

Error Analysis of Measurement and Control Techniques of Outside Air Intake Rates in VAV Systems

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ABSTRACT

This paper provides a theoretical error analysis of common airflow measurement and control techniques to maintain minimum outside air intake rates in variable air volume (VAV) systems. The results of the error analysis indicate that control strategies using direct airflow measurement from either an averaging Pitot-tube array or an electronic thermal anemometry provided the best ventilation control. Calculation of the outside airflow rate using a CO₂ concentration balance can also allow for adequate system control, except when occupancy is low or when the outside air represents a small fraction of the supply air delivered. In addition, the results show that the use of the temperature balance technique to calculate the outside air intake rate is not adequate under common building operating conditions. In the case when measurement of the outside airflow rates is not possible, plenum pressure control can provide adequate control of outside air intake rates. Finally, the use of a fixed minimum outside air damper position or a volumetric fan tracking control strategy both proved to be inadequate control techniques for maintaining minimum ventilation rates in VAV systems.

INTRODUCTION

Due to an increased concern about maintaining acceptable indoor air quality and meeting ventilation codes and standards, the accurate control and measurement of outside air intake rates has come to the forefront of attention of several HVAC engineers and designers. Unfortunately, the necessary monitoring equipment and control logic to maintain minimum outdoor intake rates are often nonexistent or are used improperly if they are installed. Consequently, several commercial

buildings, in particular those with variable air volume (VAV) systems, have been found to have inadequate ventilation (Sterling et al. 1992). Unlike constant air volume (CAV) systems, VAV systems present the additional challenge caused by varying pressure in the mixed air plenum that make techniques used to control outdoor air intake in CAV systems ineffective.

ASHRAE Standard 62-1999 establishes minimum outdoor air ventilation rates for acceptable indoor air quality (IAQ) standards within buildings. Section 5.3 of the standard requires that "when the supply of air is reduced during times the space is occupied (e.g., in variable air volume systems), provision shall be made to maintain acceptable indoor air quality throughout the occupied zone" (ASHRAE 1999). Measurement and control of the outside air intake rate is one method to address this requirement.

The use of appropriate airflow measurement or VAV control techniques is critical to maintain minimum outside air intake rates. In this paper, theoretical error analyses are presented to determine the accuracy of various techniques for outside airflow measurement and VAV control. A companion paper presents the results of experimental testing of several outside airflow measurement and VAV control techniques described in this paper.

LITERATURE REVIEW

This section presents a comprehensive literature review of airflow measurement techniques and VAV control methods. In this section, only the measurement and control techniques tested in the laboratory are described in detail. Brief descriptions of several other airflow measurement and control techniques are included for completeness.

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Airflow Measurement Techniques

Several techniques exist for measuring airflow in HVAC systems. These techniques can be divided into two categories: direct and indirect measurement techniques. The first method measures the airflow directly, using, for instance, an anemometer. The second method measures other parameters that are dependent upon the airflow, as in the energy balance method.

It would be difficult and impractical to attempt to test and compare all of the airflow measurement techniques that are available. Therefore, only the techniques tested in the laboratory for the ASHRAE RP-980 project are described in this paper (Krarti et al. 1999).

Direct Airflow Measurement Techniques

Direct airflow measurement techniques require well-developed airflow profiles for accurate readings (Drees et al. 1992). Generally, existing systems do not have sufficient space within the outside air ducts to meet manufacturer's minimum recommendations (typically 7.5 unobstructed duct lengths upstream and 3 unobstructed duct lengths downstream) for installing these flow measurement stations. This coupled with the low air velocities found in outside air ducts are the primary obstacles for accurate direct measurement of ventilation air. Table 4 in Chapter 14 of *ASHRAE Fundamentals* lists several direct airflow measurement techniques, their applications, and expected accuracy (ASHRAE 1997a). ASHRAE Standard 111-1988 also provides a comprehensive summary of direct airflow measurement techniques, their expected accuracy, and limitations (ASHRAE 1988).

Averaging Pitot-Tube Array

Averaging pitot-tube arrays are based upon the fundamental airflow measurement device, the pitot-static tube. Ower and Pankhurst (1977) found that air could be treated as an incompressible fluid with negligible error for flow rates typically encountered in building HVAC applications. Therefore, the airflow velocity can be calculated by

$$V = C \cdot \sqrt{\frac{V_p}{\rho}} \quad (1)$$

where

- V = airflow velocity, fpm (m/s);
- C = constant, 1096.7 (1.4123);
- V_p = velocity pressure, in. w.g. (Pa);
- ρ = density, lb_m/ft³ (kg/m³).

From the ideal gas law, the air density can be found from Equation 2.

$$\rho = \frac{p}{R \cdot T} \quad (2)$$

where

- p = atmospheric pressure, psia (kPa)

R = gas constant for air, 53.347 ft lb_f/(lb_m °R) [287 J/(kg K)]

T = absolute temperature, °R (K)

Equation 2 neglects any effects of the air humidity on the air density. Calculating the airflow velocity from Equation 1 while using Equation 2 to calculate the air density introduces at most 0.60% error.¹ Substituting Equation 2 into Equation 1 gives

$$V = C \cdot \sqrt{\frac{V_p \cdot T}{p}} \quad (3)$$

where

C = constant, 667,517 (0.7566).

An averaging Pitot-tube array averages both the total and static pressures throughout the duct and then calculates the airflow velocity from the difference between the two average pressures, or the velocity pressure. Some inaccuracies are inherent with this method due to the nonlinear relationship between the velocity pressure and the airflow velocity. Specifically, averaging the velocity pressure readings across the duct before calculating the velocity from Equation 4 introduces some errors in the measurement. This is especially true when a nonuniform velocity profile exists in the duct. Figure 1 shows an example velocity profile for an HVAC duct (Table 1 includes the individual point velocities). The true airflow rate is the average of the individual velocities, or 785 fpm (3.99 m/s) for the values shown in Figure 1. However, an averaging Pitot-tube array would average the velocity pressures associated with these points and then calculate the airflow velocity. Assuming standard atmospheric conditions of 70°F and 14.696 psia (21.1°C, 101.3 kPa), the average

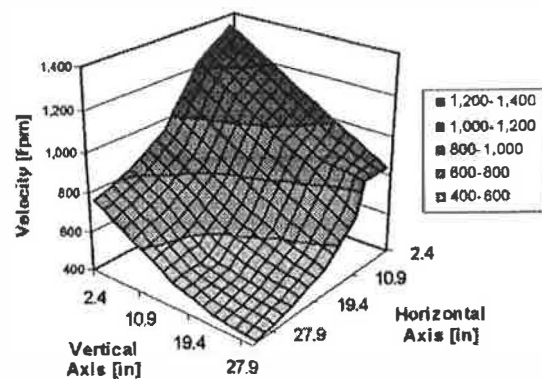


Figure 1 Example velocity profile in an HVAC duct.

