despite a number of limitations, MOISTWALL-2 can provide valuable qualitative insight into the potential effects of air leakage on moisture accumulation in walls.

NOMENCLATURE

$A = \rho c_p v R_t$	--
$B = \rho c v R_v$	--
$c = W/P$	grain/lb·in Hg (Pa^{-1})
c_p = specific heat of air	Btu/lb· F (kJ/kg·°C)
c_L = latent heat of vaporization or sublimation	Btu/grain (kJ/kg)
k = thermal conductivity	Btu/hr·ft· F (W/m·°C)
ℓ = distance from the warm side of insulation	ft (m)
p = water vapor pressure	in Hg (Pa)
R_t = thermal resistance	hr·ft^2· F/Btu (m^2·K/W)
R_v = water vapor diffusion resistance	Rep (m/s)
t = temperature	F (°C)
T = temperature	°R (K)
q_L = latent heat release	Btu/hr·ft^2 (W/m^2)
v = velocity of air	ft/hr (m/s)
w = moisture accumulation rate	grain/hr·ft^2 (kg/m^2·s)
w_H = water vapor flow rate	grain/hr·ft^2 (kg/m^2·s)
W = humidity ratio	--
x = thermal parameter	--
y = vapor diffusion parameter	--
ρ = density of air	lb/ft^3 (kg/m^3)
μ = water vapor permeability	perm·in (s)

Subscripts

dif. = diffusion

exf. = exfiltration

i = inside

inf. = infiltration

H = water vapor

L = latent

n = pertaining to layer n

o = outside

s = saturation

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APPENDIX

Derivation of Moisture Accumulation and Latent Heat Equations

This appendix contains a complete derivation of the equations for moisture accumulation (Equations 17 and 18) and latent heat release (Equations 20 and 21). Equations 22 and 23 for temperature increase are derived from the equations for latent heat release.

General Equation for Heat and Vapor Flow

If condensation is occurring simultaneously at locations $y = y_1, \ldots, y_N$ (Figure 2), vapor pressures at those locations are at saturation. Vapor flow toward the outside is

$$w_H = -\frac{1}{R_v}\frac{dp}{dy} + \rho c v p \tag{A1}$$

and heat flow toward the outside is

$$q = -\frac{1}{R_t}\frac{dt}{dx} + \rho c_p v t \tag{A2}$$

Both equations have the same form:

$$f = -\frac{1}{R}\frac{dg}{dz} + \rho c v g \tag{A3}$$

General Equation for Temperature and Vapor Pressure

The equations for temperature and vapor pressure also have an identical form. The only difference is that moisture accumulation at (x_i, y_i) equals the net vapor flow to that location

$$w_j = w_{H,j-1} - w_{H,j} = -(w_{H,j} - w_{H,j-1}) \tag{A4}$$

while the latent heat release equals the net heat flow away from that location:

$$q_{L,j} = q_j - q_{j-1} \tag{A5}$$

Both equations can still be considered to have the same form:

$$h_j = f_j - f_{j-1} \tag{A6}$$

substituting h_j for $q_{L,j}$ or $-w_j$.

Diffusion (v = 0)

The general form of temperature and vapor pressure equations, either side of (x_i, y_i), is

$$g = g_{j-1} - \frac{z - z_{j-1}}{z_j - z_{j-1}}(g_{j-1} - g_j) \quad \text{for } z_{j-1} \leq z < z_j \tag{A7}$$

$$g = g_j - \frac{z - z_j}{z_{j+1} - z_j}(g_j - g_{j+1}) \quad \text{for } z_j \leq z < z_{j+1} \tag{A8}$$

Flow to (x_j, y_i) is

$$f_{j-1} = -\frac{1}{R}\left(\frac{dg}{dz}\right)_{z=z_{j-1}} = \frac{g_{j-1} - g_j}{z_j - z_{j-1}} \tag{A9}$$

and flow from (x_j, y_j) is

$$f_j = \frac{g_j - g_{j+1}}{z_{j+1} - z_j} \tag{A10}$$

Thus, the general form of latent heat and moisture accumulation equations is

$$h_j = f_j - f_{j-1} = \frac{1}{R}\left(\frac{g_j - g_{j+1}}{z_{j+1} - z_j} - \frac{g_{j-1} - g_j}{z_j - z_{j-1}}\right) \quad \text{for } j = 2, \ldots, N-1 \tag{A11}$$

The equations for the first (x_1, y_1) location and the last location (x_N, y_N) are slightly different:

$$h_1 = \frac{1}{R}\left(\frac{g_i - g_1}{z_1} - \frac{g_1 - g_2}{z_2 - z_1}\right) \tag{A12}$$

$$h_N = \frac{1}{R}\left(\frac{g_N - g_o}{1 - z_N} - \frac{g_{N-1} - g_N}{z_N - z_{N-1}}\right) \tag{A13}$$

With only one point of condensation at (x_s, y_s) the above equations are combined:

$$h_{dif.} = \frac{1}{R} \frac{g_s - g_i + z_s(g_i - g_o)}{z_s(1 - z_s)} \tag{A14}$$

With the appropriate substitutions, Equation A14 yields Equations 17 and 20.

