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A Simple Parametric Study of Ventilation

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A simple study is described which demonstrates the relative importance of the various parameters which determine ventilation. The graphs presented can be used directly to estimate natural and mechanical ventilation rates under a wide variety of simple conditions. By adopting a non-dimensional approach which introduces the concept of the whole-house leakage Reynolds number, the basis is laid for a more general means of estimating ventilation from graphical data sheets.

NOMENCLATURE

A	area (suffix: T)
A_r	Archimedes number
B	dimensionless constant
a, b	coefficients in crack flow equation
a_T, b_T	coefficients in whole-house leakage characteristic
C_P	pressure coefficient
C_D	discharge coefficient
$C_{D\infty}$	discharge coefficient at high Reynolds number
g	acceleration due to gravity
h	maximum difference in height between the openings
L	width of crack, [3]
m	number of openings on the windward side
n	total number of openings
P	pressure (suffices: I, R)
Q	flow rate (suffices: i, M, T, W)
R_{EL}	whole-house leakage Reynolds number
T	absolute temperature
U	wind speed
U_B	Buoyancy velocity
y	height of opening, [3]
z	vertical coordinate (or depth of crack, [3])
$\hat{z}$	dimensionless vertical coordinate ($=z/h$)
β	flow exponent
ρ	density of air (suffices: I, O)
μ	dynamic viscosity
ν	kinematic viscosity

Prefix

Δ, δ	difference between two values of the same quantity

Suffices

I	relate to inside
M	relate to mechanical ventilation
O	relate to outside
R	relate to reference point
T	relate to total
i	number of opening
W	relate to wind.

1. INTRODUCTION

THE OBJECTIVE of this paper is to present the results of a simple parametric study of the natural and mechanical ventilation of dwellings. It is felt that such a study is useful for three main reasons. First, it allows the effects of the various parameters to be easily identified. Second, it is convenient to have the results of a large number of simple calculations in a compact form. Third, the results of the study point the way to a more detailed study which could be of use for design purposes, e.g. sizing mechanical systems, siting of air vents.

2. DESCRIPTION OF THE THEORETICAL MODEL

The model used for the calculations is basically the single-cell version of the British Gas multi-cell model [1]. However most of the calculations have been done with a simplified form of the flow equation. In addition, the correction for turbulence has not generally been applied so as not to confuse the effects of the main parameters. In essence, the model consists of three separate equations for describing:

(i) the pressure difference across an opening,
(ii) the relationship between flow and pressure difference and
(iii) continuity of mass.

2.1 *Pressure difference*

Referring to Fig. 1, an expression is required for the mean pressure difference, δP, acting across the surface of the dwelling at a height z.

Assuming a uniform air density, ρ_o, in the external flow, the mean absolute pressure at a point can be considered as the sum of two components; one due to gravity and the other due to fluid motion. The latter can be expressed in the form of a pressure coefficient, C_P, which can be measured in a wind tunnel.

Inside the dwelling, it is assumed that the density ρ_I is uniform and that the effects of air motion can be neglected.

It follows that δP can be expressed as

$$\delta P = P_R - P_I + \tfrac{1}{2}\rho_o U_R^2 C_P - \Delta\rho g z \qquad (1)$$

where,

P_R = absolute pressure at external reference point
P_I = internal pressure at reference height ($z = 0$)

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U_R = reference wind speed at $z = 0$
$C_P = (P_w - P_{wR})/\frac{1}{2}\rho_o U_R^2$
P_w = static pressure on external surface due to wind
P_{wR} = static pressure at reference point due to wind
$\Delta\rho = \rho_o - \rho_I$
g = acceleration due to gravity.

In the above expression the reference points for z and the wind speed have been taken as coincident.

To make use of the equation, it is further assumed that it remains valid in the presence of an opening in the fabric of the dwelling. For small openings this is a reasonable assumption. However for large openings, such as open windows, it may not be reasonable, if only because of effects on the surface pressure coefficient. A more fundamental reason why large openings are excluded from the present analysis, is that the flow equation is not intended to cope with them.

2.2 *Relationship between flow and pressure difference*

The chosen relationship between flow rate and δP is one which applies to steady flow. Thus it is assumed that mean flow rates through openings are determined by mean pressures acting across the openings. Any effects of external flow turbulence are therefore neglected. This could be a significant source of error (see, for example [1]), but it is a common assumption in ventilation calculations.

The chosen flow equation is

$$\delta P = aQ^2 + bQ \qquad (2)$$

where Q denotes the mean flow rate. It is important to note however, that for most of the present calculations, the coefficient b has been set equal to zero. This means that the openings have a discharge coefficient C_D which is independent of flow rate. This follows from the definition of C_D:

$$C_D \equiv \frac{Q}{A}\sqrt{\frac{\rho_o}{2\delta P}},$$

where A is a specified area of the opening. It also follows that the coefficients a and C_D are related by

$$a = \frac{\rho_o}{2C_D^2 A^2}. \qquad (3)$$

When the coefficient b is non-zero, the discharge coefficient of the opening becomes dependent on Q, i.e. C_D is a function of a Reynolds number. The relationship given by (3) remains valid, but C_D now denotes the limiting value at high Reynolds number. We can thus consider the results for $b = 0$ as the results for high Reynolds number, and they will be referred to in this way. Similarly the results for $a = 0$ will be referred to as low Reynolds number cases.

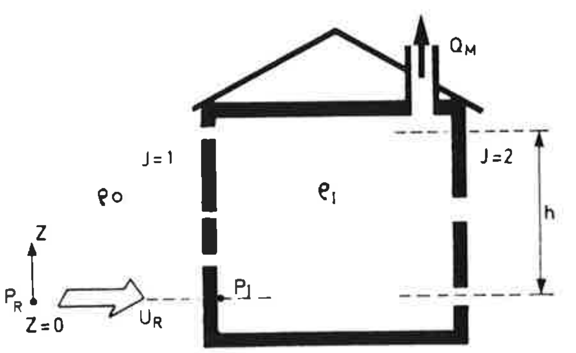

Fig. 1. Major notation.

2.3 *Continuity equation*

In the mean, the mass of air inside the dwelling is constant so that the continuity equation can be expressed by

$$\sum_n \rho Q_i = \rho Q_M$$

where

$\rho = \rho_o$ for inward flow
$\rho = \rho_I$ for outward flow
Q_M = mechanical fan flow rate

and the subscript i denotes the ith opening and n the number of openings.

For simplicity the above equation is approximated by

$$\sum_n Q_i = Q_M. \qquad (4)$$

The errors in Q_i due to the neglect of density differences will be small for moderate temperature differences (i.e. $\Delta\rho/\rho_o$ is less than 10% for temperature differences up to 27 K). Hence, in solving the complete set of equations ρ_I is replaced by ρ_o, except of course for the buoyancy term in the pressure equation.

For the calculations with mechanical ventilation, Q_M is assumed to be independent of pressures generated inside or outside the dwelling.

3. DESCRIPTION OF DWELLING AND OPENINGS

In order to simplify the presentation of results, the following assumptions have been made about the dwelling and the openings. All openings in the dwelling are identical and they are present on only two surfaces (windward and leeward) for which it is further assumed that the external pressure coefficient is uniform on each surface.

Figure 2 shows the seven basic cases which have been considered. Each case corresponds to a different configuration of openings, relative to the reference wind direction. Calculations for both natural and mechanical ventilation have been made for all of these cases, using the reduced form of the flow equation ($b = 0$).