¹ Errors introduced by neglecting the effects of humidity on air density and calculating the airflow velocity from Equation 1 were investigated using hourly weather data for the following locations: Denver, Madison, Miami, New York, Phoenix, and San Francisco.

TABLE 1
Example Velocity Profile in an HVAC Duct (fpm)

400	450	525	700	850
425	480	600	850	975
500	524	700	975	1100
625	700	825	1100	1225
750	820	950	1225	1350

velocity pressure corresponding to the points shown in Table 1 is 0.043 in.w.g. (10.707 Pa). Inserting this value into Equation 3 gives a velocity of 831 fpm (4.22 m/s), an error of almost 6% compared to the true value.

To reduce errors related to nonuniform airflow profiles, manufacturers' installation guidelines typically require straight, unobstructed duct for 7.5 duct diameters upstream and 3 duct diameters downstream from the airflow measurement station (ASHRAE 1997a).

Typically, averaging Pitot-tube arrays are not accurate for flow rates below 600-800 fpm (3.05 to 4.06 m/s) unless auto-zeroing and temperature-compensated differential pressure transmitters are used (Drees et al. 1992; ASHRAE 1988). Such instrumentation may not be practical for some building installations. Additionally, small errors in the differential pressure transmitters can result in large errors in the calculated flow rate.

Electronic Thermal Anemometry

Thermal devices, such as thermistors, hot-wires, and hot-films, have long been used to measure flow rates. They have the capability to accurately measure flow rates as low as 1 to 10 fpm (0.0051 to 0.051 m/s) (ASHRAE 1997a; Haines 1994). Other benefits include a much closer approximation to constant error as a percentage of airflow than for Pitot-tubes (Solberg et al. 1990). Additionally, the measured flow signals are linear and electronic, which allows for simple use with DDC controllers (Solberg et al. 1990).

The theory of thermal anemometry is relatively simple; a heated device loses heat at a rate determined by the temperature and velocity of the fluid being measured. Ower and Pankhurst (1977) provide a good description of the physical relationships underlying the theory. Traditionally, the flow rate is calculated from King's law (Beckwith et al. 1993):

$$e_o^2 = C + D\sqrt{\rho} \cdot V \quad (4)$$

where

- e_o = output voltage, V
- C, D = constants
- ρ = density of fluid, lb_m/ft³ (kg/m³)
- V = fluid velocity, ft/s (m/s)

Flow rates determined from King's law are typically subject to nonlinear output signals and are dependent on the ambient temperature of the fluid. For these reasons, the span

and temperature ranges of calibrated thermal anemometers are somewhat limited. There are other drawbacks for the use of thermal anemometer flow-measuring stations. These drawbacks include their high sensitivity to turbulence in the airstream and the complicated field calibration procedures that must be performed frequently (Drees et al. 1992; ASHRAE 1997a; Maki et al. 1997; ASHRAE 1988).

Electronic thermal anemometry attempts to account for some of these limitations by converting the analog signal from the sensor to a digital signal. If the ambient temperature of the fluid stream is also measured, corrections can be made and a linear output signal can be generated. Experimental results by Drees et al. (1992) indicate that calculated outside air intake rate errors as large as 50% were possible at lower outside air temperatures when using thermal anemometry. No negative effects due to low outside air temperatures (down to 35°F [1.67°C]) were found by the authors of this paper.

Indirect Airflow Measurement Techniques

Indirect methods provide an alternative for the measurement of outside airflow intake rates, which are typically low and difficult to measure directly. Two indirect measurement techniques are described in this section: the enthalpy balance and the concentration balance.

Enthalpy Balance

Assuming adiabatic mixing of the return airflow and outside airflow in an HVAC system, the outside airflow rate can be determined by performing a mass and energy balance on the airstreams as shown in Equation 5. At low outside air intake rates, the quantities on the right side of Equation 5 may be more easily measured than the outside airflow rate directly.

$$\dot{V}_{OA} = \dot{V}_{SA} \cdot \frac{\rho_{MA}}{\rho_{OA}} \cdot \left(\frac{h_{RA} - h_{MA}}{h_{RA} - h_{OA}} \right) \quad (5)$$

where

- \dot{V}_{OA} = outside airflow rate, cfm (L/s)
- \dot{V}_{SA} = supply airflow rate, cfm (L/s)
- ρ_{MA} = density of mixed air, lb_m/ft³ (kg/m³)
- ρ_{OA} = density of outside air, lb_m/ft³ (kg/m³)
- h_{RA} = enthalpy of return air, Btu/lb_{da} (kJ/kg_{da})
- h_{MA} = enthalpy of mixed air, Btu/lb_{da} (kJ/kg_{da})
- h_{OA} = enthalpy of outside air, Btu/lb_{da} (kJ/kg_{da})

The supply airflow rate in Equation 5 must still be measured directly by an averaging pitot-tube array or other direct airflow measurement technique. However, by neglecting the changes in the humidity ratio and specific heat, the enthalpy balance can be simplified to a temperature balance given by Equation 6. Errors associated with these simplifications will be addressed later in this paper.

$$\dot{V}_{OA} = \dot{V}_{SA} \cdot \frac{T_{OA}}{T_{MA}} \cdot \left(\frac{T_{RA} - T_{MA}}{T_{RA} - T_{OA}} \right) \quad (6)$$

where

T_{OA} = outside air dry-bulb temperature, °R (K)

T_{RA} = recirculated air dry-bulb temperature, °R (K)

T_{MA} = mixed air dry-bulb temperature, °R (K)

This approach has the advantage that it is easily incorporated into today's microprocessor-based control systems. Moreover, temperatures are easily measured with standard instrumentation, with the exception of the mixed air temperature. Indeed, the temperature of the mixed air must be measured before passing through any coils or fans. Hence, stratification of the air can make accurate temperature reading difficult to obtain (Drees et al. 1992).