Latent heat release causes a rise in temperature $\Delta t_{L,j}$ at (x_j, y_j). This results in a decrease Δq_{j-1} in heat flow to (x_j, y_j) and an increase Δq_j in heat flow from (x_j, y_j). Both Δq_{j-1} and Δq_j are proportional to $\Delta t_{L,j}$.

$$\Delta q_{j-1} = -\frac{\Delta t_{L,j}}{R_t(x_j - x_{j-1})} \tag{A15}$$

$$\Delta q_j = \frac{\Delta t_{L,j}}{R_t(x_{j+1} - x_j)} \tag{A16}$$

The net change in heat flow must equal the latent heat release rate $q_{L,j}$:

$$q_{L,j} = c_L w_j = \Delta q_j - \Delta q_{j-1} = \frac{\Delta t_{L,j}}{R_t} \frac{x_{j+1} - x_{j-1}}{(x_j - x_{j-1})(x_{j+1} - x_j)} \tag{A17}$$

Thus, the increase in temperature is

$$\Delta t_{L,1} = c_L w_1 R_t \frac{x_1(x_2 - x_1)}{x_2} \tag{A18}$$

$$\Delta t_{L,j} = c_L w_j R_t \frac{(x_j - x_{j-1})(x_{j+1} - x_j)}{x_{j+1} - x_{j-1}} \quad \text{for } j = 2, \ldots, N-1 \tag{A19}$$

$$\Delta t_{L,N} = c_L w_N R_t \frac{(x_N - x_{N-1})(1 - x_N)}{1 - x_{N-1}}$$

For a single condensation point (x_s, y_s) Equation 22 emerges:

$$\Delta t_L = c_L w R_t x_s (1 - x_s) \tag{22}$$

Combined Diffusion and Convection

Derivation of moisture accumulation and latent heat equations with airflow is in principle the same as for diffusion, but here the general form of vapor pressure and temperature equations is

$$g = g_{j-1} - (g_{j-1} - g_j) \frac{e^{D(z - z_{j-1})} - 1}{e^{D(z_j - z_{j-1})} - 1} \quad \text{for } z_{j-1} \leq z < z_j \tag{A20}$$

$$g = g_j - (g_j - g_{j+1}) \frac{e^{D(z - z_j)} - 1}{e^{D(z_{j+1} - z_j)} - 1} \quad \text{for } z_j \leq z < z_{j+1} \tag{A21}$$

Flows to and from (x_j, y_j) are

$$f_{j-1} = -\frac{1}{R}\left(\frac{dg}{dz}\right)_{z=z_{j-1}} + \frac{D}{R} g_{j-1} = \frac{D}{R} \frac{g_{j-1} - g_j}{e^{D(z_j - z_{j-1})} - 1} + \frac{D}{R} g_{j-1} \tag{A22}$$

$$f_j = \frac{D}{R} \frac{g_j - g_{j+1}}{e^{D(z_{j+1} - z_j)} - 1} + \frac{D}{R} g_j \tag{A23}$$

The general form of latent heat and moisture accumulation equations for combined diffusion and convection is

$$h_j = f_j - f_{j-1} = \frac{D}{R}\left(\frac{g_j - g_{j+1}}{e^{D(z_{j+1} - z_j)} - 1} - g_{j-1} - g_j \frac{e^{D(z_j - z_{j-1})}}{e^{D(z_j - z_{j-1})} - 1}\right)$$

$$\text{for } j = 2, \ldots, N-1 \tag{A24}$$

For $j = 1$, the first location, and $j = N$, the last location, the generalized equations are

$$h_1 = \frac{D}{R}\left(\frac{g_1 - g_2}{e^{D(z_2 - z_1)} - 1} - (g_i - g_1) \frac{e^{Dz_1}}{e^{Dz_1} - 1}\right) \tag{A25}$$

$$h_N = \frac{D}{R}\left(\frac{g_N - g_o}{e^{D(1 - z_N)} - 1} - (g_{N-1} - g_N) \frac{e^{D(z_N - z_{N-1})}}{e^{D(z_N - z_{N-1})} - 1}\right) \tag{A26}$$

With a single point of condensation at (x_s, y_s) the three equations reduce to

$$h = \frac{D}{R}\left(\frac{g_s - g_o}{e^{D(1 - z_s)} - 1} - (g_i - g_s) \frac{e^{Dz_s}}{e^{Dz_s} - 1}\right) \tag{A27}$$

Appropriate substitutions in Equation A27 yield Equations 18 and 21.

The rise in temperature $\Delta t_{L,j}$ caused by latent heat release results in changes in heat flows to and from (x_j, y_j).

$$\Delta q_{j-1} = -\frac{A}{R_t}\frac{\Delta t_{L,j}}{e^{A(x_j - x_{j-1})} - 1} \tag{A28}$$

$$\Delta q_j = \frac{A}{R_t}\frac{\Delta t_{L,j}\, e^{A(x_{j+1} - x_j)}}{e^{A(x_{j+1} - x_j)} - 1} \tag{A29}$$

Equations A28 and A29 can be derived by making the appropriate substitutions in Equations A22 and A23 and subtracting the heat flows without latent heat release from the heat flows with latent heat release.

