Impact and Stability of Closed Airflow Paths Inside Insulated Structures

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ABSTRACT

In insulated structures, air voids and cracks allow for airflows driven by temperature differences, i.e., natural convection. The airflow paths often exist in structures built with bad workmanship, but sometimes even with the best workmanship they are difficult to avoid. Air paths within new types of loose-fill insulation may also occur. For horizontal structures, critical channel flow Rayleigh numbers can be identified for the onset of convection. In this paper, the increase in heat loss through the structure due to convection is analyzed, and both critical channel flow Rayleigh numbers and the increase in heat loss are presented.

INTRODUCTION

The conductive heat flow in highly insulated structures is normally quite small. Airflows within the structure have the potential to cause a significant change in the total heat loss and can also change the temperature distribution considerably. In this paper, airflows caused by stack effects, i.e., the case of natural convection, are considered. A structure consisting of an evenly thick, single insulation material will be analyzed. The energy is assumed to be transferred by conduction and convection only. Other types of transport phenomena such as vapor diffusion, condensation of water, or radiation over air gaps are not considered in this paper.

An idealized two-dimensional situation is considered, with dense insulation and unintentional airflow channels. For simplicity, only rectangular air channel loops are considered in more detail. Due to the dense insulation, the convective effect vanishes inside the domain of the insulation, and airflow only occurs in the channels.

Assume that the air is flowing in well-defined air channels parallel to the x, y plane. The channels are surrounded by an impermeable insulation material; see Figure 1. Due to the two-dimensional nature of the problem, any cross section of the structure parallel to the x, y plane will look the same. The interior temperature below the structure, T_i, is greater than the exterior one, T_e. The heat is transferred by conduction only in the insulation material outside the air channel.

The channel air gap height, b, is considered in the calculations of the airflow resistance of the channel. However, in the solution of the heat conduction problem in the structure, the height of the channel is neglected. The case with constant thermal properties in the structure and negligible surface resistances, both in the channels and at the surfaces of the structure, is considered. This case is particularly realistic for well-insulated structures where the contribution to the total thermal resistance from the surface resistances is small.

Thermal problems of the type considered can be found in the literature. Kohonen et al. (1985) studied heating of air leakage in cracks. In particular, they studied air that flows straight through the construction. Their simulation technique is based on a division of the air channel into small elements of the same size as the material cells. Separate energy balances for these cells are established in order to predict the temperature along the air channel.

Lecompte (1989) studied, both numerically and experimentally, the impact on the heat loss due to airflows caused by natural convection in closed air channel loops inside an insulated cavity wall.

Hagentoft (1991a) solved an integral equation for the temperature in an air channel that goes straight through a wall. The temperature distribution and the extra heat loss caused by the airflow are tabulated.

Hagentoft (1991b, 1991c) presented studies similar to the one in this paper for vertical constructions. These should be considered as background reports for this article. The software ANHConP (Hagentoft 1991b) has been used for all numerical calculations of heat flows in this paper.

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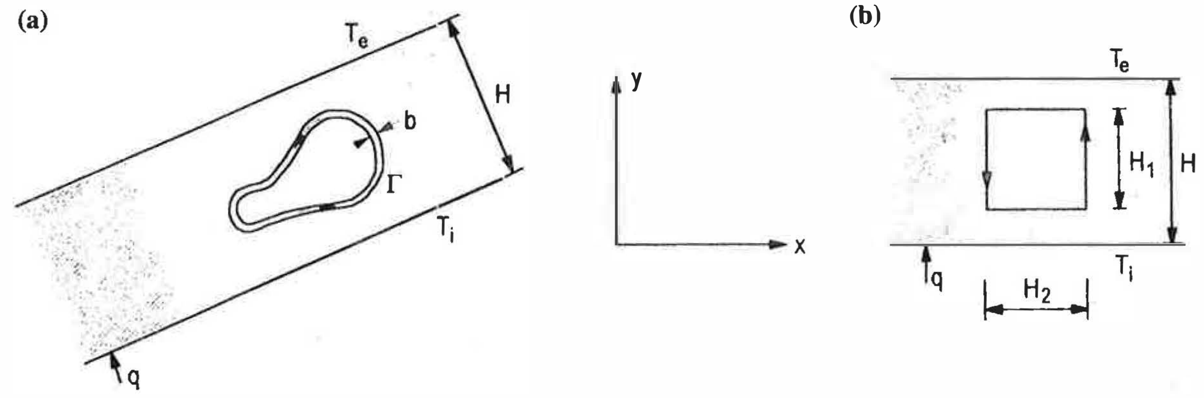

Figure 1 *(a) Cross section of an insulation layer with an airflow channel loop, denoted by Γ, inside the structure. Due to the two-dimensional nature of the analysis, the air is assumed to flow parallel to the x, y plane. (b) Cross section of a horizontal insulation layer with a rectangular airflow channel loop.*

The research results found in the literature mainly treat two-dimensional cases with the help of numerical tools. No studies of the onset of convection or the increase in heat loss are found for insulated horizontal structures. This study gives a combined analytical and numerical analysis of the problem.

