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A Computer Algorithm for Predicting Infiltration and Interroom Airflows

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ABSTRACT

This report discusses the extension of an infiltration predicting technique to the prediction of interroom air movements. The airflow through openings is computed from the ASHRAE crack method together with a mass balance in each room. Simultaneous solution of the mass balances in all rooms having both large and small openings is accomplished by a slightly modified Newton's method. A simple theory for two-way flow through large openings is developed from consideration of density differences caused by different temperatures in adjoining rooms. The technique is verified by comparison to published experimental results. The results indicate that the simple model provides reasonable results for complex two-way flows through openings. The model is as accurate as the available data, that is, about ±20%. The airflow algorithm allows infiltration and forced airflows to interact with the doorway flows to provide a more general simulation capability.

INTRODUCTION

Although numerous building thermal modeling techniques and computer programs, for example, NBSLD (Kusuda 1976), BLAST (Hittle 1979), and DOE-2 (LBL 1980), exist throughout the United States, none of the existing techniques/programs handles the following processes simultaneously:

- envelope heat transfer
- envelope air leakage
- envelope solar heat gain
- room-to-room heat transfer
- room-to-room air and moisture transfer
- intraroom air movement
- energy consumption by the heating/cooling equipment
- indoor comfort
- water vapor condensation and contaminant migration

Existing models are virtually all single-room models, where dynamic coupling between the heated and nonheated spaces and/or the cooled and noncooled spaces is ignored.

Comprehensive multiroom building simulation capabilities will be needed in the coming years for the following reasons:

1. Intraroom convection plays a significant role, not only for the transfer of heat from the interior surfaces to the room air, but also for the thermal comfort of the occupants. Yet existing computational technology for predicting the temperature stratification and air motion in the room is very inadequate.

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2. Passive solar design techniques are expected to be used to a large extent in new building design. Proper design is going to depend on understanding and being able to predict natural energy flows within the buildings.

3. Proper control of the airflow in the central air system requires accurate knowledge of room-by-room energy demand.

4. Moisture and contaminant distribution throughout a building must be capable of being predicted in order to insure adequate designs as the emphasis on "tighter" buildings increases.

5. Until such capability exists, effective design of internal partitioning with respect to natural ventilation can only be done by empirical techniques.

In response to these needs, a research-oriented computer program has been developed to allow the detailed study of simultaneous air, heat, and moisture transfer in and through a building with complex internal architecture. This program is called the Thermal Analysis Research Program (TARP). Documentation (Walton 1983) for the program has been published. Primary emphasis in the development of TARP has been on air transfer, because this is a major shortcoming of present techniques and because it is basic to further developments in moisture and contaminant analysis. A previous report (Walton 1982) describes initial results in the development of TARP and indicated five significant areas for further research:

1. Calculation of airflows through large openings in reasonable computation time,

2. Calculation of convective exchange between rooms,

3. Accurate prediction of the wind-induced pressure distribution around the envelope of the building,

4. Calculation of the effects of room air stratification, and

5. Availability of data for estimating the opening areas in the envelopes of buildings.

This report will address the first two areas.

METHODS OF CALCULATIVE

The Airflow Equation

The TARP airflow algorithm is described in detail in Walton (1982) where the program was called the Multi-Room Loads Program (MRLP). The airflow algorithm is based on the equation (ASHRAE 1977):

$$F = K * (\Delta P)^X \tag{1}$$

where

F = flow rate (kg/s)
K = a constant
ΔP = pressure difference across an opening (Pa)
X = flow exponent

Pressure differences arise from wind, air-density differences, and system-induced flows. By applying equation 1 to all openings in the building envelope and all openings between rooms and requiring a mass balance in each room, a set of simultaneous nonlinear algebraic equations is created that can be solved for the zonal pressures and the airflow through each opening.

An estimate for the value of K was made by referring to the orifice equation:

$$F = C * A * \rho * \sqrt{2 * \Delta P / \rho} \tag{2}$$

where

C = flow coefficient
A = observed opening area (m^3)
ρ = air density (kg/m^3)

When the opening is small, C equals 0.6 for a wide range of Reynolds numbers, as shown in figure 1 (ASHRAE 1981). TARP assumes this value of C as a default. TARP allows a variable flow exponent (instead of the 0.5 of the orifice equation) because building presurization measurements typically correlate to equation 1 when X equals about 0.65 (ASHRAE 1977).

