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FINITE ELEMENT CALCULATION OF NATURAL VENTILATION

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ABSTRACT

The intention of this paper is not to compare discretization schemes but to show some advantages of a stabilized finite element method for modelling natural ventilation. Based on the finite element theory we present a formulation of boundary conditions that can be used for most ventilation openings in buildings. Stationary as well as transient situations can be considered without modelling of the outdoor space. Mathematical background and implementation details are discussed. Results are presented for ventilation of a living room at typical outdoor conditions.

KEYWORDS

CFD, Finite Elements, Boundary Conditions, Natural Ventilation

MOTIVATION

Using CFD for analysing flow and temperature distribution in rooms and buildings one has often to deal with boundary conditions at large openings. In case of windows and similar ventilation openings the formulation of pressure boundary conditions seems to be reasonable.
Most of the commercial and research CFD codes offer options to specify pressure boundary conditions. But the results of using it for natural ventilation are often unsatisfactory or at least very expensive concerning computational resources.
From the mathematical point of view finite elements are very flexible concerning boundary conditions. Therefore we investigated the implementation of pressure boundary conditions within a special finite element method. We do neither intend to present the final solution of the problem nor to definitely favourite finite element calculations for ventilation problems. We simply would like to show some advantages of using finite element technology for calculation of natural ventilation.

MATHEMATICAL BACKGROUND

In order to explain what is meant with "flexibility" of a finite element formulation (or more exactly variational formulation) concerning boundary conditions we will look on the stationary Navier-Stokes equations for an incompressible fluid, dimensionless notation :

$$(u \circ \nabla)u - \nabla \circ \tau + \nabla p = f \quad \text{in}\,\Omega. \tag{1}$$

Here u denotes the velocity vector, p the pressure, f a body force, and τ stands for the stress tensor $\tau = \nu\left(\nabla u + (\nabla u)^T\right)$. A variational formulation of equation (1) is achieved by integration and multiplication with a test function υ. For a mathematical detailed definition of functions and function spaces the reader is referred to Carey&Oden (1986). Using partial integration of the term with the stress tensor and the pressure gradient the following boundary integrals arise:

$$-\int_\Omega \nabla \circ \tau \circ \upsilon \; d\Omega = \int_\Omega \tau \mathbin{\substack{\circ\\\circ}} (\nabla \upsilon)^T \, d\Omega - \int_\Gamma n \circ \tau \circ \upsilon \; d\Gamma, \tag{2}$$

$$\int_\Omega \nabla p \circ \upsilon \; d\Omega = -\int_\Omega p(\nabla \circ \upsilon) d\Omega + \int_\Gamma p n \circ \upsilon \; d\Gamma. \tag{3}$$

On one hand this is a so called weak formulation of the problem because it contains lower derivations than the original equation. On the other hand it is possible to use the boundary integrals to formulate boundary conditions in a natural way.
Using the pressure boundary condition according to Heywood et al.(1996) along the boundary Γ,

$$\sigma = p - n \circ \tau \circ n, \tag{4}$$

the momentum equation reads:

$$\int_\Omega (u \circ \nabla) u \circ \upsilon \; d\Omega + \int_\Omega \tau \mathbin{\substack{\circ\\\circ}} (\nabla \upsilon)^T d\Omega - \int_\Omega p(\nabla \circ \upsilon) \; d\Omega = \int_\Omega f \circ \upsilon \; d\Omega - \sigma \int_\Gamma n \circ \upsilon \, d\Gamma. \tag{5}$$

This means, the pressure boundary condition along the boundary Γ is included in the variational formulation.

