

### A Universally Valid Strategy for Low Energy Houses

Bruno Keller, Eugen Magyari, Tian Yuan, Sabine Bödefeld  
Chair of Building Physics, Institute of Building Technology, Swiss Federal Institute of Technology ETHZ  
Zurich, Switzerland

#### ABSTRACT

On the base of universally valid laws: energy conservation and the theorem of Fourier, the dynamic behaviour of a room is traced back to only two most important parameters. With the aid of the so-called free-run temperature a generally valid strategy for low energy houses is deduced and its transfer to practice illustrated. With the climate surfaces, a planning tool is introduced allowing the strategic planning of low energy houses based on these two parameters. Finally the obsolescence of the passive-solar rules and strategies is demonstrated.

#### KEY WORDS

Energy, Buildings, passive solar

#### INTRODUCTION

In the last 20 years, a lot of international research projects, congresses, books and papers have dealt with the way, low energy houses can be built. Modern and partially very complex simulation programs have been developed. A today's architect however still misses a clear strategy he can follow to arrive at an optimised low energy house in a given climate. For some specific situations and climates, some rules have been developed on an empirical base, but they are not, lacking a clear theoretical background, transferable to other climates. From the point of view of a physicist, this situation is not very satisfactory. Based on universally valid fundamental laws as the conservation of energy, the authors will therefore show in the subsequent chapters, that and how such a strategy can be developed and transferred into concrete measures on a building. Because of its physical background, this strategy will be valid for any thinkable building in any given climate of the world. The way it must be realised however will very much depend on the specific situation.

#### THE APPROACH

In a first step, the law of energy conservation for a room (and therefore also for a building) is considered:

$$G \cdot I(t) + P_{hc} + P_{int} - K \cdot (\vartheta_i(t) - \vartheta_e(t)) = \frac{dQ}{dt} \tag{1}$$

Input                      - Loss                      = Increase

$$K = \frac{1}{A_e} \cdot \left[ \sum_k A_{k,e} \cdot U_k + n \cdot V \cdot \frac{(c \cdot \rho)_{air}}{3600} \right] \quad n: \text{air change rate in h}^{-1} \tag{2}$$

$$G = \frac{1}{A_e} \cdot \sum_k A_{k,ext} \cdot g_k = \text{mean total solar energy transmission} \tag{3}$$

Q: heat deposit in the internal storage masses, normalised to the external surface [J/m<sup>2</sup>]  
P<sub>hc</sub>: power contribution of heating/cooling elements [W/m<sup>2</sup>]: P<sub>hc</sub>>0: heating, P<sub>hc</sub><0: cooling

$P_{int}$ : contribution of internal sources [W/m<sup>2</sup>]

For thin storage layers (walls, ceilings, floors), the heat deposited in can be expressed by means of the storage capacity  $C$ , related to the external surface:  $dQ = C \cdot d\vartheta$ , and one obtains:

$$\vartheta_i(t) + \tau \cdot \frac{d\vartheta_i(t)}{dt} = \vartheta_e(t) + \gamma \cdot I(t) + \frac{P_{hc}}{K} + \frac{P_{int}}{K} \quad (4)$$

with the abbreviations:

$$\tau = \frac{C}{K} [s, h] = \text{time constant of the room, demonstrating the length of its thermal memory or its thermal inertia} \quad (5)$$

$$\gamma = \frac{G}{K} \left[ \frac{m^2 \cdot K}{W} \right] = \text{gain-to-loss factor or solar correction factor to the external temperature, taking into account the influence of the solar radiation.} \quad (6)$$

This differential equation for the time development of the internal temperature already allows some conclusions:

1. The room is dynamically completely described by only two constants:  $\gamma$  and  $\tau$ .
2. The effect of the weather or climate is described by a simple meteo-function  $\Phi(t) = \vartheta_e(t) + \gamma \cdot I(t)$  (7)
3. The effect of the HVAC equipment is fully determined by  $\frac{P_{hc}(t)}{K}$  (and not by  $P_{hc}(t)$  alone!)
4. The HVAC equipment acts as a complementation of the weather (as well as the internal sources).

