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SOME RELATIONS REVISITED IN TRACER GAS ANALYSES USING NUMERICAL METHODS

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ABSTRACT

Tracer gas measurements have long been used to quantify the performance of ventilation systems by exploring such scales as the air exchange efficiency, the local mean age of the air, the residence time distribution and so on. The present work deals with a numerical reexamination and calibration of some relations previously derived from tracer gas analysis. The relations revisited include the approximated local air age in terms of the local, instant tracer concentration using the step-up and step-down methods, and the equations for the mean cumulative age distribution and for its frequency distribution. A mixing ventilation flow is employed in the numerical examination. The availability of these relations for practical tracer gas analyses is discussed and they are shown to be useful for efficient tracer measurements.

KEYWORDS

Tracer gas analysis, Local air age, Ventilation performance, Numerical simulation

INTRODUCTION

Tracer gas experiments have been widely employed for quantifying building ventilation performance, by which a quantitative assessment of room effectiveness can be made on the basis of a number of ventilation scales or indices. In experiments, the air flow in a ventilated space is tracked by a passive tracer gas released at the air supply opening or at an interior location. The tracer gas concentration and its time series are recorded and subsequently translated into different ventilation parameters, which are used to assess the capabilities of the ventilation system to deliver fresh air to and/or expel contaminants from the ventilated space. Different measurement methods have been employed, including the step-up method, the step-down method and the pulse method, see e.g. Etheridge and Sandberg (1996). They have been proved in both laboratory and field measurements to be very powerful approaches for analyzing ventilation performance.

Several relations were derived in a previous work based on the so-called *imaginary* tracer gas analysis using numerical and/or multi-zonal approaches (Peng *et al.*, 1997). The derivation of these relations has largely resorted to the local mass conservation, by tracking the passive tracer gas which is presumed completely to follow the ventilation flow. These relations include approximations of the local concentration in terms of the local mean age of the air, and the discrete zonal equations for the mean cumulative age distribution and its frequency distribution.

While these formulations have been approximated on a plausible basis of some *imaginary* tracer gas experiments, no comprehensive examination of them has been conducted using either experiments or numerical simulations. By means of numerical analyses, this work revisits these relations and investigates the degree to which they can reach with a certain confidence and be used as complementary formulations in practical tracer gas measurements. The relations are closely correlated with practical applications in assessing ventilation performance. It should be noted that the purpose of this work is not ultimately directed toward numerical simulations of tracer experiments. Instead, numerical methods are employed here to generate several tracer gas experiments whereby the validation of the relations inherent are calibrated. A mixing ventilation flow is used to account for such a numerical calibration.

THE RELATIONS FOR TRACER GAS ANALYSIS

The relations were approximated from the compartmental (zonal) method and formulated locally (Peng *et al.*, 1997) . They can thus be examined on any interior local cell using the CFD modeling method or at any location in the ventilated space when an experimental measurement is employed. It should be pointed out that, in the derivation, the tracer gas is assumed to be passive with no interaction with the ventilation air flow, whose concentration, C, is thus governed by the transport equation

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x_j}(u_j C) = \frac{\partial}{\partial x_j}\left(\Gamma \frac{\partial C}{\partial x_j}\right) \tag{1}$$

where Γ is the diffusivity, in which a turbulent eddy diffusivity should be included if the flow is turbulent, and u_j are velocity components.

The *step-down analysis* is taken for a room with a volume of V and a supply air flow rate of Q. The nominal time constant is then $\tau_n = V/Q$. Before carrying out a step-down measurement, an initial concentration is set up in the room. This can be done by releasing the tracer gas at a constant rate, q, through the air supply opening for such a long period that the tracer gas concentration in the room is eventually equal everywhere, and $C_p(0) = q/Q$. The step-down measurement is then switched on by stopping the release of the tracer gas at the inlet. During the step-down procedure, the tracer gas concentration is assumed to decay exponentially with time t. This suggests that the local concentration at location P can be approximated as

$$C_p(t) \approx C_p(0)\exp(-E_p t) \tag{2}$$

where E_p is a coefficient needed to be determined.