Solution of the Flow Equations

The development of a technique to solve the airflow equations has been particularly troublesome. Efforts have focused on two techniques described by Conte and DeBoor (1972). The first technique is the classic Newton's method. The mass balance requirement may be expressed as

$$\Sigma F_i = 0 \tag{3}$$

for every room. The flows, F_i, are summed over all openings, i, in the enclosing surfaces of the room. In Newton's method, successive values of room pressure, P_n, for each room, n, are calculated by

$$P_n^{(k+1)} = P_n^{(k)} - D_n \tag{4}$$

where

k is the iteration number and D is computed from the matrix relationship

$$[\ J\ ] * [\ D\ ] = [\ B\ ] \tag{5}$$

where

[B] is a column matrix, each element given by

$$B_n = \Sigma F_i \tag{6}$$

and [J] is the square (i.e., N by N for a building of N rooms) Jacobian matrix. The values of the diagonal elements of the Jacobian matrix are given by

$$J_{n,n} = \sum_i \frac{\partial F_i}{\partial P_n} \tag{7}$$

for all openings, i, into room n. The values of the other elements are given by

$$J_{n,m} = \sum_i \frac{\partial F_i}{\partial P_m} \tag{8}$$

for all openings, i, between room m and room n. Iteration proceeds until the net airflow into every room, B_n, is sufficiently close to zero. These partial derivatives are very easy to compute:

$$\frac{\partial F_i}{\partial P_n} = - X_i * F_i/\Delta P \tag{9}$$

$$\frac{\partial F_i}{\partial P_m} = X_i * F_i/\Delta P \qquad \text{(interroom surface)} \tag{10}$$

or

$$\frac{\partial F_i}{\partial P_m} = 0 \qquad \text{(envelope surface)} \tag{11}$$

Note that the term $(\Delta P)^{X-1}$ has been eliminated from these expressions, thus allowing evaluation of the derivatives by a simple division rather than a time-consuming exponential. Also note that as ΔP approaches zero, the derivative is undefined.

The second method calls for successively approximating each room pressure according to

$$P_n^{(k+1)} = P_n^{(k)} - \Sigma F_i / \Sigma \frac{\partial F_i}{\partial P_n} \tag{12}$$

where k is again the iteration number. This method is quite simple and requires less storage space than Newton's method because it does not use the Jacobian matrix.

Initial tests of these two methods showed that, although Newton's method was fastest for most test cases, it occasionally would not converge. The second method converged for the original test cases and was chosen for the initial version of TARP (Walton 1982). It was then found that this method converged very slowly when the openings between rooms were much larger than the openings in the building envelope, which is a very common condition. This problem led to a reexamination of Newton's method. A simple four-room test case was found to usually be quadratically convergent (about 4 iterations) except for a few cases where it converged verly slowly (about 30 iterations). In those cases, it was found that successive iterations were overcorrecting. That is, they were successively far above and then far below the correct solution, because successive corrections were of about the same magnitude but opposite sign. Convergence could be achieved by reducing the size of the pressure correction by about one-half. Since Newton's method is normally rapidly convergent, it is also necessary to reduce the size of the correction only when overcorrecting occurs. The reason for occasional slow convergence of Newton's method has not been found. It can occur with nothing more than a change in wind direction from what was a quardratically convergent case. It often occurs when the wind and stack pressures are about equal. Convergence is always fastest when the flow exponent for interroom openings is near one. The experimental studies below indicate, however, that the exponent should be one-half.

The airflow algorithm was incorporated into a test program that allowed various solution techniques to be studied without revising TARP, which is very large. The test program is available from the author.

Newton's method requires the simultaneous solution of the matrix equation (Walton 1982) at each iteration. Several techniques were considered for that solution. The first choice, and the one ultimately chosen, was Gauss elimination. It has the disadvantage that solving time is proportional to the cube of the number of equations, N, when N is large. The number of equations is equal to the number of rooms. The Cholesky method (Brown and Solvason 1962) is somewhat faster at large N, but is was found to be sensitive to computer truncation errors. A Gauss-Seidel iteration was also tried, since it has solving time proportional to the square of N. However, this iteration also failed for large openings in much the same way that equation 12 did. Newton's method was tested for larger numbers of rooms, and it was found that the number of iterations increased with the number of rooms. Thus, many rooms require both longer iterations and more iterations. The number of iterations was also found to increase with the size of the interroom openings, but it did not increase as dramatically as it had done with equation 12. Therefore, the Newton's method is most appropriate for a small number of rooms with large execution time penalties paid for simultaneously solving many rooms.

