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SOME EXAMPLES OF SOLUTION MULTIPLICITY IN NATURAL VENTILATION

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ABSTRACT

This paper shows that under certain conditions, multiple solutions for the flow rate exist in a natural ventilation system, induced by the non-linear interaction between buoyancy and wind forces. Under certain physical simplifications, the system is governed in steady state by a non-linear algebraic equation or a system of equations. Three examples are given here: a single-zone building with two openings, a channel with two end openings, and a two-zone building with two openings in each zone. Analytical and numerical solutions are presented. It is shown that in all three cases the flow rate exhibits hysteresis. These results have significant implications for multi-zone modelling of natural ventilation and smoke spread in buildings. An experimental investigation using a small-scale water model in a water tunnel confirms that two steady-state solutions exist for a single-zone building.

KEYWORDS

Natural ventilation, multi-zone simulation, analytical methods, winds, thermal buoyancy.

INTRODUCTION

Our recent studies of natural ventilation (Li & Delsante 1998; in press) showed that there are multiple solutions of the equations governing natural ventilation when the flow rate is determined by combined buoyancy and wind forces. Some similar results were also obtained by Nitta (1996). Linden (1999) reported analytical and experimental studies of opposing wind and buoyancy forces, and also noted the phenomenon of hysteresis experimentally.

The presence of multiple solutions is significant for understanding natural ventilation control and the interpretation of results obtained from mathematical modelling. This paper provides a summary of the key findings of our work using three examples (see Figure 1). The analytical results are supported by some new experimental results from a water tunnel.

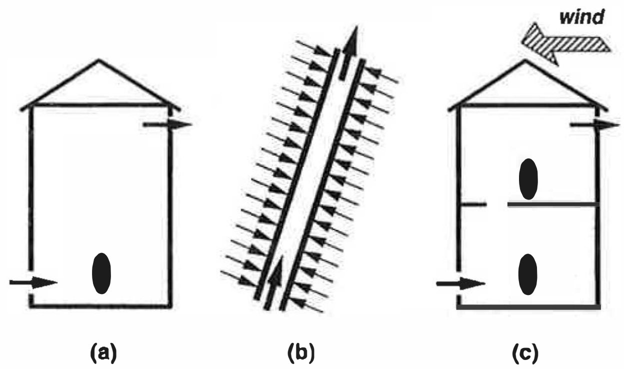

Figure 1: Three example enclosures with opposing wind. (a): single-zone building; (b): an inclined channel; (c): a two-zone building. In the channel, heat is applied to the two surfaces. In the buildings, the heat source is indicated by the solid ellipse.

A SINGLE–ZONE BUILDING

The building has two openings at different vertical levels on opposite walls, as shown in Figure 1(a). The heights of the two openings are relatively small, and the areas of the top and bottom opening are A_t and A_b respectively. There is an indoor source of heat, E. The wind force can assist or oppose the thermal buoyancy force. We assume that the indoor air is fully mixed, i.e. the air temperature is uniform, recognising that in practice the air will be stratified in some circumstances. However the fully mixed assumption is used here because it leads to relatively simple equations which nonetheless display interesting behaviour (which can then be checked by experiment), and because this assumption is used in the simpler treatments of natural ventilation.

Steady-state Solutions

Three air change parameters (α, β and γ) are used to characterise respectively the effects of the thermal buoyancy force, the envelope heat loss and the wind force:

$$\alpha = (C_d A^*)^{2/3}(Bh)^{1/3} \tag{1}$$

$$\beta = \frac{\Sigma U_j A_j}{3\rho c_p} \tag{2}$$

$$\gamma = \frac{1}{\sqrt{3}}(C_d A^*)\sqrt{2\Delta P_w} \tag{3}$$

where $A^* = A_t A_b \big/ \sqrt{A_t^2 + A_b^2}$ is the effective area; C_d is the opening discharge coefficient (assumed to be the same for both openings); B is the buoyancy flux, given by $B = Eg/\rho c_p T_o$, where g is the acceleration of gravity, ρ and c_p are the density and heat capacity of the air respectively, and T_0 is the outdoor air temperature; ΔP_w is the wind pressure difference between the two openings; and U_j and A_j

are the U-values and area of wall element j respectively. For assisting winds, the ventilation flow rate, q, is determined by (see Li & Delsante, in press):

$$q^3 + 3\beta q^2 - 3\gamma^2 q - 2\alpha^3 - 9\gamma^2\beta = 0 \tag{4}$$

For opposing winds, the flow rate is determined by:

$$q^3 + 3\beta q^2 = |-3\gamma^2 q + 2\alpha^3 - 9\gamma^2\beta| \tag{5}$$

The solution for opposing winds is complex. The behaviour of the flow rate as a function of α and γ reveals that for $\beta \neq 0$ the general form of the solutions must be as shown in Figure 2, where for clarity upward flow is indicated as positive and downward flow as negative. The flow exhibits hysteresis. It can also be shown that a solution on the A–B curve is not stable (see Li & Delsante, in press).