The other calculations which have been carried out will be introduced in a later section.

At this point it is relevant to note that there are similarities between the present calculations and those described in [2]. However they differ in two important respects.

First, the model used in [2] is based on a uniform distribution of leakage on each surface and several surfaces can be considered, each with its own distribution. In the present cases, openings are placed on only two surfaces (at discrete points). The latter distribution has the advantage that it allows a more simple presentation of results, which is the main reason for choosing it for the present study.

The second difference is that the well-known power-law

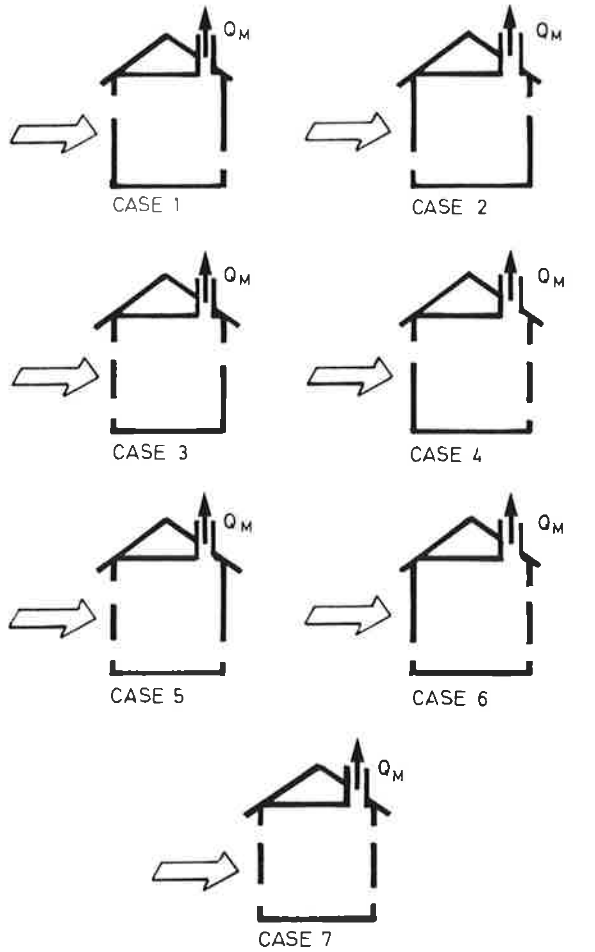

Fig. 2. Basic cases considered.

equation is used in [2] to describe the flow, i.e.

$$Q \propto \delta P^{\beta}$$

where β is the flow exponent. This is a fundamental difference between the two models. The basic argument for the quadratic equation has been given in [3] and more recently its application to whole-house leakages has been discussed in [5].

4. SOLUTION OF EQUATIONS

With the simplifications noted in the previous section, the system of equations to be solved is n equations of the form:

$$\frac{bA}{\rho_o U_B}\left(\frac{Q_i}{AU_B}\right)+\frac{aA^2}{\rho_o}\left(\frac{Q_i}{AU_B}\right)^2=\frac{P_R-P_I}{\rho_o U_B^2}+\frac{C_{P_j}}{2A_r^2}-\hat{Z}_i \tag{5}$$

and the continuity equation

$$\sum_n Q_i = Q_M. \tag{6}$$

To obtain (5), δP has been eliminated from (1) and (2) and the resulting equation has been non-dimensionalized for generality by dividing throughout by $\Delta\rho g h$. The reference height h is defined as the maximum vertical distance between two openings. In addition the following terms have been introduced.

$$U_B \equiv \sqrt{\Delta\rho g h/\rho_o},$$

$$A_r \equiv \sqrt{\frac{\Delta\rho g h}{\rho_o U_R^2}},$$

and

$$\hat{Z}_i \equiv z_i/h.$$

U_B has the dimensions of speed, A_r is the Archimedes number, and it can be seen that

$$A_r = \frac{U_B}{U_R}.$$

For the reason which will be clear below, it is convenient to define the coefficient of the overall pressure difference across the dwelling by

$$\Delta C_P \equiv C_{P_1} - C_{P_2},$$

where C_{P_1} and C_{P_2} are the windward and leeward pressure coefficients, i.e. $j = 1$ or 2 in (5).

Equations (5) and (6) can be considered as $(n+1)$ equations for the $(n+1)$ unknowns, i.e. Q_i and $(P_R - P_I)$. It should be noted that (5) has been derived by non-dimensionalizing with $\Delta\rho g h$ (i.e. $\rho_o U_B^2$). This means that solutions can not be obtained when U_B (i.e. ΔT) tends to zero. A more general equation could have been obtained by dividing by $\rho_o(U_B^2 + U_R^2)$. However by using $\rho_o U_B^2$ the solutions for $\Delta T > 0$ can be collapsed onto a single curve, which simplifies the graphical presentation of results. The solutions for ΔT equal to zero can be given by one other curve. This simplified approach is also justifiable from the practical viewpoint, because cases with non-zero values of ΔT are of most interest.

4.1 *Solution for high Reynolds number, $b = 0$*

Although setting the coefficient b equal to zero simplifies the equations, we have not been able to find a simple analytic solution to the general case of n openings. Even for specific cases the algebra becomes unwieldy. For this reason, most of the solutions have been obtained with a computer, using a program in which the internal pressure coefficient $[(P_R - P_I)/\rho_o U_B^2]$ is adjusted until the continuity equation is satisfied to a chosen precision. Curves have then been fitted by hand to the computed points. This means of course that the curves are not completely precise, but for the present purposes the errors are not significant.

For the very simple case of two openings (Case 1 below) the solution can be expressed simply as

$$\frac{Q_1}{C_D A U_B} = \frac{Q_2}{C_D A U_B} = \frac{1}{4}\sqrt{\frac{\Delta C_P}{A_r^2}+2}.$$

The parameter $(\Delta C_P/A_r^2)$ which occurs in the above equation is important, because it is a measure of the ratio of the two basic driving forces for the ventilation of the dwelling, i.e. wind pressure $(\rho_o U_R^2 \Delta C_P)$ and buoyancy $(\Delta\rho g h)$.

Although a general analytic solution has not been found, it is possible to show that the general solution is of the form

$$\frac{Q_i}{C_D A U_B} = f_i\left(\frac{\Delta C_P}{A_r^2}\right)$$

and

$$\frac{Q_T}{C_D A U_B} = f_T\left(\frac{\Delta C_P}{A_r^2}\right)$$

where Q_T denotes the ventilation rate

$$Q_T = \tfrac{1}{2}\Sigma|Q_i|.$$

The functions f_i and f_T are determined by the opening distribution. Thus the results of the calculations can be simply presented as plots of $Q/C_D A U_B$ against $\Delta C_P/A_r^2$. The reason for defining ΔC_P can also be seen, i.e. the flow rates depend only on the difference between the C_{P_j} and not on their separate values.

4.2 *Solution for low Reynolds number*, a = 0

When the coefficient a is set to zero, it is possible to obtain an analytic expression for the general case of n openings with m openings on the windward side.

For the ith opening on the windward side the flow rate is given by

$$\frac{bQ_i}{\Delta\rho gh} = \left(\frac{\sum_n \hat{Z}_i}{n} - \hat{Z}_i\right) - \frac{\Delta C_P}{2A_r^2}\left(\frac{m}{n} - 1\right). \quad (7)$$

For the leeward openings $(m/n - 1)$ is replaced by m/n.