TARP uses an hourly heat balance in estimating dynamic room energy requirements. This heat balance is solved iteratively, and at each iteration a quasi-steady solution of the airflow equations is obtained by the Newton's method described above. Techniques used to reduce the number of heat-balance iterations contribute to the overall efficiency of TARP and are described in Walton (1982).

A Theory of Flow through Large Openings

Equation 1 permits air to flow in only one direction through an opening. Large openings, such as doorways, may have two-way flow as the stack effect between two rooms may cause a positive ΔP at the bottom of the doorway and a negative ΔP at the top (or vice versa). A theoretical estimate of the stack-induced airflow through a large opening in a vertical partition (a doorway) is given by Brown and Solvason (1962). The following discussion shows that the TARP method is equivalent. Figure 2 shows a cross section of a rectangular opening of height H and width W in a vertical partition that separates two rooms at temperatures T_1 and

T_2. The absolute pressure at the centerline (z = 0) is everywhere equal. The pressure difference caused by stack effect at height z is

$$P_1 - P_2 = (\rho_1 - \rho_2) * g * z \tag{13}$$

The volumetric flow through an infinitesimal area is given by

$$dQ = C * W * dz * \sqrt{2 * \Delta P/\rho} \tag{14}$$

which can be integrated to give the flow through the top half of the opening

$$Q = \int_{Z=0}^{Z=H/2} dQ = C/3 * W * \sqrt{g * \Delta\rho/\rho * H^3} \tag{15}$$

The coefficient of thermal expansion is $\beta = -\Delta\rho/(\rho * \Delta T)$, where ρ is the average density. For computational simplicity, TARP uses the density of the incoming fluid instead, but this cannot cause a significant error at normal temperatures. Other definitions are:

heat-transfer rate: $\dot{q} = Q * \rho * c * (T_1 - T_2)$ (16a)

heat-transfer coefficient: $h = \dot{q} / [W * H * (T_1 - T_2)]$ (16b)

Nusselt number: $Nu = h * H / k$ (16c)

Prandtl number: $Pr = c * \mu / k$ (16d)

Grashof number: $Gr = \rho^2 * g * \beta * (T_1 - T_2) * H^3 / \mu^2$ (16e)

The expressions can be substituted into equation 15 to give

$$Nu = C/3 * Pr * \sqrt{Gr} \tag{17}$$

This simplified analysis has neglected viscous effects and effects of an air velocity parallel to the partition. According to Brown and Solvason, the viscous effects reduce the airflow through openings in thick partitions and an air velocity may either increase or reduce the airflow.

TARP can handle the two-way airflow through a doorway by dividing the door into an upper and a lower opening. It is easy to determine the heights of the two openings, which cause a stack effect giving the same volumetric flow as the Brown and Solvason model. These turn out to be 13/18 * H for the upper opening and 5/18 * H for the lower one.

Brown (1962) also studied openings in horizontal partitions and developed a theoretical expression for convection through such openings:

$$Nu = (0.29 \text{ to } 0.35) * Pr * \sqrt{Gr} \tag{18}$$

In this case, the Nusselt and Grashof numbers are based on the thickness of the partition. This thickness is the vertical space available for the development of a stack effect.

VALIDATION OF THE LARGE-OPENING MODEL

Flow through Openings in Vertical Partitions

Weber and Kearney (1980) give a correlation for the two-way flow through a doorway based on temperature measurements in the doorway

$$Nu = 0.26 * Pr * \sqrt{Gr} \tag{19}$$

and another based on average room temperatures

$$Nu = 0.3 * Pr * \sqrt{Gr} \tag{20}$$

These correlations are based on similitude experimental studies of the room shown in figure 3 and tests on a full-scale structure with a similar configuration. Weber and Kearney estimated that they should be dependable to within 20%. They also compare their results to two other studies (figure 4) including Brown and Solvason's. Comparing equations 19 and 17 gives a value of 0.78 for the flow coefficient, C. Equation 20, which uses average room temperatures, gives a value of 0.90 for the flow coefficient. This seems unreasonably large and is apparently due to the uneven temperature distributions occurring in real rooms, especially above the door openings. A study of a doorway between a small room and a much larger, high-heat-loss, two-story room did not agree well with the correlation (equation 20).

