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HVAC Duct System Design Using Genetic Algorithms

Y. Asiedu **Robert W. Besant, P.E.** Fellow ASHRAE **P. Gu**

A genetic algorithm technique is used to design an HVAC air duct system with minimum life-cycle cost. The approach has the capability to incorporate standard (discrete) duct sizes, variable time-of-day operating conditions and variable time-of-day utility rates. An example is used to illustrate these capabilities and results are compared to those obtained using weighted average flow rates and utility rates to show the life-cycle cost savings possible using this genetic algorithm methodology. Life-cycle cost savings are minimal for some designs, but much larger savings are possible for complex designs and operating constraints.

INTRODUCTION

Airflow ventilation is essential in interior spaces to remove airborne contaminants including odors, toxic gases, volatile organic compounds, and aerosols. Airflow is also needed for temperature and humidity control. A significant fraction of the electrical energy used in buildings goes toward the air handling fans and a significant fraction of the building interior space is needed for the air handling equipment. The life-cycle cost of air handling in buildings is important even though it is seldom investigated for the least life-cycle cost. Among the opportunities that exist for the reduction of these life-cycle costs are: improved contaminant removal, reduced airflows, and improved ducting design.

The effectiveness of contaminant, thermal energy, and water vapor removal strongly depends on the location and jet momentum of the supply air to a room as well as the location of the return grilles (Irwin et al. 1998). Compared to well mixed spaces, the contaminant removal effectiveness for most interior spaces is less than 100% (Heiselberg 1996), suggesting that opportunities exist to improve these designs by improved interior airflow patterns that include some aspects of displacement ventilation.

Reduction of airflow while still maintaining good interior air quality may, in some applications, be achieved by using some radiant ceiling cooling or slightly reduced supply air temperatures (Kirkpatrick and Elleson 1996). The life-cycle cost effectiveness of these two options has not been determined.

A third option is to improve the life-cycle cost of the ducting systems in buildings. Ducting systems are complex in terms of their layout in a building, with terminal flow rate requirements that vary from room-to-room and over time in each room. Because designers of ducting systems are faced with many constraints and requirements, as well as choices for layout, they most frequently use rules of thumb rather than minimization of the life-cycle costs.

Life-Cycle Engineering

In an attempt to improve the design of systems (products) and reduce design changes, cost, and time to market, concurrent engineering or life-cycle engineering has emerged as an effective

Y. Asiedu is a research engineer with Venmar CES, Saskatoon, Saskatchewn. **Robert W. Besant** is a professor emeritus in the Department of Mechanical Engineering, University of Saskatchewan. **P. Gu** is a professor and head of the Dept. of Mechanical and Manufacturing Engineering, The University of Calgary Canada.

approach to design in today's competitive global market. The unique aspect of life-cycle engineering is that the complete life-cycle of the system is considered and treated in each phase of the system development (Keys 1990). The approach begins by identifying the need and extends through design, production, customer use, support and finally, disposal.

The key design component of life-cycle engineering is a Life-Cycle Cost (LCC) analysis, where the LCC of the product/system is assessed and the design with the least LCC is sought, subject to the constraints of performance, size, reliability, etc. The total cost of any system from its earliest concept through its retirement will eventually be borne by the user and will have a direct bearing on the marketability of that system (Wilson 1986). Purchasers pay for the resources required to bring forth and market the system, and owners of the system, pay for the resources required to deploy, operate and dispose of the system. Studies suggest that the design of the system influences between 70% and 85% of the total LCC of a system (Dowlatshahi 1992). Designers are therefore in a position to substantially reduce the LCC of the systems they design by giving due consideration to life-cycle cost issues early in the design process. LCC analysis provides a framework for specifying the estimated total incremental costs of developing, producing, using and retiring a particular item.

The LCC concept was first applied to contracts by the U.S. Department of Defense. Its importance in defense contracts was stimulated by findings that operation and support costs for typical weapon systems accounted for as much as 75% of the total cost (Gupta 1983), and that better designs could substantially reduce these components of cost. An area in which LCC analysis can be useful is in the design of HVAC air duct systems. The duct design problem is discussed in the next section.

PROBLEM FORMULATION

HVAC air duct systems are one of the major electrical energy consumers in industrial and commercial buildings (Tsal et al. 1988a). A poorly designed air duct system will lead to energy waste and/or installation of excessive ductwork material. Both of these increase the LCC. The design could also lead to poor indoor air quality that may affect the health and productivity of the occupants of the building. In designing an air duct system, the designer usually starts off with a duct system layout and airflow rates. The design challenge is to select the materials and specify the sizes for the ducts, fittings and fan(s). Ideally, these selections should be made so that the LCC is minimized.

The LCC comprise the initial capital cost and the operating cost which is primarily the energy cost. That is,

$$E = (E_p \times \mathrm{PWEF}) + E_s \tag{1}$$

where

E = present worth owning and operating cost (LCC)
E_p = first year energy cost
E_s = initial cost
PWEF = present worth escalation factor

Note: The calculation of the terms is shown in the appendix.

Many constraints are associated with this problem. These include the fact that only standard duct sizes can be used, total path pressure losses need to be the same for all the paths, velocity must be limited to reduce duct noise, restrictions due to building architecture, etc. The duct size optimization problem can thus be stated as follows

Minimize $$E = (E_p \times \mathrm{PWEF}) + E_s \quad (2)$$

Subject to the constraints

Pressure balancing for each flow path

$$\sum_{x \in I} \Delta P_x = P_{fan} \qquad \text{for } \iota \in S_s \quad (3)$$

Size and flow limitations

$$d_x^{min} \le d_x \le d_x^{max} \qquad \text{for } x = 1,2,\ldots,X \quad (4)$$

$$d_x \in SD \qquad \text{for } x = 1,2,\ldots,X \quad (5)$$

$$V_x^{min} \le V_x \le V_x^{max} \qquad \text{for } x = 1,2,\ldots,X \quad (6)$$

where

ΔP_x = total pressure losses in duct section x
P_{fan} = total fan pressure
S_s = set of paths in a (sub)system s
I = set of duct sections in path ι
SD = set of standard duct sizes
X = total number of duct sections in the system
x = index (number) of the duct section
d_x = size of duct section x
d_x^{min} = minimum allowable size for duct section x
d_x^{max} = maximum allowable size for duct section x
V_x^{max} = maximum allowable velocity for duct section x
V_x^{min} = minimum allowable velocity for duct section x

The set of constraints above is not exhaustive and other problem specific constraints might exist. Generally, the pressure balancing constraint, Equation (3), is the most difficult constraint to deal with because large adjustments may be required from one airflow path to another in the ducting system.

In the past, designers tended to pay more attention to capital or first costs even though life-cycle operating costs are usually larger (Carrier et al. 1998). Generally, the operating cost decreases while the capital cost increases with increasing duct size. The set of functional or workable designs for a given design problem is usually very large or infinite (Stoecker 1989), but the set of good designs for which the life-cycle costs are minimized is a small subset or even a unique one. The purpose of any duct design methodology should be to come up with a procedure that enables the designer to balance these conflicting cost elements to determine the duct sizes that minimizes the LCC of the entire duct system.

Unfortunately this is not the case in some used methodologies. The design methodologies that are widely used are the Equal Friction Method, Static Regain Method, and the T-Method (ASHRAE 1997). The Equal Friction and Static Regain methods are non-optimizing methods that rely on heuristics that do not explicitly take into consideration the LCC of the system. The Equal Friction method requires ducts to be sized so that there is a constant pressure loss per unit length of each duct in the system. The objective of the static regain design method is to reduce air velocity in the direction of flow so that the increase in static pressure of each transition balances with the pressure losses in the following section. These approaches result in designs that are functional or workable but not necessarily cost efficient.

The T-Method is an optimizing method that is gaining increased acceptability in the HVAC community. It consists of three main steps: system condensation, fan selection, and system expansion. In the first step, the entire duct system is replaced with a single duct section having the same pressure drop and economic characteristics. An optimal fan pressure is selected in the second step. This pressure is then distributed throughout the system in the third stage. A detailed discussion and illustration of this method can be found in Tsal et al. (1988a,b). Although the T-Method is currently the most widely used optimization approach in air duct design, this method does not treat the constraints of standard duct sizes very well. This constraint is relaxed throughout the design procedure and incorporated at the very end through the use of a heuristic. For a ducting system with a large number of components, this does not ensure that the design is optimal. The process of system condensation and expansion requires a lot of computations that can cause complications in large systems. In Asiedu et al. (2000a) a simple design methodology that eliminated the need to condense and expand the system was suggested. Another shortcoming of the T-Method is that it is restricted to a specific objective function with fixed input parameters (unit energy cost, duct flow rates, etc.). Currently the T-Method cannot be used to optimize a system that has, for example, time variable duct flow rates or utility rates.