4.3 *Solution for* a ≠ 0, b ≠ 0

When both terms are retained in the flow equation, the solution becomes more difficult to obtain analytically, so all the results presented here have been obtained using a computer.

However it can be shown that for a given distribution of openings, the individual and the total flow rates are given by the functional relationship

$$\frac{Q}{C_{D_\infty} A U_B} = f\left(\frac{\Delta C_P}{A_r^2}, \frac{bAC_{D_\infty}}{\rho_o U_B}\right) \quad (8)$$

where C_{D_∞} is the discharge coefficient at high Reynolds number.

The term $bA/\rho_o U_B$ can be identified as the reciprocal of a Reynolds number, if one assumes that the bQ term in the flow equation corresponds to the viscous term in the crack flow equation of [3]. This is not an unreasonable assumption and it will be made here. In the terminology of [3] the coefficient b is given by

$$b = \frac{\mu B z L^2}{8A^3}$$

and it follows that

$$\left(\frac{bA}{\rho_o U_B}\right)^{-1} = \frac{8}{B}\frac{\rho_o}{\mu}\frac{y^2}{z}U_B$$

where B is a dimensionless constant, μ is viscosity and the term y^2/z has the dimension of length. It can be seen that the right-hand side of the above expression is a Reynolds number, based on the geometry of the opening and the equivalent air speed U_B.

If the whole-house leakage characteristic is

$$\delta P = a_T Q^2 + b_T Q$$

then we can define a whole-house leakage Reynolds number by

$$R_{E_L} \equiv \frac{\rho_o U_B}{b_T A_T}.$$

When all the openings are identical the coefficients b_T and a_T are simply related to a and b by

$$b_T = \frac{b}{n}, \qquad a_T = \frac{a}{n^2}$$

and A_T is simply equal to nA. It then follows that

$$R_{E_L} = \left(\frac{bA}{\rho_o U_B}\right)^{-1}.$$

By way of clarification it should be noted that R_{E_L} is not the same as a Reynolds number based on the flow rate through the openings, such as the crack Reynolds number of [3]. R_{E_L} depends on the geometry of openings in the house and the temperature difference ΔT, so that as far as a designer is concerned it forms part of the input required for a solution. The form of R_{E_L} stems from the fact that (5) has been obtained by non-dimensionalizing with $\rho_o U_B^2$. With this definition R_{E_L} tends to zero when U_B tends to zero, but high crack Reynolds numbers will still occur if the wind speed U_R is high so that a low value of R_{E_L} does not necessarily mean that the effects of viscosity are dominant. In this respect it would have been better to base the leakage Reynolds number on the resultant wind speed $(U_B^2 + U_R^2)^{1/2}$, but this has not been done for the reason noted at the end of Section 4.

Nevertheless a change in R_{E_L} does produce effects which are analogous to the conventional scale effect associated with Reynolds number, e.g. if C_{D_∞} and $\Delta C_P/A_r^2$ are kept constant in (8), an increase in R_{E_L} will lead to an increase in $Q/C_{D_\infty} A U_B$ (this will be seen below in Fig. 14). Of course as far as the solutions of (8) are concerned it is the parameter C_{D_∞}/R_{E_L} which is important. This can be expressed as:

$$\frac{C_{D_\infty}}{R_{E_L}} = \frac{1}{\sqrt{2\rho_o}} \cdot \frac{1}{U_B} \cdot \frac{b}{\sqrt{a}}$$

and it can be seen that a low value of R_{E_L} can mean that $b \gg a$ (the low Reynolds number case considered here). Similarly a high value of R_{E_L} implies that $a \gg b$ (the high Reynolds number case).

5. PRESENTATIONS OF RESULTS FOR HIGH REYNOLDS NUMBER

The main results are given in Figs. 3–9 which show the individual flow rates Q_i and the ventilation rate Q_T. The individual flows are only shown when they are inward, because fresh air entry is generally of more interest than outward flow.

The form of presentation is slightly unusual and requires some clarification. Normally one is interested in the variation of ventilation rate with either wind speed or temperature difference ΔT. For a fixed ΔT, the graphs show directly how Q_T changes with U_R^2. When U_R is fixed, the graphs essentially show the variation of $Q/\sqrt{\Delta T}$ with

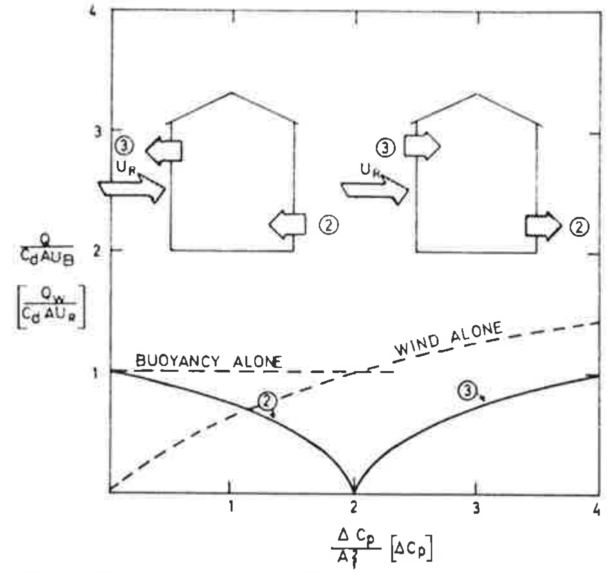

Fig. 3. Case 1. Natural ventilation. Solid lines show total ventilation rate for all wind speeds with non-zero temperature difference. Lines with circled numbers show the corresponding rates at which fresh air enters the openings. The total ventilation rate is also shown for buoyancy alone and wind alone. For wind alone the scales of the graphs are those in square brackets.

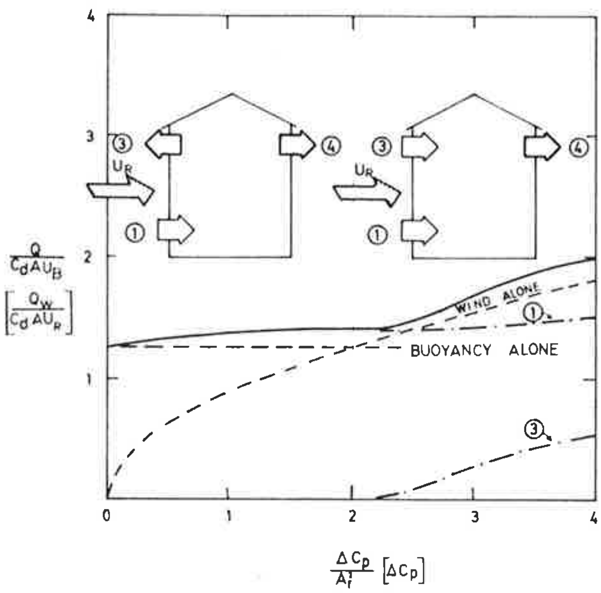

Fig. 5. Case 3. Natural ventilation. See notes to Fig. 3.

ΔT^{-1}. When U_R is equal to zero, the flow rates are proportional to $\sqrt{\Delta T}$ as shown by the intercepts of the curves with the vertical axis.

For the case when ΔT is equal to zero, the total ventilation rate is shown by the 'wind alone' curve, using the scales given in square brackets in the graphs. Individual flow rates are not shown, but they can be deduced.

Appendix 1 illustrates the use of the graphs.