The impact on the heat loss due to a prescribed airflow rate in the channel is investigated in the next section. In the following section, the driving force behind natural convection is analyzed. The stability problem of natural convection in horizontal structures resulting in critical numbers for the onset of natural convection is investigated next. Then, in the section "Examples," the Nusselt number is calculated for a few cases with a rectangular airflow channel. Also, the required air gap height for the onset of convection is estimated.

EXTRA HEAT LOSS

Figure 1a shows a general air channel loop, denoted by Γ, in an insulation layer of constant thickness. The steady-state temperature in the structure is denoted by $T(x,y)$. It is assumed that the air is in good contact with the surrounding material, i.e., there is no temperature difference between the air and the channel walls. Let the x and y coordinates along the channel loop, Γ, be given by the functions $x(s)$ and $y(s)$. The starting point, $s = s_0$, then corresponds to the x and y coordinates $x(s_0)$ and $y(s_0)$. The end point of the loop is defined by $s = s_1$, i.e., the point defined by $x(s_1)$ and $y(s_1)$ in the x, y plane. For a closed loop, the x and y coordinates will be identical for the start and end points. The curve length from the start point to the point on the curve defined by s will be equal to $s - s_0$ since s is a curve length parameter. Formally, we can define the x and y coordinates along the curve Γ from the length parameter s by the following relation:

$$\Gamma(s): \qquad x(s), y(s) \qquad s_0 < s < s_1 \tag{1}$$

The temperature in the air channel is denoted by $T(s)$, i.e., a function of the length coordinate s. The thermal conductivity of the surrounding material is k. The total heat loss through the structure is denoted by q. The heat loss over a certain part the structure, with the width D including the air channel considered.

Temperature Distribution

The temperature in the structure can be scaled w respect to the temperature difference over the constructi We introduce the nondimensional temperature T'. It va between zero and one. By using dimensional analysis, we

$$T = T_e + (T_i - T_e) \cdot T'(x/H, y/H, \mathbf{Pe}_c, \ldots)$$

where the dots refer to all the dimensionless ratios betw geometrical dimensions, defining the loop and the heigh the material layer. For the case of Figure 1b, they will co spond to the dimensionless parameters: H_1/H, H_1/H_2. We introduce the important dimensionless channel P number, $\mathbf{Pe}_c$:

$$\mathbf{Pe}_c = \frac{m_a \cdot c_a}{k}$$

where m_a is the air mass flow rate in the channel, and c_a i heat capacity of air. The channel Peclet number gives the between the strength of convective and conductive heat fl It represents a dimensionless airflow rate. The influence the airflow can be neglected if the Peclet number is very s much less than one. The channel temperature is given b following expression:

$$T = T_e + (T_i - T_e) \cdot T'(s/H, \mathbf{Pe}_c, \ldots)$$

Heat Loss Factor h_e

The airflow in the air channel could cause an extra loss, compared with the case without airflow, $m_a = 0$, i region of the structure where the air is flowing. The extr loss, denoted by q^e, is defined by

$$q^e = q - q|_{m_a = 0}$$

Using dimensional analysis, the extra heat loss can be written as

$$q^e = k(T_i - T_e) \cdot h^e(\mathbf{Pe}_c, \ldots) \tag{6}$$

where h^e is a dimensionless heat loss factor.

Analytical Formula for Small Peclet Numbers

For small Peclet numbers, much less than one, the temperature in the air channels is approximately equal to the one obtained for zero airflow rates, i.e., a linear temperature distribution. Figure 2 illustrates an airflow channel with an entrance at $s = s_0$ and an exit at $s = s_1$. The channel can either be closed, i.e., air is circulating in a loop, or open with an external entrance and exit, as illustrated in Figure 2.

The thermal influence of the airflow in the channel corresponds to the influence obtained from a distribution of line heat sources (perpendicular to the plane of the paper) along the channel (Hagentoft 1991a). The magnitude of the line heat sources is equal to

$$m_a \cdot c_a \cdot \frac{\Delta y}{H} \cdot (T_i - T_e) \tag{7}$$

where $\Delta y/H \cdot (T_i - T_e)$ represents the temperature drop over a part of the airflow channel. Airflows going upward give positive Δy values that can be represented by line heat sources, and flows going downward can be represented by line heat sinks. Using the channel Peclet number (Equation 3), the magnitude of the line heat source can be written as

$$k \cdot \frac{\Delta y}{H} \cdot (T_i - T_e) \cdot \mathbf{Pe}_c . \tag{8}$$

The following formula, derived by Hagentoft (1998), gives the extra heat flow through the structure.

$$q^e = k(T_i - T_e) \cdot \frac{\mathbf{Pe}_c}{2} \cdot (T'^2(s_1/H) - T'^2(s_0/H)) \qquad |\mathbf{Pe}_c| \ll 1 \tag{9}$$