The TARP model dividing the doorway into halves was used for a wide range of parameters and the results compared to equation 19. The interroom mass and energy flows were computed for five values of ΔT (2, 4, 6, 8, and 10 C) each at five doorway heights (1.0, 1.5, 2.0, 2.5, and 3.0 meters). A flow coefficient of .78, a flow exponent of 0.5, and stack heights at 5/18ths and 13/18ths of the doorway height give TARP computed mass and energy flows that agree with equation 19 to within 0.1% for all cases.

Flow through Openings in Horizontal Partitions

Figure 5 shows the experimental results (H = thickness; L = width of square opening) of Brown's (1962) study of openings in horizontal partitions. In this configuration, there is a significant frictional effect in the thicker partitions. Brown incorporated this into a single equation:

$$Nu = 0.0546 * Pr * Gr^{.55} * (L/H)^{.33} \tag{21}$$

For a TARP evaluation, it is better to consider the thickness effect as a modifier to the stack height. A TARP model of an opening in a horizontal partition would again divide it into two openings, one a distance Z above the center of the partition and one an equal distance below it. Then, for C = 0.78, the values of Z for different H/L would be:

H/L	Z
.0833	.168*H
.167	.120*H
.333	.093*H
.667	.074*H

These almost exactly duplicate the Brown results.

There are still several questions about this model. The effects beyond the studied ranges of Gr and H/L are not known. The model predicts that the flow should go to zero as H approaches zero! There should be some flow. It is possible that two separate openings would behave differently from a single opening of equivalent area, because one-way flow could develop in each. These questions indicate room for further experimental work. In addition, a TARP model would have to allow for no flow between rooms when the upper is warmer than the lower.

CONCLUSIONS AND RECOMMENDATIONS

In this paper it has been shown that the convective flows through large openings can be predicted by use of the simple orifice equations (equation 2) and a technique for solving the mass balance equations for multiple rooms. The orifice equation is used by dividing the opening into two equal areas and applying the appropriate flow coefficients and effective opening heights. Appropriate values are given in the paper. The TARP algorithm was developed for predicting airflows due to stack, wind, and mechanical efforts. Since no generality was lost in developing the coefficients for flow through large openings, TARP should properly allow these forces to interact with the natural convection through the openings. Although more work must be done for other building configurations, the close match between the TARP algorithm on the experimental correlations is encouraging.

Because of the rapid increase of calculation time with the number of rooms simulated, it is recommended that large buildings with many rooms be simulated by dividing the building into groups of closely coupled rooms. First, treat each group of rooms as a single room to solve for the infiltration through the building envelope and airflows between the groups of rooms. Then, while treating those airflows as constant gains or losses to the appropriate rooms, solve for the airflows between the individual rooms in each group.

Further study is needed on the simulation of room air stratification, both for its direct effects on comfort and energy requirements as well as for its effect on interroom air movement. Study is also needed on the calculation of wind pressure on the building envelope. This would include the interesting question of simplifying the effect of a pressure distribution across the surface acting on many small openings in the surface. Then detailed validation should be performed with carefully selected full-scale tests. After successful validation, it will still be necessary to develop data on air-openings for typical building components and construction techniques to permit the analysis and design of buildings.

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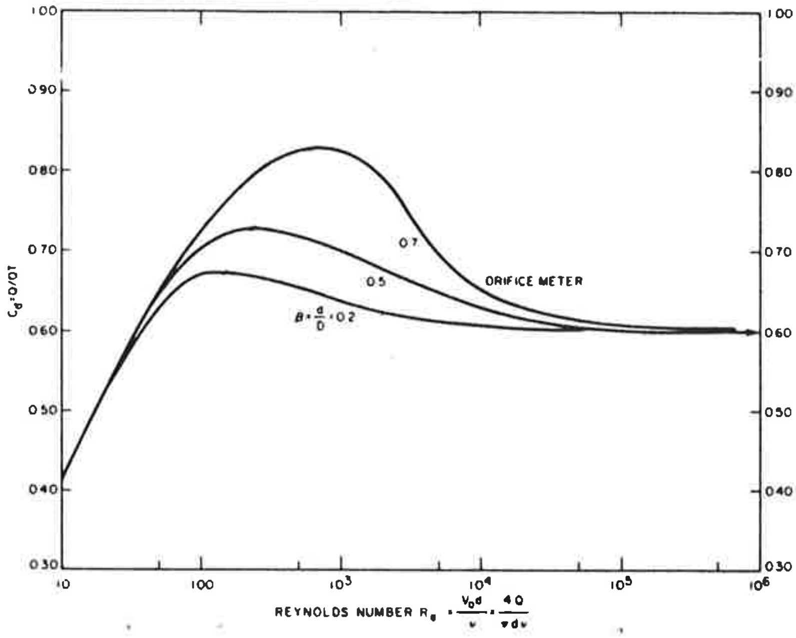