Genetic algorithms are in the class of optimization methods known as evolutionary algorithms. These are 0-order methods that can handle non-linear problems defined on discrete, continuous, or mixed search spaces; and on some unconstrained or constrained (Michalewicz et al. 1996). This approach has the capability to handle standard duct sizes, complex objective functions, and to incorporate other problem specific constraints. The approach tends to be faster than other methods that have been used for optimal duct design.

Genetic Algorithms

Genetic Algorithms (GAs) are an optimization strategy in which points or states in the design space are analogous to organisms in a process of evolution by natural selection (Chapman et al. 1993). Each candidate solution or state in the optimization problem is represented by a coded representation of design attributes that is analogous to a *chromosome*. Thus a chromosome completely defines one functional design. The goodness of an individual chromosome as a solution to the problem is evaluated as its *fitness*. Initially, GAs use problem knowledge to randomly generate a *population* of functional designs or chromosomes. Operations such as *selection*, *crossover* and *mutation* are then performed on the chromosomes to produce the next generation of designs with improved fitness. This process of creating new designs for a new generation is

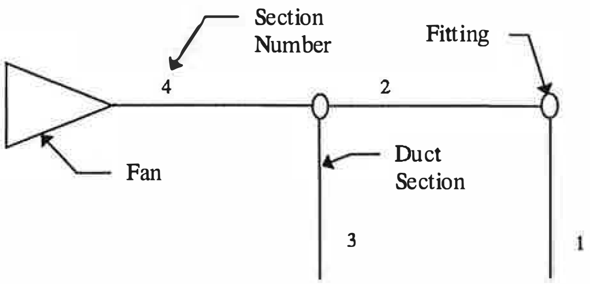

Figure 1. Simple duct system

analogous to biological reproduction. The population of design alternatives evolves over a series of generations until a terminal criterion is met (i.e. until a good solution is obtained). Unlike most optimization algorithms, GAs work with a collection of design solutions rather than with a single solution with the search proceeding along different paths simultaneously. In this way, GAs can find optimal or relatively good sub-optimal solutions in a short computation time.

A schematic for a very simple HVAC air duct system is shown in Figure 1. This system has four duct sections or elements numbered 1 through 4 and two airflow paths comprised of sections (4 and 3), and (4, 2 and 1). Duct elements 4, 3, and 1 are specified round ducts and duct 2 is rectangular. This duct system design problem is used below to explain some of the terminology introduced above and illustrate the genetic algorithm method of design.

Chromosome

Each design or chromosome is comprised of a string or set of *genes*, which may be visualized as boxes arranged in a linear fashion as shown in Figure 2. The position of each gene is called the *locus* of the gene and its value, the parameter being optimized, is called the *allele*. The five gene values (alleles) in this example correspond to the sizes of the four duct elements. Because duct element number 2 is rectangular, it requires two genes to specify its size (height and width), giving a total of five genes. This value can be any real number (integers are used in Figure 2) or a binary representation. Thus, Figure 2 can also be represented as in Figure 3 where each parameter is now represented by 4 binary digits, or one group of four genes.

In this simple air duct system, the issues of material and fan selection are not treated. A chromosome should thus represent the specification of the duct sizes and must be of size equal to the number of sizes to be determined. If it is assumed that the height of the rectangular duct is predetermined by architectural constraints, then the set of genes in each chromosome is defined to be equal to the total number of duct sections with each gene value representing the size of that duct

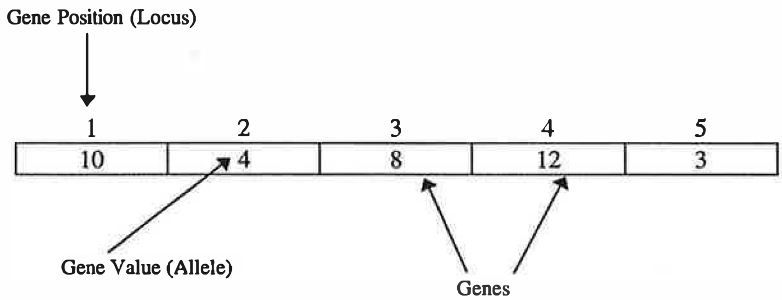

Figure 2. Chromosome representation as a string

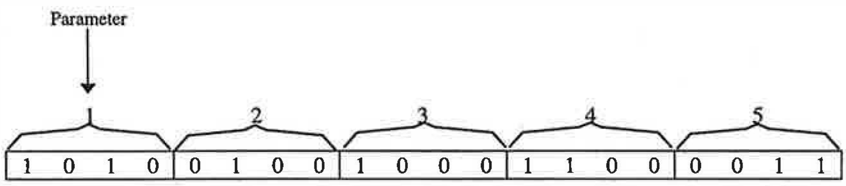

Figure 3. Binary representation of allele for genes of chromosome in Figure 2

section. It is not necessary that the gene position correspond to the duct section number (this is discussed further in Section Coding Scheme), but that the value represent the true duct size in some manner. Two representations are proposed. The first maps numbers onto a table containing the duct sizes. For the duct system in Figure 1, a table of acceptable duct sizes for round ducts (Table 1) and rectangular ducts (Table 2) are developed with each duct in each table having a unique index (ducts in different tables can have the same index).

Table 1. Available Round Duct Sizes

Index	1	2	3	4
Diameter, m	0.100	0.125	0.150	0.175

Table 2. Available Rectangular Duct Sizes

Index	1	2	3	4
Width, m*	0.150	0.175	0.200	0.225

*Assuming that the height of the duct section is predetermined.

The alleles represent these indices. Thus given any chromosome, the duct size is uniquely determined by the tables. For the chromosome shown in Figure 4, the duct sizes are, Duct 4: 0.125 m; Duct 3: 0.175 m; Duct 2: 0.225 m; and Duct 1: 0.150 m. (the gene positions do not correspond to the duct numbers)

The second representation deals with the case where the duct sizes are in a given interval and increase by jumps of a constant value at the end of each interval. In this case, the tabular representation is not needed since a gene value can be transformed to a corresponding duct size by Equation (7). This is the representation adopted in the implementation of GA for the duct design problem in this paper.

$$\text{Duct Size} = \text{Lower Limit} + (\text{Size Jump} \times \text{Gene Value}) \tag{7}$$

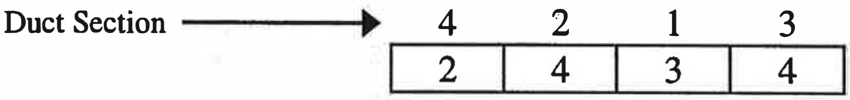

Figure 4. Sample chromosome

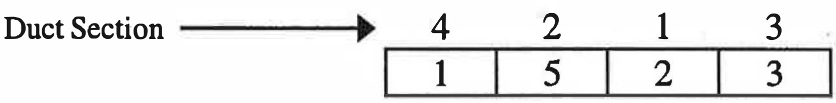

Figure 5. Sample chromosome based on Equation (7)

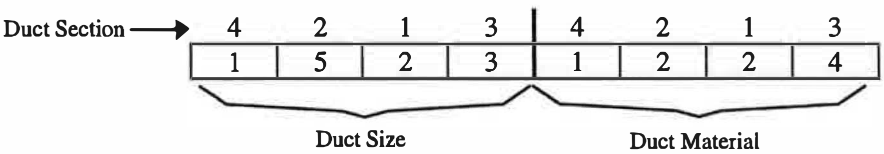

Figure 6. Sample two-parameter chromosome

Using this representation, and assuming that the *Lower Limit* equals 0.1 m and the *Size Jump* equals 0.025 m, the chromosome corresponding to the same sizes as that of Figure 4 is as shown in Figure 5.