In order to keep the internal temperature at a constant value  $\vartheta_{min}$  (=20°C as example for winter), the change of the internal temperature has to be kept at zero:

$$\vartheta_{min} = \vartheta_e(t) + \gamma \cdot I(t) + \frac{P_{hc}(t)}{K} \quad \text{thus:} \quad P_h(t) = K \cdot \{\vartheta_{min} - [\vartheta_e(t) + \gamma \cdot I(t)]\} \quad (8)$$

(the internal sources left apart without limiting the validity).

Putting the HVAC equipment and the internal sources in a next step apart, one obtains a differential equation for the time development of the internal temperature under the pure influence of the climate: the free-run temperature:

$$\vartheta_i(t) + \tau \cdot \frac{d\vartheta_i(t)}{dt} = \vartheta_e(t) + \gamma \cdot I(t) \quad (9)$$

Its solution for thin layers is:

$$\vartheta_i(t) = \frac{1}{\tau} \cdot \int_{-\infty}^t e^{-(t-t')/\tau} \cdot [\vartheta_e(t') + \gamma \cdot I(t')] \cdot dt' \quad (10)$$

a convolution integral of the meteo function  $\Phi(t)$  with a memory function, demonstrating the exponentially decaying thermal memory of the room. For layers of finite thickness, the solution is more complicated but principally very similar, see Appendix.

As a result, one concludes:

- The free-run temperature of a room can exactly be computed based on the weather data.
- It depends only on the two constants of the room:  $\gamma$  and  $\tau$

### THE STRATEGY

From the above mentioned results, the strategy can be developed in a straight forward way:

- The time development of the internal temperature is completely determined by the weather and  $\gamma$  and  $\tau$ .
- As long as the free-run temperature remains within the comfort limits  $\{\vartheta_{min} \leq \vartheta_i(t) \leq \vartheta_{max}\}$  no intervention of the HVAC equipment is necessary:  $P_{hc}(t)=0$ .
- As soon as the free-run temperature attains the comfort limits and risks to cross them, the weather has to be complemented by the HVAC equipment:

$$P_h(t) = K \cdot \{\vartheta_{min} - [\vartheta_e(t) + \gamma \cdot I(t)]\} = K \cdot [\vartheta_{min} - \vartheta_e(t)] - G \cdot I(t) \quad (11)$$

$$P_c(t) = K \cdot \{[\vartheta_e(t) + \gamma \cdot I(t)] - \vartheta_{max}\} = K \cdot [\vartheta_e(t) - \vartheta_{max}] + G \cdot I(t) \quad (12)$$

- The crossing of the comfort limits by the free-run temperature thus determines the switching on and off of the HVAC equipment.
- From this facts the strategy immediately follows (Fig.1):

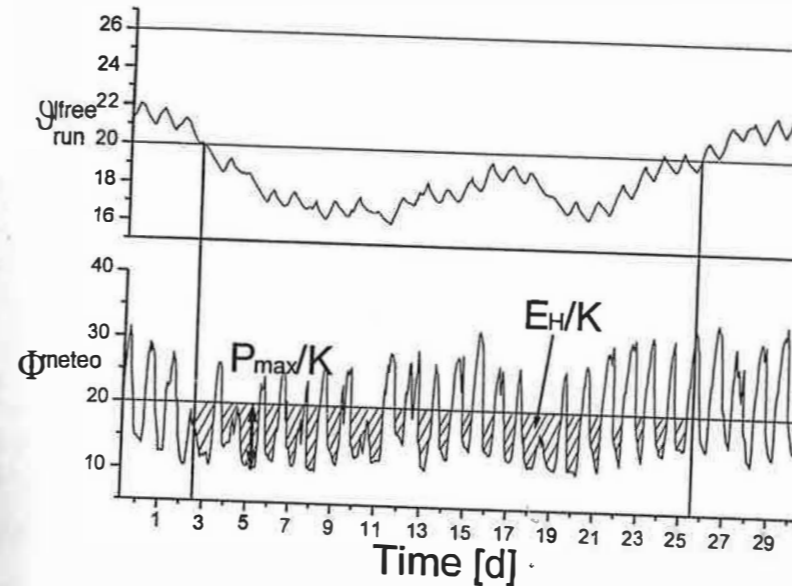


Fig. 1: Relation between free-run-temperature, meteo-function and heating power and energy.