Figure 1. Orifice flowmeter coefficient (ASHRAE 1977)

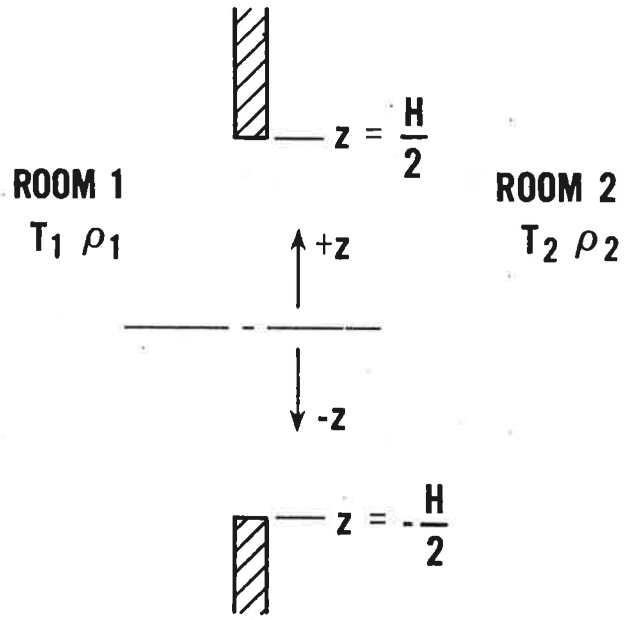

Figure 2. Schematic of opening in a vertical partition

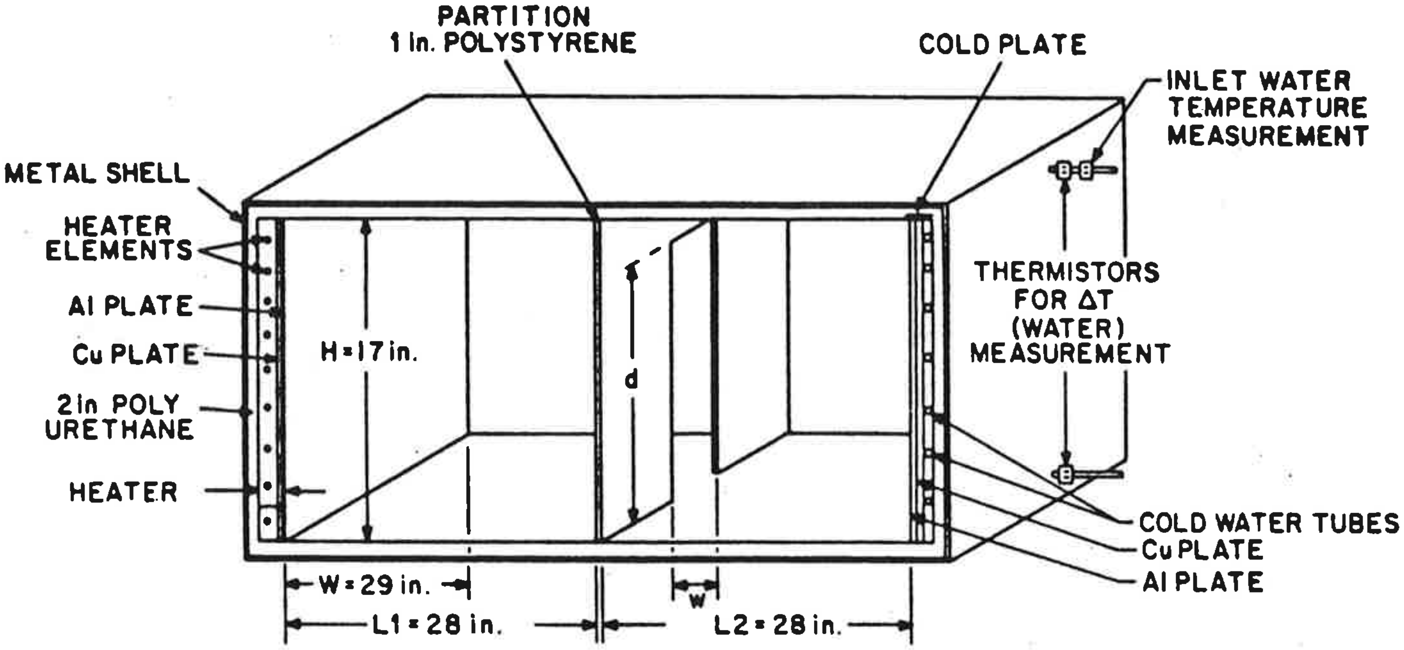

Figure 3. Two room model for determining natural convective transfer through the doorway (Weber and Kearney 1980)

