SIMULATION OF BUOYANCY AND WIND INDUCED VENTILATION

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1. INTRODUCTION

Ventilation and air exchange in buildings and industrial plants can be induced by external winds, which create pressure differences between various opennings of the buildings, and by buoyancy forces created by temperature-density differences between the inner and outer air (a stack effect). When the outer air motion is very slow, the warmer air in the buildings will leave it through upper opennings and will be replaced by cooler air entering the building from lower opennings.

Assuming that the characteristic temperature rise in the building is ΔT and that the vertical distance between the upper and lower opennings is L the exit velocity of the warm air will be determined by the balance of the pressure losses at the opennings $[O(\rho U^2/2)]$ and the buoyant pressure $[O(\Delta\rho gL)]$ so that

$$u_{exit} \sim ((\Delta\rho/\rho)gL)^{\frac{1}{2}} = ((\Delta T/T)gL)^{\frac{1}{2}} \qquad (1)$$

The heat flux through the upper opennings, whose area is A, would be

$$H \sim \rho C_p \Delta T\, u_{exit} A \sim \rho C_p \Delta T^{3/2} g^{1/2} L^{1/2} A/T^{1/2} \qquad (2)$$

When the outer wind speed U increases, the air exchange pattern will be changed, as pressures of the order of $\pm\,\rho U^2/2$ will be built on the envelope of the buildings. The ventilation will either increase or decrease, depending on the particular building geometry and the position of the opennings relative to the wind direction. At high wind speeds $U \gg u_{exit}$, or

$$U/((\Delta T/T)gL)^{\frac{1}{2}} \gg 1 \qquad (3)$$

the effect of buoyancy will be negligible and the air exchange will be determined primarily by the wind induced pressures.

The dependence of the air exchange and heat transfer on a large number of factors, including the detailed configuration of the building and surroundings, makes an analytical or numerical analysis of practical design problems impractical, particularly when both the buoyancy and the wind induced pressures are of the same order of magnitude. It would thus be convenient if the combined effect of the wind motion and buoyancy in a particular geometry could be simulated in small scale wind tunnel models.

This paper discusses the requirements for such simulations. It is shown that in many cases only approximate simulations can be obtained in small wind tunnel models. Their scaling laws are specified and some of their limitations are discussed. The modelling of a chemical plant, which produces a considerable amount of heat and pollution, is described as an example.

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CUP, 1982.
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2. CRITERIA FOR MODELLING

Consider a large heat source within a building. The air motion and heat transfer is determined by the following independent dimensional parameters:

- H – the heat flux from the inner source
- L – a characteristic length of the building, such as its height
- ρ – the density of the ambient air
- T – the temperature of the ambient air
- C_p – the specific heat of the air (at constant pressure)
- U – a characteristic velocity of the ambient wind field
- ν – the kinematic viscosity of the air
- α – the molecular thermal diffusivity of the air

and a large number of dimensionless parameters describing the relative geometrical configuration, the relative velocity and turbulence distribution in the wind field and the boundary conditions. These dimensional parameters should be the same in the model and the prototype.

It follows from dimensional considerations that dependent dimensionless parameters in the field, such as the relative temperature rise at a given point of the prototype and the corresponding point in the model, would be a function of the set of the independent dimensionless parameters which can be grouped from the above list of variables. Accordingly one may write that a relation exists

$$F\left(\frac{\Delta T}{T}\ ;\ \frac{H}{\rho C_p T U L^2}\ ;\ \frac{U^2}{C_p T}\ ;\ \frac{U^2}{gL}\ ;\ Re,\ Pr\right) = 0 \qquad (4)$$

where Re is the Reynolds number UL/ν and Pr is the Prandtl number ν/α. To ensure a complete similarity between the model and prototype, the values of all the independent dimensionless parameters in (4) must be equal in model and prototype [1]. Obviously this requirement makes modelling impossible. Fortunately, it is recognized that under certain conditions the effect of several dimensionless parameters on the phenomenon is insignificant and they can be neglected.

