Capture Envelope of an Exhausted Opening Under Cross Draft – A Numerical Approach

Chen Y K[1], Huang R-F[2], Chen C W[3]
[1]Department of Occupational Safety and Hygiene, Chang Jung Christian University
Kway Jen, Taiwan
[2]Department of Mechanical Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan
[3]Institute of Occupational Safety and Health, Council of Labor Affairs, Taipei, Taiwan

Introduction

The performance of an exterior hood is known to be affected by the cross draft(1, 2). Based on the knowledge from a classical "Rankin's nose" or "semi-infinite body" problem, in which the opening shrinks to a sink instead of a finite opening, the exhausted airflow combining the cross draft forms a capture envelope in front of the hood(3). All streamlines within the envelope lead to the hood opening, those outside of the envelope lead to infinity. Therefore, contaminant released inside the envelope tends to be captured by the hood; otherwise, it tends to escape beyond capture.

In this study, the hood is modelled by a circular opening on an infinite wall. The capture envelopes formd by the combination of the exhaust airflow and cross draft are computed numerically based on the potential flow theory. The general geometric pattern of the envelope then is determined by compiling the results from extensive computations. A recent study conducted in a wind tunnel also justified the existence of capture envelopes and their consistency with computation based on the potential flow theory(4).

Coordinate System and Flow Field

The center of a circular opening with a diameter D is located at the origin. The opening lies on an infinite wall defined by $z = 0$. An uniform cross draft with velocity V_c directs along x-direction. Hence, the z-axis is the central axis of the opening. The airflow is exhausted through the opening with a flow rate Q (Figure *1*). The normal component of the exhaust flow is uniform on the opening face. Therefore, the normal airflow on the opening is $V_f = 4Q/(\pi D^2)$. Both cross draft and exhaust are assume to be potential flow. Only the flow field within $z \geq 0$, or in front of the opening, is considered.

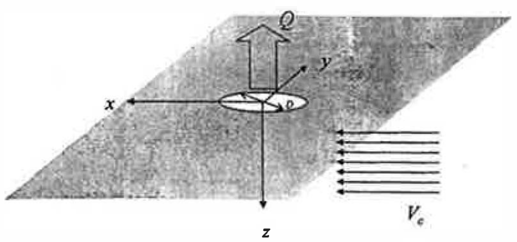

Figure 1. Coordinate system, the circular opening on an infinite wall.

The flow field is established by direct superposition of the uniform cross draft on the exhausting flow. With any given starting point (x, y, z) in front of the opening, the streamline, or the trajectory, can be computed numerically, by using Runge-Kutta 4-th order scheme, from(5)

$$\frac{dx}{dt} = V_c + V_o \quad , \tag{1}$$

where x is the space vector, and V_c and V_o are the velocity vector of the cross draft and the exhausted airflow, respectively. The capture envelope will be revealed after sufficient starting points are used for streamline computation. A point is inside the envelope if a stremline starting from it leads to the opening; otherwise, it is outside the envelope. Therefore, with given Q, D, V_c, x and y, by judging the destination of the streamline starting from (x, y, z), the bounday of the envelope $z = f(x, y)$ can be found by iterative computation. The streamline on the boundary is a limiting streamline.

Geometry of a Capture Envelope

As modifying from the Rankine's nose, the *xy*- and *xz*- cross sections of the envelope on and for a finite opening can be written as

$$\frac{x}{D} = \frac{x_s}{D} \frac{1 - \left(\frac{y, z_c / D}{y_0 / D} \right)^2}{\sqrt{1 - \left(\frac{y, z_c / D}{y_\infty / D} \right)^2}} \quad , \tag{2}$$

In above equation, $y_c = y_c(x)$ is the cross section of the envelope on *xy*-plnae, $z_c = z_c(x)$ is the cross section of the envelope on *xz*-plnae, y_0 and z_0 are the extents of the envelope along y- and z-axes, respectively, y_∞ and z_∞ are the extent of the envelope at $x \to -\infty$ in *y*- and *z*-directions, respectively. The cross section of an envelope normal to *x*-axis, assumed to be an ellipse.

Through extensive computations by varying V_f / V_c, the normalized parameters resemble

$$\frac{x_s}{D} = \frac{1}{2} \sqrt{\frac{2.77899 / 2}{\exp(2.77899 V_c / V_f) - 1} + 1} \quad , \tag{3}$$

$$\frac{y_0}{D} = \frac{3(V_f / V_c)^{7/2} + \sqrt{2}(V_f / V_c)^2 + 1}{6(V_f / V_c)^3 + 2} \quad , \tag{4}$$

$$\frac{y_\infty}{D} = \frac{4\sqrt{2}(V_f / V_c)^{5/2} + 3(1 + V_f / V_c)}{8(V_f / V_c)^2 + 6} \quad , \tag{5}$$

$$\frac{z_0}{D} = \frac{1}{2}\sqrt{\frac{1}{2}\left[-1+\sqrt{1+4\left(\frac{V_f}{V_c}\right)^2}\right]} \quad , \qquad (6)$$

and

$$\frac{z_\infty}{D} = \frac{R_\infty^2}{Dy_\infty} = \frac{V_f / V_c}{2y_\infty / D} = \frac{4(V_f / V_c)^3 + 3V_f / V_c}{4\sqrt{2}(V_f / V_c)^{5/2} + 3(1 + V_f / V_c)} \quad , \qquad (7)$$

respectively. Eqs.(3) to (7) are compared with the corresponding computed results in Figure *2*.

