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QUANTIFICATION OF UNCERTAINTY IN THERMAL BUILDING SIMULATION - PART 1: STOCHASTIC BUILDING MODEL

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ABSTRACT

In order to quantify uncertainty in thermal building simulation stochastic modelling is applied on a building model. An application of stochastic differential equations is presented in Part 1 comprising a general heat balance for an arbitrary number of loads and zones in a building to determine the thermal behaviour under random conditions. Randomness in the input as well as the model coefficients is considered. Two different approaches are presented namely equations for first and second order time-varying statistical moments and Monte Carlo Simulation. A simple test case is presented showing the mean value process and the standard deviation process (pursuing a confidence interval) for a naturally ventilated atrium during a winter week and during a summer week. Part 2 considers the generation of appropriate stochastic load input.

KEYWORDS

Thermal Building Simulation, Stochastic Differential Equations, Uncertainty, Stochastic, Deterministic, Natural Ventilation, Randomness, Monte Carlo Simulation, Statistical Moments.

INTRODUCTION

In the design of buildings, prediction of long term system performance with regard to energy consumption and thermal comfort is traditionally performed with thermal building simulation programmes based on deterministic building models and loads.

A deterministic approach implies that all input parameters and model coefficients are 100% certain with zero spread. In practice this is not the case, for instance inhabitant behaviour and internal loads may vary significantly and external loads as wind, external temperature and solar radiation are obviously stochastic in nature. One reason for ignoring randomness is the fact that mechanically ventilated heavy buildings are often highly "damped" and shielded toward external loads. This kind of buildings will also control the influence of the internal load effectively by means of the building energy management system and the HVAC system. However, lighter constructions that are naturally or hybrid ventilated are more sensitive to stochastic variations in the loads. The behaviour of the building depends thoroughly on natural driving forces characterised by external temperature, wind speed and direction, and the internal load of the building, etc. Due to the fact that this kind of buildings are increasing fast in number either as new built or retrofit stresses the importance of considering the effect of randomness in the design phase.

The present work is divided in two parts. Part 1 comprises the outline of a stochastic building simulation model and a simple test case. Part 2 considers the determination of stochastic loads.

STOCHASTIC BUILDING MODEL

A system of linear Stochastic Differential Equations (SDE) is formulated describing the temporal variation of the zone temperatures in a building model. The procedure of solving the SDE system can be regarded as an operator transforming the stochastic input quantities to stochastic output quantities (Haghighat et al., 1987). The input of the SDE system is modelled by a number of independent time varying stochastic processes representing internal and external loads applied on the building.

The stochastic building model comprises a general heat balance for an arbitrary number of zones, surfaces and construction parts. There are n nodes with unknown temperatures, θ_i, (output) and effective thermal capacities, C_i. There are m nodes with known temperatures, θ_j^b; denoted boundary nodes. There are k_i independent heat flux components, Φ_{ij}, applied on the i'th node with an unknown temperature. H_{ij} and H_{ij}^b are the specific heat losses related to the unknown temperatures and the boundary nodes, respectively. For node i at the time t the following heat balance apply

$$C_i \frac{d\theta_i(t)}{dt} = \sum_{\substack{j=1 \\ j\neq i}}^{n} H_{ij}(t)\left(\theta_j(t)-\theta_i(t)\right) + \sum_{j=1}^{m} H_{ij}^b(t)\left(\theta_j^b(t)-\theta_i(t)\right) + \sum_{j=1}^{k_i} \Phi_{ij}(t) \quad , i = 1, 2, \dots, n \tag{1}$$

In the heat balance, Eqn. 1, all or some of the input parameters θ_j^b, H_{ij}, H_{ij}^b and Φ_{ij} can be regarded as time varying stochastic processes, the rest being deterministic. Each stochastic input component, denoted by $z(t)$, is modelled by a time varying mean value function, $\overline{Z}(t)$, superimposed by a fluctuating component comprising a time varying standard deviation function, $\sigma_Z(t)$, multiplied by a standard white noise process, $w(t)$, i.e.

$$z(t) = \overline{Z}(t) + \sigma_Z(t)w(t) \tag{2}$$

The white noise process is usually regarded as an appropriate model for rapidly random fluctuating phenomena, when correlation in time becomes small rapidly (Arnold, 1974). The white noise process, $w(t)$, is defined as the time derivative of the so called Wiener process, $W(t)$, such that $dW(t) = w(t)dt$.

