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QUANTIFICATION OF UNCERTAINTY IN THERMAL BUILDING SIMULATION - PART 2: STOCHASTIC LOADS

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ABSTRACT

In order to quantify uncertainty in thermal building simulation stochastic modelling is applied on a building model. Part 1 deals with the stochastic thermal building model and a test case. This paper deals with the determination of the stochastic input loads. The importance of obtaining a proper statistical description of the input quantities to a stochastic model is addressed and exemplified by stochastic models for the external air temperature and the solar heat gain.

Each of the external climate parameters is modelled as a stochastic process with time varying mean value function superimposed by a time varying standard deviation function. The statistics of the external air temperature is obtained by means of Fast Fourier Transform (FFT). A model of the solar heat gain is presented, considering the obvious fact that solar radiation is present only during daytime. The Danish Design Reference Year (DRY) is used as experimental data.

KEY WORDS

Thermal Building Simulation, Fast Fourier Transform, External Air Temperature, Solar Radiation, Solar Heat Gain, Design Reference Year, Experimental Data.

INTRODUCTION

In the design of buildings, prediction of long term system performance with regard to energy consumption and thermal comfort is traditionally performed with thermal building simulation programmes based on deterministic building models and loads.

A deterministic approach implies that all input parameters and model coefficients are 100% certain with zero spread. In practise, this is not the case, for instance, inhabitant behaviour and internal loads may vary significantly and external loads as wind, external air temperature and solar radiation are obviously stochastic in nature.

The purpose of this work is to quantify the uncertainty in thermal building simulation by means of a stochastic approach considering randomness in the loads as well as the model coefficients. In this connection, it is crucial to be able to quantify the uncertainty of the input parameters.

Part 1 comprises the outline of a stochastic building simulation model and a test case (Brohus et al., 2000). The stochastic thermal model presented in Part 1 requires that the loads be modelled by time varying stochastic processes. Hereby, the uncertainty in the load parameters is quantified in shape of time varying mean value and standard deviation functions. This paper considers the determination of those functions. As an example, statistics for the external air temperature and the solar heat gain are determined for instance by means of Fast Fourier Transform (Press et al., 1989).

Load data for the stochastic model is obtained from the Danish DRY (Jensen & Lund, 1995). DRY is an artificial data set, which is used in thermal building simulation to predict parameters like energy consumption and thermal response of buildings. In DRY a typical yearly weather climate is recorded in terms of hourly values of external climatic parameters, gathered from selected monthly data from a 15-year period. Although DRY is an artificial data set it will be used here to illustrate how measured weather climate parameters can be used for stochastic modelling.

EXTERNAL AIR TEMPERATURE

The mean value function, $\mu_{\theta ext}(t)$, of the external air temperature, $\theta_{ext}(t)$, is modelled by the sum of a constant and two cosine functions accounting for the systematic yearly and daily variations, respectively. Fast Fourier Transform (FFT) of the DRY data obtains the model coefficients

$$\mu_{\theta_{ext}}(t) = 7.76 + 8.93\cos(2\pi f_1 t + 2.74) + 2.45\cos(2\pi f_{365} t + 2.73) \text{ in °C} \quad (1)$$

where t is the time in seconds from the beginning of the year, f_1=1/(365*24*3600) Hz is the frequency corresponding to the yearly variation and f_{365}=1/(24*3600) Hz is the frequency corresponding to the daily variation. When Eqn. 1 is subtracted from the original data the fluctuating part of $\theta_{ext}(t)$ is obtained. Based on visual inspection, the standard deviation function is assumed to be time independent and can be expressed as

$$\sigma_{\theta_{ext}}(t) = 3.42 \quad \text{in °C} \quad (2)$$

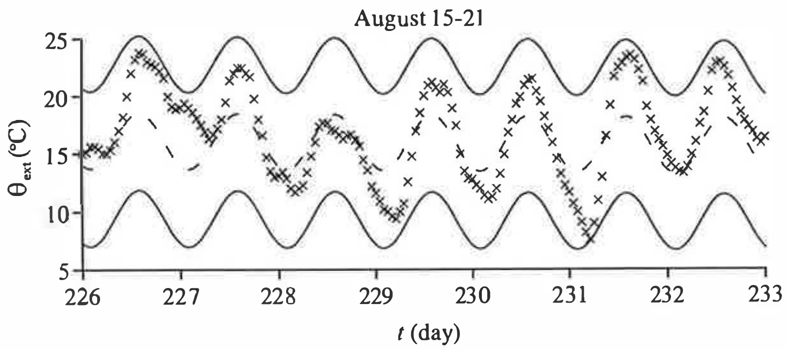

Figure 1: External air temperature data from DRY shown together with the modelled mean value function (dashed line) and the 95 % confidence interval (solid lines) during a week in August.

Figure 1 shows the DRY data for the external air temperature together with the mean value function and the 95 % confidence interval corresponding to the mean value function ± 1.96 times the standard deviation function.