6. DISCUSSION OF RESULTS FOR HIGH REYNOLDS NUMBER

One of the most important features of ventilation is the relative effect of wind and buoyancy. The form of

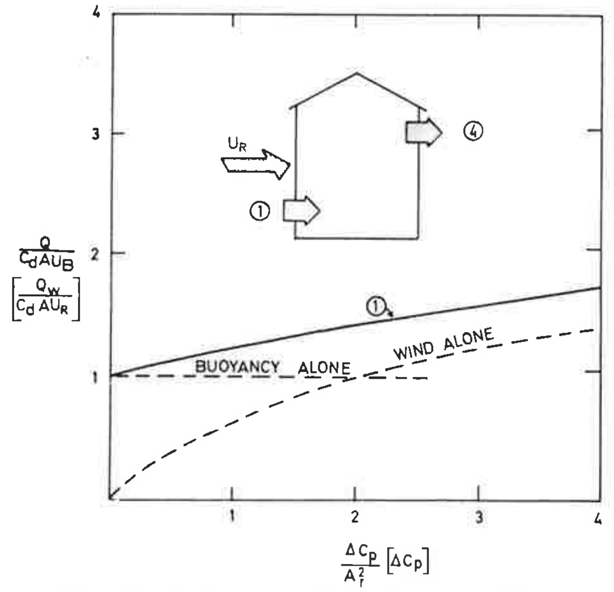

Fig. 4. Case 2. Natural ventilation. See notes to Fig. 3.

presentation shows this feature clearly, because $\Delta C_P/A_r^2$ is the ratio of the two driving forces. The importance of A_r alone has been discussed in earlier work, notably [2] and [4], but A_r does not give a complete picture because it does not include the direction of the wind. A_r is the ratio of freestream wind pressure and buoyancy pressure, whereas it is the surface pressures generated by the wind which are directly relevant to ventilation.

For the present study, the complete effect of meteorological conditions is given by the single parameter $\Delta C_P/A_r^2$, because the effect of wind direction is primarily incorporated in ΔC_P (for opening distributions which are not symmetric it is also necessary to consider complementary cases such as Cases 1 and 2).

The influence of the above parameter is particularly noticeable for individual openings, because it causes changes in flow direction (these changes are referred to as 'turning points'). It also has an influence on the ventilation rate, but to a lesser extent. These two influences are discussed separately.

6.1 *Turning points*

The turning point of an opening is defined as the value of $\Delta C_P/A_r^2$ for which a change in flow direction occurs (i.e. δP_i and Q_i are equal to zero).

For the seven opening distributions considered here it is fairly easy to calculate exactly the turning points for high Reynolds number. As can be seen from the results these have integer values. Furthermore it is interesting to note that the values are the same as those for low Reynolds number. The turning points for this type of flow can be expressed in a general form by putting $Q_i = 0$ in (7), i.e.

$$\frac{\Delta C_P}{A_r^2} = 2\,\frac{\frac{1}{n}\sum_n \hat{Z}_i - \hat{Z}_i}{\left(\frac{m}{n}-1\right)} \quad \text{for windward openings}$$

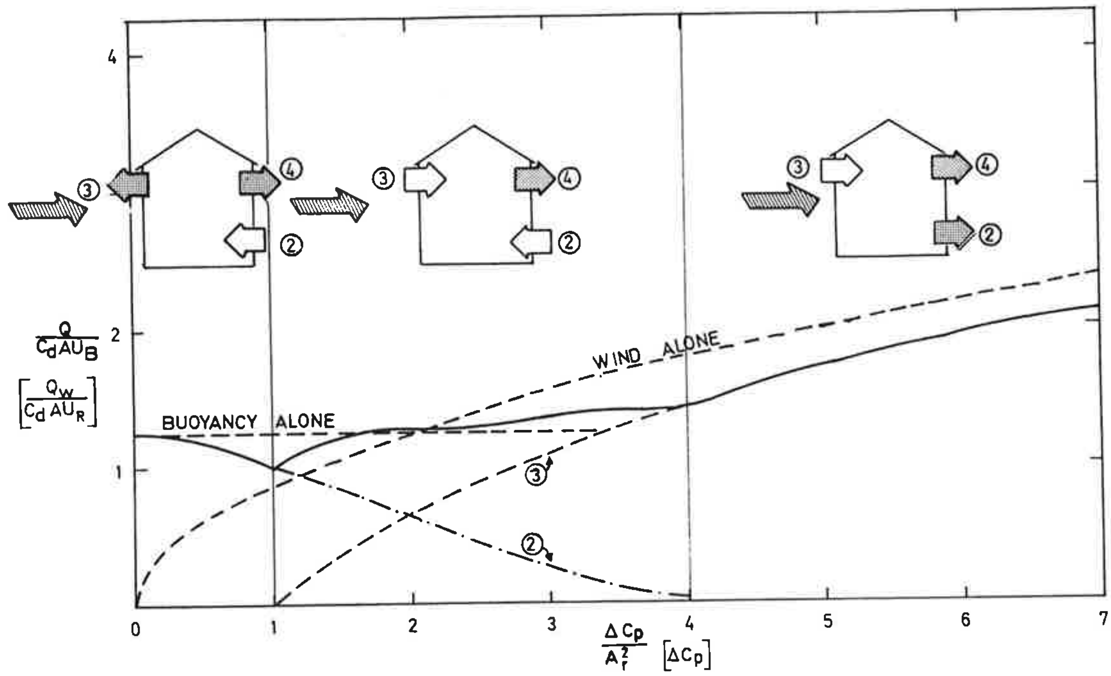

Fig. 6. Case 4. Natural ventilation. See notes to Fig. 3.

and

$$\frac{\Delta C_P}{A_r^2} = 2\frac{\sum \hat{Z}_i - n\hat{Z}_i}{m} \quad \text{for leeward openings.}$$

Although this does not hold for the general case, it indicates that the turning points are more closely related to the distribution of openings than to their flow characteristics.

6.2 *Influence of wind and buoyancy on ventilation rate*

An important aspect of the results is the difference between the ventilation rates which occur when buoyancy and wind act together and those which occur due to buoyancy or wind alone. It is much easier to calculate ventilation rates for buoyancy alone or wind alone, than it is when they are both present. The range of values of $\Delta C_P/A_r^2$ for which either buoyancy alone or wind alone gives an acceptable approximation to Q_T when they are combined can be seen directly (for buoyancy alone) from Figs. 3–9.

For Case 1 which is one of the simplest cases, with only two openings, neither buoyancy alone nor wind alone gives a good approximation to the combined result. However, when the wind direction is reversed, as in Case 2,