### Strategy:

- To minimise the energy need for heating and/or cooling in a any given climate, a room (or building) has to be designed by its time constant  $\tau$  and solar factor  $\gamma$  such that the free-run temperature remains most of the time within the comfort limits.
- Keeping the loss factor  $K$  low almost always helps to keep the power needed low. In warm climates a minimising of the solar transmission  $G$  helps, whereas in cold climates a limited (not too much!) increase of  $G$  helps also.

This strategy is valid everywhere, from a space shuttle to the igloo of Eskimos to a hotel in Saudi Arabia etc. The way however, how it can be transposed into reality depends very much on the climate and the other boundary conditions of a building: available materials, living habits, user requirements etc..

### THE REALISATION

For a transposition to reality, one must know what effect a change of  $\gamma$  and  $\tau$  has on the free-run temperature. For this to be investigated, a harmonic representation of the climate values is used. This is based on the theorem of Fourier according to which any reasonably well behaving function can be represented by infinite series or an integral of harmonic functions. In fact, the Fourier-analyses of real weather data easily shows, even the first order representation to be a ver good approximation of reality:

$$\vartheta_e(t) = \bar{\vartheta}_e + \Delta \vartheta_e \cdot \cos(\omega \cdot t) \quad I(t) = \bar{I} + \Delta I \cdot \cos(\omega \cdot t + \varphi) \quad \text{with } \omega = \frac{2 \cdot \pi}{T} \quad T = 24h \quad (13)$$

Using the daily mean values  $(\bar{\vartheta}_e, \bar{I})$  and the daily amplitudes  $(\Delta \vartheta_e, \Delta I)$ . Using this analytical model weather one obtains for the free-run temperature:

$$\vartheta_i(t) = \bar{\vartheta}_i + \Delta \vartheta_i \cdot \cos(\omega \cdot t + \varphi') \quad (14)$$

with  $\bar{\vartheta}_t = \bar{\vartheta}_e + \gamma \cdot \bar{I}$  and  $\Delta \vartheta_t = \frac{1}{\omega \cdot \tau} \cdot [\Delta \vartheta_e + \gamma \cdot \Delta I]$  (15)

From these expressions the effect of changing  $\gamma$  and/or  $\tau$  turns out immediately:

- The solar factor  $\gamma$  shifts the mean value of the free-run temperature and influences also its amplitude,
- the time constant  $\tau$  however influences only the amplitude of the free-run temperature.

(Fig. 2)

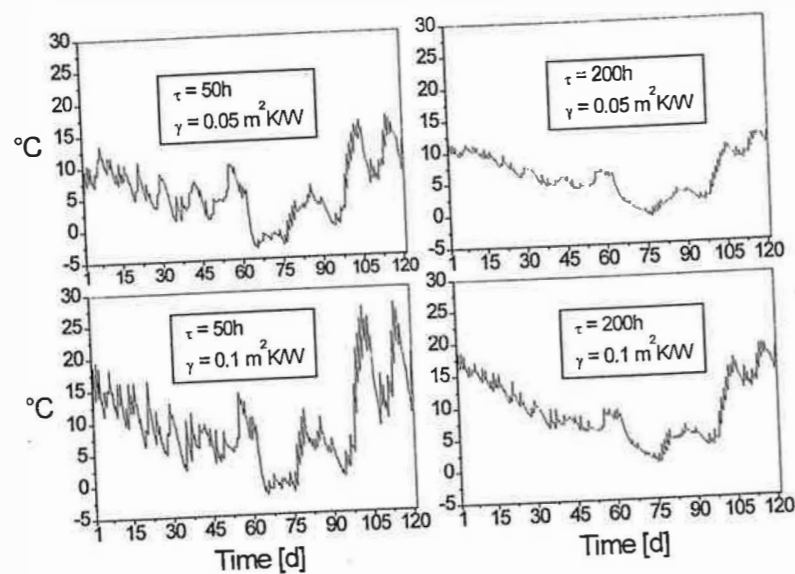


Fig.2: The effect of  $\gamma$  and  $\tau$  on the free-run temperature

The realisation of the strategy consists thus  
In the cold part of the year:

- First in shifting the mean value of the free-run temperature upwards into the comfort range with  $\gamma$ ,
- this also increases the daily amplitude,
- secondly in limiting this amplitude, if necessary, by means of a large enough time constant  $\tau$ .