SOLAR HEAT GAIN

The following model, partly adapted from Lund (1997), determines the solar heat gain, $\Phi_{sun}(t)$, as a function of reflection ratio of surroundings, window orientation, inclination, window area and effective reduction factor. The angles are defined in Figure 2.

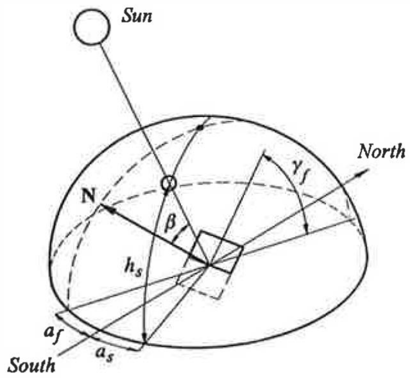

Figure 2: Definition of angles in the solar radiation model. β is the angle of incidence, h_s is the solar altitude, a_s is the solar azimuth, a_f is the surface azimuth, γ_f is the surface inclination, **N** is the normal vector to the surface (Lund, 1997).

A number of coefficients are defined and used in the calculation in order to facilitate the determination of the mean value function, $\mu_{\Phi sun}(t)$, and the standard deviation function, $\sigma_{\Phi sun}(t)$. Here, both direct, diffuse and reflection solar radiation are considered.

$$\cos(\beta(t)) = \cos(a_s(t) - a_f)\cos(h_s(t))\sin(\gamma_f) + \sin(h_s(t))\cos(\gamma_f) \tag{3}$$

$$c_1(t) = \begin{cases} \cos(\beta(t)) & \text{if} \quad \cos(\beta(t)) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$c_{aux}(t) = \begin{cases} 0.55 + 0.437\cos(\beta(t)) + 0.313(\cos(\beta(t)))^2 & \text{if} \quad \cos(\beta(t)) > -0.2 \\ 0.4751 & \text{otherwise} \end{cases} \tag{5}$$

$$c_2(t) = c_{aux}(t)(1 - \cos(\gamma_f)) + \cos(\gamma_f) \tag{6}$$

$$c_3(t) = 0.5\rho\,(1 - \cos(\gamma_f))\sin(h_s(t)) \tag{7}$$

$$c_4 = 0.5\rho\,(1 - \cos(\gamma_f)) \tag{8}$$

$$c_5 = A_f F_s \tag{9}$$

The coefficient c_5 considers the surface area, A_f, and the effective reduction factor, F_s, accounting for the frame-glass ratio and all kinds of solar shading.

In Eqn. 10 and Eqn. 11, statistical calculation rules for mean values and variances (and thus standard deviations) are applied on the deterministic formula. The standard deviation function, $\sigma_{\Phi sun}(t)$, is determined by assuming independence between $E_d(t)$ and $E_0(t)$

$$\mu_{\Phi_{sun}}(t) = c_5\left[(c_1(t)+c_3(t))\mu_{E_0}(t) + (c_2(t)+c_4)\mu_{E_d}(t)\right] \quad \text{in W} \tag{10}$$

$$\sigma_{\Phi_{sun}}(t) = c_5\sqrt{(c_1(t)+c_3(t))^2\sigma_{E_0}^2(t) + (c_2(t)+c_4)^2\sigma_{E_d}^2(t)} \quad \text{in W} \tag{11}$$

Both the diffuse solar radiation, $E_d(t)$, and the direct solar radiation, $E_0(t)$, exhibit the characteristics that both processes are zero during night and obtain a maximum at noon. The two processes are treated similarly, and only the results for the diffuse solar radiation will be shown here.

The mean value function, $\mu_{Ed}(t)$, is modelled by a parabola for each day as shown in Figure 3

$$\mu_{E_d}(t) = \begin{cases} \dfrac{4\mu_{E_{d,\max}}(k)}{(t_{do}(k)-t_{up}(k))^2}\left[(t_{up}(k)+t_{do}(k))t - t_{up}(k)t_{do}(k) - t^2\right] & \text{for } t_{up}(k)\le t\le t_{do}(k) \\ 0 & \text{otherwise} \end{cases} \quad \text{in W/m}^2 \tag{12}$$

where $t_{up}(k)$ and $t_{do}(k)$ are the sunrise and sunset times, respectively, in seconds after the beginning of the year for day k, obtained from the DRY data by FFT analysis

$$t_{up}(k) = 2.42\cdot 10^4 + 8.89\cdot 10^3\cos(2\pi f_1 86400(k-1)+0.12) + 86400(k-1) \quad \text{in s} \tag{13}$$

$$t_{do}(k) = 7.04\cdot 10^4 + 8.95\cdot 10^3\cos(2\pi f_1 86400(k-1)-2.88) + 86400(k-1) \quad \text{in s} \tag{14}$$

The mean value function of the maximum diffuse solar radiation process, $\mu_{Ed,max}(k)$, is found by conducting a FFT-analysis on the data series consisting of the maximum values of $E_d(t)$ for each day to give

$$\mu_{E_{d,\max}}(k) = 206.26 + 139.74\cos(2\pi f_1 86400(k-1)-2.92) \quad \text{in W/m}^2 \tag{15}$$