At location P, the mean cumulative age distribution, Φ_p, can be expressed in terms of C_p as

$$\Phi_p = 1 - \frac{C_p(t)}{C_p(0)} \tag{3}$$

Using Eqn. (3), the local mean age of the air can be derived from

$$\tau_p = \int_0^\infty t\left(\frac{\partial \Phi_p}{\partial t}\right) dt \tag{4}$$

Substituting Eqn. (2) and (3) into (4) yields $E_p = 1/\tau_p$, which reasonably makes the concentration a function of the location through the local air age, τ_p. The local concentration in the step-down measurement is then approximated as

$$C_p(t) \approx C_p(0)\exp\left(-\frac{t}{\tau_p}\right) \tag{5}$$

In the *step-up measurement*, the initial tracer concentration at any location in the room is zero, i.e. $C_p(0) = 0$. During the measurement, the tracer gas keeps releasing through the air supply opening at a constant rate, q. As the local tracer gas concentration is recorded for a sufficiently long period, it will eventually become equal at all locations and $C_p(\infty) = q/Q$. Using an approximation analogous to the step-down analysis, the local concentration during the step-up procedure is formulated as

$$C_p(t) \approx C_p(\infty)\left[1 - \exp\left(-\frac{t}{\tau_p}\right)\right] \tag{6}$$

In addition, it was shown previously that the discrete zonal equations for the cumulative air-age distribution, Φ, and for its frequency distribution, ϕ, can be derived from tracer gas analyses based on the zonal model (Peng *et al.*, 1997). Here, these equations are reformulated in differential forms for numerical use. This can readily be done using the mass transport equation for tracer gas. In the step-up method, $\Phi_p = C_p(t)/C_p(\infty)$, and $C_p(\infty) \equiv q/Q$ at an arbitrary location P. Dividing $C_p(\infty)$ on both sides of the concentration equation, (1), gives

$$\frac{\partial \Phi}{\partial t} + \frac{\partial}{\partial x_j}(u_j \Phi) = \frac{\partial}{\partial x_j}\left(\Gamma \frac{\partial \Phi}{\partial x_j}\right) \tag{7}$$

For the age frequency distribution, ϕ, we have $\phi = \frac{\partial \Phi}{\partial t}$. Differentiating Eqn. (7) with respect to t yields the governing equation for ϕ,

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x_j}(u_j \phi) = \frac{\partial}{\partial x_j}\left(\Gamma \frac{\partial \phi}{\partial x_j}\right) \tag{8}$$

These equations can also be derived from similar arguments using the step-down method. They are generally available for indicating the statistical characteristics of air distribution related to the local air age for internal flow systems, although the derivation presented here is based on the tracer gas analysis. Eqn. (7) and (8) always hold in time-dependent forms since they are essentially the governing equations for time-related distribution quantities. Therefore, they should be solved transiently with proper initial values. In this work, only the derivation is given, while their use incorporated into the numerical simulations is not further explored.

The emphasis here is instead placed on the calibration of the availability of Eqn. (5) and (6) and their use in tracer gas measurements. If the two equations hold (conditionally), the local air age at an arbitrary location, P, can be determined by an instant sample of the local concentration, $C_p(t_m)$, at a measurement time t_m in the step-down or step-up procedure by rewriting them, respectively, as

$$\tau_p \approx \frac{t_m}{\ln[C_p(0)] - \ln[C_p(t_m)]}, \quad \text{with the step-down method.} \tag{9}$$

$$\tau_p \approx \frac{t_m}{\ln[C_p(\infty)] - \ln[C_p(\infty) - C_p(t_m)]}, \quad \text{with the step-up method.} \tag{10}$$

If the tracer gas is released at the inlet, $C_p(0)$ and $C_p(\infty)$ in the above expressions are equal to q/Q. When the step-up measurement is undertaken with a tracer released at an interior position, $C_p(\infty)$ in (10) varies locally. Furthermore, it should be pointed out that Eqn. (5) and (6) are not general solutions to the concentration equation under the step-up and step-down conditions. The main purpose of this work is to numerically verify to what degree these relations can be used in tracer gas analyses.