Equation 6 indicates that large errors in the calculated outside air intake rate are possible when the value of $T_{RA} - T_{OA}$ becomes small. Drees et al. (1992) found experimentally that the calculated outside air intake rates were within 10% of full-scale reading using an averaging pitot-tube array reading. These results were somewhat limited since they are based on only two days of measurements.

Concentration Balance

Another indirect method for outside air intake rate measurement is using a concentration balance based on CO₂ concentration levels. Numerous papers have been published dealing with this topic, including Drees et al. (1992), Elovitz (1995), Janu et al. (1995), Ke and Mumma (1997a), Ke et al. (1997b), Meckler (1994), and Persily (1993). In the CO₂ concentration balance model, the outside air intake rate is based on a volume balance of the airstreams and is given by Equation 7. Similar to the enthalpy balance method, when the value of $CO_{2RA} - CO_{2OA}$ becomes small, errors in the calculated outside air intake rate become very large (Janu et al. 1995).

$$\dot{V}_{OA} = \dot{V}_{SA} \cdot \left(\frac{CO_{2RA} - CO_{2SA}}{CO_{2RA} - CO_{2OA}} \right) \quad (7)$$

where

CO_{2OA} = outside air CO₂ concentration, ppm

CO_{2RA} = recirculated air CO₂ concentration, ppm

CO_{2SA} = supply air CO₂ concentration, ppm

Other Airflow Measurement Techniques

Several other methods exist to measure airflow. Brief descriptions of several of these techniques are presented below. The reader is referred to the references for further information regarding these measurement techniques.

- **Fan inlet:** In this technique, flow rates are measured directly at the fan inlet mounted in the intake bell of the fans. This is not a new technique since flow rates are usually measured with an existing technology, such as pitot tube or thermal anemometer, but merely a new measurement location. The fan inlet is the location with

the highest airflow velocity, but the profile is extremely nonuniform at this point, with higher velocities located at the outside edges of the duct and lower velocities at the center (Kettler 1995).

- **Rotating vane and propeller anemometers:** These anemometers contain wind-driven wheels with mechanical, electrical, or magnetic pickups for measuring flow rates. Typical vane anemometers have an accuracy of 2%-5% for velocities from 100 fpm to 3000 fpm (0.51 m/s to 15.2 m/s) (ASHRAE 1997a; Dols and Persily 1995; Ower and Pankhurst 1977). However, they are very susceptible to changes in flow rates and the instrumentation is very delicate, which limits its usefulness for a typical air distribution system. Additionally, these devices require frequent calibration. Use of these anemometers is not recommended for measurements within ducts due to their relatively large surface area (ASHRAE 1988).
- **Swinging vane anemometers:** These devices contain a pivoted vane attached to a resistive hairspring. Deflections of the spring correlate to a reading displaced on an indicating scale. Total pressure readings are possible for a velocity range of 50-10,000 fpm (0.25-50.8 m/s) with a typical error of ±10% (ASHRAE 1988). Readings tend to be high on the suction side and low on the discharge side of a fan. Swinging vane anemometers are not recommended for critical measurements (ASHRAE 1988).
- **Vortex shedding meters:** These meters use the eddy-shedding principle to measure the velocity of airstreams. To create the eddies, or "Karmen Vortices," an object is inserted in the flow stream and piezoelectric pressure transmitters measure the periodic pressure fluctuations, which are then converted to velocities using calibration constants (Ower and Pankhurst 1977). While high accuracy can be achieved for flows with large Reynolds numbers (i.e., 0.5% at $Re = 10^4$), at lower flow rates this accuracy drops off to the point where these meters are no longer useful. The accuracy of the pressure transmitters used to measure the fluctuations further reduces the accuracy of the meter. As a rule of thumb, this method is not recommended for airflow rates of less than 500 fpm (2.54 m/s).
- **Integrated damper and measuring devices:** Typically, these units contain a pressure-based measurement device similar to an averaging pitot-tube array built into a damper. The measured flow can then be used to control the damper to maintain a constant flow rate. Errors and limitations on the use of these devices are similar to that of the averaging pitot-tube array.
- **Laser doppler anemometry (LDA):** LDA and a similar operating fiber optic system are very accurate measurement techniques but prohibitively expensive for both the required equipment and operation. Mease et al. (1992) provide a detailed description of the measure-

ment technique. Errors less than 1% at flow rates as low as 15 fpm (0.076 m/s) are possible with this measurement technique. Typically, these systems are used for calibrating other calibration systems (ASHRAE 1997a).

- **Orifice Meters:** Airflow rates can be measured using orifice meters. ASME Standard MFC-3M (ASME 1989) describes measurement of flow through pipes, ducts, and plenums for all fluids using the orifice, nozzle, and venturi. ASME Standard PTC 19.5 specifies their construction. The measurement of flow rates by orifice meters is determined by measuring the pressure difference across the orifice. The accuracy of orifice meters is 1% as long as the Reynolds number is above 500. The main limitation of the orifice meters is that the determination of the discharge coefficient and the accuracy of the measurement depend on the installation conditions.

Accuracy of Airflow Measurement Techniques

Based on the literature review and manufacturers' declared specifications, Table 2 summarizes the accuracy and the range of the direct measurement techniques for airflow rates discussed above. As will be discussed later, the accuracy of the indirect measurement techniques (e.g., enthalpy balance and concentration balance) depends on various factors, including the accuracy of both a direct measurement technique (to measure the supply airflow rates) and a measurement technique of the temperature (for the enthalpy balance) or of the CO₂ (for the concentration balance).