It is well established [1,2] that when the Reynolds number of a flow is sufficiently large, namely

$$Re > Re_{minimum} \qquad (5)$$

the flow will be turbulent and its primary features would be independent of the Reynolds number. The effect of the Prandtl number is also negligible because the flow is turbulent and its value is the same in the model and prototype, so that one may write

$$F\left(\frac{\Delta T}{T}\ ;\ \frac{H}{\rho C_p T U L^2}\ ;\ \frac{U^2}{C_p T}\ ;\ \frac{U^2}{gL}\right) = 0 \qquad (6)$$

The term U^2/C_pT is recognized as the ratio of the kinetic energy to the thermal energy of the gas and is significant only when the Mach number of the flow is large, [3]. Neglecting this term in Eq. 4 gives:

$$F\left(\frac{\Delta T}{T} \; ; \; \frac{H}{\rho C_p T U L^2} \; ; \; \frac{U^2}{gL}\right) = 0 \tag{7}$$

Denoting by x_m and x_p the value of any variable in the model and prototype respectively and defining the scaling of x as $\lambda(x) = x_m/x_p$, it follows from (7) that the necessary conditions for similarity, in addition to Eq.(5), are

$$\lambda\,(H/\rho C_p TUL^2) = 1 \tag{8}$$

$$\lambda\,(U^2/gL) = 1 \tag{9}$$

The scaling of the relative temperature rise in such a model would be

$$\lambda\,(\Delta T/T) = 1 \tag{10}$$

Simulations which satisfy these scalings laws are usually termed <u>exact simulations</u>.

Before proceeding with the analysis, it is worthwhile to stress that physical arguments which lead to neglection of terms in general functional forms, like Eq. (6), must be used carefully and the results must be critically examined [4]. Assume for example that the functional dependence described in Eq. 6 is expressed in a different form, such as

$$F\left(\frac{\Delta T}{T} \; ; \; \frac{H}{\rho U^3 L^2} \; ; \; \frac{U^2}{C_p T} \; ; \; \frac{U^2}{gL}\right) = 0 \tag{11}$$

which is fully equivalent of course to (6). If the earlier arguments about the term U^2/C_pT are applied to the new equation, the resultant equation is

$$F\left(\frac{\Delta T}{T} \; ; \; \frac{H}{\rho U^3 L^2} \; ; \; \frac{U^2}{gL}\right) = 0 \tag{12}$$

which is not equivalent to Eq. (7) and gives different scaling laws. The only way to check out the validity of the proposed approximation is to examine whether dimensional or dimensionless parameters which are physically significant in the problem have been omitted, and whether the remaining dimensionless parameters can describe the problem correctly. Such an examination will easily show that Eq. 12 is not a valid approximation. It is known that convective motion is induced by buoyancy and it is thus expected that the buoyancy flux, which is proportional to $H/\rho C_p$, should appear in one of the dimensionless parameters. Obviously any form, such as Eq. (12), which does not include the specific heat C_p

has to be rejected. Clearly the wrong result was obtained in our case by formulating the interdependence between the various variables in terms of dimensionless parameters which are not significant in this problem, as $H/\rho U^3L^2$.

3. APPROXIMATE SIMULATIONS

The use of significant dimensionless parameters is also very helpful in drawing additional conslusions from the dimensional analysis. It is clear for example that a typical heat flux in our problem is $\rho C_p \Delta TUL^2$ rather than $\rho C_p TUL^2$.