The net change in heat flow must equal the latent heat release:

$$q_{L,j} = c_L w_j = \Delta q_j - \Delta q_{j-1} = \frac{A}{R_t}\Delta t_{L,j}\frac{e^{A(x_{j+1} - x_{j-1})} - 1}{\left(e^{A(x_{j+1} - x_j)} - 1\right)\left(e^{A(x_j - x_{j-1})} - 1\right)} \tag{A30}$$

The increase in temperature is

$$\Delta t_{L,1} = \frac{R_t c_L w_1}{A}\frac{\left(e^{A(x_2 - x_1)} - 1\right)\left(e^{Ax_1} - 1\right)}{e^{Ax_2} - 1} \tag{A31}$$

$$\Delta t_{L,j} = \frac{R_t c_L w_j}{A}\frac{\left(e^{A(x_{j+1} - x_j)} - 1\right)\left(e^{A(x_j - x_{j-1})} - 1\right)}{e^{A(x_{j+1} - x_{j-1})} - 1} \quad \text{for } j = 2, \ldots, N-1 \tag{A32}$$

$$\Delta t_{L,N} = \frac{R_t c_L w_N}{A}\frac{\left(e^{A(1 - x_N)} - 1\right)\left(e^{A(x_N - x_{N-1})} - 1\right)}{e^{A(1 - x_{N-1})} - 1} \tag{A33}$$

For a single location (x_s, y_s) Equation 23 emerges:

$$\Delta t_L = \frac{R_t c_L w}{A}\frac{\left(e^{Ax_s} - 1\right)\left(e^{(1 - x_s)A} - 1\right)}{\left(e^A - 1\right)} \tag{23}$$

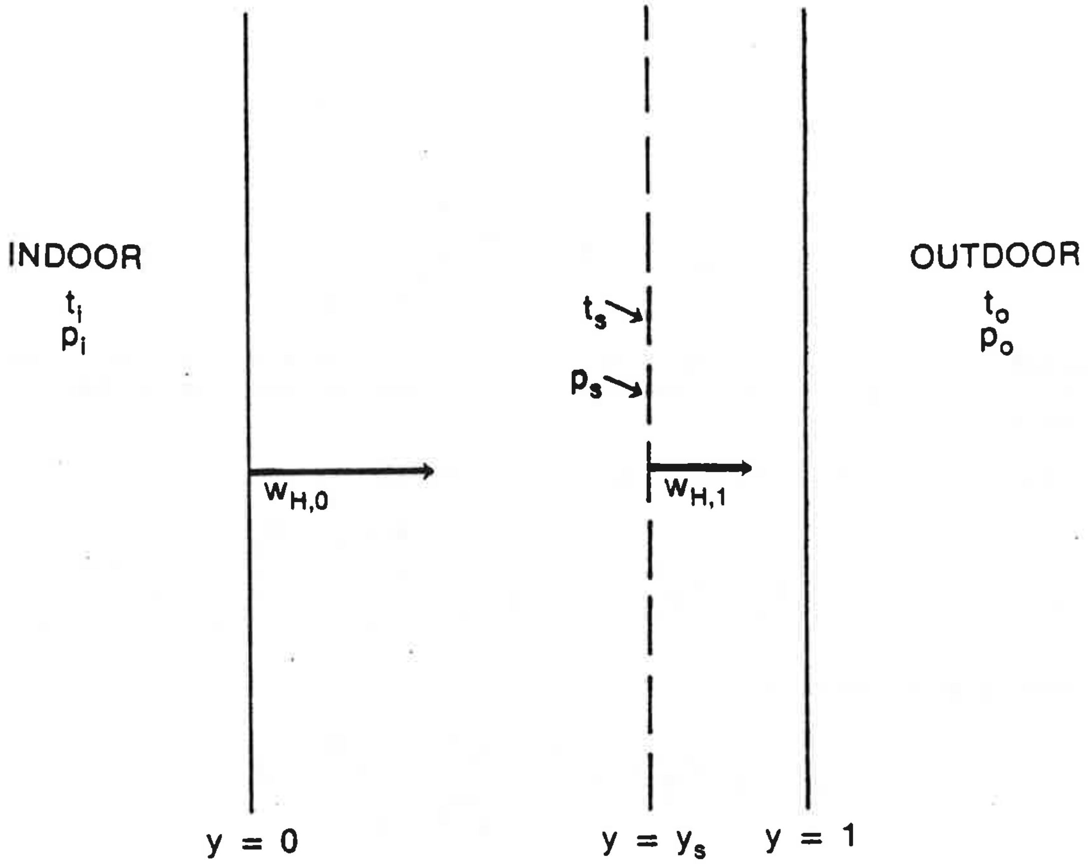

Figure 1. Condensation at a single location, $y=y_s$, with continuity of vapor flow in the rest of the wall