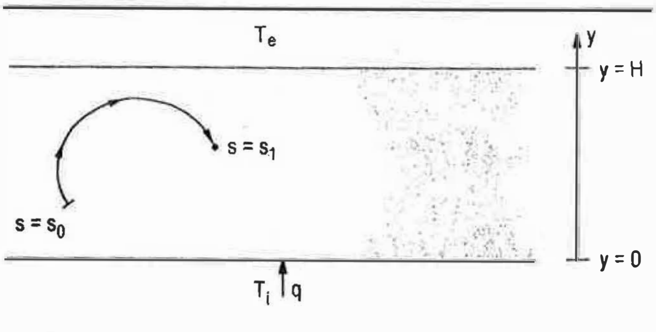

Figure 2 *Cross section of a horizontal insulation layer with an airflow channel. The channel is defined according to the definition in Equation 1. The entrance (air is entering the layer) of the airflow channel is given by the length coordinate, $s = s_0$. The exit (air is leaving the layer) is defined by $s = s_1$.*

where

$$T' = 1 - y/H. \tag{10}$$

From this very general formula, it can be concluded that closed air channels or channels with an entrance and exit at the same level, i.e., the same y value, do not give any extra heat loss for small Peclet numbers. Each small segment of the channel where air is flowing upward, which thermally can be represented by an internal heat source, will be matched by a corresponding heat sink at the same y coordinate, but of course at another x coordinate. The net effect on the heat loss due to these pairs of sources and sinks will vanish.

Example with a Rectangular Air Channel Loop

Figure 3 shows the numerically calculated (ANHConP) heat loss factor for a symmetrically placed rectangular airflow loop, with $H_1/H_2 = 1$ and $H_1/H = 0.5$, as a function of the Peclet number.

For low airflow rates, i.e. small Peclet numbers, the heat loss is very small, as expected from the results in the previous subsection. For large Peclet numbers, the heat loss approaches an asymptotic value. For very large channel Peclet numbers, the air temperature in the channel will be the same everywhere in the enclosed area. This final constant temperature will only depend on the shape of the loop and its location. Since the temperature approaches a constant value, the heat loss factor will also approach a constant one.

DRIVING FORCE BEHIND NATURAL CONVECTION

The airflow rate in the channel due to natural convection depends on the airflow resistance and the geometry. Furthermore, it depends on the temperature difference over the structure. Temperature differences of the air at different locations of the channel give differences in density. This results in a

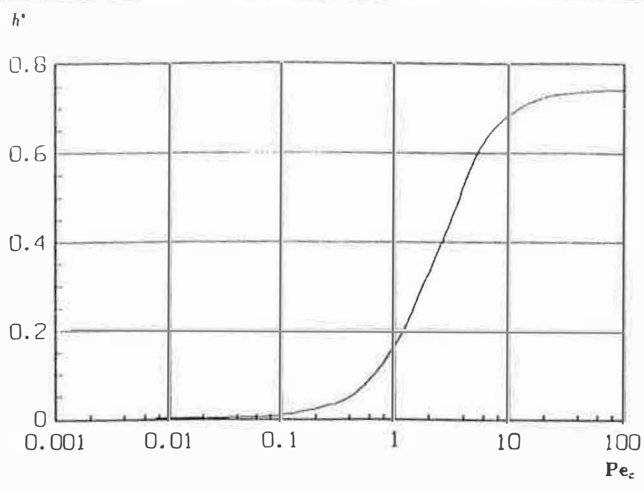

Figure 3 *Numerically calculated heat loss factor for a rectangular airflow loop. Symmetrically placed loop with $H_1/H_2 = 1$ and $H_1/H = 0.5$.*

stack effect that is analogous to an internal buoyancy pressure difference, Δp_B. Over a closed curve Γ (loop), the total pressure difference Δp_B becomes

$$\Delta p_B = g\beta\rho_a \oint_\Gamma (T(\vec{r}) - T_{ref})\hat{y} \cdot \vec{dr} \tag{11}$$

where $\hat{y}$ is a unit vector pointing upward, parallel to the field of gravity, $\vec{dr}$ is a curve element pointing in the tangential direction of the curve (counterclockwise), and β is the coefficient of volumetric expansion. For an ideal gas, it is equal to $1/T$ where T (K) is the absolute temperature. The density, ρ_a, of air at atmospheric pressure is used.

Equation 11 gives the driving pressure difference in the loop. A temperature distribution giving a positive value of Δp_B means that the buoyancy force wants to move the air counterclockwise. If the sign is negative, it wants to move the air clockwise instead. The contribution from a constant temperature term, such as T_{ref}, in the tangent line integral vanishes. This can be found directly from Green's formula.

Stack Effects for Peclet Number Equal to Zero

Consider the airflow loop in Figure 4, which is tilting the angle θ to the horizontal plane. The considered angle can vary between 0 and $\pi/2$.