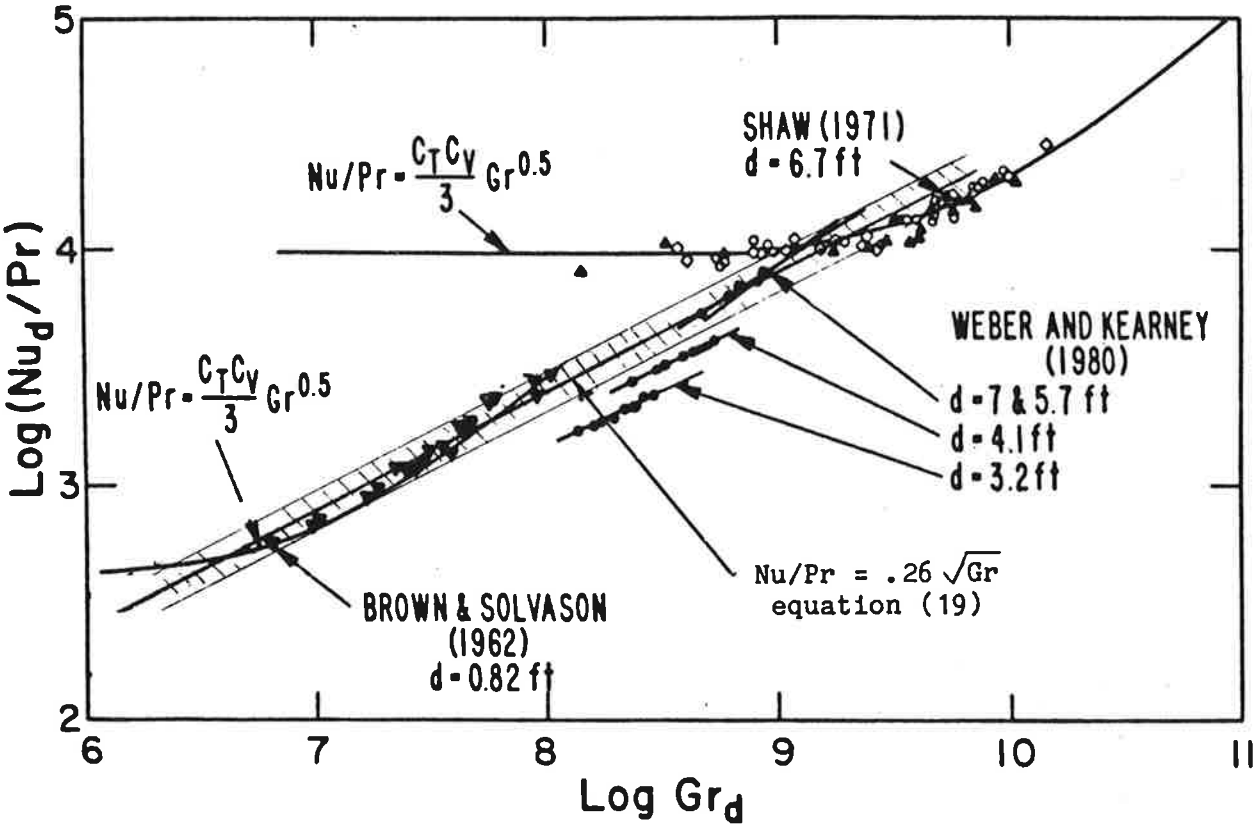

Figure 4. Comparison of experimental results for convection through openings in vertical partitions (Weber 1981). Note that symbol "d" is the opening height that is referenced by sumbol "H" in this paper. Shaded area covers ±20% from equation 29.