VAV CONTROL TECHNIQUES

Several techniques for controlling minimum outdoor intake rates in VAV systems have been used in the field or suggested in literature. In this section, detailed descriptions of control strategies tested in the laboratory are presented. For completeness, brief descriptions of several other control techniques are included. It is important to note that the following

descriptions and schematic diagrams apply to VAV system operation during minimum outside air intake mode. Control at other times may differ, especially during economizer use.

VAV Control Techniques for Economizer Systems

In economizer systems, the size of the outside air duct must be large enough to safely provide 100% of the design flow. This large size, however, results in very low airflow velocities during minimum outside air intake rate mode, which can make measurement difficult with pressure-based airflow measurement devices. Labeling of system diagrams presented here will follow the approach adopted by Kettler (1998) for consistency.

a. Fixed Minimum Outdoor Air Damper Position

A fixed outside air damper position is a common method used to meet minimum outside airflow intake rates in VAV systems. Under design flow conditions, the outside air damper is positioned to meet the minimum outside air requirements. This predetermined damper position is then used when only minimum outside airflow is required, even as the supply fan speed is reduced in the VAV system.

In VAV systems, this control method will not deliver the minimum outside air intake due to the variation in the static pressure of the mixing plenum (Drees et al. 1992; Mumma and Wong 1990). Outside air intake rates will be much closer to a constant percentage of supply air than a constant volume flow rate, a fundamental flaw of this control method (Janu et al. 1995). Another problem with this method is stack and wind effects on the outside air intake rate (Solberg et al. 1990). Because of the limitations inherent to the technique of using a fixed minimum outside damper position, Ke et al. (1997b) found through simulation that this method was the least effective control strategy to maintain a minimum outside airflow rate among those commonly used.

TABLE 2
Accuracy and Range of Airflow Measurement Techniques

Technique	Range	Accuracy	Comments
Pitot Tube	200 fpm - 9,000 fpm (1.0 m/s - 45.7 m/s)	1% - 5%	For low flows (200-600 fpm [1.0 - 3.0 m/s]), high accuracy DP is needed.
Thermal Anemometer	>1 fpm (0.005 m/s)	2% - 5%	Sensitive to turbulence. Needs frequent calibration.
Rotating Vane Anemometer	100 fpm - 3,000 fpm (0.5 m/s - 15.2 m/s)	2% - 5%	Susceptible to changes in flow rates. Needs periodic calibration.
Swinging Vane Anemometer	50 fpm - 10,000 fpm (0.2 m/s - 50.8 m/s)	10%	Not sufficiently accurate for OA measurements.
Vortex Shedding Meter	>500 fpm (2.5 m/s)	1% - 5%	Not accurate for low flow rates.
Integrated Damper/Measuring Device	200 fpm - 9,000 fpm (1.0 m/s - 45.7 m/s)	1% - 5%	Same errors and limitations as for Pitot-tube.
Laser Doppler Anemometer	1 fpm - 5,000 fpm (0.005 m/s - 25.4 m/s)	1% - 3%	Accurate at low rates. Too costly for field applications.
Orifice Meter	>20 fpm (0.1 m/s)	1% - 5%	Accuracy is affected by installation conditions.

b. Volumetric Fan Tracking

Figure 2 shows a schematic of the volumetric fan tracking system. The flow measurement stations, AFS-1 and AFS-2, measure the supply and return airflow rates, respectively. The return fan speed is controlled (AFC-1) to maintain a fixed differential in the return airflow rate compared to the supply airflow rate. The preset fixed differential in the return and supply airflow rates must then be made up by outside air. Damper positions for the return, exhaust, and outside air are generally set to fixed positions during minimum outside air intake mode. The fixed flow differential is typically based upon the initial system air balancing. The outside air provided to the space maintains a slight positive static pressure within the building to reduce unwanted infiltration.

With this control strategy, the differential air flow between supply air and return air is technically what is required to achieve the desired level of minimum building pressurization to minimize infiltration; however, it is not necessarily equal to the minimum outdoor air rate. In fact, this control strategy only works conceptually when the minimum outdoor air rate is equal to the amount of air desired for pressurization. The control strategy does not work if the exhaust air rate is not equal to zero during minimum outdoor air operation since the exhaust air rate is not measured. For instance, in an application requiring a large fraction of outdoor air, such as an assembly space, the minimum outdoor air rate may exceed that which will mildly pressurize the building. Some exhaust air is required to prevent overpressurization. In this case, the differential between supply air and return air will be less than the required outdoor air rate. Since the outdoor air and exhaust air rates are not directly measured, it is not possible to control the outdoor air at or above the minimum required level using the fan tracking control strategy.

Volumetric tracking is one of the more common control methods used in VAV systems today (Kettler 1995; Avery 1992). The benefit of this method is that airflow rates in the supply and return ducts are generally large enough that standard flow-measuring techniques can be sufficiently accurate. However, several authors have alluded to weaknesses in this control method. Elovitz (1995) states that even small measure-

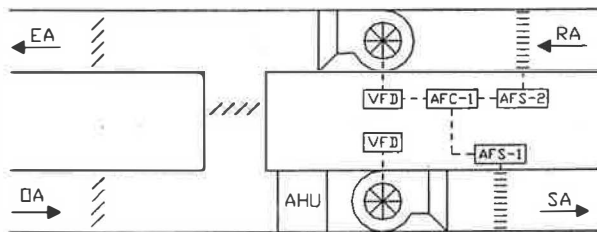


Figure 2 Outside airflow rate control schematic for a system using volumetric fan tracking control strategy.

ment errors in large flow rates can translate to large errors in the calculated outside air intake rates and that a fixed differential flow is not versatile enough to account for exhaust and leakage flow rate changes.

Using a fixed position for the outside air damper also limits the flow rates of outside air for space pressurization. If the damper is not sufficiently open, it is possible that there will not be enough outside air available (Janu et al. 1995). Janu et al. (1995) also recommend that online measurement of outside air intake rates be provided and that the differential flows vary to compensate for operation of variable exhaust flows and the opening and closing of windows and doors. Finally, Kettler (1995) makes the argument that when typical measurement errors are accounted for, the outside air intake rate can vary by as much as 35%.

c. Measurement and Control of Outside Airflow Rate with Economizer

A typical arrangement for this type of system is shown in Figure 3. The outside air duct is sized to allow for economizer control of the system. During minimum outside airflow intake mode, a flow measurement station (AFS-1) records the flow of outside air and controls the return and outside dampers (M-1 and M-2, respectively) to maintain the required minimum outside airflow intake rate.