Hagentoft (1998) derived the following formula for the pressure difference for cases with a Peclet number equal to zero:

$$\Delta p_B = g\beta\rho_a(T_i - T_e) \cdot \sin(\theta) \cdot \frac{S}{H} \tag{12}$$

where S refers to the area enclosed by the air channel loop. For a tilting angle greater than zero, there will exist a buoyancy pressure difference other than zero, i.e., there will exist an airflow in the channel. However, for horizontal structures the pressure difference is zero.

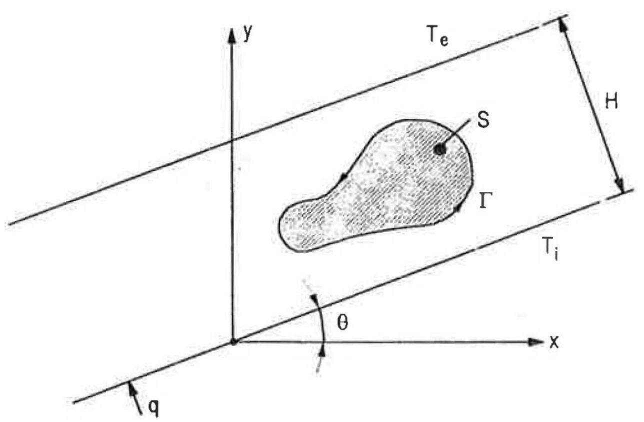

Figure 4 *Airflow channel loop in a structure tilting the angle θ to the horizontal plane.*

Stack Effect for Peclet Numbers Greater than Zero

In order to establish a nonzero pressure difference for horizontal structures, the Peclet number must be greater than zero, i.e., there must exist an airflow in the channel. For instance, in a rectangular loop (see Figure lb), the airflow can cause a temperature difference between the vertical parts of the airflow channel, which in turn generates a pressure difference.

Using the technique of scaling, the buoyancy pressure difference in the airflow channel becomes

$$\Delta p_B = g\beta\rho_a(T_i - T_e) \cdot H_1 \cdot \Delta T'(\mathbf{Pe}_c, \ldots). \tag{13}$$

Here, an average dimensionless temperature difference in the channel loop has been introduced:

$$\Delta T'(\mathbf{Pe}_c, \ldots) = \frac{1}{H_1(T_i - T_e)} \oint_\Gamma (T(\vec{r}) - T_{ref})\hat{y} \cdot \vec{dr} \tag{14}$$

For a rectangular loop, it will represent an average dimensionless temperature difference in the two vertical channel segments.

The numerically calculated temperature difference is shown in Figure 5 for the case with a symmetrically placed loop with H_1/H_2 = 1and H_1/H = 0.5.

Pressure Drop Due to Friction

Laminar theory for the flow can be used at the onset of convection since the airflow rates are very small. The Reynolds number gives the upper limit for laminar flow between two infinite parallel planes:

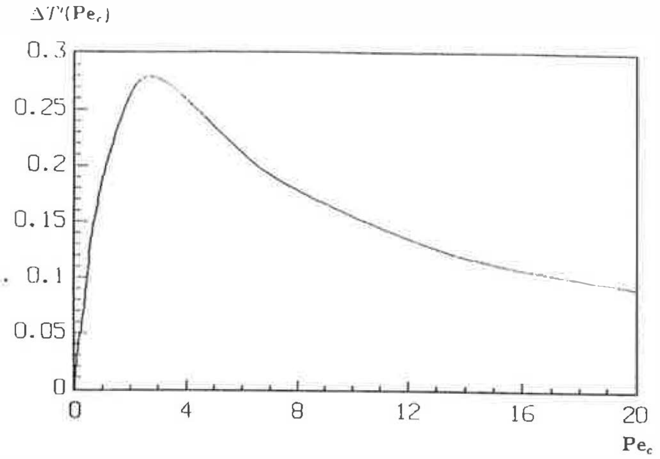

Figure 5 *Numerically calculated average dimensionless temperature difference, $\Delta T'$, as a function of the channel Peclet number. Symmetrically placed loop with H_1/H_2 = 1 and H_1/H = 0.5.*

$$\frac{2 \cdot m_a}{\eta} < 2000 \quad \text{or} \quad \frac{m_a}{\rho_a} < 0.014 \text{ m}^3/\text{ms} \quad \text{at } 10°\text{C} \quad (15)$$

where η is the dynamic viscosity.

The airflow resistance R_{fc}(Pa/(m^3/ms)), always greater then zero, is defined by the following relation:

$$R_{fc} = \frac{\Delta p_F}{|m_a/\rho_a|} \quad (16)$$

where Δp_F is the pressure drop in the channel due to friction. According to Idelchik (1966), the airflow resistance for a closed rectangular loop (see Figure lb) becomes

$$R_{fc} = \frac{24\eta}{b^2} \cdot \left(\frac{H_1 + H_2}{b} + 159.3\right). \quad (17)$$

The first term between the parentheses represents the friction in the channel segments between the four corners, and the second one the extra friction due to the corners. For small air gap heights, i.e., b is much smaller than H_1 and H_2, the corner friction term is marginal. However, for wider gaps the corner terms might be of importance.