In the warm part of the year:

- First in shifting the mean value of the free-run temperature downwards into the comfort range with  $\gamma$ ,
- this also decreases the daily amplitude,
- secondly in limiting this amplitude, if still necessary, by means of a large enough time constant  $\tau$ .

According to the nature of the two principal parameters  $\gamma$  and  $\tau$ , there exist many ways of realising these measures:

Increase of  $\gamma = \frac{G}{K}$ : Increase of G: Window size, glazing type (total solar energy transmission)  
or decrease of K: U-values of external parts, air infiltration.  
Decrease of  $\gamma$ : in a corresponding way, including sun shading.

Increase of  $\tau = \frac{C}{K}$ : Increase of C: internal surfaces, material (thermal effusivity  $b = \sqrt{\lambda \cdot c \cdot \rho}$ ),  
or decrease of K: see above.

In general, there will be contradicting requirements between the warm and the cold part of the year. They can be solved by an overall year compromise or by adaptable elements as sun shading, variable air change etc.

The specific technical properties of the building elements involved limit the optimisation process. The strategy and its realisation, however, as explained above, allows in any kind of climate to find the best solution. The graphic representation of the free-run temperature for different values of  $\gamma$  and  $\tau$  helps a lot for a first orientation.

### A SIMPLE PLANNING TOOL: CLIMATE SURFACES

In order to work in practice with this strategy, it could be helpful to have a direct representation of the above relations. This can in fact be obtained in different ways:

#### The Most Concise Representation

According to the above relations the power needed for heating and/or cooling can directly be computed from the weather-data. The necessary corrections by the HVAC equipment to the weather is given by:

$$\frac{P_h(t)}{K} \equiv \Pi_h(t; \gamma, \tau) = \vartheta_{min} - [\vartheta_e(t) + \gamma \cdot I(t)] \quad \text{and} \quad \frac{P_c(t)}{K} \equiv \Pi_c(t; \gamma, \tau) = [\vartheta_e(t) + \gamma \cdot I(t)] - \vartheta_{max} \quad (16)$$

The time constant does not appear explicitly in these equations but it is relevant for the switching on and off of the HVAC equipment.

The maximum value of the temperature (weather-) corrections over a year can be registered and represents the design temperature difference for heating or cooling:  $\Pi_{h,c}(\gamma, \tau)$ . As a function of two parameters it can easily be represented by 3D-surface directly indicating this value for any kind of room: Power-surfaces.

The integration of the momentary temperature or weather corrections between the switching on and the off-times directly yields the energy used for heating or cooling:

$$E_h(\gamma, \tau) = \int_{t_1(\gamma, \tau)}^{t_2(\gamma, \tau)} P_h(t') \cdot dt' = K \cdot \int_{t_1(\gamma, \tau)}^{t_2(\gamma, \tau)} \Pi_h(t'; \gamma, \tau) \cdot dt' = K \cdot \Omega_h(\gamma, \tau) \quad (17)$$

$$E_c(\gamma, \tau) = \int_{t_1(\gamma, \tau)}^{t_2(\gamma, \tau)} P_c(t') \cdot dt' = K \cdot \int_{t_1(\gamma, \tau)}^{t_2(\gamma, \tau)} \Pi_c(t'; \gamma, \tau) \cdot dt' = K \cdot \Omega_c(\gamma, \tau) \quad (18)$$

The functions  $\Omega_{h,c}(\gamma, \tau)$  represent as time integral of temperature differences so-called generalised heating-degree-days. Instead of one number for the traditional heating degree days, they represent functions of two parameters and they include the effect of the solar radiation and of the thermal inertia. These functions can also be represented as 3D surfaces, directly indicating these values for any kind of room: Energy-surfaces.