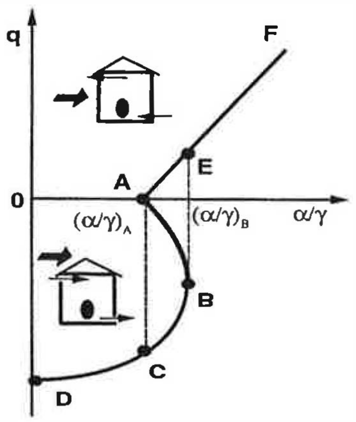

Figure 2: Analytical sketch of the opposing wind situations

A CHANNEL

Examples of airflows in a channel (Figure 1(b)) can be found in road tunnels, solar chimneys and the air gaps behind photovoltaic panels (Sandberg & Moshfegh 1998). Again, an ideal channel is considered with constant total heat flux E applied to the channel walls. We assume a linear temperature profile in the channel for both upward and downward flows. Equations for the flow rate can be derived which are the same as those for the one-zone building, but with minor changes in some parameters. Firstly, the effect of friction is now included in the definition of the effective area, which becomes

$$C_d A^* = \left(\frac{1}{(C_{d1}A_1)^2} + \frac{1}{(C_{d2}A_2)^2} + \frac{K_f}{(A_c)^2} \right)^{-1/2} \tag{6}$$

where C_{d1} and C_{d2} are the discharge coefficients of the two end openings with opening areas A_1 and A_2, A_c is the cross-sectional area of the channel, and K_f is the channel friction loss coefficient, which is proportional to the length of the channel. Secondly, because of the linear temperature profile, the definition of the thermal air change parameter is also slightly changed:

$$\alpha = (C_d A^*)^{2/3} (\tfrac{1}{2} Bh)^{1/3} \tag{7}$$

With these changes, the hysteresis behaviour is the same as for the one-zone building.

A TWO–ZONE BUILDING

The two-zone building has two openings at different vertical levels in each zone, as shown in Figure 1(c). Again, we assume that the height of the vertical openings is relatively small. There is an indoor source of heat, E_i, in each zone i. Again we assume that the indoor air is fully mixed, i.e. the air temperature is uniform in each zone. Additionally, we assume that the partitions between the zones are perfectly insulated, i.e. heat loss only takes place through the external walls, roof and floor.

The flow is governed by a quartic equation (see Delsante and Li, 1999, for details). Figures 3(a)–3(d) show schematically some possible solution behaviours for the upward and downward flows as a function of a buoyancy force parameter α_D. Figure 3(a) shows the simplest case: the slope at $q = 0$ is positive for upward flow and negative for downward flow, and the flows are monotonic. Thus, for any value of α_D there is a unique solution for the flow. Note that there is no analogue for this behaviour in the single-zone case, where the slope is always positive at $q = 0$. Figure 3(b) shows similar hysteresis behaviour to that found for the single-zone case: for certain values of α_D there are three possible solutions – one upward flow (S1), and two downward flows (S2 and S3). Figure 3(c) also shows three possible solutions for certain values of α_D, but with two upward flows and one downward flow. Finally, Figure 3(d) shows five possible solutions – two upward and three downward. This set of behaviours is not exhaustive.

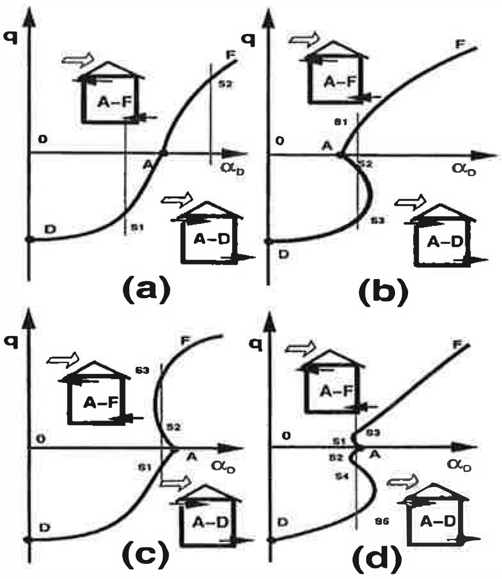

Figure 3: Some possible behaviours of the flow rate q for opposing winds in a two-zone building.