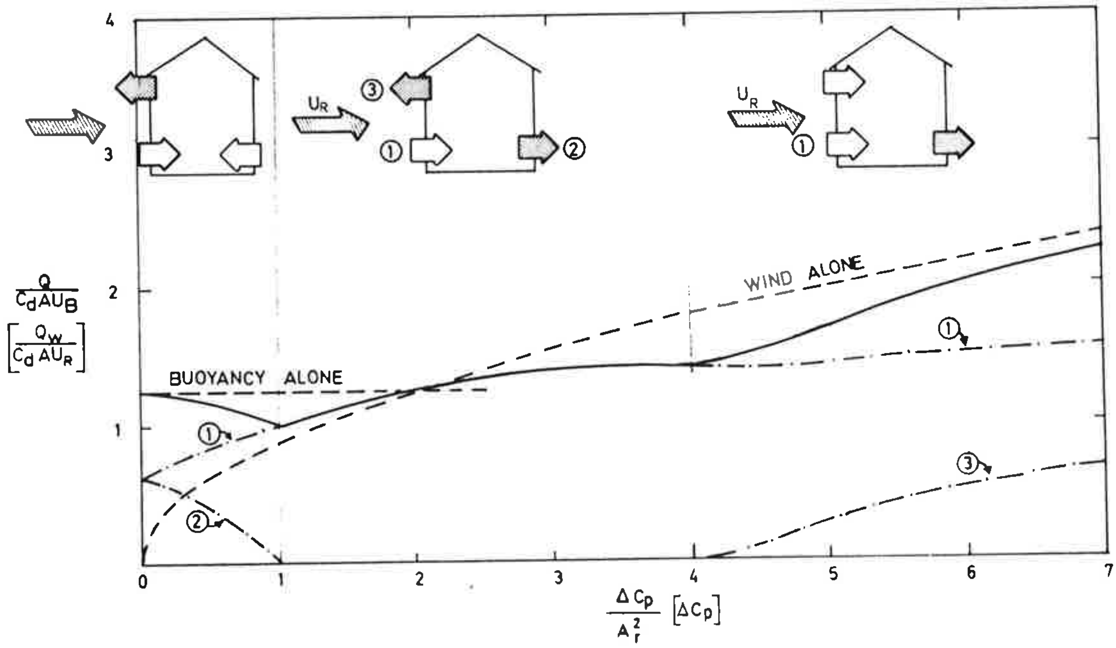

Fig. 7. Case 5. Natural ventilation. See notes to Fig. 3.

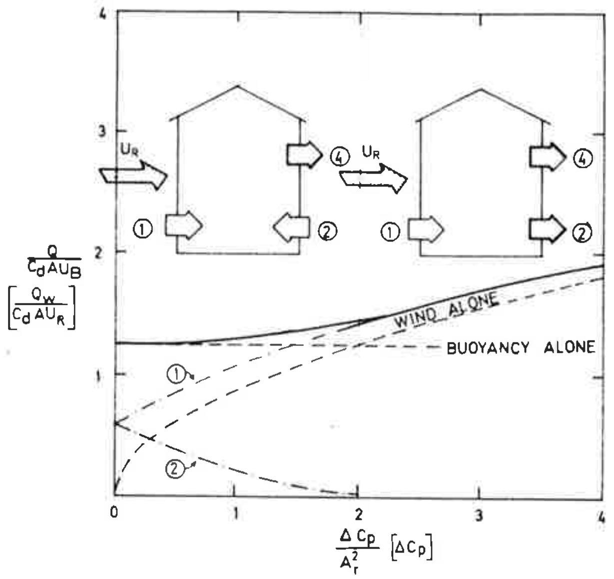

Fig. 8. Case 6. Natural ventilation. See notes to Fig. 3.

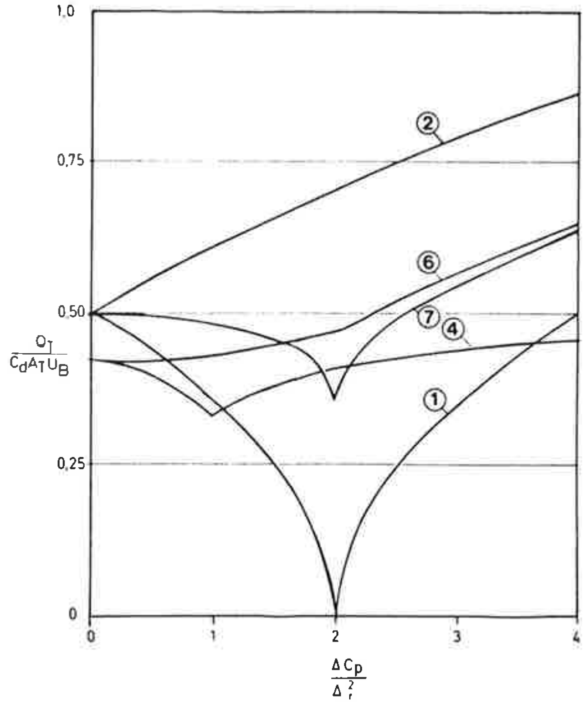

Fig. 10. Influence of opening distribution on natural ventilation for Cases 1, 2, 4, 6, and 7.

better approximations are apparent. The buoyancy alone results give closer agreement for $\Delta C_P/A_r^2 < 2$ and above this the wind alone results are better. A similar behaviour is apparent for Case 3, and for Case 6. For Cases 4 and 5 however, buoyancy alone gives better agreement up to $\Delta C_P/A_r^2 \simeq 5$. Finally, Case 7 is similar to Cases 3 and 6.

6.3 *Influence of opening distribution on ventilation rate*

One aspect of ventilation which is often discussed is the dependence of ventilation rate on the distribution of openings, i.e. for a given total area of openings, to what extent does Q_T depend on their distribution?

For simple opening distributions considered here, the dependence can be quite strong. Figure 10 shows the ventilation rates for four of the seven cases non-dimensionalized with A_T, the total area of openings. It can be seen that there are large differences between the curves. However when Cases 1 and 2 are neglected (the cases with only two openings), the differences become much less marked. The implication of this is that as the number of openings increases such that the total area becomes more evenly distributed the effect of the distribution is, not surprisingly, reduced.

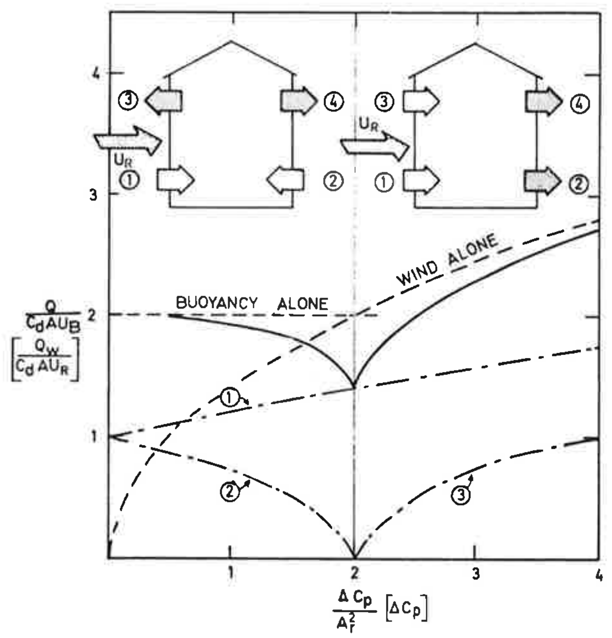

Fig. 9. Case 7. Natural ventilation. See notes to Fig. 3.

6.4 *Mechanical ventilation*

Figure 11 shows the ventilation rates arising when a mechanical flow rate Q_M is applied to Case 7. For this case, the results are the same for both supply alone and extract alone, because the opening distribution is symmetrical. The results for a balanced system (equal supply and extract rates) are not shown, because they can be obtained simply by adding Q_M to the values of Q_T for natural ventilation (Fig. 9). This is possible, because for the dwellings considered the balanced system will cause no change to the internal pressure distribution and the flow through each opening will be unaltered.

One of the most important features of the results is the ability or otherwise of the system to provide a constant ventilation rate independent of meteorological conditions. For example, from Fig. 11 it can be seen that there is a value of $Q_M/C_D A U_B$ below which the system cannot operate at its design condition. That is, the ventilation rate with the system operating, Q_{T_M}, is greater than Q_M for all meteorological conditions, except for the special condition where both U_R and ΔT are equal to zero.