#### The Easier to Interpret Representation

The functions  $\Pi_{h,c}(\gamma, \tau)$  and  $\Omega_{h,c}(\gamma, \tau)$  are the most concise possible representations of the energy and power relations, but not so easy to read because of the factor K which has to be multiplied to get the real power and energy need. If one accepts the fact that the specific storage capacity of rooms falls in about 3 to 4 classes from very light weight to very heavy construction, one can separate this value out and directly give representations showing the specific power and energy need as a function of G and K. The latter are easier to understand for the practitioner and the power and energy need can directly be read out. The only procedure still needed is its transposition from values related to the external surface to those related to the floor area to arrive at traditionally comparable values.(Fig.3)

### CONCLUSIONS

With the aid of only two universally valid laws, the energy conservation and the theorem of Fourier, the authors hope to have definitely shown, the thermal dynamic behaviour of a room and therefore also of a whole building can be understood from first principles. Only three parameters are necessary for a complete description. It turns out to be possible to respect the solar contributions in a way corresponding to the dynamics of the room and

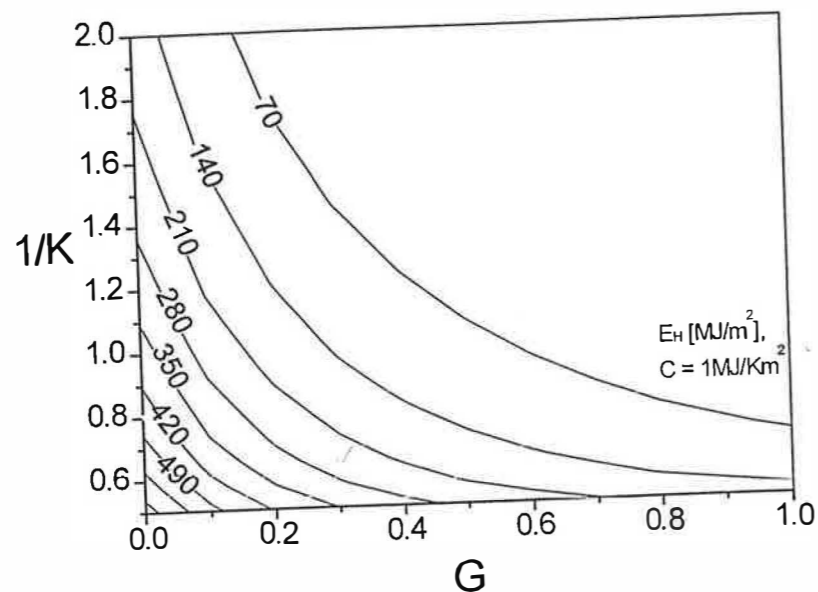


Fig.3: The heating energy (per sqm of external surface!) as a function of  $G$  and  $1/K$  for a typical mid heavy structure ( $C=1 \text{ MJ/Km}^2$ )

respecting the comfort limits by means of generalised heating-degree-days. The results of this work has been tested extensively by comparison with the measured energy consumption of several different real buildings in operation with very good results.

The strategy itself is of a very simple form which can easily be transposed onto any kind of specific situations. The free-run temperature plays a key role. Instead of pretending a high accuracy by means of many parametric simulation tools at a late phase of planning, the strategy can already be applied in the very early states of planning, guiding the designer in the right direction.

As a nice illustration for the power of these tools, the dependence of the need for heating energy on the window partition and on the glazing quality has been investigated (for the climate of Zurich). It turns out, for high level glazings, the energy need becoming almost independent on the partition on the south-side (but a cooling need emerging above about 40%!) and even on the north side being much lower than the need for a very well insulated wall. This opens new perspectives of design freedom for architects and proves some passive-solar rules to be wrong.

The whole passive-solar exercise seems to the authors to become definitely obsolete now, because it never managed to give any clear strategies neither quantitatively nor even qualitatively, nor could it develop any rules of general enough validity to be transferred to different climates. Software tools allowing the work with climate surfaces are available on request from the authors for many locations of the world.