Stack Effect vs. Friction

The airflow rate will increase if the driving buoyancy pressure difference is larger than the corresponding pressure loss due to friction in the channel:

$$|\Delta p_B| > \Delta p_{fF} \quad (18)$$

When the pressures are equal, an equilibrium can be reached, i.e., a final steady-state airflow rate in the channel can be established. The onset of convection has occurred if equality is obtained for Δp_B different than zero. Figure 6 shows the two pressure differences for a specific case as a function of the Peclet number. The straight dashed and dotted lines represent the friction pressure drop for different airflow resistances. The solid curve represents the buoyancy pressure difference. In order to get any airflow greater than zero, the slope for low Peclet number for the friction pressure loss must be equal to or lower than the buoyancy pressure difference.

Equation 18, together with Equations 3, 13, and 16, gives

$$\Delta p_F = \frac{m_a}{\rho_a} \cdot R_{fc} = \frac{R_{fc} \cdot k}{\rho_a c_a} \mathbf{Pe}_c \le g\beta\rho_a(T_i - T_e) \cdot H_1 \cdot \Delta T'(\mathbf{Pe}_c, \ldots) = \Delta p_B \quad (19)$$

Reformulated, it becomes

$$\frac{\mathbf{Pe}_c}{\Delta T'(\mathbf{Pe}_c)} \le \mathbf{Ra}_c. \quad (20)$$

Here, the channel flow Rayleigh number is introduced:

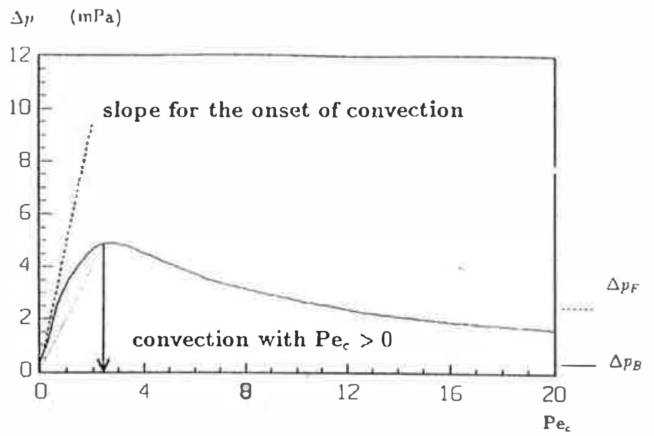

Figure 6 *Driving pressure difference due to stack effect and the corresponding friction pressure loss as a function of the Peclet number. The straight dashed and dotted lines represent the friction pressure drop for different airflow resistances. The drawn curve represents the numerically calculated buoyancy pressure difference. Symmetrically placed loop with $H_1/H_2 = 1$ and $H_1/H = 0.5$.*

$$\mathbf{Ra}_c = \frac{\rho_a^2 c_a g\beta(T_i - T_e) \cdot H_1}{k \cdot R_{fc}} \quad (21)$$

In Equation 20, the buoyancy and friction terms are compared at different Peclet numbers. A stable airflow rate is obtained if the ratio between these two terms is equal to the channel flow Rayleigh number. The ratio $\mathbf{Pe}_c/\Delta T'(\mathbf{Pe}_c)$ is a kind of signature, a dynamic response of the thermal system; see the numerical example in Figure 7. This ratio, which determines if there will be any convection, will be analyzed further in the following section. The channel flow Rayleigh number (Equation 21) is a fixed number, not depending on the Peclet number. For a too low Rayleigh number, let us say below 3.7, according to the example in Figure 7, no $\mathbf{Pe}_c/\Delta T'(\mathbf{Pe}_c)$ ratio will match, i.e., no convection occurs.

CRITICAL CHANNEL FLOW RAYLEIGH NUMBERS FOR THE ONSET OF CONVECTION

As investigated in the previous section, an airflow rate other than zero will create a buoyancy pressure difference. In the section "Extra Heat Loss," it was concluded that the thermal influence of the airflow corresponds to the one obtained from a distribution of line heat sources and sinks along the channel.