By rearranging Eqn. 1 and applying Eqn. 2 for each input process and by removing higher order terms of fluctuating components the following linear SDE system is obtained

$$d\theta = [\mathbf{X}\theta + \mathbf{x}]dt + \sum_{k=1}^{VAR} [\mathbf{Y}_k\theta + \mathbf{y}_k]dW_k \tag{3}$$

where VAR is the number of stochastic processes. Assembly routines for $\mathbf{x}$, $\mathbf{X}$, $\mathbf{y}_k$ and $\mathbf{Y}_k$ can be found in Brohus et al., 1999. The SDE system, Eqn. 3, can be solved either for the statistical moments or for a number of response realisations of the output.

Statistical Moment Equations

A convenient description of the stochastic output parameters, i.e. the unknown temperatures, is their first and second order statistics given by the time varying expected values, $E[\theta_i]$, and the second order statistical moments, $E[\theta_i\theta_j]$. The corresponding standard deviations are easily derived from the first and second order moments by means of a statistical standard formula.

Arnold (1974) shows that the following deterministic differential equations for the first and second order statistical moments of the zone temperatures correspond to the SDE system Eqn. 3. Eqn. 4 and Eqn. 5 express the statistical moments and the initial conditions for the first and the second order moments, respectively. The first order linear differential equations can be solved by means of standard tools like the fourth order Runge-Kutta method.

$$\begin{cases} \dfrac{dE[\theta]}{dt} = \mathbf{X}E[\theta] + \mathbf{x} \\ E[\theta(t=0)] = E[\theta^0] \end{cases} \tag{4}$$

$$\begin{cases} \dfrac{dE[\theta\theta^T]}{dt} = \mathbf{X}E[\theta\theta^T] + E[\theta\theta^T]\mathbf{X}^T + \mathbf{x}E[\theta]^T + E[\theta]\mathbf{x}^T \\ +\sum_{k=1}^{VAR}\left[\mathbf{Y}_k E[\theta\theta^T]\mathbf{Y}_k^T + \mathbf{Y}_k E[\theta]\mathbf{y}_k^T + \mathbf{y}_k E[\theta]^T \mathbf{Y}_k^T + \mathbf{y}_k \mathbf{y}_k^T\right] \\ E[\theta(t=0)\theta(t=0)^T] = E[\theta^0\theta^{0T}] \end{cases} \tag{5}$$

Response Realisation

The SDE system Eqn. 3 has an infinite number of solutions. Every solution corresponds to one realisation of the solution process. The alternative approach of Monte Carlo Simulation uses realisations of the stochastic input processes generated according to their joint density functions. The corresponding output is then calculated from a deterministic model, which expresses the response of the building model. If the procedure is repeated a large number of times the resulting output data can be treated statistically. Eqn. 3 can be rewritten to obtain the following expression

$$d\theta = \mathbf{f}(\theta,t)dt + \mathbf{G}(\theta,t)d\mathbf{W} \tag{6}$$

The vector $\mathbf{f}(\theta,t) = \mathbf{X}\theta + \mathbf{x}$, the matrix $\mathbf{G}(\theta,t) = [\mathbf{Y}_1\theta + \mathbf{y}_1 \cdots \mathbf{Y}_{VAR}\theta + \mathbf{y}_{VAR}]$ and the $d\mathbf{W}$ vector is given as $d\mathbf{W}^T = [dW_1 \cdots dW_{VAR}]$. The components of the $d\mathbf{W}$ vector are generated as realisations of independent normally distributed variables with zero mean and variance dt. Eqn. 6 can be solved for instance by a stochastic version of the fourth order Runge-Kutta method, see Arnold (1974).