In the foregoing, it was assumed that the design decision in this case was only the duct sizes. However, material selection could also have been included. In that case, two four-gene chromosomes end-to-end are used with the first set of genes dedicated to the duct sizes and the other to the materials. Given that 1, 2, 3, and 4 represent aluminum, fiber glass, galvanized steel, and PVC plastic pipe, respectively; then the chromosome in Figure 6 represents a design with the following specifications: Duct 4: 0.12 m aluminum duct, Duct 3: 0.175 m PVC plastic pipe duct, Duct 2: 0.225 m fiber glass duct, and Duct 1: 0.150 m fiber glass duct. Likewise, the chromosome can be expanded to incorporate other attributes of the duct section such as external insulation.

Fitness Function

The fitness is an expression of how well a particular chromosome satisfies the constraints of a problem and the designer's requirements. The fitness is used to screen the chromosomes in one generation set and decide which ones will proceed to the next generation and which ones will be used for crossover (breeding) and gene mutation. In most instances, the value of the objective function is a good measure of the fitness of the chromosomes. Unfortunately, the objective function value of a chromosome alone is not always useful for guiding a genetic search. For example, in combinatorial optimization problems where there are many constraints, most points in the search space often represent invalid chromosomes and hence have zero "real" values. For a GA to be effective in this case, a fitness function where the fitness of an invalid chromosome is viewed in terms of its potential to lead to a valid chromosome must be invented (Beasley et al. 1993).

If the objective function given by Equation (2) is a good fitness function, the fitness of the chromosome of Figure 5 is determined be calculating the LCC (usually the fitness function is chosen such that higher values of the function will correspond to better solutions. However, in the case of a minimization problem, a lower value of the fitness function corresponds to a better solution). However, the design might not be feasible because one or more of Equations (3) through (6) may not be satisfied or the design may lack certain characteristics desired by the designer and thus the LCC may not be able to measure the value of the design. This issue is discussed later, in the context of an explicit HVAC duct design problem.

Selection, Crossover, and Mutation

In GAs, design solutions (organisms) are generated and tested in succeeding generations with offspring designs arising from parent designs. Individuals for the next generation are selected according to their fitness values and the next generation is generated through the processes of crossover and mutation. An individual may persist across several generations (and experience longevity) or be replaced in the very next generation (and experience early death) depending on the generation-gap policy effected by the modeler (Chan et al. 1996). That is, the choice of killing off or retaining members of one generation in the succeeding generation depends on the designer. Crossover is an operation in which two chromosomes are combined to produce one or two new offspring chromosomes. This allows offspring chromosomes or designs to retain traits from parent designs.

A crossover operation is shown in Figure 7. Table 3 shows the corresponding duct sizes of the two parents and two offspring designs. The two original designs (parents) in this crossover would have come from a population of size 10, for example (i.e., 10 designs). As discussed, the choice (selection) of a chromosome to undergo these operations depend on its fitness. Generally, the fitter the chromosome, the higher the chances that it will be selected. However, lesser fit chromosomes are sometimes kept to ensure that some of their desirable properties are main-

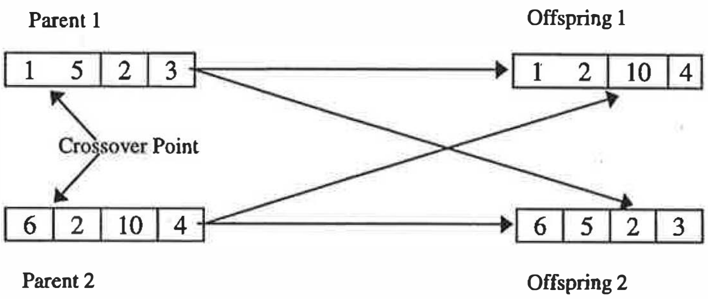

Figure 7. Crossover operation on chromosomes

Figure 8. Mutation operation on a chromosome

Table 3. Duct Sizes Corresponding to Crossover Operation

	Duct Sizes			
Duct Section	**Parent 1**	**Parent 2**	**Offspring 1**	**Offspring 2**
1	0.150	0.350	0.350	0.150
2	0.225	0.150	0.150	0.225
3	0.175	0.200	0.200	0.175
4	0.125	0.250	0.125	0.250

tained in the population pool. Because the total number of designs should be maintained through each generation, the two new designs (offspring) will have to replace (1) both parents, (2) one parent and one of the other 8 designs, or (3) two of the other 8 designs.

Figure 8 depicts the mutation operation. Mutation involves randomly changing one or more allele of a chromosome. This is less frequent than crossover but has the potential to yield radically improved designs over parent designs and also to allow the search to escape from points that may be local optimums but not necessarily a global optimum. In this case, the new design is obtained by changing the size of duct section 1 from 0.150 m to 0.175 m.

Application to HVAC Duct Sizing

Most of the steps in the traditional GA can be implemented using a number of different algorithms. The choice of chromosomes for crossover and mutation, how these operations are done, and the product of these operations depend on the particular algorithm adopted by the designer. The basic mechanism of a GA is robust enough that within fairly wide margins, parameter settings (i.e. crossover and mutation probabilities, population size, etc.) are not critical. What is critical in the performance of a GA, however, is the fitness function and the coding scheme used.

In this section, the fitness function and coding scheme used for the HVAC air duct system and the exact GA implementation adopted are presented.

Coding Scheme

The power of the GA lies in it being able to find building blocks. Building blocks, are schemata of short defining lengths consisting of bits that work well together and tend to lead to improved performance when incorporated into individual solutions (Beasley et al. 1993). A successful coding scheme is therefore one that encourages the formation of building blocks by ensuring that related genes are close together on the chromosome, while there is little interaction between genes. Interaction between genes means that the contribution of a gene to the fitness of the chromosome depends on the value of other genes in the chromosome. In most applications, interaction cannot be eliminated.

Given the relationship between ducts in the same path, and "children" of a "parent" duct, the building of blocks can be encouraged by putting ducts in the same path together and children of the same parent relatively close to each other in the chromosome. Because duct sections might be present across paths, it is not possible to put all the ducts in each path together. The following is a procedure developed for arranging the genes in the chromosomes.

1. Identify all duct paths in the system.
2. Arrange the paths such that the one with the most duct sections is at the top of the list and the one with the least number of sections is at the bottom of the list.
3. Starting from the path at the top of the list, assign the first duct section to the first gene, and the second to the second gene and so on until all the ducts in the path have been assigned gene positions.
4. Cross out any duct section that was assigned a gene position in step 3 from any other path.
5. Remove all paths that have no remaining duct sections from the list.
6. Starting from the path at the top of the list, select the first path with at least one duct section crossed out. If no such path exists, select the path at the top of the list. Assign the first duct to the first available gene, and the second to the second available gene and so on until all the ducts in the path have been assigned gene positions.
7. Repeat steps 4 through 6 until all of the ducts sections have been assigned gene positions.

If the system consists of both supply and return systems, steps 1 through 7 must be applied to each subsystem separately. The chromosome representation used in Section Chromosome was based on the above procedure.

Evaluation of Candidate Solutions

The objective function can often be used to evaluate the fitness of chromosomes. In the example HVAC air duct design problem, the LCC was used as the fitness function. Given the coding scheme adopted, it is possible to have infeasible solutions. Using chromosome representations and operators to ensure that infeasible solutions are not generated in the first place could eliminate this problem. However, that is not the approach that was adopted in this paper. Infeasible solutions may be discarded once they are generated (death penalty), repaired after they are generated, or given a fitness value that is less than the lowest fitness value for a feasible solution by the use of a penalty function. The "death penalty" approach simplifies the algorithm since infeasible solutions do not need to be evaluated. This method works reasonably well when the feasible search space is convex and constitutes a reasonable part of the whole search space. Otherwise, the approach has serious limitations. Sometimes, the values of genes can be changed to make an infeasible chromosome feasible (i.e., repaired). In some problems, the effort required to do this may be excessive.