Let the disturbance in the temperature field caused by the airflow be denoted by $\tilde{T}$. The temperature in the structure can then be written as

$$T = T_e + \frac{y}{H} \cdot (T_i - T_e) - \tilde{T}. \quad (22)$$

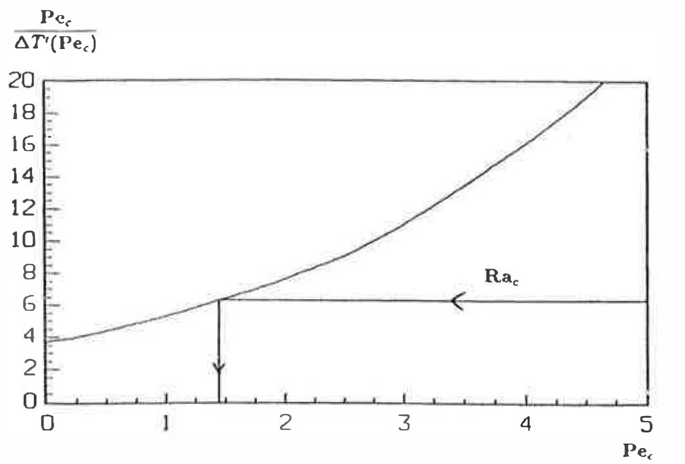

Figure 7 *Numerically calculated ratio between the channel Peclet number and the average dimensionless temperature difference in the channel. For a given channel flow Rayleigh number, a Peclet number can be found (follow the arrows). Symmetrically placed loop with* $H_1/H_2 = 1$ *and* $H_1/H = 0.5$.

The two first terms, representing the channel temperature for a Peclet number equal to zero, do not give any buoyancy pressure difference, which is shown in the subsection "Stack Effects for Peclet Number Equal to Zero." Using Equation 22, the dimensionless temperature difference in Equation 14 becomes

$$\Delta T' = \frac{1}{H_1 \cdot (T_i - T_e)} \cdot \oint_{\Gamma} \tilde{T}(\vec{r})\hat{y}\,\vec{dr}\,. \tag{23}$$

The temperature disturbance $\tilde{T}$ can be determined by the use of fundamental solutions for a line source in an infinite strip. The temperature at a point (x, y) from a line source (see Equation 8) in the point (z',y') is given by

$$\frac{k\mathbf{Pe}_c \Delta y'}{H}(T_i - T_e) \cdot u_p(x, y, x', y')\,. \tag{24}$$

The analytical expression for u_p, which is the temperature distribution from a unit line source, is given by Hagentoft (1991a):

$$u_p(x, y, x', y') = -\frac{1}{2\pi k} \cdot \Re\left\{ \ln\left(\frac{\sinh(\pi(z - z'))/(2H)}{\sinh(\pi(z - \bar{z}'))/(2H)} \right) \right\}$$
$$z = x + i \cdot y$$
$$z' = x' + i \cdot y' \tag{25}$$

where i is the imaginary unit $\sqrt{-1}$, and $\Re$ denotes the real part of the complex-valued expression.

The temperature $\tilde{T}$ becomes

$$\tilde{T}(x, y) = (T_i - T_0)\frac{k\mathbf{Pe}_c}{H} \cdot \oint_{\Gamma'} u_p(x, y, x', y')\hat{y}' \cdot \vec{dr'} \tag{26}$$

The temperature difference $\Delta T'$ then becomes

$$\Delta T' = \frac{k\mathbf{Pe}_c}{H \cdot H_1} \oint_{\Gamma}\left(\oint_{\Gamma'} u_p(x, y, x', y')\hat{y}' \cdot \vec{dr'} \right)\hat{y} \cdot \vec{dr}\,. \tag{27}$$

From the results in a previous subsection of this paper, we have found that the ratio $\mathbf{Pe}_c/\Delta T'(\mathbf{Pe}_c)$ for small Peclet numbers, at the onset of convection, determines the critical level of the Rayleigh number. We thereby have a definition of the critical airflow channel Rayleigh number:

$$\mathbf{Ra}_c^{cr} = \lim_{\mathbf{Pe}_c \to 0} \frac{\mathbf{Pe}_c}{\Delta T'(\mathbf{Pe}_c)}\,. \tag{28}$$

Using this stability criterion, we can state

$$\mathbf{Ra}_c \geq \mathbf{Ra}_c^{cr} \qquad \text{Convection in the structure}\,. \tag{29}$$

With Equation 28 combined with Equation 27, the critical channel flow Rayleigh number can be obtained from

$$\frac{1}{\mathbf{Ra}_c^{cr}} = \frac{k}{H \cdot H_1} \oint_{\Gamma}\left(\oint_{\Gamma'} u_p(x, y, x', y')\hat{y}' \cdot \vec{dr'} \right)\hat{y} \cdot \vec{dr}\,. \tag{30}$$

Inserting Equation 25 into Equation 30, we get the following very general and explicit expression for the critical channel flow Rayleigh number for a horizontal layer:

$$\frac{1}{\mathbf{Ra}_c^{cr}} = \tag{31}$$
$$\left(-\frac{1}{2\pi \cdot H \cdot H_1}\right) \oint_{\Gamma}\left(\oint_{\Gamma'} \Re\left\{ \ln\left(\frac{\sinh(\pi(z - z'))/(2H)}{\sinh(\pi(z - \bar{z}'))/(2H)} \right) \right\}\hat{y}' \cdot \vec{dr'} \right)\hat{y} \cdot \vec{dr}$$

Rectangular Channel

For rectangular airflow channel loops, it is possible to derive direct analytical expressions for the critical airflow channel Rayleigh number. Neglecting the temperature influence from one vertical segment to the other, a special Fourier series technique can be used (see Hagentoft 1998). From comparisons with numerical calculations, it is found that the formulas are approximately valid for $H_2/H > 0.1$.