Due to the relatively large size of the outside air duct in this system, the measurement of the outside air intake rate at the flow measurement station (AFS-1) can be difficult with pressure-based airflow measurement devices. The accuracy of this control technique depends directly upon the accuracy with which the outside air intake rate can be measured.

d. Plenum-Pressure Control

This method relies upon additional instrumentation, such as a manometer or differential pressure transmitter, to measure the pressure drop across a fixed orifice. By maintaining a constant pressure drop, the minimum outside airflow requirements can be met (Janu et al. 1995; Haines 1994; Elovitz 1995). It can be implemented either in a dedicated ductwork or in an existing economizer duct. The fixed orifice in this case is the combination of the outside air louver (L-1) and the

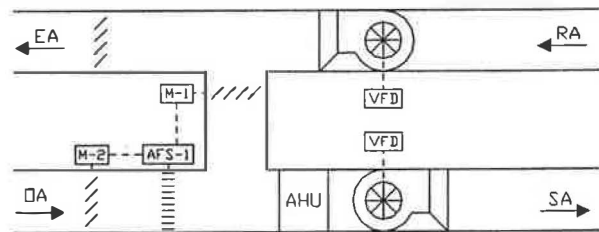


Figure 3 Outside airflow rate control schematic for a system with economizer damper.

damper installed in the outside air duct as suggested by Mumma and Wong (1990). This system is shown schematically in Figure 4. The pressure drop must be large enough so it can be accurately measured but not so large to create an excessive energy penalty (Ower and Pankhurst 1977; Kettler 1998). The differential pressure transmitter (DP-1) measures the pressure drop and the return air damper is controlled to maintain a constant value determined during an air balance test that results in the desired minimum outdoor air flow. Obviously, if an actuator is not located on the return air damper, one must be added.

For a fixed damper position, the value of the loss coefficient, C , for the damper is constant. The outside airflow intake rate is related to the pressure drop across the damper by Equation 8 (ASHRAE 1997b).

$$V = D \sqrt{\frac{\Delta p_j}{\rho \cdot C}} \quad (8)$$

where

- V = velocity, fpm (m/s)
- D = constant, 1096.7 (1.4123)
- Δp_j = total pressure loss, in. w.g. (Pa)
- ρ = density, lb_m/ft³ (kg/m³)
- C = local loss coefficient [-]

To maintain a constant airflow rate, the ratio of the pressure loss to the density of the airstream must remain constant. In practice, the effects of changing densities are usually neglected and only a constant pressure drop is maintained.

Moreover, it should be noted that the accuracy of the airflow measurement using the fixed damper position depends on whether a modulating damper or a separate minimum outdoor air damper is used (see the next section, item c). The pressure differential measurement is more accurate when using a separate minimum outdoor damper with only two positions (fully closed or fully open). The fully open position provides a reliable fixed orifice. However, an accurate fixed position achieved with a modulating damper can be difficult to obtain due to hysteresis and linkage slip.

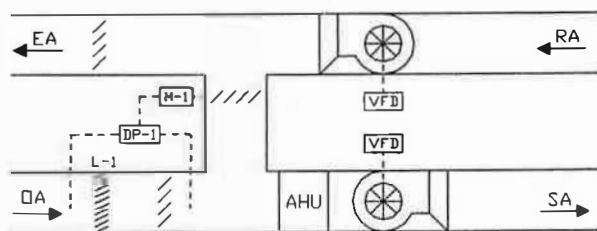


Figure 4 Plenum-pressure control schematic.

VAV Control Techniques for Systems with a Dedicated Outside Air Duct

The next two control strategies attempt to remedy the main disadvantage of an HVAC system equipped with only one outside air duct. By adding another duct through which only the minimum outside air must flow, the size can be made much smaller, thereby increasing the airflow velocities and making them easier to measure. Typically, the larger duct is used only during economizer control mode and is closed when minimum outside air intake rates are required.

a. Measurement and Control of Dedicated Minimum Outside Duct Airflow Rate

This system is shown schematically in Figure 5. In economizer mode, the damper on the larger outside air duct is controlled to regulate the outside air intake rate. During minimum outside air intake mode, the dedicated outside airflow intake duct is opened while the damper in the larger outside air duct is closed. A flow measurement station (AFS-1) records the outside airflow rate and controls the return (M-1) and the dedicated outside air dampers (M-2) to maintain the minimum outside airflow intake rate. The exhaust air damper can be left in a fixed position during minimum outside airflow intake mode or, alternatively, can also be controlled from the flow measurement station (exhaust damper control not shown in Figure 5).

b. Outside Air Injection Fan

This system is illustrated in Figure 6. A dedicated minimum outside airflow intake duct contains a fan used to control outside airflow whenever it is required. A flow measurement station (AFS-1) installed in the dedicated outside airflow duct measures the outside airflow rate. To maintain minimum outside airflow intake rates, the air flow in the dedicated duct can be controlled by either

- a variable-frequency drive (VFD) on the injection fan as shown in Figure 6, or
- a dedicated outside air damper (M-2) as illustrated in Figure 5. It may be argued that the VFD fan is not sufficient for two reasons: (1) the air will be pulled through the fan even when the fan is not operating since the

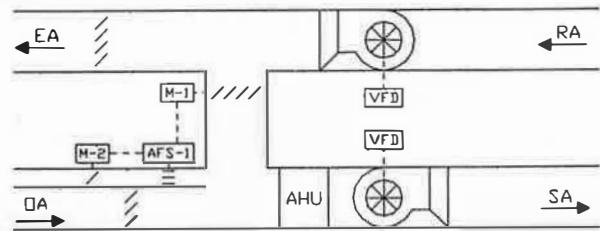


Figure 5 Outside airflow rate control schematic for a system with dedicated minimum outside airflow duct.