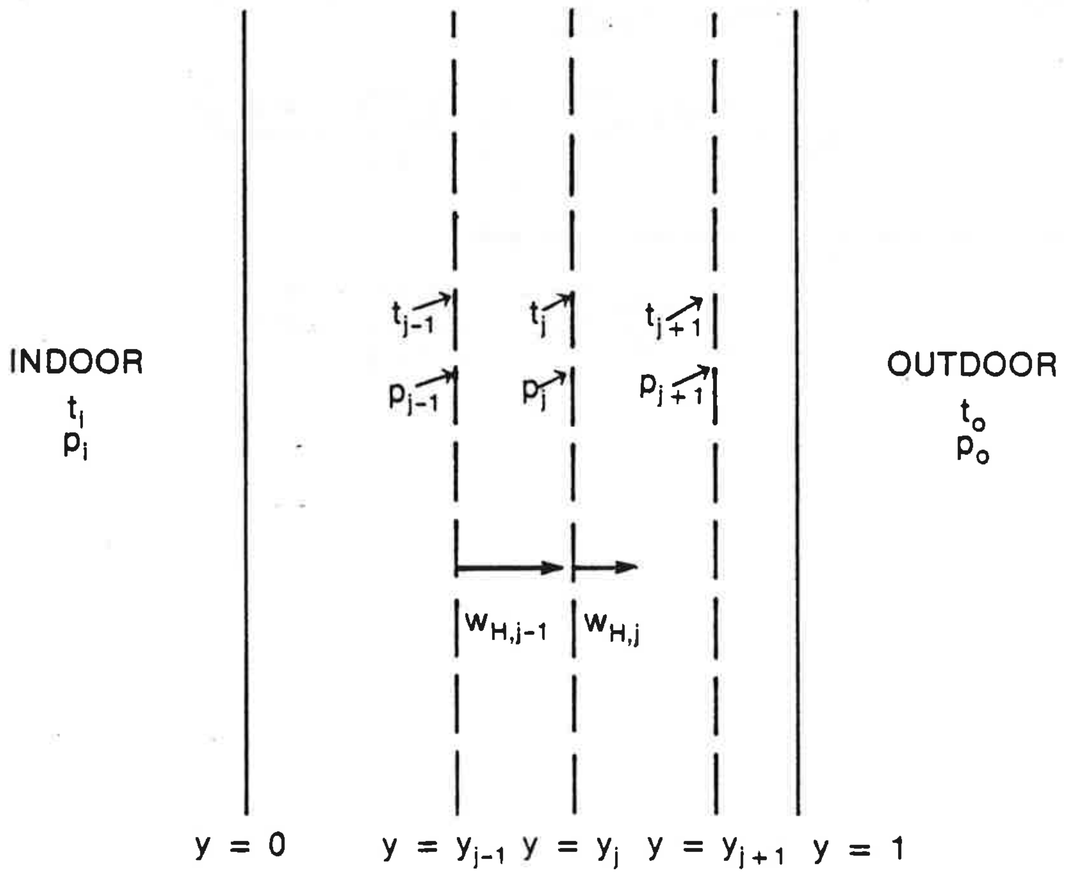

Figure 2. Condensation at several locations, $y=y_1$, y_2, ..., y_N, with continuity of vapor flow in the rest of the wall