The most successful way of dealing with infeasible solutions is by penalizing chromosomes for being infeasible. Generally, penalty functions that represent the amount by which the constraints are violated are better than those which are based simply on the number of constraints that are violated (Beasley et al. 1993). The problem with penalty functions is that often, the feasible and infeasible regions are not precisely known. Thus there is the risk of specifying a penalty function that is either too severe or too lax. If the amount of penalty for being infeasible is too small, the search may yield infeasible solutions, while the flaw of heavily penalizing infeasible designs is that it limits the exploration of the design space to feasible regions, precluding short cuts through the infeasible domain. A variable penalty function applies a very small penalty in the beginning of an algorithm run but increases the penalty with time or each generation. Another approach is to maintain two populations; one using a severe penalty and the other using a lax penalty with the two populations interacting. This is the so called *Segregated Genetic Algorithm* (SGA) (Michalewicz et al. 1996).

The segregated genetic algorithm is an attempt at desensitizing the GA to the choice of the penalty parameters. The population is split into two coexisting and cooperating groups that differ in the way the fitness of their members are calculated. Each group uses a different value of the penalty function and corresponds to the best performing individuals with respect to one penalty parameter. The two groups interbreed, but they are segregated in terms of rank. The advantages are that because the penalty parameters are different, the two groups will have distinct trajectories in the design space, and because the two groups interbreed, they can help each other out of a local optima. The SGA is thus expected to be more robust than the GA with one fitness function.

This is the approach that was adopted for duct design in this paper. However, the SGA is used only to treat the pressure balancing constraint. Constraints related to duct noise levels, architectural requirements, etc., just impose minimum and maximum duct size ranges on the ducts and these can be easily incorporated using the second method (i.e., repair of the chromosome).

In penalizing a design that does not satisfy the pressure balancing constraints, the amount of penalty is assumed to be directly proportional to the magnitude of the pressure imbalance in each path. The penalty functions comprise 2 components; one assigns a penalty based on the magnitude of the largest path pressure imbalance and the other assigns a penalty based on the magnitudes of all path pressure imbalances. For a system with 5 paths, a design with 4 paths having a pressure imbalance of, for example, 5 Pa each (for a total of 20 Pa) is preferable to a design with only one path having a pressure imbalance of 20 Pa. The second component of the penalty function is thus multiplied by a factor u (<1), to reflect this preference. The value of u should be closer to 0 if the value of the maximum pressure imbalance is of far greater interest to the designer and closer to 1 if the pressure imbalance in all the paths are of interest. The penalty function is thus defined as

$$PF = w\left[\sum_{g=1}^{G}\left(\sup_{\iota \in S_s}\left[\psi_g(P_g^s - P_{\iota,g})I_{(P_g^s - P_{\iota,g} > \hat{P})}\right] + \sum_{\iota \in S_s} u \cdot \psi_g(P_g^s - P_{\iota,g})I_{(P_g^s - P_{\iota,g} > \hat{P})}\right)\right] \quad (8)$$

where

G = number of different system operation modes
w = penalty weighting parameter
ψ_g = fraction of time system operates in mode g
P_g^s = maximum subsystem path pressure during operation mode g
$\hat{P}$ = maximum allowable path pressure imbalance

$P_{\iota,g}$ = path pressure during operation mode g
$I_{(\cdot)}$ = indicator function

This function is for the general situation where there might be variable time of day duct flow rates and/or variable time of day utility rates. The first term is intended to account for the magnitude of the largest path pressure imbalance and the second term for the magnitudes of all path pressure imbalances.

The fitness function is therefore given by

$$FF = E_s + PF + \sum_{g=1}^{G} \frac{Q_{fan,g}(E_d + E_{c,g}\Psi_g T)P_g^s}{10^3 \eta_f \eta_m} \tag{9}$$

where

$Q_{fan,g}$ = fan flow rate during operation mode g
$E_{c,g}$ = unit electrical energy cost in operation mode g

The fitness function was selected to include both the objective and penalty functions. The smaller the fitness function, the better the design. This fitness function will be used in the example problem section as part of the genetic algorithm in an illustrative example of designing a ducting system.

Specific Implementation Algorithm

Several implementation algorithms are in the literature, but the Segregated Genetic Algorithm (SGA) is used for the example duct design problem because it offers a way to handle infeasible chromosomes while avoiding the problems associated with selecting penalty parameters.

The implementation of the SGA starts with the generation of 2 m designs at random (m is any suitable large integer, e.g. 100 or 500). These designs are then evaluated using two different penalty parameters ($w = w_1$ and $w = w_2$, with $w_1 >> w_2$) in Equation (8) to create two ranked lists. In ranking the chromosomes, duplicated individuals are pushed to the bottom (low rank) of the lists. This is a protection against premature uniformization of the population. From the two lists of 2 m ranked individuals, one single population of m individuals is built that mixes the relative influences of the two lists. The best individual from the list established using the highest penalty parameter w_1 is selected. Then, the best individual of the other list (based on the lower penalty parameter w_2), is selected. The process is repeated alternatively on each list until m individuals have been selected. Then reproduction occurs by application of linear ranking selection, crossover, mutation, and swapping to the combined list, creating m offspring. They are added to the m parents and the entire process is repeated until the stopping criterion is met.

Most studies have used a generation gap of 1; that is, complete population replacement. The opposite of this is steady-state replacement. In this case, only a small number of deaths and births occur at the same time. Thus at each generation, only a small number (typically two) individuals are replaced with offspring. The implementation of SGA in this paper was slightly different from that discussed previously with regards to the generation gap policy. The steady-state replacement approach was used because an initial study on the HVAC air duct design problem showed that it converges faster than the original SGA. A κ-chromosome tournament was used to select the parents for mating (where κ is a small integer). In this approach, a predetermined number κ, of chromosomes were selected at random from the population and the one with the best fitness function value was chosen. All the members were put back into the population to take part in further tournaments. This eliminated the need to sort (rank) the population. Using this procedure, two parents were selected at each generation to undergo crossover. One half of the

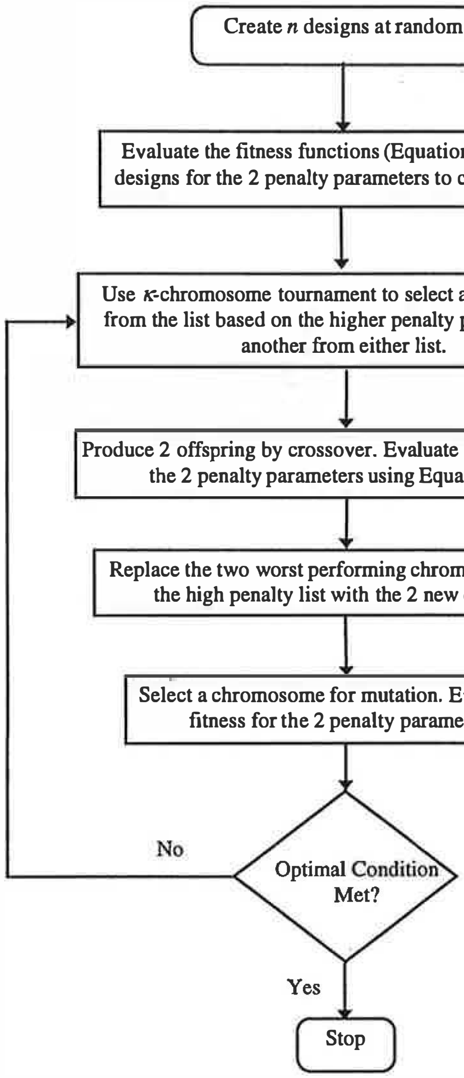

Figure 9. Flow chart of implemented segregated genetic algorithm

time, the parents were selected from the list based on the high penalty parameter, and during the other half, to obtain the effect of interbreeding, a parent was selected from each list. The two worst performing chromosomes with respect to the fitness function with the higher penalty parameter were replaced with the two offspring. The complete algorithm is shown in Figure 9. The population size is represented by n (where $n = 2m$).