IMPLEMENTATION IN A CFD CODE

The pressure boundary condition, equation (4), was implemented in the finite element code ParallelNS, a common research code of Göttingen University and Dresden University of Technology. This code was originally developed for testing a non-overlapping domain decomposition method, see Lube et al. (1998), Auge et al. (1998). Furthermore the code works with a stabilized finite element method (FEM) because standard methods are not suitable for calculating convection dominated flows. The method is called *Galerkin Least-Squares FEM* and includes stabilization to overcome the problems resulting from convection and velocity pressure coupling, Hughes et al. (1986), Hughes et al. (1989). The basic concept of the Least-Squares FEM can be used for a wide range of applications, Jiang (1998).
In co-operation with mathematicians from the Institute of Numerical and Applied Mathematics (University of Göttingen) the code was extended by a k-ε turbulence model especially for calculating flows within rooms and buildings. Basic equations were the time dependent Navier-Stokes equations for

incompressible fluids. Effects of buoyancy were modelled based on the Boussinesq-Approximation. Wall functions for room air flows were implemented, see Neitzke (1998). Beside the boundary condition of the flow field for the temperature and the turbulence quantities appropriate boundary conditions at the openings have to be formulated. Thus a mix of boundary conditions of first and second kind is applied depending on the flow direction. That means, the incoming flow is specified by fixed values of temperature and turbulence quantities and for the outgoing flow the boundary condition is included in the variational formulation similar to equation (5).

EXAMPLES OF CALCULATION

We started our computations with simple test cases, see figure 1. A cube of 1 m x 1 m x 1 m with different openings and a temperature difference of 10 K was assumed. Thereby the structure of the flow at the openings and global balances were investigated. Horizontal openings are also possible to calculate but the highly unsteady flow is difficult to evaluate.

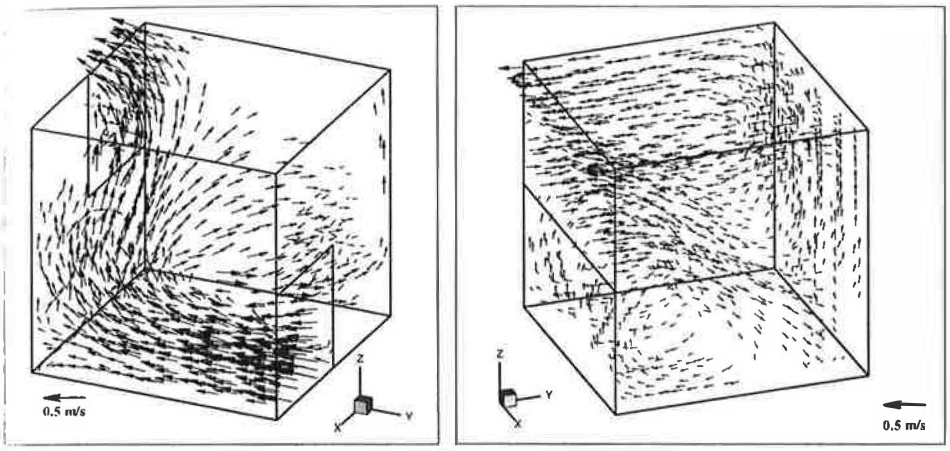

Figure 1: Velocity distributions in a cube with different openings

Our main efforts were dedicated to the computation of the flow through open windows at calm. In this case the pressure boundary condition as in (4) is equivalent to a stress free outlet, sometimes called "do nothing" condition.

Figure 2 shows a room of about 120 m^3 volume after a ventilation period of 20 seconds. The initial indoor temperature is 20 °C, the outdoor temperature is 10 °C. The walls are regarded as adiabatic. The illustrated cloud shows the temperature distribution field below 18 °C.

Figure 3 illustrates the above temperature distribution field 20 seconds later from another viewpoint. Reflecting and mixing of the cold air can be seen because the incoming air has reached the wall opposite to the window.