Similarly since the body force on a heated element is proportional to $\Delta\rho g$, which for small $\Delta T/T$ is proportional to $\rho g \Delta T/T$. Thus, one expects that $U/\sqrt{(\Delta T/T)gL}$ would be a more significant dimensionless parameter than $U/\sqrt{gL}$. These arguments suggest that it is more meaningful to rewrite Eq. 12 in the form

$$F\left(\frac{H}{\rho C_p \Delta TUL^2}\ ;\ \frac{U^2}{\Delta TgL/T}\ ;\ \frac{\Delta T}{T}\right) = 0 \tag{13}$$

One further expects, by analogy to similar problems that T would not be a significant parameter in this problem, except for its effect on the body force. It thus follows that for small $\Delta T/T$ one may write that

$$F\left(\frac{H}{\rho C_p \Delta TUL^2}\ ;\ \frac{U^2}{\Delta TgL/T}\right) = 0 \tag{14}$$

or

$$F\left(\frac{HT^{1/2}}{\rho C_p g^{1/2} L^{5/2} \Delta T^{3/2}}\ ;\ \frac{U^2}{\Delta TgL/T}\right) = 0 \tag{15}$$

Based on this conclusions, the criteria for achieving simulation in small scaled models are

$$\lambda\ (H/\rho C_p \Delta TUL^2) = 1 \tag{16}$$

$$\lambda\ (U^2/(\Delta T/T)gL) = 1 \tag{17}$$

which imply of course that

$$\lambda\ (HT^{1/2}/\rho C_p g^{1/2} L^{5/2} \Delta T^{3/2}) = 1 \tag{18}$$

Simulations which satisfy these requirements are usually termed approximate simulations [3]. The scaling of $\Delta T/T$ in such models can be set arbitrarily to any desired value, provided $\Delta T/T$ is not a large number. When $\lambda(\Delta T/T) = 1$, these requirements become identical to those specified for exact simulations.

It is interesting to note that when the outer velocities are very small, Eq. 15 becomes

$$F\,(HT^{1/2}/(\rho C_p g^{1/2} L^{5/2} \Delta T^{3/2})) = 0 \tag{19}$$

or

$$HT^{1/2}/(\rho C_p g^{1/2} L^{5/2} \Delta T^{3/2}) = \text{constant} \tag{20}$$

which is consistent with Eq.2. When U becomes very large, on the other hand, the effect of buoyancy is negligible and the last term in Eq. 14 can be eliminate. One finds for this case that

$$H/\rho C_p \Delta T U L^2 = \text{constant} \tag{21}$$

The relaxation of the requirement $\lambda(\Delta T/T) = 1$ in approximate simulations has two benefits. When one uses higher temperature differences in the model, the velocity scaling, which according to Eq.(17) is equal to

$$\lambda(U) = \lambda((\Delta T/T)gL)^{1/2}\ ,$$

becomes larger and the Reynolds number of the flow becomes larger. It is thus easier and manv times the only possible way to satisfy the requirement $Re > Re_{minimum}$. The larger temperature differences in the model are are more easy to measure.

4. LIMITATIONS OF THE PHYSICAL MODELLING

The limitations of both the exact and the approximate simulations are primarily related to the accuracy of the assumption of Reynolds-number-independence. There is evidence that above $Re = 4 \times 10^4$ [1,2] the effect of the Reynolds number in external flows around bluff bodies is small. Measurements of the internal velocities and turbulence in a model of a house suggest that this rule might also hold for internal flows [5]. One recalls that the flow of air through windows is basically similar to a jet flow, which becomes turbulent at even lower Reynolds numbers, and thus this conclusion is not surprising. In case of buoyant flows Reynolds-number independence is expected to start even earlier [6, p. 512].

It must be realized, however, that the appropriate Reynolds number for flows near boundaries is a local Reynolds number which is based on the distance from the wall. Close to the wall this Reynolds number is always small and one cannot expect processes which are controlled by the wall region to be Reynolds-number-independent. This is particularly true of heat transfer from the wall. Consider for example a simulation of a room in which one section of a vertical wall is heated to a temperature $T + \Delta T_s$.