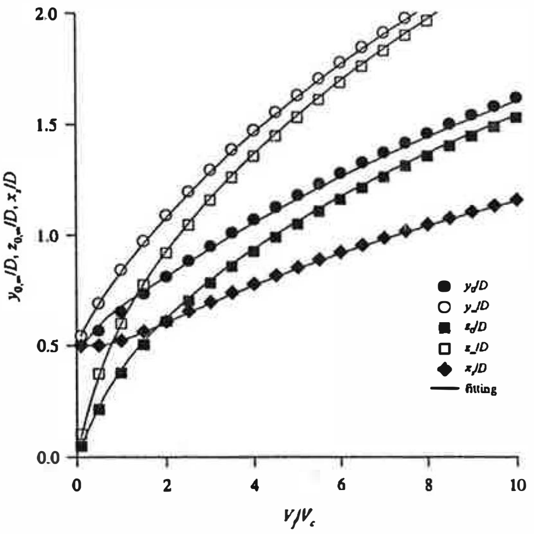

Figure 2. The computed results of x_s / D, y_0 / D, y_∞ / D, z_0 / D, and z_∞ / D in term of V_f / V_c and the corresponding fitting curves defined by Eqs.(3), (4), (5), (6), and (7), respectively.

As observed in the experiments conducted in a wind tunnel. The emperical extents of the capture envelope on the central plane were found (4). Figure *3* shows above parameters obtained emperically and those fit from the computation. The comparison reveals a significant over-estimation on z_∞ / D by computation. However, an extent of a capture envelope at the infinite upstream of the cross draft is very difficult to be determined accurately in a real flow. Figure *3* also shows a slight over-estimation of z_0 / D by computation, the computational result is still acceptable for the purpose of hood desgin. While $V_f / V_c = 0$, the computation gives $x_s / D = 0.5$ due to infinite exhausted velocity at the edge of the opening, but the fitting from the experiments, which were conducted at $V_f / V_c > 3$, gives $x_s / D = 0$. The differences between both fittings of x_s / D becomes significantly at $V_f / V_c < 2$, which is too low for a common hood operation.

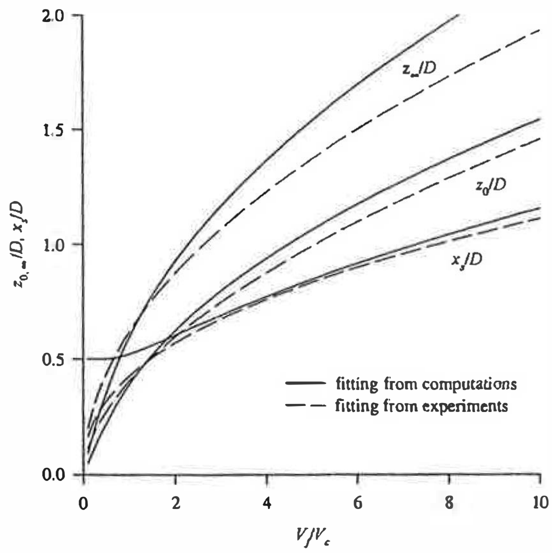

Figure 3. Extents of the capture envelope at the central plane. The solid lines indicate the fitting from the computation in this study (Eqs.(3), (6) and (7)); the dashed lines indicate the empirical relation determined from the experiment(4).

Conclusions

The capture envelope formed by the exhausted airflaw induced by an openning combined with an uniform cross draft is computed based on the potential theory. The geometry of the capture envelope normalized to the diameter of the opening is found to be the function of V_f / V_c, the ratio of the face velocity at opening to the uniform cross draft velocity. While $V_f / V_c > 2$, the geometry of the envelope can de described by the pattern modified from Rankin's nose.

Acknowledgement

The work described by this paper has been financed by the Institute of Occupational Safety and Health, Council of Labor Affairs (Taiwan).

References

1. Flynn MR, Ellenbecker, M. Capture Efficiency of Flanged Circular Local Exhaust Hoods. Ann. Occup. Hyg. 1986:30, 497–513.
2. Flynn MR, Ellenbecker M. Empirical Validation of Theoretical Velocity Fields into Flanged Circular Hoods. Am. Ind. Hyg. Assoc. J. 1987:48, 380–389.
3. Yuan SW. Foundation of Fluid Mechanics, 1967.
4. Chen YK, Chen JL, Chen CW. Capture Envelope of an Exhausted Opening Under Cross Draft-An Experimental Approach (in preparation). 1999
5. Press HP, Flannery BP, Teukolsky SA, Vetterling, WT. Numerical Recipes in C: The Art of Scientific Computing. 1988.