For symmetrical and rectangular air channel loops (see Figure 1b), we have

$$\mathbf{Ra}_c^{cr} = \frac{\pi^3}{8} \frac{1 - 2\alpha}{\sum_{m=0}^{\infty} \cos^2((2m + 1) \cdot \pi a)/(2m + 1)^3}\,.$$
$$\frac{H_1}{H} = 1 - 2\alpha \tag{32}$$

The critical Rayleigh number will not depend on H_1/H_2 as long as $H_2/H > 0.1$. Numerical evaluation of the formula is given in Table 1.

TABLE 1
The Critical Channel Flow Rayleigh Number (Equation 32) as a Function of H_1/H for an Air Channel Loop with Rectangular Form with $H_2/H > 0.1$

H_1/H	$\mathbf{Ra}_c^{cr}$
1.0	3.68491
0.9	3.45204
0.8	3.36691
0.7	3.37818
0.6	3.47874
0.5	3.68491
0.4	4.04445
0.3	4.67543
0.2	5.92235
0.1	9.37894
0.08	10.98993
0.06	13.56007
0.04	18.40608
0.02	31.66722
0	∞

EXAMPLES

In this section, the impact due to convection is analyzed in terms of Nusselt numbers for the case with a rectangular channel in a horizontal layer. The stability criterion for the onset of convection is also interpreted in terms of maximum allowed air gap height.

The Nusselt Number

Using Equation 6 for the extra heat loss due to the airflow channel, we get the following expression for the Nusselt number:

$$\mathbf{Nu} = 1 + \frac{q^e}{kD(T_i - T_e)/H} = 1 + \frac{H}{D} \cdot h^e(\mathbf{Pe}_c, \ldots) = \mathbf{Nu}(\mathbf{Ra}_c, \ldots) \tag{33}$$

where D is the considered width of the structure. The Nusselt number is calculated numerically (ANHConP) and presented in Figure 8 for three cases. From Figure 8, it can be seen that the onset of convection starts approximately at the same critical Rayleigh number; according to Table 1, it should be at the numbers 5.92, 3.68, and 3.37. The magnitude of the impact on the heat loss is rather different between the three cases.

From the Nusselt numbers shown in the example, it is obvious that the thermal transmittance of a building envelope, locally at the region of the air channel, might increase dramatically. Air channels, connecting colder regions of the building envelope with warmer ones, short-circuiting the thermal insulation, give a significant increase in heat loss.

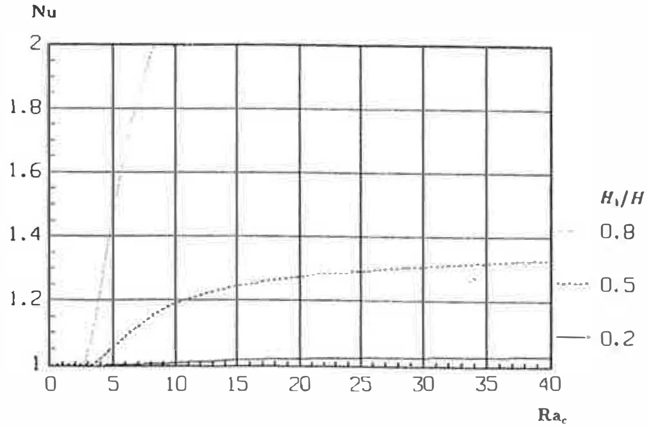

Figure 8 *Numerically calculated Nusselt number as a function of the Rayleigh number, for a symmetrically placed rectangular loop, $H_1/H_2 = 1$, $H/D = 0.5$, and $H_1/H = 0.2$, 0.5, or 0.8.*

Relation to the Air Gap Height

It is of interest to study the relation between critical channel flow Rayleigh numbers and the heights of the air gaps for rectangular air channels. Using Table 1 and Equations 21 and 17, a maximum height of the air gap can be calculated, below which no natural convection occurs.

$$\frac{b^2 \rho_a^2 c_a g \beta (T_i - T_e) \cdot H_1}{24\eta k \cdot ((H_1 + H_2)/b + 159.3)} \le \mathbf{Ra}_c^{cr} \tag{34}$$

Neglecting the corner friction term 159.3, which is reasonable for small air gap heights since the number 159.3 will be small compared with the term $(H_1 + H_2)/b$, we get

$$\frac{b^3}{1 + H_2/H_1} \cdot \frac{\rho_a^2 c_a g \beta (T_i - T_e)}{24\eta k} \le \mathbf{Ra}_c^{cr}. \tag{35}$$

The requirement for b, then, becomes approximately

$$b \le \sqrt[3]{\left(1 + \frac{H_2}{H_1}\right) \cdot \frac{24\eta k}{\rho_a^2 c_a g \beta (T_i - T_e)} \cdot \mathbf{Ra}_c^{cr}}. \tag{36}$$

The required air gap height is presented in Table 2 for some parameter cases using the approximate Equation 36.