TEST CASE

A simple test case is chosen in order to demonstrate the SDE approach, see Figure 1. The test case comprises a naturally ventilated atrium surrounded by building parts exposed to internal and external loads. The thermal capacity of the atrium corresponds to a medium heavy building. The atrium is naturally ventilated with two equally sized openings fixed without control. The specific heat loss due to natural ventilation, H_{vent}, in this case is expressed by

$$H_{vent} = \rho c_p C_D A (g\Delta h)^{1/2} \left[\frac{(\theta_1 - \theta_{ext})}{\theta_1 + 273.15} \right]^{1/2} \tag{7}$$

where ρ is the density, c_p is the specific heat, C_D is the discharge coefficient, A is the opening area, g is the gravitational acceleration and Δh is the stack height. H_{vent} is found by assuming a constant value for the term $\rho c_p C_D A (g\Delta h)^{1/2} = 6 \cdot 10^4$ W/K during the simulations. If the $[(\theta_1 - \theta_{ext})/(\theta_1 + 273.15)]^{1/2}$ term is introduced directly in Eqn. 1 the system of differential equations becomes non-linear in θ. In order to avoid non-linearity, the values of θ_1 and θ_{ext} in Eqn. 7 are adapted throughout the simulations from the previous time step. Figure 3 shows the corresponding air change rate.

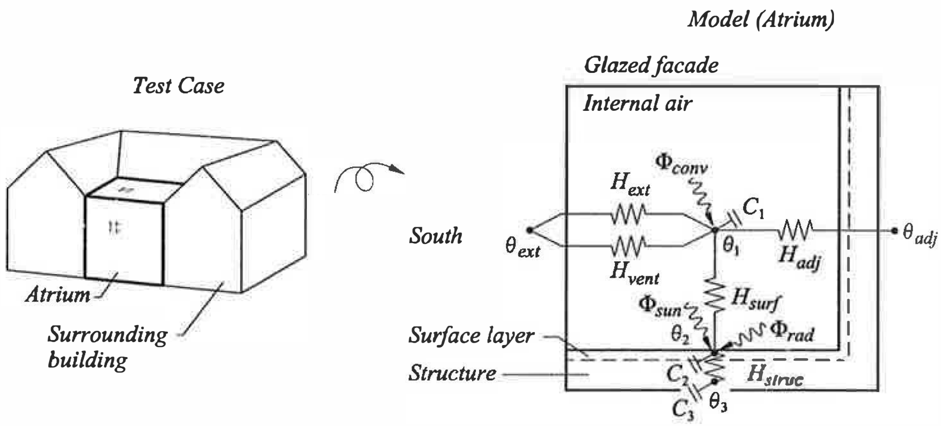

Figure 1: Test Case. An atrium surrounded by an adjacent zone and the external climate. Three unknown temperatures are to be determined, i.e. θ_1 - θ_3. The surface layer accounts for one fourth of the thermal capacity of the building and the "structure" accounts for the rest.

TABLE 1

TEST CASE PARAMETERS. θ_{ext} AND Φ_{sun} ARE DISCUSSED IN BROHUS ET AL., 2000

Parameter	Unit	Mean value	Standard deviation	Stochastic ?
θ_1 , θ_2 , θ_3	°C	Output	Output	Yes
θ_{ext} , Φ_{sun}	°C, W	Data from Danish DRY	Data from Danish DRY	Yes
θ_{adj}	°C	20	2	Yes
C_1 , C_2 , C_3	J/K	$6 \cdot 10^5$, $1 \cdot 10^7$, $3 \cdot 10^7$	0	No
H_{vent}	W/K	Calculated or 6000	0.3 times mean value	Yes
H_{ext}	W/K	800	0	No
H_{adj}	W/K	150	45	Yes
H_{surf}, H_{struc}	W/K	1400, 4800	0	No
Φ_{conv}, Φ_{rad}	W	Assumed data sets	0.3 times mean value	Yes

An assumed 24-hour load profile for the mean value of the internal sensible heat load is applied in the simulation. The sensible heat is divided into 50% convective heat, Φ_{conv}, and 50% radiative heat, Φ_{rad}. The convection heat flow is assumed to influence the internal air, i.e. θ_1, and the radiation heat flow is assumed to influence the surface layer, i.e. θ_2.

The calculations are performed during a warm week in summer and a cold week in winter, which can be thought of as a kind of design load periods. The simulations are started three days before the week in question in order to avoid unrealistic values due to the initial guess.

RESULTS AND DISCUSSION

Figure 2 shows the mean value process for the internal air during the winter week (left) and during the summer week (right). In addition the 95% confidence interval is shown corresponding to the mean value process ±1.96 times the standard deviation process. Due to a considerable air change rate, see Figure 3, the internal temperature is only slightly higher than the external air temperature, e.g. 3–6 °C.