The value of ΔT_s is related to the heat flux H. Assuming that the heat flux is by free convection, the Nusselt number, Nu, which is proportional to H, will be a function of the Grashof number $G = gh^3\, \Delta T_s/(T\nu^2)$ where h is the height of the heated wall. When the product of the Grashof number and the Prandtl number $GP < 10^8$ the flow is laminar and Nu ∝ $(GP)^{1/4}$. When GP > 10^{10}, the flow is turbulent and Nu ∝ $(GP)^{2/5}$. Taking h = 2m, ΔT_s = 30°C one gets in the prototype for Pr = 0.7, GP > 10^{10} whereas in a 1:10 scale model GP will be reduced by $\lambda(L)^3$ giving GP < 10^8. Clearly in a 1:100 model the flow will be totally laminar. Thus for a given H, ΔT_s will be highly dependent on the viscosity, and the temperature rise ΔT_s between the wall and the environment will not be scaled as the temperature rise of the air in the room above the ambient temperature. Fortunately, the thermal boundary layer on the wall is very small, so that one could therefore build the model according to the scaling derived earlier, using as an independent parameter H, rather than ΔT_s, calculating the H_p from known correlations.

The temperature rise in the building itself in such a model would scale according to Eq. 18, except that near the wall, where the temperature rise will be larger. An estimate of the additional temperature rise near the wall can be made using heat transfer correlations. The same approach can be used in cases where the heat transfer from the internal sources, is by forced convection. Deviation from the model scaling is expected to exist only near the heat sources.

5. A SIMULATION OF HEAT DISPOSAL FROM A CHEMICAL PLANT.

A plant for manufacturing chlorine and sodium hydroxide by electrolysis is being built by Makhteshim Chemical Works Ltd. in Israel. The process, which is performed in 18 large cells located above large opennings on the first floor (40 × 40 m^2) of a single building, produces a relatively high flux of heat (550 KW). The air heated by the cells is expected to rise and leave the building through a large openning along the centerline of the roof. The ground floor is partially open to the atmosphere and fresh air can easily enter the building. The designer of the plant has originally proposed to build a rather elaborate roof which would avoid the entrance of rain water and secure the disposal of the heat through the roof, independent of the ambient wind, See Fig. 1 (a). The cost of the original roof was very high and the question rose whether one can not use a much simpler and less expensive design of the type shown in Fig. 1(b).

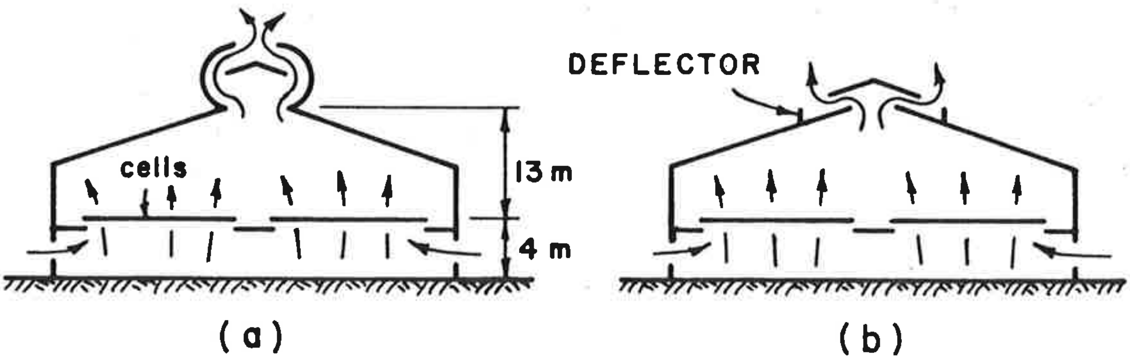

Fig. 1 Schematic description of the tested plant
(a) Original roof configuration
(b) Final roof configuration

The ventilation and heat disposal from the plant have been investigated in the M. David Lipson Environmental Wind Tunnel at the Technion - Israel Institute of Technology. The cross section of the 15m long wind tunnel is 2m × 2m.