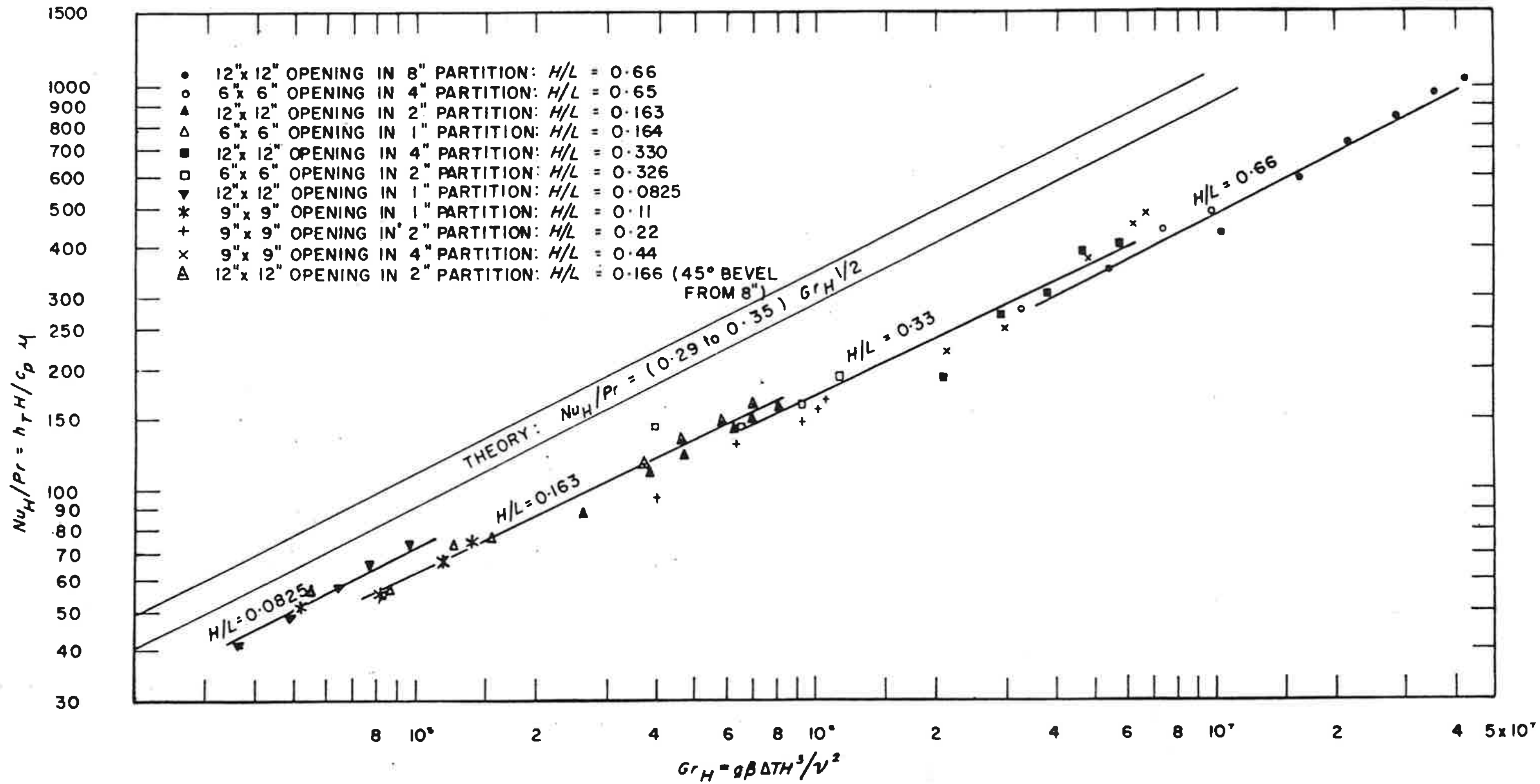

Figure 5. Experimental results for airflow through a square opening in a horizontal partition (Brown 1962)