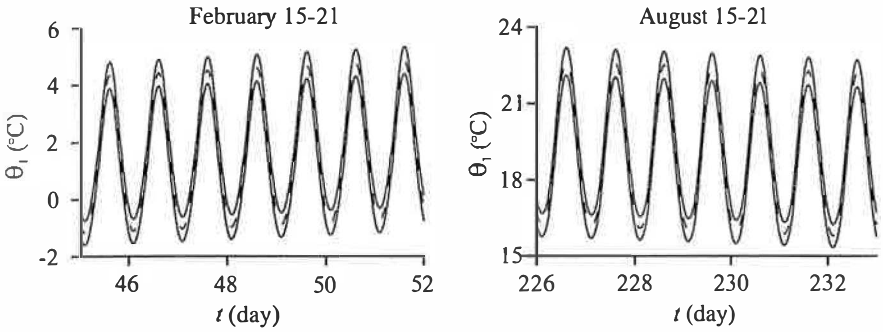

Figure 2: Internal air temperature in the atrium, θ_I, for a week during winter (left) and during summer (right). The figure shows the mean value and the 95% confidence interval (solid lines).

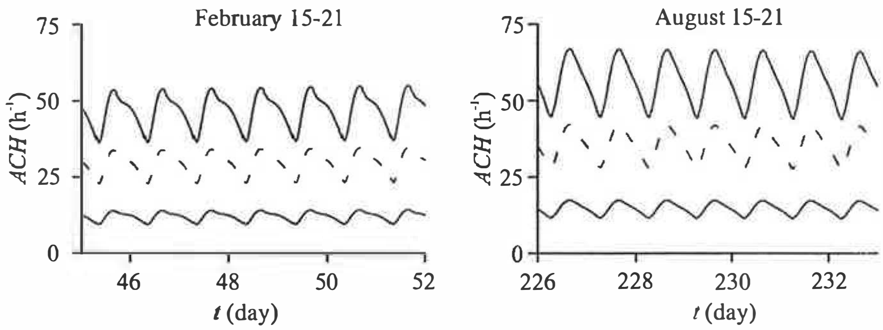

Figure 3: Mean air change rate and 95% confidence interval (solid lines) during a winter week (left) and a summer week (right). A fixed standard deviation of 0.3 times the mean value of H_{vent} is assumed.

In Figure 4 a realisation of the stochastic output process is presented together with the 95% confidence interval for the same temperature as shown in the right part of Figure 2. In this case a constant value of H_{vent} = 6000 W/K is chosen. The random fluctuations in this application are relatively small when compared with the mean values.

In this paper, the SDE approach is applied on a heat balance in order to examine the thermal behaviour of a relatively heavy building. However, if the approach is applied on a mass balance in order to determine for instance the contaminant concentration in a relatively light and hybrid or naturally ventilated building it is expected that randomness would play a more significant role.

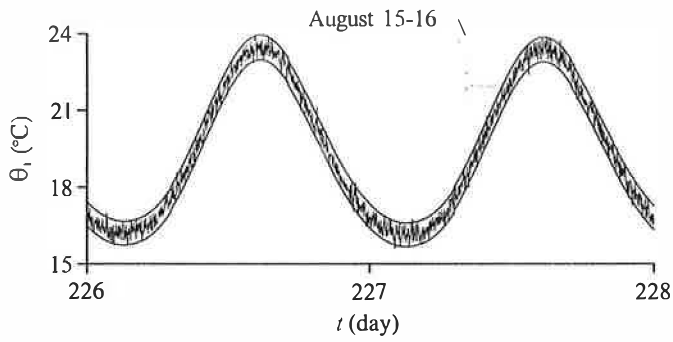

Figure 4: Stochastic realisation of internal air temperature in the atrium, θ_I. The figure also shows the corresponding 95% confidence intervals. In order to illustrate the stochastic fluctuations more clearly the plot is limited to show 48 hours of the summer week.

Future work may include a whole year case where it is possible to calculate the cumulative distribution and assess the energy consumption. Another further development is the inclusion of various control strategies in order to simulate realistic conditions both regarding thermal comfort and indoor air quality as well as energy consumption.

Probabilistic methods establish a new approach to the design of ventilation systems, which, apart from more realistic modelling, enable designers to include stochastic parameters like inhabitant behaviour, operation, and maintenance to predict the performance of the systems and the level of certainty for fulfilling design requirement under random conditions.

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