Example Problem

To illustrate the use of SGA for air duct design, the duct layout from Chapter 32 of ASHRAE (1997) was used. The layout of the air duct system is shown in Figure 10, along with the eco-

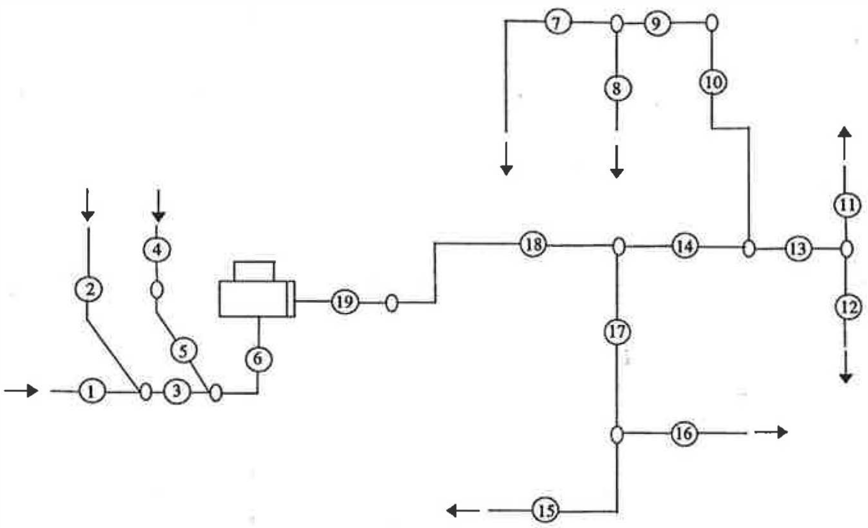

Figure 10. Duct layout for the *ASHRAE Handbook* (1997) sample problem

nomic and general system parameters. Table 4 gives input parameters for each duct section. Unlike the ASHRAE example, economic factors are introduced. In addition, there are two different airflow rate modes for the system: High Volume and Low Volume airflow rates. This is depicted in the daily load profile shown in Figure 11.

The total operating time is 6000 h/yr. The system operates in the high volume mode half of the year and in the low volume mode in the remainder. The corresponding flow rates are shown under the "High Volume" and "Low Volume" columns of Table 4. The higher flow rates are taken from ASHRAE (1997). Furthermore, as shown below, the electrical energy rates are higher for the peak than the non-peak periods of electricity use. The billing policy is shown in Figure 12. From Figures 11 and 12, it can be deduced that 2750 hours (about 91.7%) of the High Volume operation occurs during the peak utility rate period yearly and in the case of the Low Volume mode, it is 500 hours (16.7%).

Economic and General Data

Absolute roughness	0.09 mm
Air density	1.204 kg/m^3
Unit electrical energy cost	
Non-peak period	0.06 \$/kWh
Peak period	0.1 \$/kWh
Ductwork cost	43 \$/m^2
Fan efficiency	0.75
Motor and drive efficiency	0.8
Fan flow rate	
High volume	1.9 m^3/s
Low volume	0.95 m^3/s
Total operating time	6000 h/year
PWEF	9.01

Table 4. Duct Input Data

Duct Section	Type	Height, mm	Length, m	Flow Rate, m^3/s High Volume	Flow Rate, m^3/s Low Volume	Extra Pressure Loss, Pa	Fittings*
1	Round	—	4.6	0.7	0.35	—	ED1-3,ED5-1,CD9-1
2	Round	—	18.3	0.25	0.125	—	ED1-1,CD3-6,ED5-1, CD6-1, CD9-1
3	Round	—	6.1	0.95	0.475	—	ED5-2,CD9-1
4	Rectangular	600	1.5	0.95	0.475	25	CR9-4,ER4-3
5	Round	—	18.3	0.95	0.475	—	CD3-17,ED5-2,CD9-1
6	Round	—	9.1	1.9	0.95	—	CD3-9,ED7-2,Cd9-3
7	Rectangular	250	4.3	0.275	0.1375	25	CR3-3A,CR9-1,SR5-13
8	Rectangular	250	1.2	0.275	0.1375	25	SR5-13,CR9-4
9	Rectangular	500	7.6	0.55	0.275	—	SR3-1
10	Rectangular	400	13.7	0.55	0.275	—	CR9-1,CR3-6,SR5-1, CR3-10
11	Rectangular	250	3.0	0.475	0.2375	—	CR9-1,SR5-14,SR2-1
12	Rectangular	250	6.7	0.475	0.2375	—	CR9-1,SR2-5,SR5-14
13	Rectangular	350	10.7	0.95	0.475	—	CR9-1,SR5-1
14	Rectangular	660	4.6	1.5	0.75	—	CR9-1,SR5-13
15	Rectangular	200	12.2	0.2	0.1	—	CR3-1,SR2-6,CR9-1, SR5-1
16	Rectangular	200	6.1	0.2	0.1	—	SR2-3,CR6-1,CR9-1, SR5-1
17	Rectangular	250	4.2	0.4	0.2	—	CR9-1,SR5-13
18	Rectangular	800‡	7.0	1.9	0.95	—	CR6-4,SR4-1,CR3-17, CR3-17
19	Rectangular	450	3.7	1.9	0.95	15	SR7-17,CR9-4

*See 1997 *ASHRAE Handbook* for complete description of fittings. ‡ Width

Problem Specific Constraints

A number of constraints need to be satisfied for this problem. Some are to ensure that the duct fittings are the same type as for the original problem.

Fixed Duct Sizes

Duct sections 4 and 19 were assumed to be 600 mm × 600 mm and 800 mm × 450 mm rectangular presized ducts, respectively, corresponding to the sizes indicated in ASHRAE (1997). The heights (width for duct 18) of the supply ducts were also assumed to be equal to those in ASHRAE (1997).

Wye and Tee Fittings

The size of the ducts to which wyes and tees are connected satisfied the following relations:

$A_b = A_s \le A_c$ for fitting Sr5-14,

$A_b + A_s \ge A_c$ for fitting Sr5-1,

$A_b \le A_c$ and $A_s \le A_c$ for fittings Ed5-1, Ed5-2 and Sr5-13,

where A_ι is the area of duct $\iota = b, s, c$ (b: branch; s: straight; c: common).

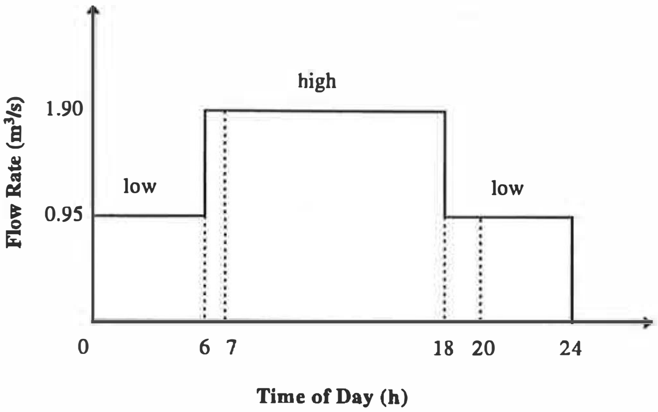

Figure 11. Daily load profile

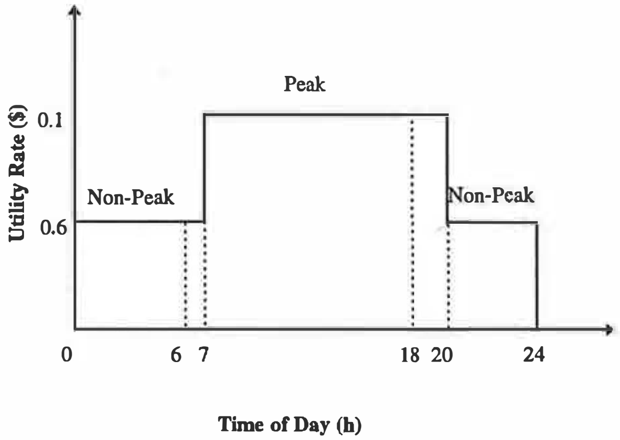

Figure 12. Electric utility billing policy

Path Pressures

Because it is difficult to come up with duct sizes that achieve equal path pressure losses for all the paths, the system was developed to achieve a reasonable path pressure imbalance. The difference between the path with the maximum pressure and that with the lowest pressure did not exceed a predetermined pressure. The aim was to achieve a zero pressure imbalance for this sample problem, and any path pressure imbalance was penalized.

Allowable Duct Sizes

A limit was placed on the allowable sizes of the ducts. The sizes were limited to between 100 mm and 800 mm with size increments of 10 mm as recommended in ASHRAE (1997). For ducts 12 and 16, maximum widths are 425 mm and 375 mm respectively. Also, ducts 11 and 12 were required to be equal in size because they are connected to a dovetail fitting.