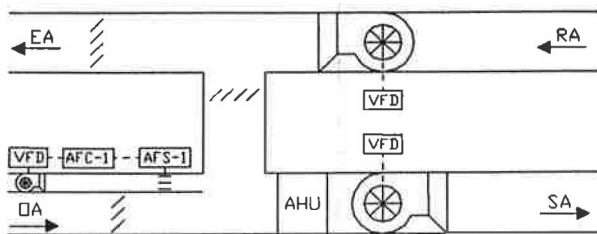


Figure 6 Outside airflow rate control schematic for a system with dedicated minimum outside airflow injection fan.

pressure in the mixing air plenum is negative, and (2) a damper is required by Standard 90.1 to prevent infiltration through the outdoor air intake when the air-handling unit is not operating. With the damper (M-2), the VFD is not required to control airflow (thus this control strategy is the same as the one illustrated in Figure 5).

The fan is chosen such that it has a very flat fan curve and operates almost as a constant volume fan over the expected range of pressures (Elovitz 1995; Avery 1989). In cases where the fan curve is very flat or the expected variation in the plenum pressure is small relative to the overall fan operating pressure, the volume rate of the fan may be sufficiently constant that an active outdoor air volume measurement and control system may not be necessary.

c. Plenum-Pressure Control

This system is identical to that described in the previous section (item a) except that the fixed orifice is a dedicated two-position minimum outdoor air damper (possibly in combination with the outdoor air louver) rather than a fixed damper position on the large economizer outdoor air damper. In cases where there is no outdoor air economizer and a return air damper does not exist, one must be added.

This design is generally more accurate than that described previously. This design is similar to that illustrated in Figure 6 (but without the AFS and with a controller modulating the return air damper). This particular design was used in the error analysis described in later parts of this paper.

Other VAV Control Techniques

Control strategies that were not tested in the laboratory are briefly described in this section. For more information on these techniques, the reader is directed to the cited references.

- **Minimum outside air damper position reset:** This control strategy attempts to compensate for the main limitation of the fixed outside air damper control strategy by allowing the damper position to be reset based upon the supply fan speed. The position of the outside air damper can be found from either a linear relationship

with the supply fan speed or a higher order polynomial equation. As with the fixed minimum outside damper position control method, online measurement of the outside air intake rates are not required. However, the minimum ventilation rate may not be met if the supply airflow rate falls too low (Ke and Mumma 1997a). Additionally, since the damper and duct are often the same size, small changes in damper position translate to large changes in flow rates (a highly nonlinear relationship) and normal hysteresis can significantly affect the outside air intake rates (Drees et al. 1992). Finally, this control strategy can not account for wind and stack effects on the system. See Solberg et al. (1990) for additional details regarding these errors.

- **Supply/return fan speed or vane position matching:** The supply and return fan speeds are controlled, often off the same control signal, to match each other with a fixed differential to maintain a slight positive pressurization. The outside airflow rate is then equal to the difference between the supply and return airflow rates. However, similar to the volumetric tracking control strategy, this is only true when there is no exhaust airflow. Whenever the exhaust flow is greater than zero, the outside air intake rate will be increased and an energy penalty may result. While this control method is inexpensive and easy to implement on existing systems, it has generally been unacceptable due to mismatched fan flow characteristics over the typical range of operation (Janu et al. 1995). Elovitz (1995) has also stated that this method is not versatile enough to account for all possible circumstances encountered in building operation, such as fume hoods and the opening and closing of windows and doors.
- **Characterization of flow through a modulated outside air damper:** By characterizing the outside air intake rate as a function of both the position and pressure drop across the damper, accurate control of ventilation air can be obtained over a wide range of operating conditions. However, this process requires significant amounts of time to properly characterize the airflow rates. In addition, this method is subject to calibration drifts in transmitters and positioners, as well as looseness and hysteresis, any of which can cause substantial errors (Janu et al. 1995).

ERROR ANALYSIS

Performing detailed error analyses for airflow measurement and outside air intake rate control techniques in variable air volume systems allows for a theoretical comparison of their practicality. Techniques that prove in theory to be invalid may not need to be tested further in a laboratory or implemented in a real building environment.

A search of existing literature on the topic of airflow measurement and VAV control error analysis provided relatively little on the subject. Solberg et al. (1990) have

performed detailed analyses regarding the effects of wind loading and stack effects on various control techniques commonly used today. Kettler (1995) also provides some rough error estimates for the volumetric fan-tracking control method.

Drees et al. (1992) provide graphs of predicted error for the indirect outside airflow measurement techniques using temperature and concentration balances based on return, mixed, and supply flows. Several simplifying assumptions are made in Drees's analysis that will be addressed in this paper.

Uncertainties

Propagation of uncertainty is the primary method used throughout this report to perform the error analysis. A brief description of this method is presented here for the reader.

Uncertainties in calculated and predicted results depend upon the uncertainty in the values used to find the result. If the value y is a function of independent variables (x_1, x_2, \dots, x_n), then the uncertainty in y can be determined by analyzing the propagation of uncertainty. Stated mathematically,

$$y = f(x_1, x_2, \dots, x_n). \quad (9)$$

The uncertainty in y, u_y , can be approximated from the uncertainties in the x values, (u_1, u_2, \dots, u_n), by Equation 10 (Taylor 1982). This method assumes that all the uncertainties are independent and occur with equal probability.

$$u_y = \sqrt{\left(\frac{\partial y}{\partial x_1} u_1\right)^2 + \left(\frac{\partial y}{\partial x_2} u_2\right)^2 + \dots + \left(\frac{\partial y}{\partial x_n} u_n\right)^2} \quad (10)$$

According to the ANSI/ASME standard on measurement uncertainty, there are three sources of uncertainty in airflow measurements: calibration, data acquisition, and data reduction (ASME 1983). Uncertainties associated with data reduction (curve fits and rounding) are neglected in the following analyses as these errors are often negligible.

Airflow Measurement Technique Error Analyses

Included in this section are error analyses for the proposed airflow measurement techniques to be tested in the laboratory. The method for calculating the expected error for each measurement strategy is described.