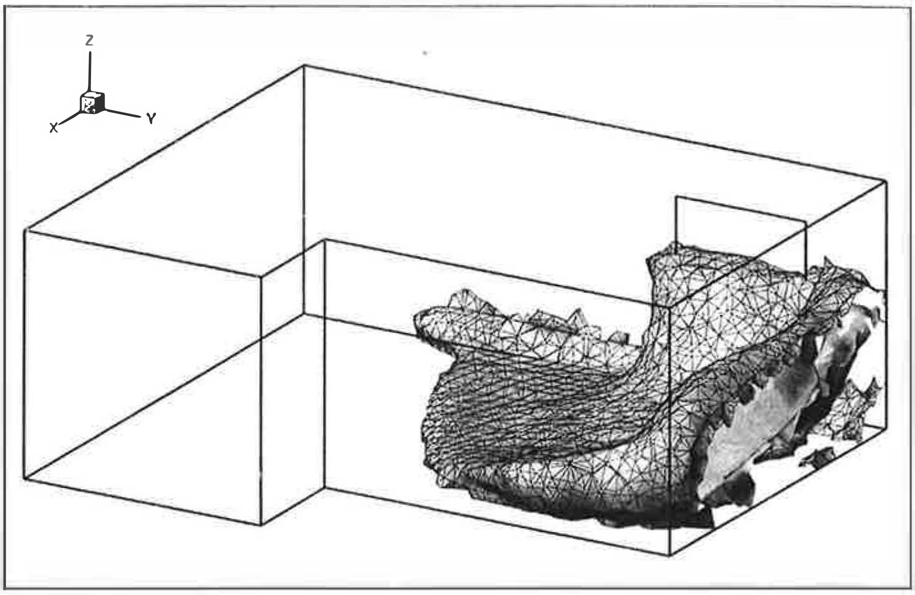

Figure 2: Region of temperature below 18 °C after a ventilation period of 20 seconds, view from the wall opposite to the window

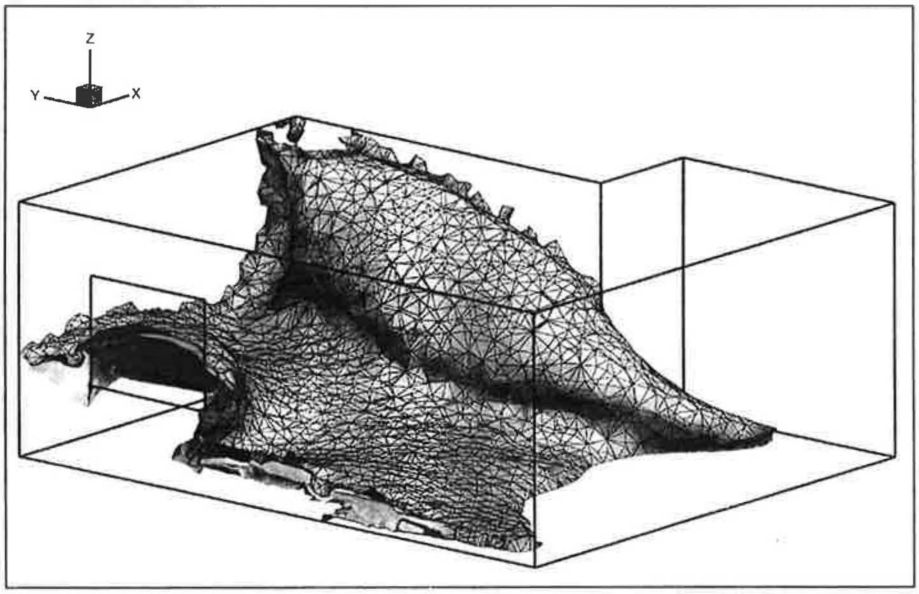

Figure 3: Region of temperature below 18 °C after a ventilation period of 40 seconds, view from the wall with the window

Beside the permanent testing of balances we investigated the computational expense to carry out those calculations. For configurations shown in figures 1 to 3 we found a similar expense as working with specified velocity and temperature profiles at the openings.

OUTLOOK

To get some experimental data for comparison with the computational results a small test chamber (1m x 1m x 2 m) was built, based on the configuration used to develop special wall functions for natural convection, compare Neitzke (1998). In the next once experimental data are gained, this situation will be numerically investigated in detail.
In addition the code was extended by special procedures to calculate time dependent values of air exchange efficiency and the age of air. To get more realistic boundary conditions it is possible to interact with a thermal building simulation program by use of the procedures of PVM (Parallel Virtual Machine).

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