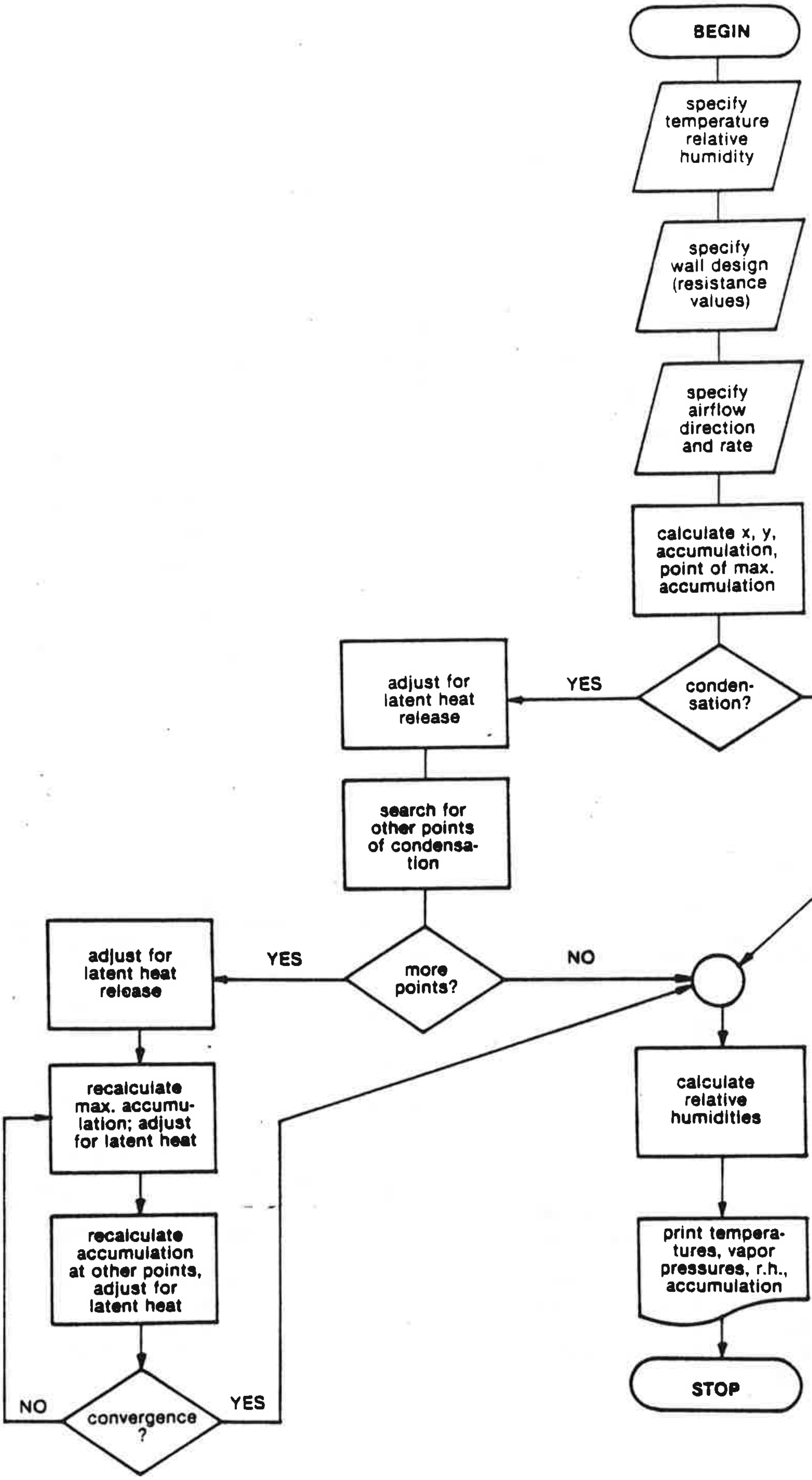

Figure 3. Flow diagram of the MOISTWALL-2 computer program

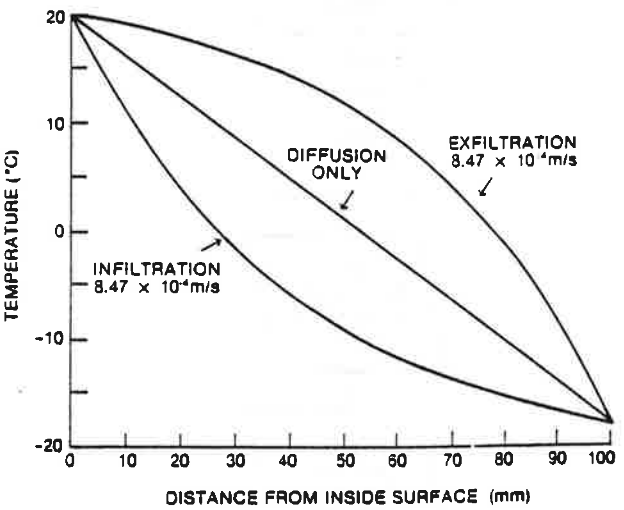

Figure 4. Temperatures with and without airflow in a single layer of insulation (example 1)

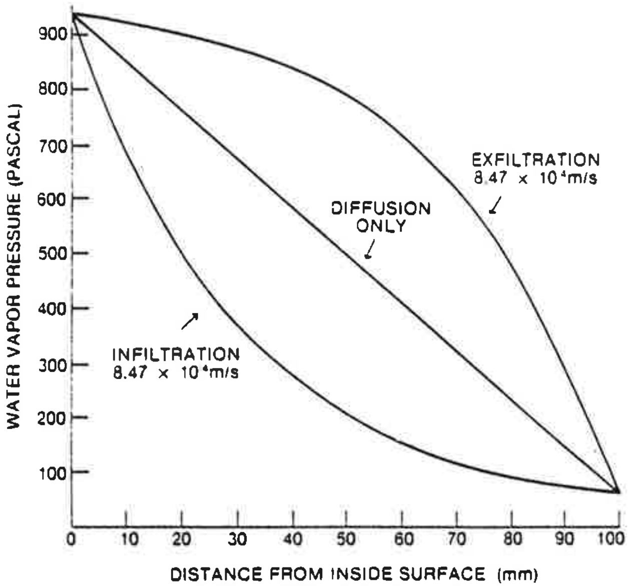

Figure 5. Water vapor pressures with and without airflow in a single layer of insulation (example 1)

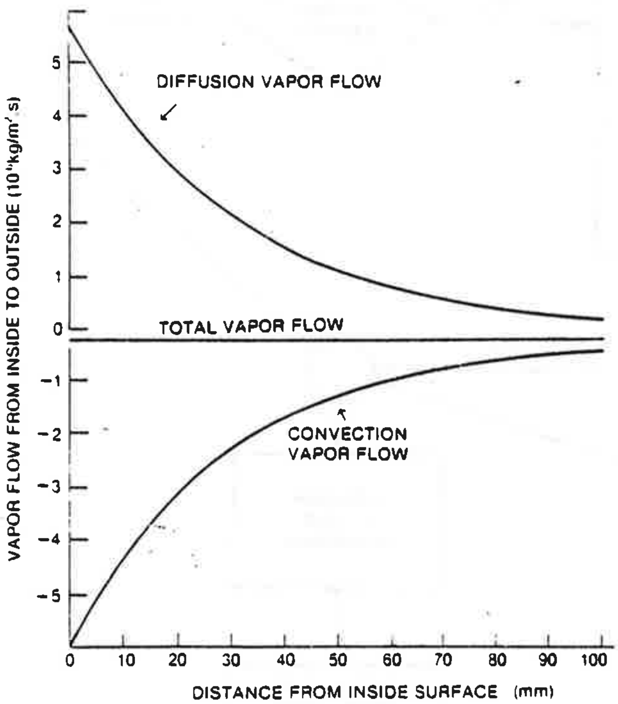

Figure 6. Vapor flow by diffusion and convection with an infiltrative airflow (8.47 x 10^{-4} m/s) in a single layer of insulation (example 1)