The values of $\Delta C_P/A_r^2$ at which Q_{T_M} just exceeds Q_M are here called 'critical points' and it is a fairly simple matter to evaluate $[\Delta C_P/A_r^2]_{CRIT}$ as a function of $Q_M/C_D A U_B$. For the extract system, say, it is basically a matter of deciding by

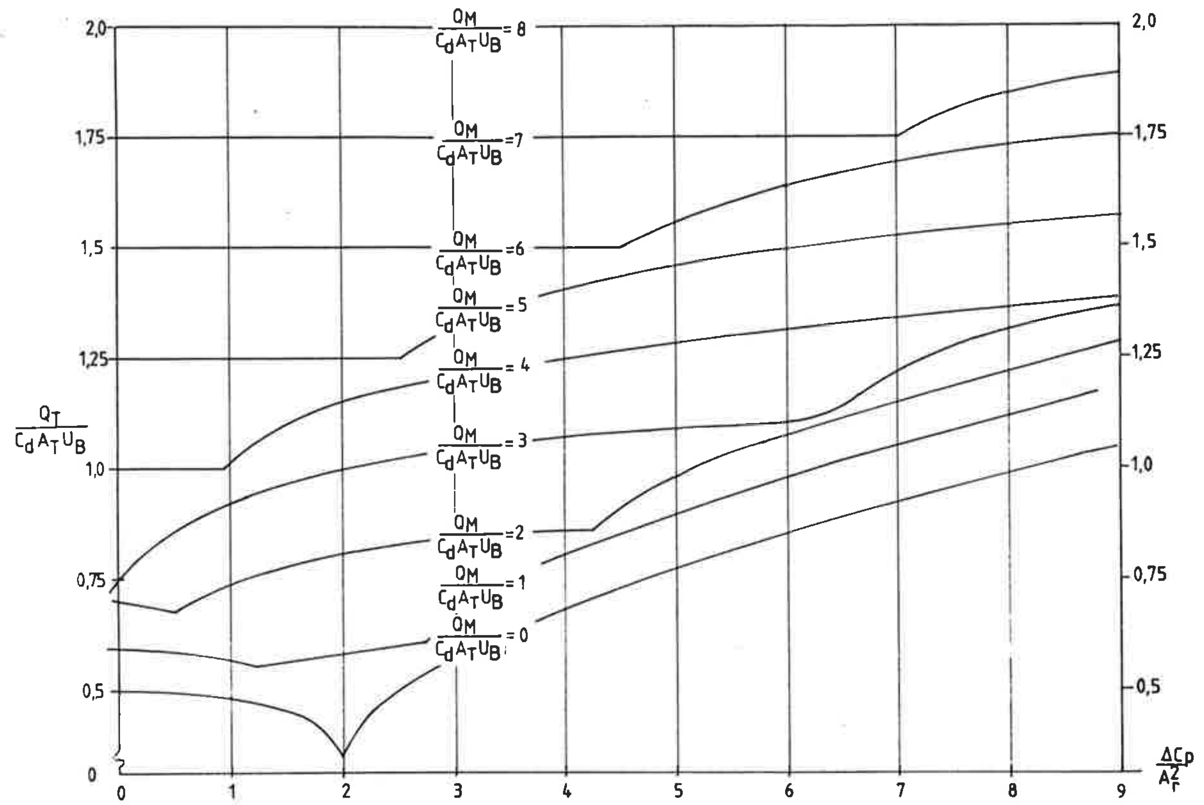

Fig. 11. Ventilation rates for Case 7 with mechanical extract (or supply).

inspection which of the openings will first experience a change of flow direction from inwards to outwards.

This has been done for the seven cases and the results for the supply and extract systems are shown in Fig. 12. For most cases it can be seen that $Q_M/C_D A_T U_B$ increases monotonically with $[\Delta C_P/A_r^2]_{CRIT}$ (the behaviour under buoyancy alone conditions is given by the intercept of the curves with the vertical axis). For some cases the curve has two distinct parts. For these it is possible for the system not to operate properly with buoyancy alone, but to operate properly when the wind speed increases.

6.5 *Practical implications of the results*

One cannot draw general conclusions from the above results, primarily because of the simple cases which have been treated, but also because of the influence of Reynolds

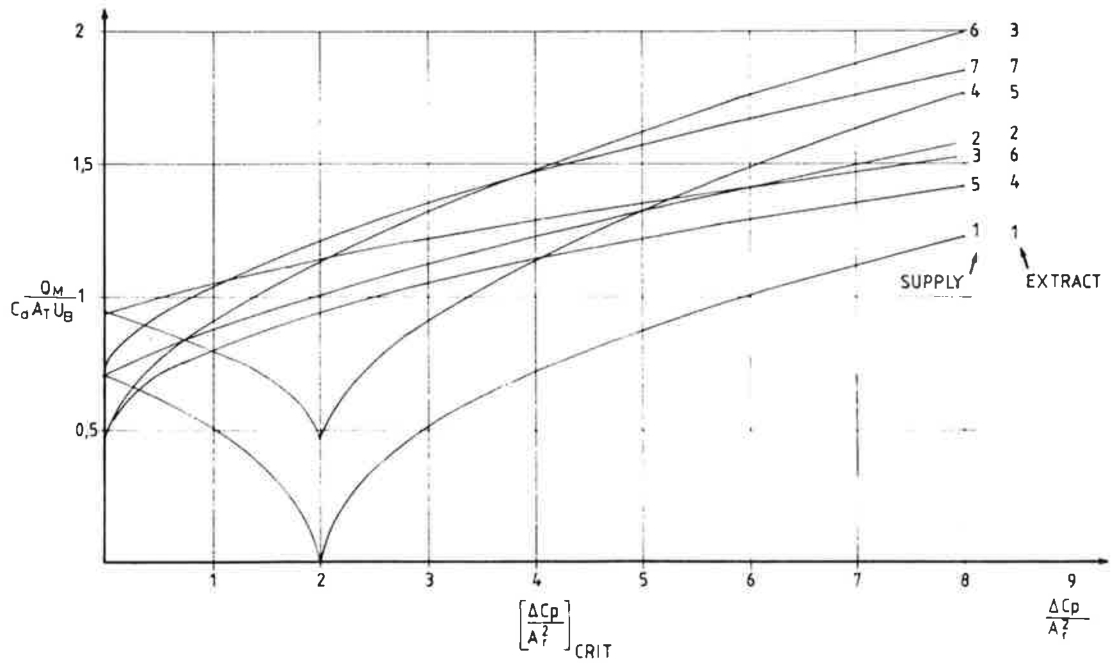

Fig. 12. Critical points for proper operation of mechanical (extract or supply) systems. Cases 1–7.

Table 1. Typical values of $\Delta C_P/A_r^2$

	$\Delta C_P/A_r^2$					
	$\Delta C_P = 0.2$			$\Delta C_P = 1.0$		
ΔT	10	20	30	10	20	30
U_R						
1	0.15	0.07	0.05	0.75	0.37	0.25
2	0.60	0.30	0.20	2.99	1.49	0.99
5	3.74	1.87	1.25	18.7	9.34	6.23

U_R in m s^{-1}; ΔT in K; $h = 4$ m.

number (see below). However the results have practical implications.

One example is the indication that the turning points of the openings are relatively insensitive to their flow characteristics, i.e. the ventilation pattern of the dwelling in qualitative terms is mainly determined by $\Delta C_P/A_r^2$ and by the opening distribution. For $\Delta C_P/A_r^2 < 1$, the ventilation pattern of all the cases considered is dominated by buoyancy, i.e. air enters through the lowest openings.