Implementation of Algorithm

The algorithm discussed was implemented using the language C. A population size of 800 chromosomes (i.e., n = 800) and a tournament size of 5 (i.e. κ = 5) were used. At each generation, a coin toss was used to determine if a chromosome should be selected at random to undergo mutation. The computer implementation of a coin toss consisted of generating a random number from a uniform distribution on the interval [0,1]. A head was assumed to be the outcome of the toss when the number was less than 0.5, and a tail occurred when the number is more than or equal to 0.5. Mutation in the problem involved selecting a gene at random and using a coin toss to determine whether to increase or decrease the value by 1 (corresponding to a duct size change of 10 mm).

Figure 13 shows the chromosome representations for the return and supply duct subsystems separately obtained using the scheme. These chromosomes contain all the duct sections with the exception of duct sections 4 and 19. It is possible to reduce the number of duct sections that are encoded in a chromosome to reduce the computation time. For example, if there is a dependency between any duct sections, only one duct need to be sized to determine the other duct sizes. Therefore, only that particular duct section need to be included in the chromosome. Also, the supply and return sub-system can be treated separately. The dynamic loss coefficients are calculated at run time by interpolation using tables provided by ASHRAE. A severe penalty parameter (w_1) of 500 and a soft penalty parameter (w_2) equal to 1.9 $(E_d + E_cT)/10^3\eta_f\eta_m$ = 1.9 (with T = 6000 and E_c = 0.1) were used. The algorithm was stopped when 98% or more of the chromosomes had the same fitness function value.

Return Subsystem

Gene Position	1	2	3	4	5
Corresponding Duct Section	6	3	2	1	5

Supply Subsystem

Gene Position	1	2	3	4	5	6	7	8	9	10	11	12
Corresponding Duct Section	18	14	10	9	7	8	13	12	11	17	15	16

Figure 13. Chromosome representation for sample problem

RESULTS

The results obtained by applying SGA to the problem are shown in Table 5 for each duct section and Table 6 for each airflow path. These are based on the best results obtained in 10 runs of the program, in which 10 different seeds were used for the random number generator and the best solution was selected. The duct sizes and associated pressure losses for both operational modes are in Table 5. The path pressures are indicated in Table 6 together with the path pressures from ASHRAE (1997).

For the high flow rates, the reference paths for calculating the pressure differences were path 2 for the return system and path 7 for the supply system. For the low flow rates, path 1 and 8 were used. For the ASHRAE design, paths 2 and 8 were used. At the high flow rates, the pressure drop imbalances among the various flow paths are excellent for the new design. Even at the low flow rates, the pressure imbalances are reasonable and activated dampers in flow paths 2, 3, 4, 5, 6, and 7 will achieve excellent flow rates as specified in the design.

The system total pressure for this design is 323 Pa and 131 Pa during the high volume and low volume operation periods respectively and an associated cost of $11,618 (material cost: $8575, energy cost: $3043). In the case of the ASHRAE Fundamentals design, the system total pressures are 679 Pa and 213 Pa while the total life-cycle cost would be $13,367 (material cost: $7123, energy cost: $6244) for the same operating conditions and utility rates shown in Figure 11 and Figure 12. For this example, the optimal design using the SGA resulted in energy costs much less than the materials cost. In contrast, for the ASHRAE design, they are nearly equal but

Table 5. Optimal Duct Sizes and Pressure Losses

Duct Section	Size, mm	High Volume Operation Friction Loss, Pa	High Volume Operation Dynamic Loss, Pa	High Volume Operation Total Loss, Pa	Low Volume Operation Friction Loss, Pa	Low Volume Operation Dynamic Loss, Pa	Low Volume Operation Total Loss, Pa
1	370	5.7	9.3	15.0	1.6	2.3	3.9
2	250	23.2	−8.1	15.1	6.6	−2.0	4.6
3	370	13.3	32.6	45.9	3.7	8.1	11.8
4	600 × 600	0.2	28.2	28.4	0.1	25.8	25.8
5	480	11.0	20.7	31.7	3.1	5.2	8.3
6	570	8.5	28.6	37.1	2.4	7.1	9.5
7	250 × 250	4.2	27.7	31.9	1.2	25.7	26.9
8	580 × 250	0.2	31.5	31.6	0.0	26.6	26.7
9	580 × 250	3.7	8.6	12.3	1.1	2.2	3.2
10	680 × 250	4.7	29.8	34.5	1.4	7.4	8.8
11	350 × 250	3.5	34.9	38.5	1.0	8.7	9.7
12	350 × 250	7.9	31.2	39.1	2.2	7.8	10.1
13	380 × 250	37.7	2.0	39.7	10.4	0.5	10.9
14	570 × 250	15.0	5.3	20.4	4.1	1.3	5.4
15	210 × 150	36.2	13.3	49.6	10.3	3.3	13.6
16	230 × 150	14.6	34.1	48.7	4.2	8.5	12.7
17	270 × 150	25.0	24.5	49.6	7.0	6.1	13.1
18	800 × 760	0.9	28.7	29.6	0.3	7.5	7.8
19	800 × 475	1.8	93.9	95.7	0.5	34.7	35.2

Table 6. Path Pressure Losses

Path	Ducts in Path	SGA High Volume Path Loss, Pa	SGA High Volume Pressure Difference, Pa	SGA Low Volume Path Loss, Pa	SGA Low Volume Pressure Difference, Pa	ASHRAE Path Loss, Pa	ASHRAE Pressure Difference, Pa
1	6-5-4	97	1	44	0	234	28
2	6-3-2	98	0	26	18	262	0
3	6-3-1	98	0	25	18	240	22
4	19-18-17-16	224	1	69	18	417	0
5	19-18-17-15	224	1	70	17	404	13
6	19-18-14-13-11	224	1	69	18	412	5
7	19-18-14-13-12	225	0	69	18	412	5
8	19-18-14-10-9-8	224	1	87	0	417	0
9	19-18-14-10-9-7	224	1	87	0	408	9

in total 15% higher. This difference may be attributed in part to the different assumptions used for the costs and operating conditions for the two designs.

The algorithm converged after 2501 generations for the return subsystem and 3359 generations for the supply subsystem. However, as can be seen from the chart in Figure 14 most of the design improvements in the supply system were made in the earlier generations and the average and best fitness values approach each other as the algorithm progresses. The same phenomenon occurs for the return subsystem and for the low penalty functions.

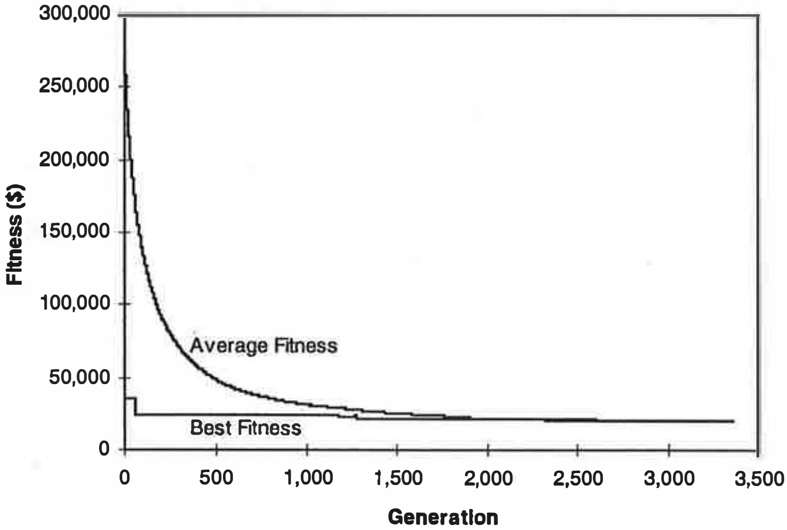

Figure 14. Progress of algorithm measured by average and minimum fitness for supply system using high penalty function (w_1=500)

One important feature of GAs is their ability to handle complex objective functions involving variable utility rates and duty cycles for the equipment. If the problem above were to be solved using other existing duct design methodologies, it would require simplification. Either the higher or time weighted average values of the time variable parameters would be needed, and a design that may not be optimal in the sense of minimum LCC and performance when operated at actual operational conditions might be produced.