It was decided to simulate the phenomena in a 1:83.3 scale model of the building and the surroundings.
Since T, C_p and g are equal in the model and prototype Eqs.(16)-(18) imply that:

$$\lambda(\Delta T) = \lambda(H^{2/3})/\lambda(L^{5/3})$$

and

$$\lambda(U) = \lambda(H^{1/3})/\lambda(L^{1/3})$$

Most of the tests were made using H_m = 630 W, which gives $\lambda(H)$ = 1:870.
The scaling of the temperature rise and velocities were therefore

$$\lambda(\Delta T) = (870^{-2/3})(83.3)^{5/3} = 17.4:1$$

and

$$\lambda(U) = (870^{-1/3})(83.33)^{1/3} = 1:2.18$$

The temperature rise in the building was later found to be around 4°C, which corresponds to a 70°C rise in the model. The maximum value of $\Delta T/T$ in the model was therefore of the order 0.25. Note that $-\Delta\rho/\rho = T/(T + \Delta T) - 1 = 0.23$ whereas $\Delta T/T = 0.25$, suggesting a 10% error in the scaling laws due to the large value of $\Delta T/T$ in the model.

The exit velocity in the model is expected to be of the order of $\sqrt{0.23 \cdot 9.81 \cdot 10/83.3} = 0.525$ m/sec, giving a Reynolds number of the order of 4×10^3, however, flow visualization showed that the flow was turbulent, and the effect of the Reynolds number was apparently small.

To examine the validity of the scaling laws the values of the temperature rise ΔT_m for different values of H_m were measured at different points in the building. Typical results are shown in Fig. 2 for two roof configurations. It appears that the 2/3 power dependency of ΔT_m on H_m is confirmed in the model. The points for H = 840 Watt appear to be slightly above the 2/3 lower law which fits the measurements with smaller H_m, probably due to effective lower density differences at this value. Since the values of the temperature rise in the prototype are one order of magnitude lower, the accuracy of the model appears to be satisfactory for design purposes.

The temperature rise at different points was measured for different roof configurations for different wind speeds and wind directions. The detailed results, which are of little value to the reader will not be described in this paper. These results showed, however, that when the wind blew normal to the openning in the roof, the simpler design (b) did not perform as well as the original design. However, a short vertical deflector placed along the roof, see Fig. 1, cured this deficiency completely.