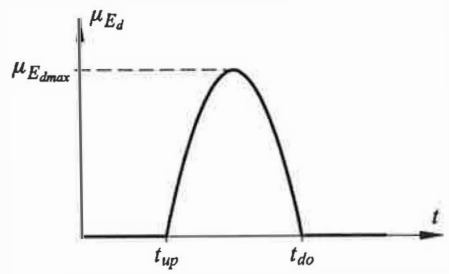

Figure 3: Outline of $\mu_{Ed}(t)$ parabola model shown for a single day

The standard deviation function of the diffuse solar radiation, $\sigma_{Ed}(t)$, is modelled as a scaled version of the mean value function as

$$\sigma_{E_d}(t)=\frac{\sigma_{E_{d,\max}}(k)}{\mu_{E_{d,\max}}(k)}\mu_{E_d}(t) \quad \text{in W/m}^2 \tag{16}$$

$\sigma_{Ed,max}(k)$ is the standard deviation of the maximum process of $E_d(t)$. This is obtained by sampling on a time window using the 11 points located closest to the time in question and FFT-analysis

$$\sigma_{E_{d,\max}}(k)=62.91+30.59\cos(2\pi f_1 86400(k-1)-2.65) \quad \text{in W/m}^2 \tag{17}$$

Figure 4 shows the DRY data for the diffuse solar radiation together with the mean value function and the 95 % confidence interval.

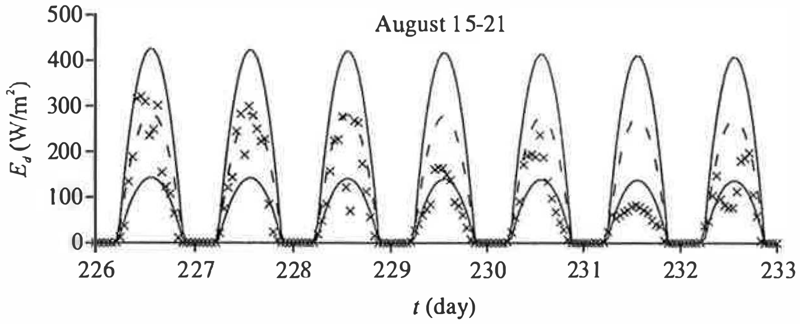

Figure 4: Diffuse solar radiation from DRY shown together with the modelled mean value function (dashed line) and the 95 % confidence interval (solid lines) for a week in August.

In Figure 5 an example is shown for one week in August for the solar gain corresponding to a vertical window, (A_f= 1 m^2, γ_f = 90 °) at south (a_f = 0 °) with a reflection ratio ρ = 0.25 and a shading ratio F_s = 0.5. Data for the Danish solar altitude, $h_s(t)$, and solar azimuth, $a_s(t)$, are found in Lund, 1997.

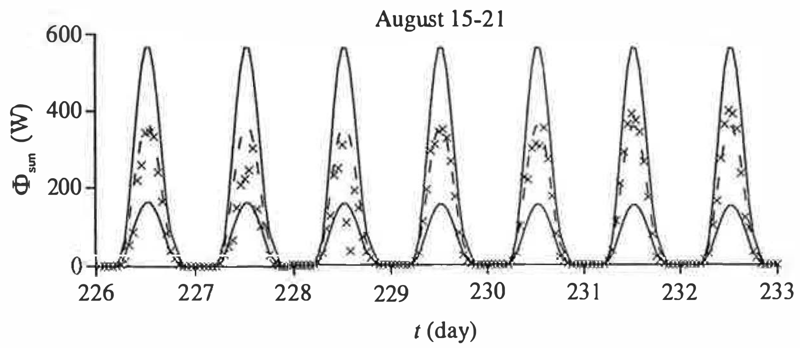

Figure 5: Solar heat gain for a 1 m^2 window facing south for a week in August. Results based directly on solar radiation data from DRY are shown together with the modelled mean value function (dashed line) and 95 % confidence interval (solid lines).

DISCUSSION

Stochastic models for the external air temperature and the solar heat gain have been developed on the basis of data from the Danish Design Reference Year. The results are applied directly in a stochastic thermal building simulation model by means of a Stochastic Differential Equation (SDE) approach as described in Part 1. Alternatively, the stochastic load models can be used for other kinds of probabilistic modelling.

The external air temperature and the solar heat gain are modelled as a time dependent mean value function superimposed by a time varying standard deviation function. Reasonable agreement between the DRY data and the stochastic models is obtained.

The fluctuating part of the processes are assumed independent both regarding mutual correlation and individual correlation in time, i.e. auto correlation, in order to be able to use the white noise assumption which is applied in the SDE approach presented in Part 1. In reality, the processes will to some extent be correlated both in time and mutually. Future modelling may include auto correlation functions, describing the time dependency of the parameters, and cross correlation functions, describing the mutual dependence.

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