NUMERICAL CALIBRATION OF THE RELATIONS

A mixing ventilation flow is used to carry out the verification of the above relations. The air is supplied through a slot below the ceiling and exhausted from an opening on the opposite wall above the floor. The room is configured with dimensions of 9 × 3, and the nominal time constant is $\tau_n = 353.4$ s. The air flow is simulated using the standard $k - \varepsilon$ model, which is shown in Figure 1. Figure 1 (c) presents the

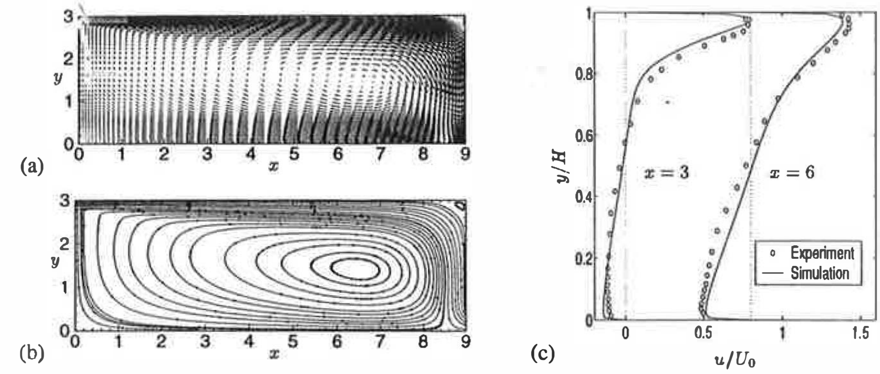

Figure 1: Simulation of the mixing ventilation flow. (a). Flow field; (b). Flow streamlines; (c). Computed velocity compared with experimental data.

velocity profiles at $x = 3$ and $x = 6$ in comparison with the experimental data. The results show that the flow computation is in reasonable agreement with the experiment.

The predicted local air age for this flow is shown in Figure 2 (a). Eight points, $pi(x, y)$ $(i = 1 - 8)$, within the room and one point, pe, at the exhaust opening are used for monitoring the relations, as sketched in Figure 2 (b).

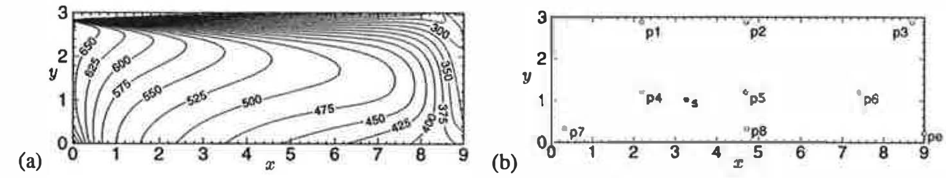

Figure 2: Simulated local air age and sketch of the monitoring points. (a). Contour lines of the simulated local air age; (b). Monitoring points used for numerical analysis.

The calibration is carried out in the following ways: computing the time-dependent concentration from its transport equation, Eqn. (1), based on the ventilation flow; comparing this simulated concentration with the one calculated from approximations (5) or (6) based on the local age. Figure 3 shows the comparison using the step-up method at the monitoring locations tabulated in Figure 2 (b). It suggests that the quality of the agreement between the two depends on the local flow features. The approximation agrees rather well with the simulation in regions with strong mixing and recirculation (at locations p4 – p8), while relatively poor agreement is found in the wall-jet where the flow is of a plug-flow type (at locations p1 – p3). In the initial stage (usually with $t/\tau_n < 1$), the simulated concentration does not even increase exponentially, as also confirmed in previous experiments (Sandberg, 1981). Without showing the results here, we pointed out that similar approximations can be reached using the step-down method, Eqn. (5), or using the step-up method by releasing the tracer in the room (at point s in Figure 2 (b)) for which $C_p(\infty)$ in Eqn.(6) should be replaced with the local, saturated tracer concentration which is often not equal to q/Q.