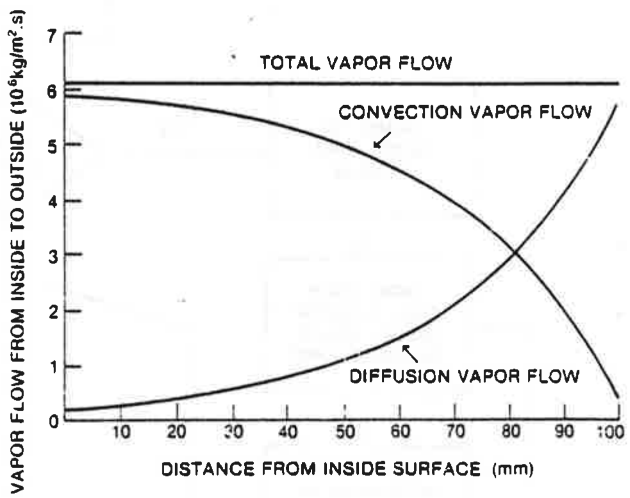

Figure 7. Vapor flow by diffusion and convection with an exfiltrative airflow (8.47 x 10^{-4} m/s) in a single layer of insulation (example 1)

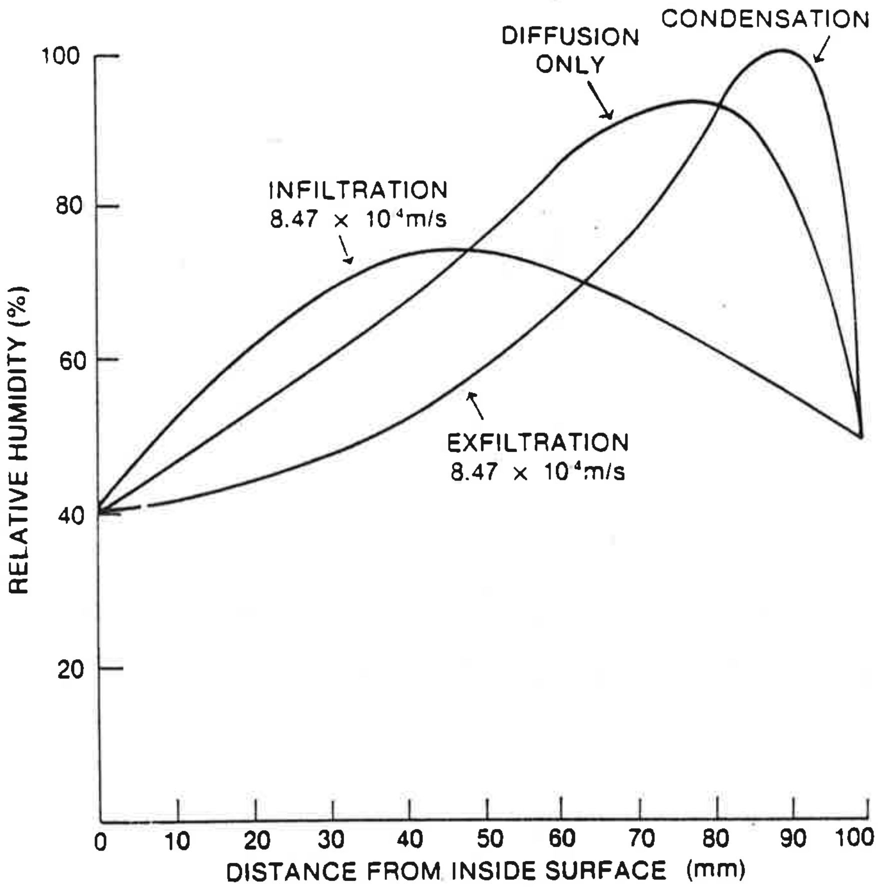

Figure 8. Relative humidities in a single layer of insulation with infiltrative and exfiltrative airflow, and without airflow (example 1)