Direct Airflow Measurement Techniques

Nonuniform flow profiles downstream of disturbances such as elbows in the ductwork can severely affect the accuracy of direct airflow measurement techniques. To account for this effect, measurement devices must be installed according to manufacturers' recommendations or enough measurements must be made across a traverse to get the true airflow velocity.

a. Averaging Pitot-tube arrays

The airflow velocity using an averaging pitot-tube array is calculated from Equation 3. In buildings where the atmo-

spheric pressure is not measured, using the annual average atmospheric pressure in Equation 3 introduces errors of less than 0.30% on average. The error in the calculated airflow velocity, u_V , is found by substituting Equation 3 into Equation 10:

$$u_V = \sqrt{\left(\frac{\partial V}{\partial p} u_p\right)^2 + \left(\frac{\partial V}{\partial V_p} u_{V_p}\right)^2 + \left(\frac{\partial V}{\partial T} u_T\right)^2}. \quad (11)$$

An important consideration when using an averaging pitot-tube array measurement station is the proper selection of the differential pressure transmitter. By far, the largest source of error in this analysis is that introduced by the accuracy of the differential pressure transmitter. The range of the transmitter should closely match the expected range of velocity pressures for the duct. Additionally, the higher the accuracy of the transmitter, the larger the range of velocity pressures that can be measured with reasonable accuracy. Figure 7 shows predicted errors for airflow measurements using three different differential pressure transmitters, each with an accuracy of 1% of full-scale reading. As illustrated in Figure 7, a smaller range for the transmitter allows for accurate measurements of lower airflow rates.

b. Electronic Thermal Anemometry

Uncertainties associated with measurement of the airflow velocity using electronic thermal anemometry are also shown in Figure 7 using the manufacturers' declared accuracy for the calibration error. More accurate values for this error were unavailable due to the proprietary factory calibration procedure of these devices. Table 3 lists the values for these uncertainties. For airflow rates between 200 fpm (1.02 m/s) and 400 fpm (2.03 m/s), the electronic thermal anemometers provide significantly more accurate measurements than the averaging pitot-tube arrays unless high-accuracy, auto-zeroing differential pressure transmitters are used.

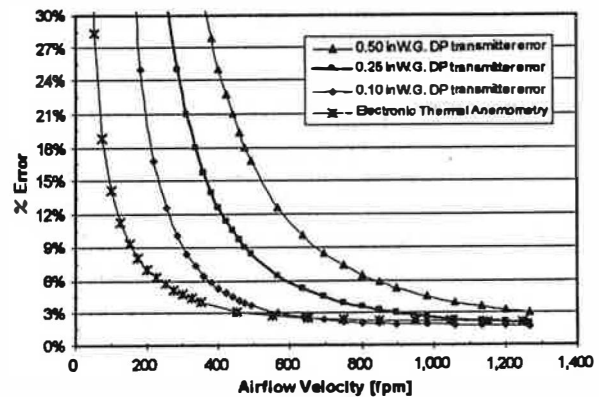


Figure 7 Predicted errors for direct airflow measurement techniques.

TABLE 3
Electronic Thermal Anemometer Uncertainties

$u_{calibration}$	± 10 fpm (0.051 m/s) for flows < 500 fpm (2.54 m/s), $\pm 2.0\%$ of reading for flows > 500 fpm (2.54 m/s)
$u_{resolution}$	$\pm 0.4\%$ of full-scale reading (2500 fpm [12.7 m/s])

Indirect Airflow Measurement Techniques

This section includes the expected errors for the two different indirect airflow measurement techniques; an enthalpy balance and a CO₂ concentration balance.

a. Enthalpy Balance

As stated earlier, some errors are introduced by approximating the enthalpy balance shown in Equation 5 with the temperature balance shown in Equation 6. Errors up to 4% can be obtained when the effects of humidity are neglected, but errors for typical building operating conditions are usually less than 1.5%. The measurement of the outside and return air temperatures can typically be accomplished with sufficient accuracy using sensors commonly available today. The measurement of the mixed air temperature, however, can be quite difficult. Due to inadequate mixing of the airflows, stratification can occur in the mixed airflow, making temperature measurement difficult even when averaging temperature sensors are used. Quantifying the error associated with the measurement of the mixed air temperature is difficult and is the focus of an ongoing research project, ASHRAE RP-1054. For purposes of this analysis, the uncertainty in the mixed air temperature measurement is assumed to be 3% of the measured value. However, this error may be less when the outside and return air temperatures are close to one another or greater when the temperature difference is large.

Measurement of the supply airflow rate is required to calculate the outside airflow rate in Equation 6. In most HVAC systems, it is easier to locate a suitable location for the direct measurement of the supply airflow rate than for the outside

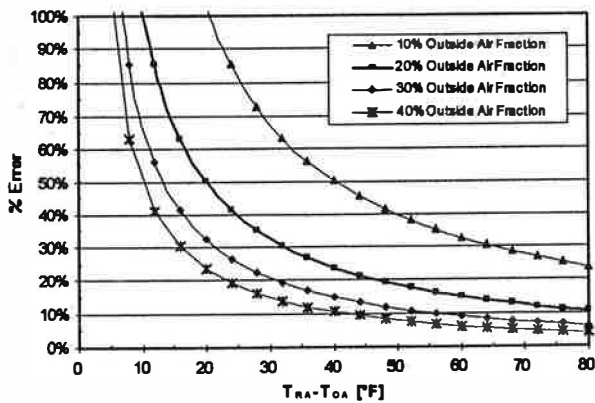


Figure 8 Predicted errors of temperature balance airflow measurement technique.

airflow rate. Errors associated with the supply airflow measurement must be accounted for when considering the accuracy of the temperature balance airflow measurement technique. Figure 8 shows the predicted errors for the temperature balance measurement technique but neglects errors associated with the supply airflow measurement. Despite this assumption, errors for the calculated outside airflow rate are still quite high. The large errors during times of low outside air fractions and small temperature differences between the recirculated and outside airflows show that this airflow measurement technique is not valid under typical building operation conditions.

b. Concentration balance

The concentration balance airflow measurement technique expressed by Equation 7 is performed using one sensor to measure all three CO₂ concentration values. Using multiple CO₂ sensors to determine the outside airflow rate is not possible due to the relatively large error associated with the absolute accuracy of commonly available sensors. When only one sensor is used, however, the absolute errors cancel out of Equation 7. The only source of error associated with the sensor then becomes its repeatability. The use of only one sensor, however, increases the time required to calculate the outside airflow rate. Each airflow must be sampled by the sensor before the outside airflow rate can be calculated, and each airflow typically requires two to three minutes to be measured with reasonable accuracy.² However, this requirement for relatively stable CO₂ concentrations limits the applicability of the concentration balance technique. In spaces where large, abrupt changes in occupancy (and, hence, CO₂ levels) can occur, this method may prove unreliable. This fact may rule out the use of this control strategy in spaces such as conference rooms and auditoriums or any building where large transient effects are possible. Typical office space should present a suitable application of the control technique using CO₂ balance.