Figure 3 illustrates that the approximation in (5) or (6) is able to reasonably represent the exponential, transient tendency of the tracer gas concentration change after a short initial period ($t < \tau_n$) in the step-up or step-down measurement. To make these relations practically useful, they are further analyzed to

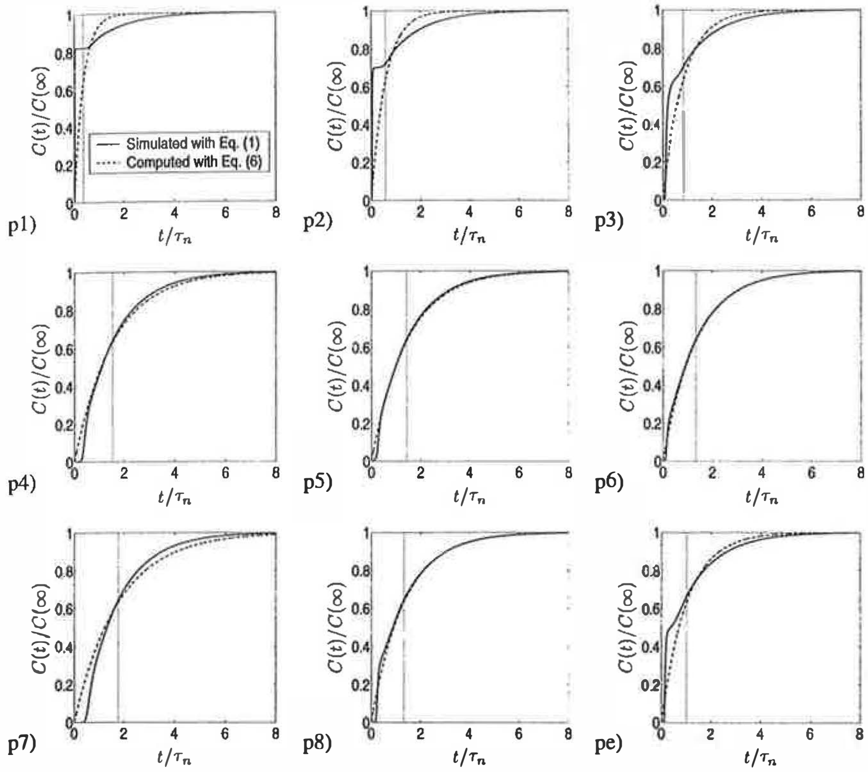

Figure 3: Local concentration approximated by Eqn. (6) using the step-up method in comparison with the numerical simulation with Eqn. (1). The vertical dotted line is the local air age, τ_p/τ_n, at the location explored.

verify whether they can be used as simple approaches to determine the local air age in terms of the local, instant tracer concentration. The local age is usually obtained by a transformation of the transient concentration series measured at the same location over a sufficiently long period. Using approximations (9) and (10), this can instead be done instantly (or in a short period in practical measurements).