TABLE 2
Maximum Air Gap Height, b (mm), to Avoid Natural Convection in a Symmetrically Placed Rectangular Air Channel Loop According to Equation 36*

H_1/H	$T_i - T_e = 30°C$	$T_i - T_e = 20°C$	$T_i - T_e = 10°C$
0.2	4.9	5.6	7.1
0.5	4.2	4.8	6.1
0.8	4.1	4.7	5.9

* The temperature difference $(T_i - T_e)$ is varying between 10°C and 30°C, the thermal conductivity k is 0.04 W/m·K, and $H_1/H_2 = 1$.

The results show that the ratio H_1/H has only a weak influence on the critical air gap heights as long as the shape is the same. This is due to the fact that both the buoyancy force and the friction are proportional to the size of the channel. This can be seen in Equation 36, where the dependence of the dimensions H_1/H has vanished except in the critical Rayleigh number. The results show that convection, in typical thermal insulation material exposed to temperature difference in the region 10°C to 30°C, starts if the gaps have the heights in the interval 4 mm to 7 mm or more.

DISCUSSION

Single airflow channel loops have been investigated in this paper for two-dimensional cases. However, in reality, a complex pattern of airflow paths usually exists. Normally, an increase in the degree of freedom will result in an increase in the heat loss through the structure. The different airflow paths can be connected thermally through an exchange of heat through the material surrounding the channels and also directly through an exchange of air. The case with true three-dimensional airflow patterns will probably also increase the total heat loss through the structure.

CONCLUSIONS

This paper gives the theoretical base for the understanding of the mechanisms behind natural convection in closed airflow channel loops in horizontal structures. In particular, the two-dimensional steady-state case with airflow channels, such as narrow cracks, imbedded in an insulation layer with constant thickness has been treated.

The channel flow Peclet and Rayleigh numbers have been introduced as dimensionless parameters governing the thermal process. General expressions for the critical channel flow Rayleigh number have been found, from which the conditions for the onset of convection can be found. A simple analytical expression for the critical Rayleigh number for rectangular air channel loops is presented in the paper. The expression has been compared with numerical calculations, and the correlation was acceptable.

The critical Rayleigh number can be interpreted in terms of a critical air gap height, i.e., a maximum air gap height below which no convection occurs. For rectangular air channels in typical thermal insulation layers, the maximum air gap height is in the range of 4 mm to 7 mm for temperature differences varying between 10°C and 30°C. The thickness of the insulation layer is of minor importance.

The increase in heat loss due to natural convection is analyzed and presented in terms of Nusselt numbers. The impact of the airflow in the channels, in terms of increase of the heat loss, varies between a few percent to 100%, depending on the size of the air channel loop. Channels with the greatest ratio between the height of the channel and the thickness of the insulation layer give the greatest increase in heat loss.

NOMENCLATURE

b	= height of air gap (m)
c_a	= heat capacity of air (J/kg·K)
D	= width of the considered structure (m)
g	= gravitational acceleration, 9.81 m/s^2
h^e	= dimensionless heat loss factor for extra heat loss
H	= thickness of insulated structure (m)
$H_1, H_2, ...$	= width and height of rectangular airflow channel (m)
k	= thermal conductivity (W/m·K)
m_a	= air mass flow rate (kg/ms)
Nu	= Nusselt number, ratio between heat losses with and without natural convection
$\mathbf{Pe}_c$	= channel Peclet number
q^e	= extra heat loss (W/m)
$\mathbf{Ra}_c$	= channel flow Rayleigh number
$\mathbf{Ra}_c^{cr}$	= critical channel flow Rayleigh number for the onset of convection
R_{fc}	= total airflow resistance for a closed air channel (Pa/[m^3/ms])
s	= curve length coordinate (m)
S	= area enclosed by a closed air channel loop (m^2)
T	= temperature in the structure (°C)
T'	= dimensionless temperature, $T = (T - T_e)/(T_i - T_e)$
$\tilde{T}$	= temperature disturbance in the structure due to airflow (°C)
T_i	= interior temperature (°C)
T_e	= exterior temperature (°C)
x	= horizontal coordinate (m)
y	= vertical coordinate (m)
β	= volumetric expansion coefficient of air (1/K)
Δp_B	= buoyancy-induced internal pressure difference in the air channel (Pa)
Δp_F	= pressure drop in the air channel due to friction (Pa)
η	= dynamic viscosity (Ns/m^2)
ρ_a	= air density (kg/m^3)

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