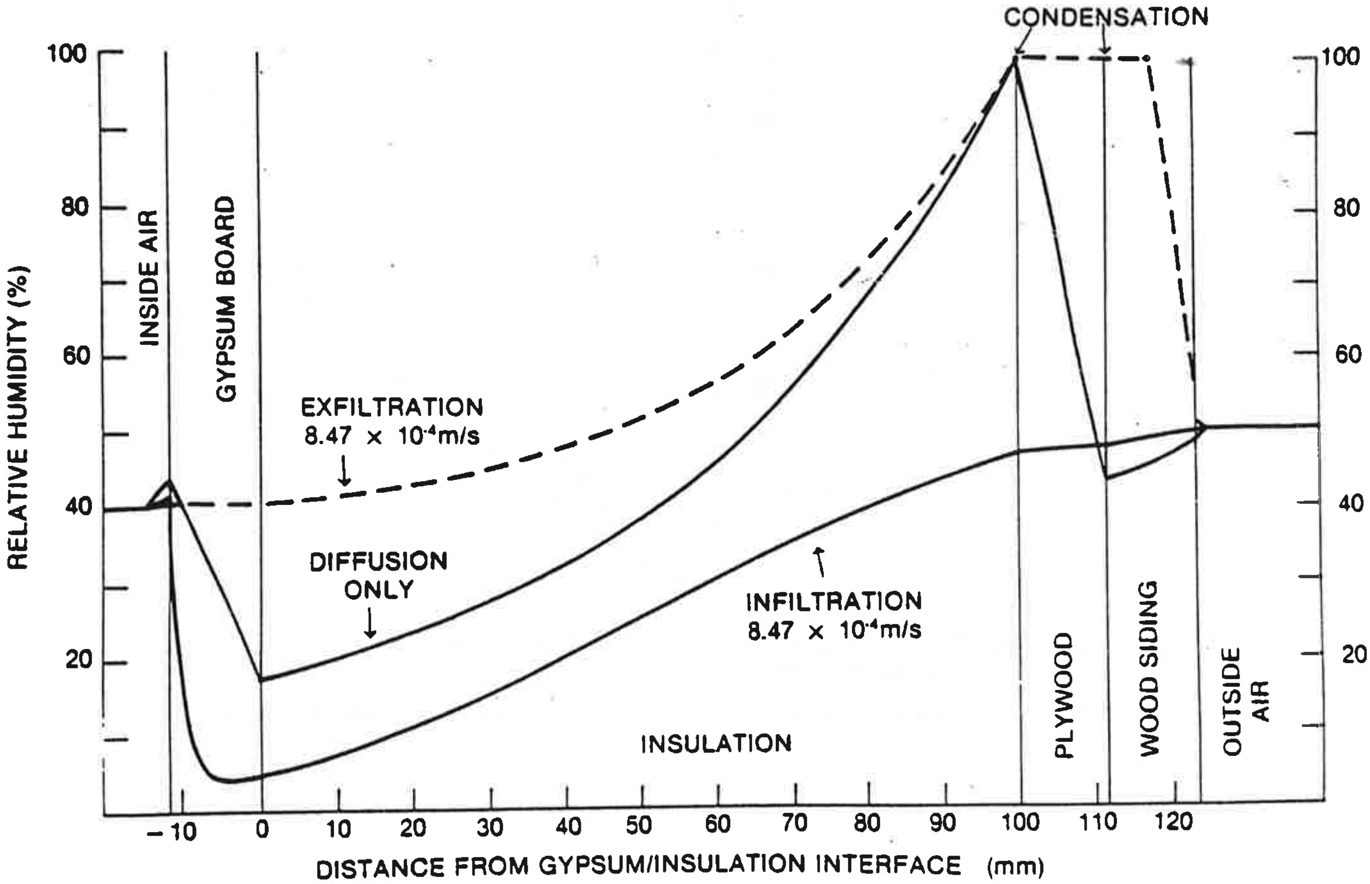

Figure 9. Relative humidities in an insulated light-frame wall with infiltrative and exfiltrative airflow, and without airflow