Figure 4 shows numerical solutions of the flow rate for a particular set of parameters. It is interesting to note that for small values of β/γ the solution is of the form shown in Figure 3(d), but as β/γ increases the form becomes that of Figure 3(c).

Thus, the hysteresis behaviour found in the single-zone building also exists in the two-zone building, and is indeed considerably more complex. Again we find that for opposing winds and a given set of heat gains, wind speeds and U-values, there appear to be several solutions for the flow. The full spectrum of solution behaviours has not yet been fully analysed. Furthermore, the stability of the multiple solutions has also not yet been resolved. A two-zone building presents a two-dimensional non-linear system. Further analysis of the system dynamics and identification of stable solutions will be carried out in the near future.

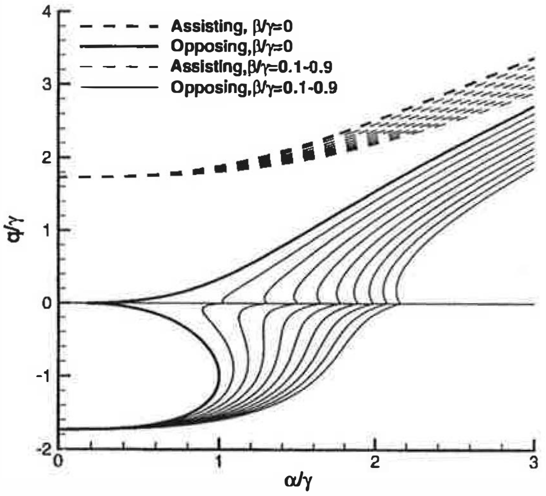

Figure 4: Scaled flow rate in a two-zone building as a function of a scaled buoyancy parameter.

SOME EXPERIMENTAL RESULTS

A small-scale water tunnel laboratory model of the single-zone building with opposing wind was constructed in order to test whether multiple solutions exist in reality. The details of this work are reported in Andersen *et al.*, (2000). One of the key results was to experimentally demonstrate the hysteresis predicted for the single-zone building. With a constant opposing wind the buoyancy flux was increased from zero (point D on Fig. 2, but note that in the experiment there was no heat flow through external surfaces, i.e. point A lies at the origin), and steady-state flows established at various points along the D-B curve. In particular a downward flow rate was established for a buoyancy flux lying between C and B. When the buoyancy flux was increased beyond E, the flow direction reversed and became buoyancy-dominated (i.e. upward) and continued to increase as the buoyancy flux was increased to point F. Crucially, as the buoyancy flux was then progressively decreased from F, an upward flow was established for a buoyancy flux lying between E and A, which was the same flux as was used to establish a downward flow between C and B. This demonstrates the hysteresis. The experimental results are shown in Fig. 5.

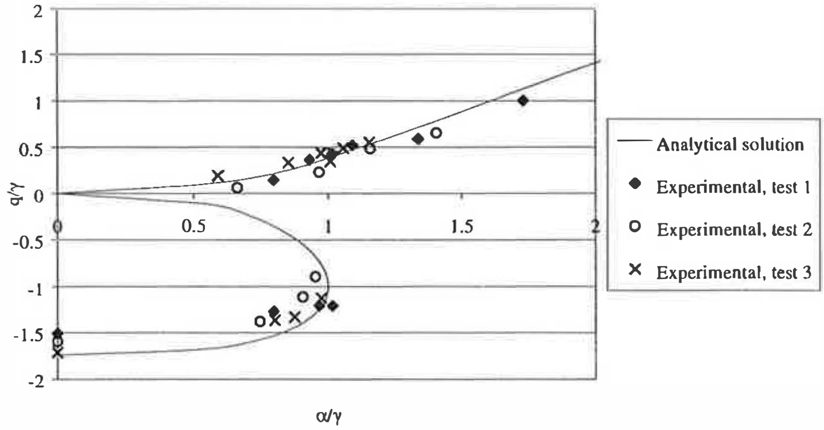

Figure 5: Experimental results from a water tunnel laboratory model of the one-zone building. Tests 1-3 refer to different wind speeds. Note that for a given wind speed, a buoyancy flux between 0.5 and 1.0 resulted in two possible flows, depending on whether the flux was increased from a low value or decreased from a high value. (Taken from Andersen *et al.*, 2000).

CONCLUSIONS

Natural ventilation flows can be quite complex even when calculated for very simple systems. For three examples it can be shown analytically that, for a certain range of parameters, multiple solutions exist in natural ventilation systems when the wind opposes thermal buoyancy. Even though the calculations assume fully mixed spaces, the existence of multiple solutions has been confirmed in small-scale laboratory tests where stratification was observed with buoyancy-dominated flows. More analytical and experimental analysis will be reported in the near future.

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