The availability of Eqn. (10) (and, similarly, of Eqn. (9) with the step-down method) is calibrated through a comparison in which the local age calculated from (10) at different time instants, t_m, is compared with the numerical solution of the transport equation for τ. At $t_m = 1.5\tau_n$, Figure 4 (c), the local air age calculated from Eqn.(10) shows very good agreement with the numerical solution shown in Figure 4 (a). In general, the approximation at all *measured* time gives an age distribution in the recirculation zone rather close to the simulated one. At $t_m = \tau_n$, Figure (b), the age of the air flow near the exhaust opening is however estimated to be somewhat *younger* by the approximation than by the numerical simulation, while *older* in the region below the inlet. As t_m increases (for $t_m \geq 2.5\tau_n$), this tendency changes inversely with larger values near the exhaust and lower values below the inlet, as compared with the simulation. The results show that appropriate estimation can be achieved in the period of $1.5\tau_n \leq t_m \leq 2\tau_n$ using the approximation for regions where the flow is characterised by mixing and recirculation. Caution should however be taken for regions where the flow is of a one-way type, e.g. in regions near walls and near air supply openings.

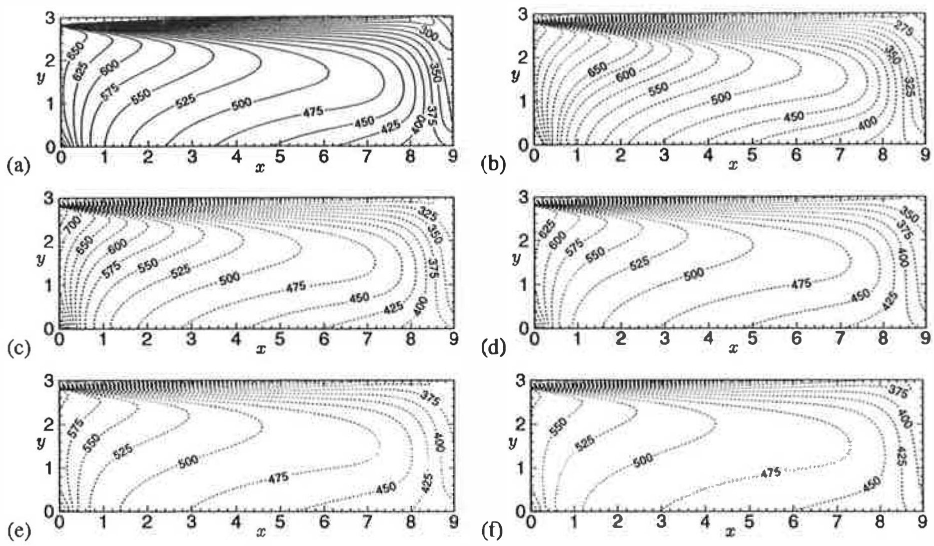

Figure 4: Comparison of the local air age: (a). Numerical simulation; (b). Calculated from Equation (10) at $t_m = \tau_n$; (c). At $t_m = 1.5\tau_n$; (d). At $t_m = 2\tau_n$; (e). At $t_m = 2.5\tau_n$; (f). At $t_m = 3\tau_n$.

CONCLUSIONS

Several relations derived previously from tracer gas analysis have been reexamined by means of numerical analyses. The governing equations for the cumulative age distribution and its frequency function are useful for numerical analysis with no need to specify a tracer gas. The approximation for the local air age formulated in terms of the instant local tracer concentration is expected to be a simple approach in realistic tracer gas measurements to efficiently determine the local air age, particularly for mixing ventilation flow as considered in this work.

The availability of the approximation was explored with a mixing ventilation flow system. It was found that the exponential growth or decay of the concentration with time holds reasonably well after a short period of about τ_n using the step-up or step-down method. This is particularly true in the recirculating flow region. The local air age approximated from these relations is in rather good agreement with the numerical solution of the local-age transport equation, provided that the tracer concentration is sampled at a time within $1.5\tau_n \leq t_m \leq 2\tau_n$. Erroneous estimations may however arise in regions where the flow is of a *one-way* type, e.g. in near-wall boundary layer and in the region near a displacement ventilation diffuser. More comprehensive investigation and calibration of these relations are needed by means of tracer gas experiments and in more complex ventilation configurations to achieve more general conclusions.

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