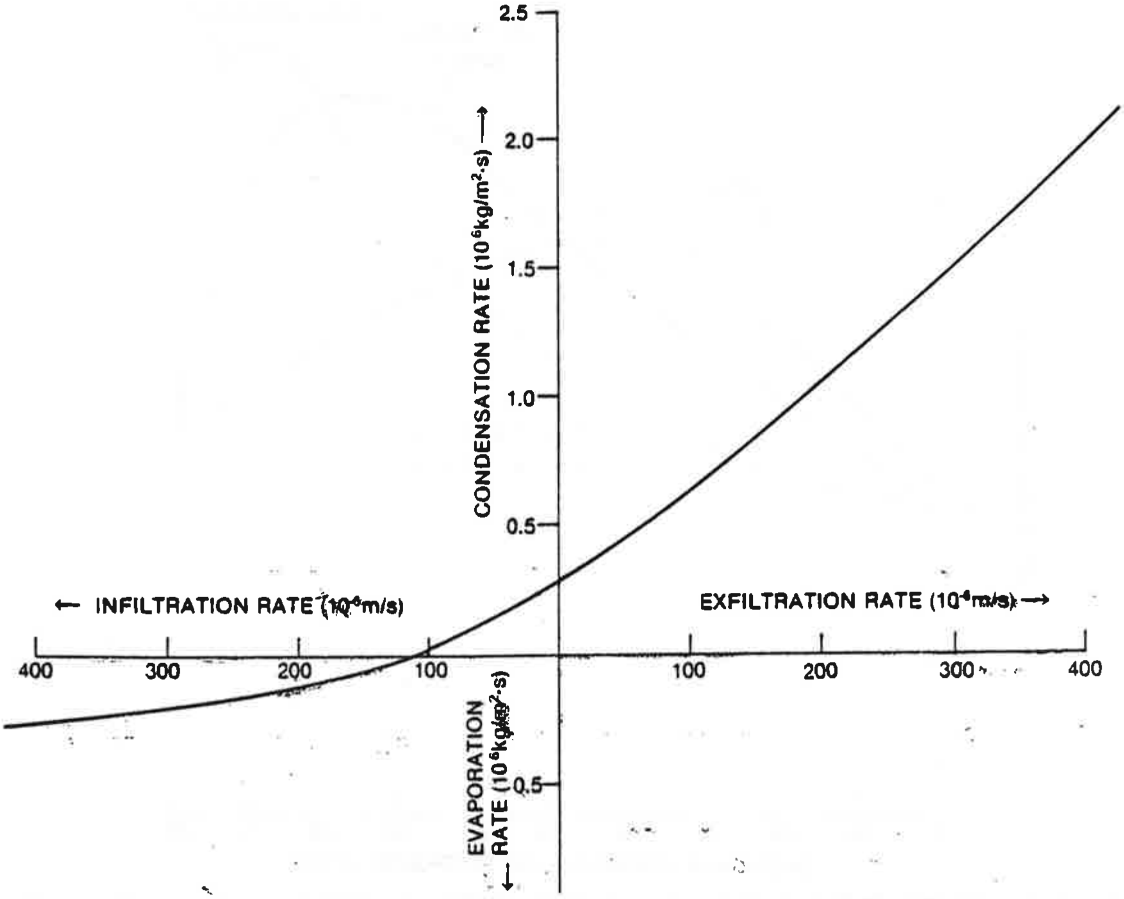

Figure 10. Moisture accumulation and evaporation on plywood sheathing in an insulated light-frame wall as a function of the air leakage rate (example 2)

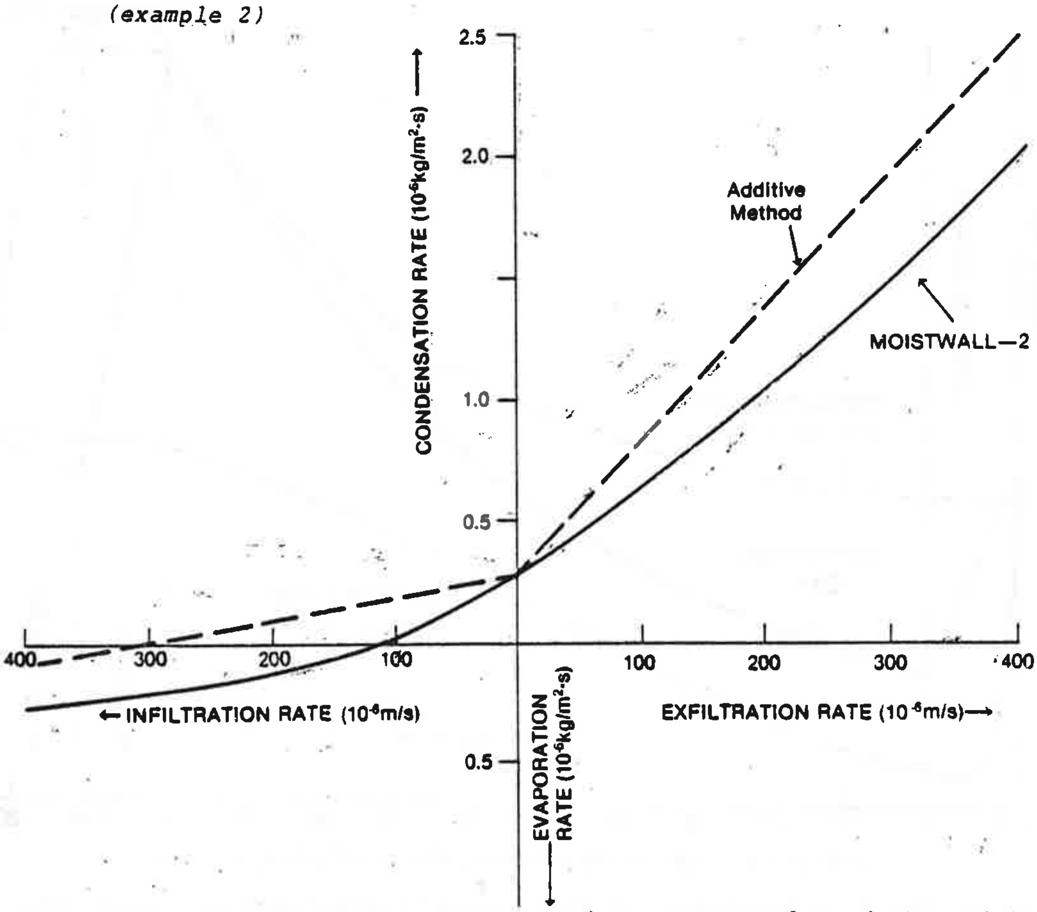

Figure 11. Moisture accumulation and evaporation rates on plywood sheathing in an insulated light-frame wall as calculated with MOISTWALL-2 and with the additive method