Table 7 and Table 8 show a comparison between the SGA solution and solutions obtained using the simplifications for the return and supply subsystems respectively. All the energy cost figures are for the costs of energy under normal operation with the daily load and utility rates shown in Figure 11 and Figure 12. As can be seen, the designs based on the use of these design simplifications result in higher life-cycle costs than the design approach presented in this paper. The actual energy cost is lower when the high flow rates are used for the design due to the fact that the high volumes necessitate the selection of larger duct sizes to reduce the energy cost

Table 7. Comparison with Simplified Design Strategies-Return Subsystem

	Cost, $			Relative Change in Cost, %		
Simplification Assumption	**Material**	**Energy**	**Total**	**Material**	**Energy**	**Total**
Basecase	3196	936	4132	—	—	—
High flow rate/ Peak utility rate	3799	494	4293	18.9	−47.2	3.9
High flow rate/ Weighted average utility rate	3762	544	4307	17.7	−41.8	4.2
Weighted average flow rate/ Peak utility rate	3442	773	4215	7.7	−17.4	2.0
Weighted average flow rate/ Weighted average utility rate	3443	902	4346	7.7	−3.6	5.2

Table 8. Comparison with Simplified Design Strategies-Supply Subsystem

Simplification Assumption	Cost, $ Material	Cost, $ Energy	Cost, $ Total	Relative Change in Cost, % Material	Relative Change in Cost, % Energy	Relative Change in Cost, % Total
Basecase	5379	2107	7486	—	—	—
High flow rate/ Peak utility rate	5676	1960	7636	5.5	−7.0	2.0
High flow rate/ Weighted average utility rate	5672	1985	7657	5.4	−5.8	2.3
Weighted average flow rate/ Peak utility rate	5529	2269	7798	2.8	7.7	4.2
Weighted average flow rate/ Weighted average utility rate	5455	2209	7663	1.4	4.8	2.4

Table 9. Pressure Losses–Weighted Average Flow Rate/Weighted Average Utility Rates

Path	Ducts in Path	High Volume Path Loss, Pa	High Volume Pressure Difference, Pa	Low Volume Path Loss, Pa	Low Volume Pressure Difference, Pa
1	6-5-4	78	18	39	0
2	6-3-2	95	1	25	14
3	6-3-1	96	0	25	14
4	19-18-17-16	236	1	72	14
5	19-18-17-15	229	8	71	15
6	19-18-14-13-11	237	1	72	14
7	19-18-14-13-12	237	0	73	13
8	19-18-14-10-9-8	219	18	86	0
9	19-18-14-10-9-7	219	18	86	0

reflected in the higher material cost (18.9% and 17.7% in the case of the return subsystem and 5.5% and 5.4% for the supply subsystem). Once the ducts have been over-sized, the pressure losses under the actual operational conditions, which include low flow rates periods, results in a lower energy cost (47.2% and 41.8% in the case of the return subsystem and 7.0% and 5.8% for the supply subsystem) resulting in a relatively small overall increase in cost. These results however, should not be seen as supporting the use of these simplifying assumptions except for simple design problems. The results are more a reflection of the nature of this particular problem; more complex problems may give much larger differences. In the case of the weighted flow rate, the low increase in cost is achieved at the expense of greater pressure imbalance as is evident by the values in Table 9 and Table 6. Such large pressure imbalances would have to be adjusted by the use of additional duct dampers which are activated differently for the daytime and nighttime operating conditions if the specified flows are to be maintained.

Table 10 shows the path pressures obtained when the system is assumed to be operating in the high volume mode and the utility rates are fixed at the peak value all the time. This illustrates the power of GA in finding a good solution to the HVAC design problem. As can be seen, the GA is able to obtain the right combination of duct sizes to virtually achieve complete pressure balance in all the paths. One parameter that affects the performance of a GA is the population size. Usually, the solution improves with increasing population size as illustrated in Table 11, which shows the optimal LCC obtained to be sensitive to different population sizes. However, there is a point where this improvement ceases to be significant.

Table 10. Path Pressure Losses–High Volume Rates/Peak Utility Rates

Path	Ducts in Path	Path Loss, Pa	Pressure Difference, Pa
1	6-5-4	48	1
2	6-3-2	49	0
3	6-3-1	48	1
4	19-18-17-16	208	0
5	19-18-17-15	208	1
6	19-18-14-13-11	207	2
7	19-18-14-13-12	208	1
8	19-18-14-10-9-8	208	0
9	19-18-14-10-9-7	208	0

Table 11. Effect of population Size on Performance of SGA

Population Size	Return Subsystem Cost, $	Supply Subsystem Cost, $	Total Cost, $
100	4363	7852	12,215
400	4355	7730	12,085
1600	4200	7537	11,736
2000	4154	7433	11,587

Uncertainty in LCC

In the example, it was assumed that the parameters of the cost model, PWEF, air density, fan flow rates, ductworks cost, etc., were known with exact certainty and were sufficient for the analysis. Although this assumption simplifies the evaluation of the costs, it is unlikely that the actual values of the parameters will be exactly what was used for the analysis. It is, therefore, important to assess the sensitivity of each parameter on the LCC cost of the duct system. Figure 15 shows the percentage changes in the LCC corresponding to changes in different parameters. The changes in the parameter values is relative to the values used above. The greatest changes in the LCC were associated with changes in the ductwork cost and high flow rate. Changes in the fan overall efficiency had an equal but opposite effect on the LCC as changes in the PWEF, total yearly operating time and air density. Each of these terms are shown in the appendix to have a similar effect on the LCC ($\eta_f \times \eta_m$ is the overall fan efficiency). The LCC is shown to be insensitive to the low flow rate and the non-peak utility rate for this particular problem.

This uncertainty analysis evaluated the effect of one parameter at a time. However, it is possible to have changes in multiple parameters simultaneously and have a parameter not vary uniformly in its assumed range. It is more appropriate to represent these variations by other probability distributions as discussed in Asiedu et al. (2000b).

The effect of changes in the load profile and billing policy, shown in Figure 11 and Figure 12 respectively, on the LCC were evaluated. Figure 16 shows the percentage changes in the LCC corresponding to changes in the high volume period and peak utility rates period. These changes are accompanied by corresponding changes in the low volume period and non-peak utility rate period, respectively. A one hour change in the high volume period in Figure 16 means that, the high volume period shown in Figure 11 now starts at 0530 hrs and ends at 1830 hrs. Similarly, a –1 hour change in the peak utility rates period in Figure 16 means that, the peak utility rates period shown in Figure 12 now starts at 0730 hrs and ends at 1930 hrs. Increases in the lengths of the high volume period and the peak utility rates period were accompanied by increases in the LCC and vice versa. The LCC is more sensitive to changes in the high volume period than to the

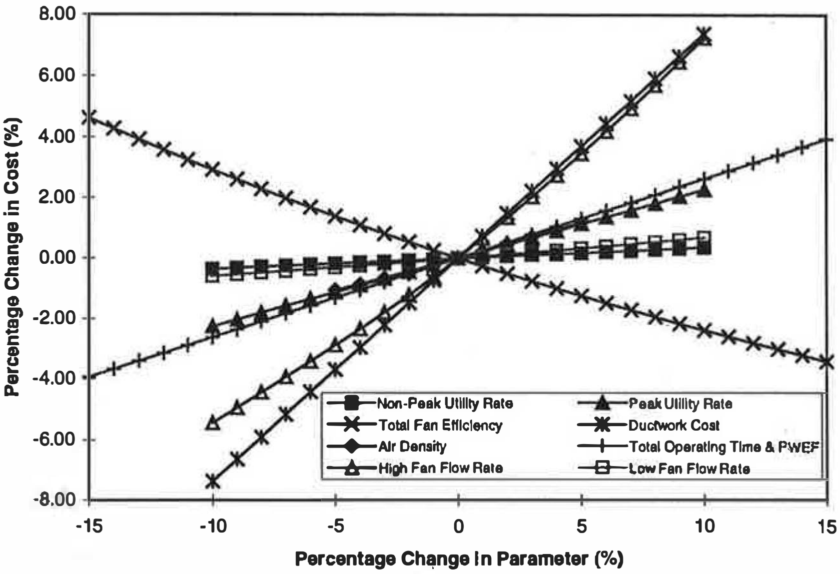

Figure 15. Assessing LCC sensitivity to changes in several parameters

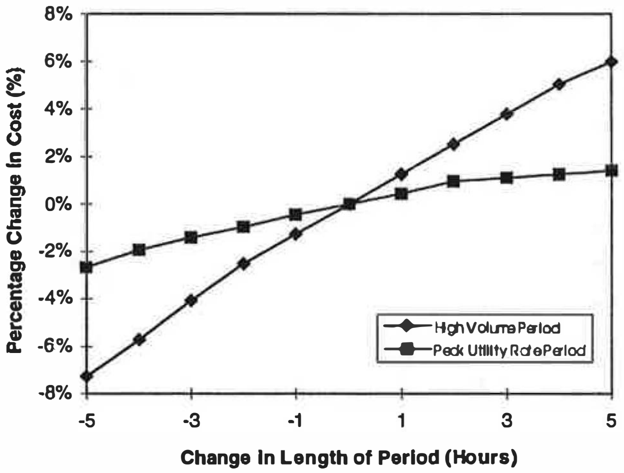

Figure 16. Effect of changing load profile and billing policy

duration of the peak utility rates. For both cases, reductions in the lengths of these periods resulted in greater percentage changes in the LCC than corresponding increases.