Some representative values of $\Delta C_P/A_r^2$ for $h = 4$ m are given in Table 1. Two values of ΔC_P have been used, to illustrate the effects of wind direction and/or sheltering. For $\Delta C_P = 1.0$, values of $\Delta C_P/A_r^2$ less than 1.0 only occur at very low wind speeds, so the range of dominance of buoyancy on the ventilation is quite small. However the total ventilation rate can still be approximated by buoyancy alone over a wider range of meteorological conditions (see Section 6.2).

Another example can be seen in the 'critical points' for mechanical systems (Fig. 12). For the U.K., where $\Delta T =$ 20 K is roughly the design temperature difference, the value of $Q_M/C_D A_T$ required to achieve proper operation is (also using Table 1, with $U_R = 5$ m/s and $\Delta C_P = 1.0$, and noting that $U_B = U_R A_r$) about 3.2 for Case 7. The corresponding value for a design temperature typical of Sweden ($\Delta T = 30$ K) is 3.4. Taking this a stage further, it is a simple matter to calculate the pressurization corresponding to these values of $Q_M/C_D A_T$. For the U.K. it is 6 Pa, and for Sweden it is 7 Pa. These values are quite low, but to be achieved with an acceptable value of Q_M, the leakage of the dwelling would need to be below a certain level. For example, taking the volume of the dwelling to be 200 m^3 and the desired air change rate to be 0.75 h^{-1} the leakage of the dwelling at 50 Pa would need to be about 2 house-volumes/h. This level of tightness can be achieved with Swedish construction techniques, but U.K. dwellings are usually much leakier. Of course, in practice one would probably tolerate some departures from proper operation, so a lower degree of tightness would be acceptable. Furthermore, it will be seen in the following that the critical points are sensitive to the leakage Reynolds number.

7. INFLUENCE OF REYNOLDS NUMBER

As noted in Section 4, calculations have also been made with the flow equation in its very low Reynolds number form ($a = 0$) and in its complete form ($a \neq 0$, $b \neq 0$). The latter form is more important, but it is interesting to consider the first equation briefly.

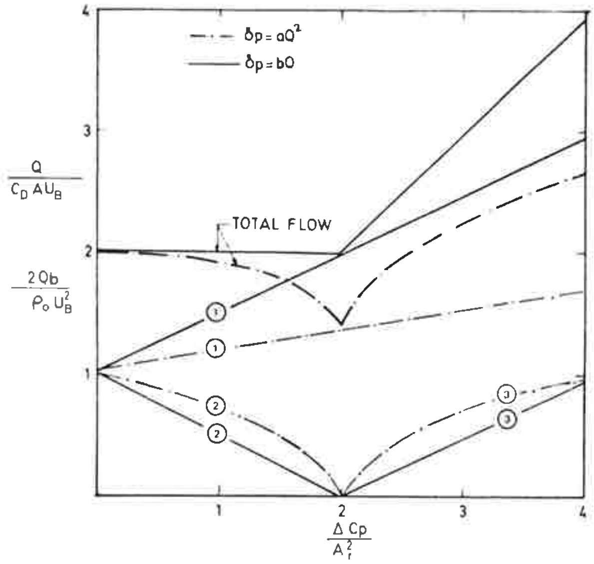

Fig. 13. Comparison of natural ventilation for Case 7 at high and low Reynolds numbers.

Figure 13 compares the results for Case 7 with the results for the high Reynolds number equation. There is a considerable similarity between the two sets of curves, and the fact mentioned earlier that the turning points are the same can be seen. However it should be noted that the scale of the plots differ.

Figure 14 shows the same comparison but for the general form of the equation (only the total flow rate is shown). The scales of the graphs are the same and the comparison is illuminating. It can be seen how different curves are obtained for different Reynolds numbers, and

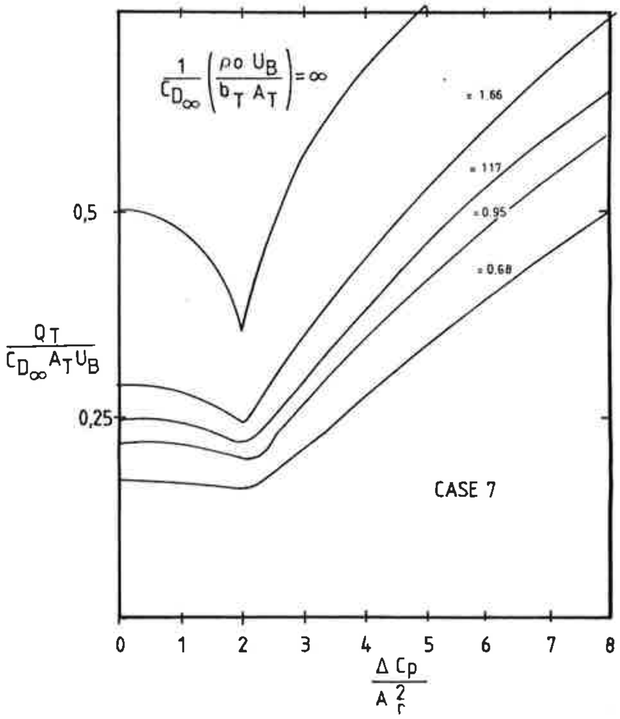

Fig. 14. Influence of leakage Reynolds number on natural ventilation for Case 7.

how the high Reynolds number results are simply a limiting case of the more general results.

Calculations have also been made to show the effect of Reynolds number on the critical points for a mechanical supply system. The results for Case 1, obtained analytically, are shown in Fig. 15. The indication from this and from Fig. 14 is that the leakage Reynolds number parameter can have a large influence on ventilation characteristics. This presupposes, of course, that values of $\rho_o U_B/aAC_{D_\infty}$ as low as 0.6 will be encountered in practice. This is considered to be feasible, because one can put

$$\frac{\rho_o U_B}{bAC_{D_\infty}} = \frac{\sqrt{a}}{b} U_B \sqrt{2\rho_o}$$

and taking $U_B = 1.5$ m/s gives a value for $\sqrt{a}/b$ of 0.25. Roughly speaking, this corresponds to a power law coefficient β of 0.68 (see [5]) which is not an uncommon value.

7.1 *A more general parametric study*

The above results indicate that to be of more general use, a parametric study should include a Reynolds number based on a reference wind speed and some characteristic of the whole-house leakage. The Reynolds number parameter C_{D_∞}/R_{E_L} in (8) should suffice for this purpose. It is useful in such a study to assume that the openings which give rise to the total leakage are identical, so that the leakage distribution is varied simply by altering the number and position of openings. This reduces the number of possible dwelling types and leads to a simple relation between the flow coefficients of the openings and the whole-house leakage.

Such a study could prove useful for estimating ventilation rates from leakage measurements. This was the main aim of the work described in [2] and is implicit in the methods included in [6], most of which are also based on the power law. The present approach can be considered as a logical extension of the concept of a crack Reynolds number to complete dwellings. For individual openings this is considered to be a more rigorous approach. Of course, the effects of Reynolds number are implicit in the power-law, however there is evidence that substantial differences can occur between the two approaches (see [5]) so further work along the present lines is warranted.