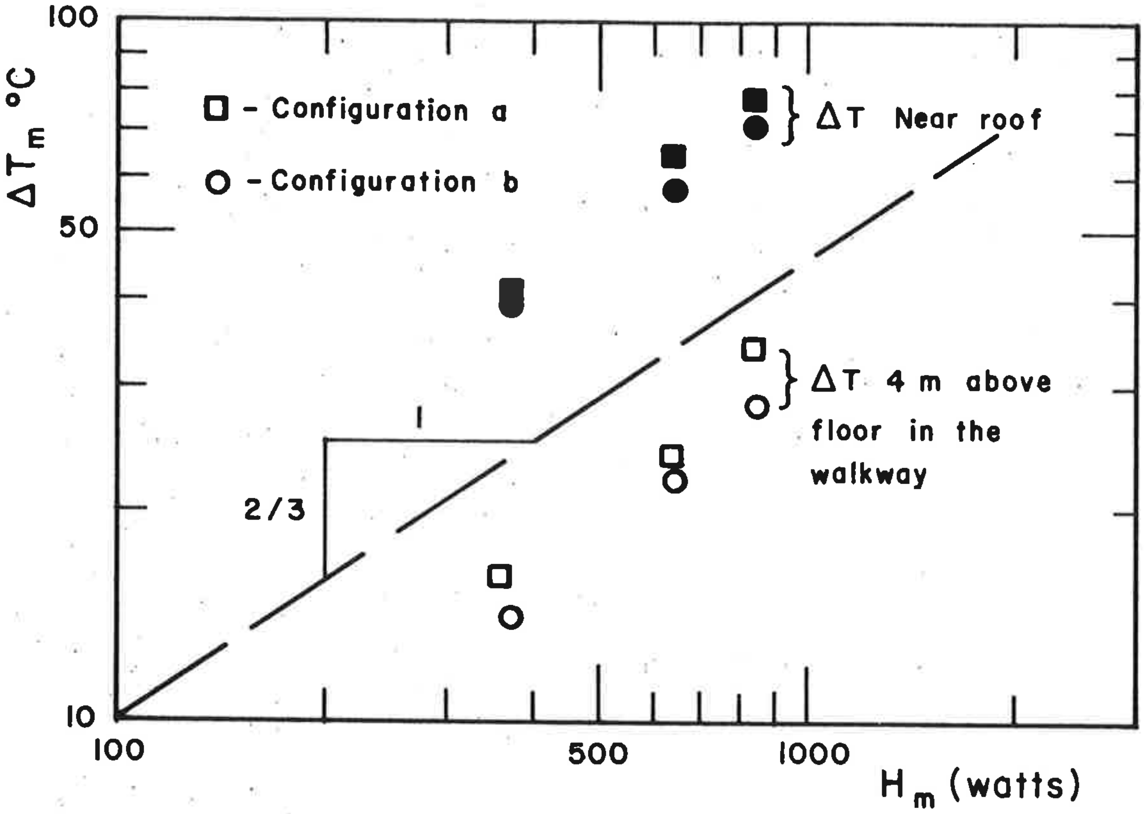

Fig. 2 The temperature rise in the model for different values of heat flux.

The model can also be used to estimate the concentration of pollutants emitted during the process. Assuming that the discharge of a given pollutant is Q, then

$$Q = C \cdot A \cdot u_{exit}$$

where C is the concentration of that pollutant at the exit. According to Eq. 2

$$H = \rho C_p \Delta T \cdot A\, u_{exit}$$

and thus it is expected that

$$\frac{C}{\Delta T} = \frac{Q}{(H/\rho C_p)}$$

The similarity between heat and mass transfer in turbulent flows suggests that this relation holds at any point in the model, except of course close to the cell where both the temperature rise and the concentrations can not be accurately simulated.

The help of Mrs. Z. Vider and Mr. E. Gantz in the experimental work is gratefully acknowledged.

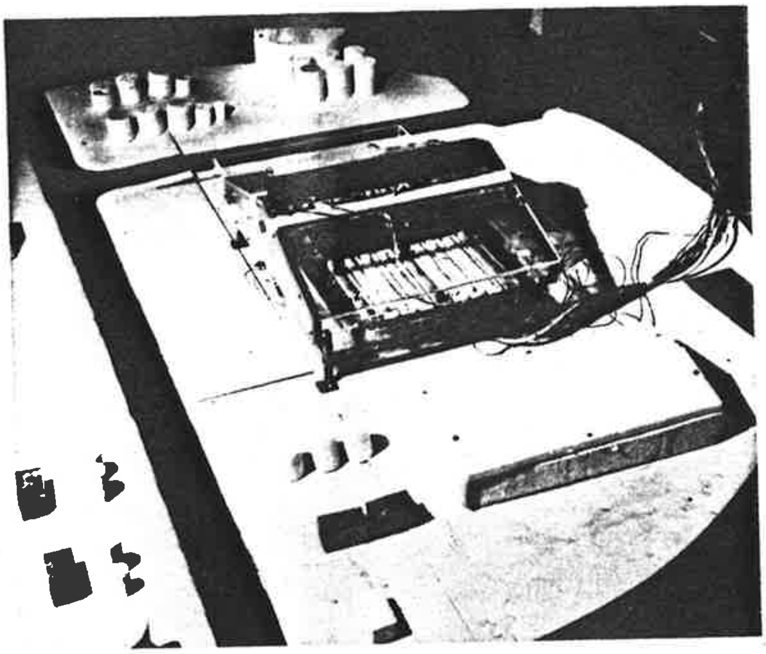

Fig. 3. Photograph of the model in the wind tunnel

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