CONCLUSION

The use of the segregated genetic algorithm provides a general design methodology that enables the design of economically efficient HVAC systems. The power of this approach is its simplicity, flexibility and the fact that most problem specific constraints and expert knowledge can be easily incorporated into the algorithm.

The illustrative example shows that the method leads to very good pressure balances among each of the many flow paths. While the T-Method might be computationally more efficient for simple duct design problems, the SGA approach is less restrictive in the kind of problems it can treat. Furthermore, it avoids the duct size rounding problem encountered in the T-Method and unlike the Static Regain and Equal Friction methods, it treats explicitly life-cycle cost implications of design choices.

Although this methodology was applied to a particular air duct system, it can be extended to other ducting and piping problems. Many HVAC&R design problems with their many components and nonlinear performance factors and constraints may be solved using genetic algorithms to reduce the life-cycle costs.

ACKNOWLEDGEMENT

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NOMENCLATURE

Acronyms

GA genetic algorithm
LCC life-cycle cost
PWEF present worth escalation factor
SGA segregated genetic algorithm

Symbols

A_i duct area connected to section ι of fitting; ι = b, s, c (b: branch; s: straight; c: common), m^2
ΣC summation of dynamic friction loss coefficients for duct fittings
d_x size of duct section x, m
d_x^{max} maximum allowable size for duct section x, m
d_x^{min} minimum allowable size for duct section x, m
D duct diameter, m
D_h hydraulic diameter, m
E present worth owning and operating cost –LCC, $
E_c unit electrical energy cost, cost/kWh
$E_{c,g}$ unit electrical energy cost in operation mode g, cost/kWh
E_d energy demand cost, cost/kW
E_p first year energy cost, $
E_s initial cost, $
f friction factor
FF fitness function
G number of different system operation modes
H duct height, m
i interest rate, %
I set of duct sections in path ι
$I_{(\cdot)}$ indicator function
j escalation rate for energy cost, %
L duct length, m
m amortization period, years
P_{fan} fan total pressure, Pa
$P_{\iota,g}$ path pressure during operation mode g, Pa
P maximum allowable path pressure imbalance
P_g^s maximum subsystem path pressure during operation mode g
ΔP_x total pressure losses in duct section x
Q duct flow rate, m^3/s
Q_{fan} fan airflow rate, m^3/s
$Q_{fan,g}$ fan flow rate during operation mode g, m^3/s
Re Reynolds number
S_s set of paths in a (sub)system s
S_d unit duct cost, cost/m^2
SD set of standard duct sizes
T operational time, hours/year
$u (<1)$ factor reflecting the relative importance of pressure imbalance
V airflow velocity in duct, m/s
V_x^{max} maximum allowable velocity for duct section x, m
V_x^{min} minimum allowable velocity for duct section x, m
w penalty weighting parameter
W duct width, m
x index (number) of the duct section
X total number of duct sections in system
ρ air density, kg/m^3
ε absolute roughness factor, m
υ kinematic viscosity of the air, m^2/s
κ number of chromosomes used in tournament selection
η_f fan total efficiency, %
η_m motor drive efficiency, %
ψ_g fraction of time system operates in mode g

Subscripts

ι duct path index
g index of duct system operation mode
x index of duct section

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APPENDIX

The Life Cycle Cost (LCC) of a duct system comprise the initial capital cost and the operating cost, which is usually the energy cost. This is expressed in Equation (A1).

$$E = E_p \times \mathrm{PWEF} + E_s \qquad \text{(A1)}$$

where

E	present worth owning and operating cost (LCC)
E_p	first year energy cost
E_s	initial cost
PWEF	present worth escalation factor

Generally, the energy cost is determined by

$$E_p = \frac{Q_{fan}(E_d + E_c T)P_{fan}}{10^3 \eta_f \eta_m} \tag{A2}$$

where

- Q_{fan} fan airflow rate, m^3/s
- E_c unit energy cost, cost/kWh
- E_d energy demand cost, cost/kW
- T operation time, hours/year
- P_{fan} fan total pressure, Pa
- η_f fan total efficiency
- η_m motor drive efficiency

However, for the problem in this paper with different operations modes and different electric rates for specific times of the day, it is calculated by

$$E_p = \sum_{g=1}^{G} \frac{Q_{fan,g}(E_d + E_{c,g}\psi_g T)P_g^s}{10^3 \eta_f \eta_m} \tag{A3}$$

where

- G number of different system operation modes
- $Q_{fan,g}$ fan flow rate during operation mode g, m^3/s
- $E_{c,g}$ unit electrical energy cost in operation mode g, cost/kWh
- w penalty weighting parameter
- ψ_g fraction of time system operates in mode g
- P_g^s maximum subsystem path pressure during operation mode g, Pa

The present worth escalation factor is calculated as follows

$$\text{PWEF} = \begin{cases} \dfrac{[(1+j)/(1+i)]^m - 1}{1 - [(1+i)/(1+j)]} & \text{if } i \neq j \\ m & \text{if } i = j \end{cases} \tag{A4}$$

where

- m amortization period
- i interest rate
- j escalation rate for energy cost

The initial cost of duct system is given by the following equations

Round ducts

$$E_s = S_d \pi L D \tag{A5}$$

where

- D duct diameter, m
- L duct length, m
- S_d unit duct cost, cost/m^2

Rectangular ducts

$$E_s = 2S_d(H+W)L \tag{A6}$$

where

H	duct height
W	duct width

This cost includes the materials, labor, shop drawings, shipping, and a mark up for overhead, maintenance, and insurance (Tsal et al. 1988a). The cost associated with indoor air quality may be included in the cost model, but they are difficult to quantify so they are not specified in this paper. Although it may also include the initial cost of the air-handling unit, this cost can be added to Equation (A1) as an additional term. This is more appropriate since for any given system pressure, there are a number of different types of fans that can be used. Furthermore, the same type of fan can be designed in different ways by the same manufacturer or other manufacturers. The initial cost of the fan depends on the particular fan selected and this cost does not influence any of the results. If this cost term is included in Equation (A1) and subsequently in the fitness function of a genetic algorithm, there should be strict rules as to how the selection of the fan is made so that for every unique chromosome (duct design), one and only one particular fan will be selected and this fan is selected each time that chromosome occurs in the algorithm. This term was not included in this work because the selection of the fans was not treated.

The fan pressure depends one the duct pressure drops ΔP which is calculated using the Darcy-Weisbach equation

$$\Delta P = \left(\frac{fL}{D_h} + \sum C\right)\frac{\rho V^2}{2} \tag{A7}$$

where

f	friction factor
D_h	hydraulic diameter, m
$\sum C$	summation of dynamic friction loss coefficient for duct fittings in the duct section
ρ	air density
V	airflow velocity in duct

The equation used to find the dimensionless friction factor was developed by Altshul and modified by Tsal (1988),

$$f' = 0.11\left(\frac{\varepsilon}{D_h} + \frac{68}{\text{Re}}\right)^{0.25} \tag{A8}$$

$$f = \begin{cases} f' & \text{if } (0.018 \le f') \\ 0.85\, f' + 0.0028 & \text{otherwise} \end{cases} \tag{A9}$$

where

ε	absolute roughness factor
Re	Reynolds number

The Reynolds number can be calculated from Equation (A10)

$$\text{Re} = \frac{D_h V}{\upsilon} \tag{A10}$$

where

υ kinematic viscosity of air