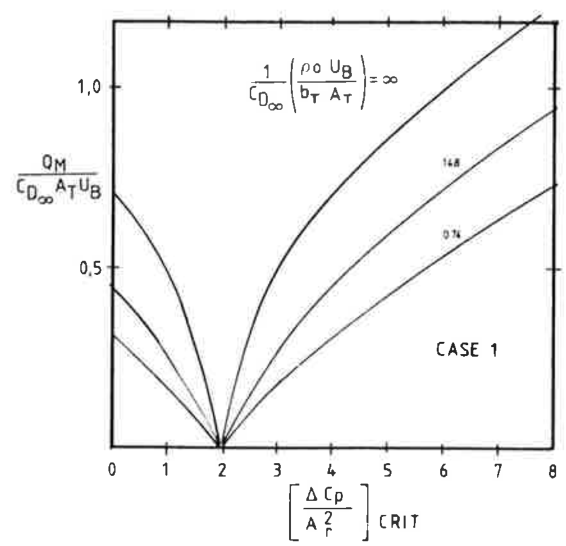

Fig. 15. Influence of leakage Reynolds number on the critical points for a mechanical supply system. Case 1.

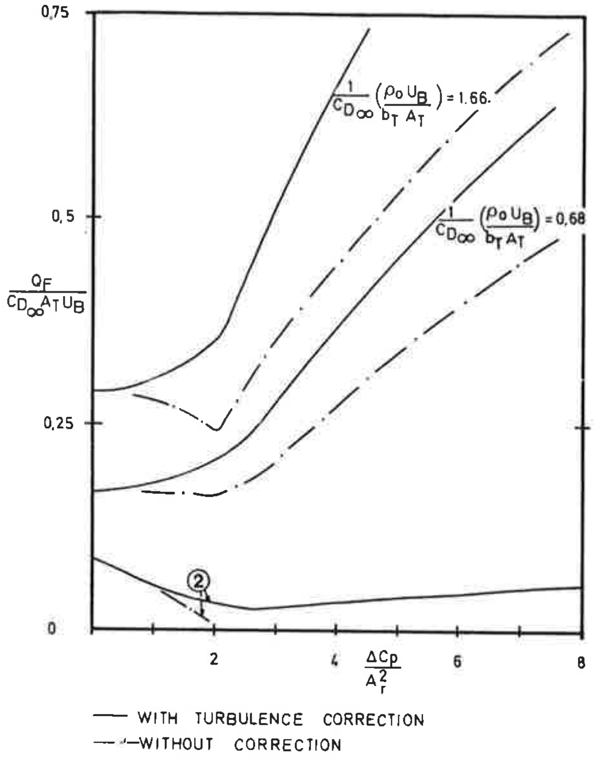

Fig. 16. Comparison of results for Case 7, with and without turbulence correction.

7.2 *Inclusion of turbulence correction*

In common with other models the results presented above do not include any effects of turbulent pressure fluctuations. When the correction for turbulence described in [1] is included in the calculation large increases in ventilation rates are observed. Figure 16 compares results for Case 7 with and without the correction. The whole-house rates for two Reynolds numbers are shown and the flow rates through one opening at the lower Reynolds number.

When assessing the comparisons it must be remembered that several assumptions are inherent in the correction [1]. Also, the coefficient of the pressure fluctuations has been taken as 0.3 and it has been assumed that the total open area of the openings is affected by the fluctuations. If only half the area were affected, then the extra ventilation ascribed to turbulence would be halved. Nevertheless it does seem desirable to apply the correction and further studies are in progress.

8. CONCLUSIONS

The simple parametric study which has been carried out for the limiting case of high Reynolds number, illustrates the influence of wind and buoyancy on the natural and mechanical ventilation of very simplified representations of dwellings. It does this in a compact way, because the simplifications allow the relative effects of the two meteorological driving forces to be described by the single parameter $\Delta C_P/A_r^2$. Although one should not draw generalized conclusions from the study, the following results have practical implications.

The ventilation pattern (i.e. the points at which air enters and leaves the dwelling) is insensitive to the flow characteristics of the openings. It is mainly determined by $\Delta C_P/A_r^2$ and the distribution of the openings. For $\Delta C_P/A_r^2 < 1.0$, the ventilation pattern is dominated by buoyancy for all the cases considered, and estimates of whole-house ventilation rates for buoyancy alone can give reasonably accurate results for a wide range of wind conditions.

The performance of mechanical supply or extract systems is influenced by meteorological conditions and by distribution of openings. The graphs presented enable estimates to be made of the mechanical flow rate (or house tightness) required for proper operation of the system. Different values will be obtained for the supply and extract systems when the opening distribution is not symmetric.

In order to investigate the more general case of low Reynolds numbers, a Reynolds number parameter C_{D_∞}/R_{E_L} based on whole-house leakage has been introduced. It appears that this can be as important a parameter as $\Delta C_P/A_r^2$.

By using the non-dimensional parameters described, a single graph can cover a very wide range of conditions. This leads to a very simple graphical way of estimating ventilation rates, which would be of use to designers and research workers.

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APPENDIX

Examples of using Figs. 3–9

Suppose one wishes to obtain whole-house ventilation rates for Case 7 (Fig. 9) for the following meteorological conditions:

(i) $T_o = 0°C$, $U_R = 2$ m/s^{-1}
(ii) $T_o = 0°C$, $U_R = 0$
(iii) $T_o = 20°C$, $U_R = 2$ m s^{-1}

with the internal temperature $T_I = 20°C$. One needs to specify the wall pressure coefficients, which will be taken as 0.7 and −0.3, so that $\Delta C_P = 1.0$. It is also necessary to specify the height h, which will be assumed to be 4 m.

(i) Knowing T_o, T_I, U_R and h the Archimedes number is given by

$$A_r = \sqrt{\frac{\Delta\rho g h}{\rho_o U_R^2}} = \sqrt{\frac{\Delta T g h}{T_I U_R^2}}$$

$$= \sqrt{\frac{20 \times 9.8 \times 4}{293 \times 4}} = 0.818.$$

The parameter $\Delta C_P/A_r^2$ is thus equal to 1.495 and from Fig. 9 the corresponding value for the parameter $Q/C_D A U_B$ is 1.83. This value can be used to evaluate Q, by substituting values for C_D, A and U_B. For the type of opening considered, C_D will be approximately equal to 0.6. The value of U_B is simply given by ($A_T \times U_R$) when U_R is non-zero. If the open area of each opening A is taken as 0.01 m^2, then

$$Q = 1.83 \times 0.6 \times 0.01 \times 0.818 \times 2$$

$$Q = 0.0180 \text{ m}^3 \text{ s}^{-1}$$

$$Q = 64.7 \text{ m}^3 \text{ h}^{-1}.$$

(ii) For this case U_R is equal to zero, so $\Delta C_P/A_r^2$ is equal to zero and $Q/C_D A U_B$ is equal to 2.0 (see Fig. 9). U_B is obtained from its definition $\sqrt{\Delta\rho g h/\rho_o}$ to be 1.64 m s^{-1}.

Thus

$$Q = 2.0 \times 0.6 \times 0.01 \times 1.64$$

$$Q = 0.0196 \text{ m}^3 \text{ s}^{-1}$$

$$Q = 70.7 \text{ m}^3 \text{ h}^{-1}.$$

(iii) For the case with ΔT equal to zero, the parameters in square brackets in Fig. 9 have to be used. Thus for ΔC_P equal to 1.0 the 'wind alone' curve shows that $Q_w/C_D A U_R$ is equal to 1.4, so

$$Q_w = 1.4 \times 0.6 \times 0.01 \times 2$$

$$Q_w = 60.5 \text{ m}^3 \text{ h}^{-1}.$$