Available sensors typically have repeatability errors on the order of 20 to 40 ppm, but this repeatability value is typically based on a one-year period. The repeatability within the time frame required for calculation of the outside airflow rate from Equation 7 is closer to 2 to 5 ppm. This significantly reduces the error in the calculated outside airflow rate. Errors associated with the supply airflow measurement and data acquisition must still be considered.

Figure 9 shows the predicted errors in the calculation of the outside airflow intake using the CO₂ concentration balance airflow measurement technique. These errors include a 5% uncertainty in the supply airflow measurement and a repeatability error of ± 3 ppm for the CO₂ sensor. The predicted errors indicate that the concentration balance airflow measurement technique may be valid except when occupancy is low or when the difference in the recirculated and outside air

². Testing in the laboratory showed sensor repeatability to be ± 3 ppm when three minutes were allowed for the CO₂ concentration measurement of each airflow.

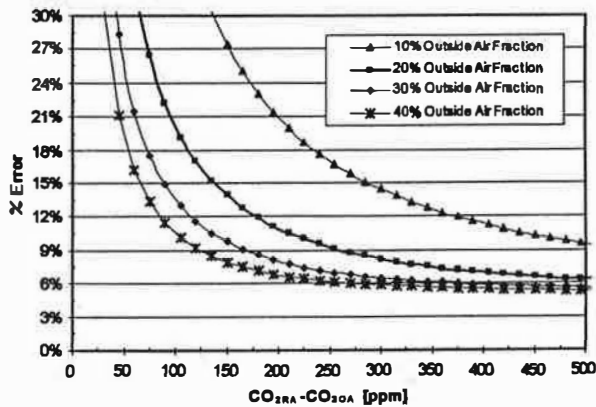


Figure 9 Predicted errors for CO_2 concentration balance airflow measurement technique.

CO_2 concentrations levels is small. Additionally, when the outside air represents a small fraction of the total supply air provided, errors in the calculated outside airflow may become too large for reliable and accurate use.

VAV Control Techniques Error Analyses

Presented in this section are error analyses for the VAV control techniques tested in the laboratory (Krtati et al. 1999).

Fixed Minimum Outside Air Damper Position

In a system where all damper positions are fixed, the outside airflow rate is proportional to the square root of the pressure difference across the outside air damper. In a VAV system, as the supply fan speed is reduced, the pressure in the mixing plenum is also reduced. Due to this reduction in pressure difference across the outside air damper, the outside airflow intake rate is also reduced. In the best case scenario, the outside airflow intake rate in a fixed minimum outside air damper system will be a constant percentage of the supply airflow. Several other authors have identified additional drawbacks to this type of control system (Janu et al. 1995; Solberg et al. 1990; Mumma and Wong 1990). Figure 10 shows the theoretical outside airflow intake rate vs. the supply airflow rate under ideal conditions. For this analysis, no calibration or acquisition errors were considered. Additionally, stack and wind effects on the system were neglected. Obviously, the use of a fixed minimum outside air damper position as a technique for maintaining minimum ventilation rates in a VAV system is impractical.

Volumetric Fan Tracking

The inadequacy of this control technique to maintain minimum outside air intake rates has been addressed by other authors (Kettler 1998; Janu et al. 1995). A system identical to

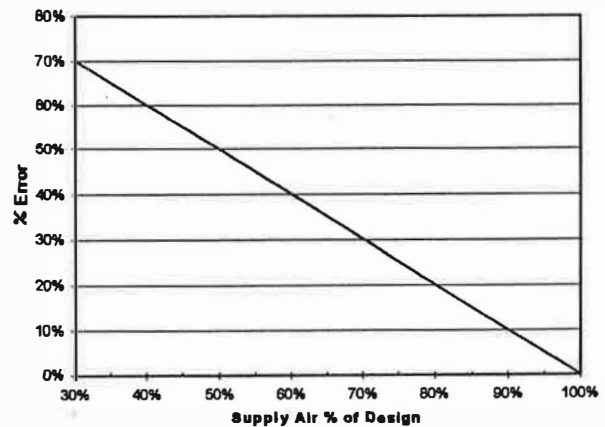


Figure 10 Predicted errors for the fixed damper position control technique.

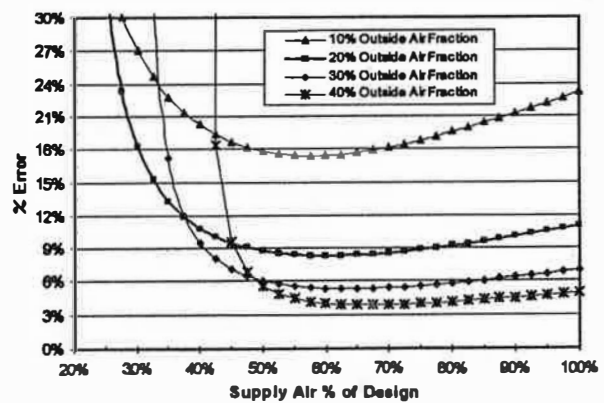


Figure 11 Predicted errors for volumetric fan tracking.

that described by Kettler (1998) will be analyzed here to illustrate these findings. Predicted errors for a system with design airflow rates of 4000 fpm (20.3 m/s) for the supply duct and 2000 fpm (10.2 m/s) for the return duct are shown in Figure 11.

Errors illustrated in Figure 11 assume that the airflow measurements were made with averaging pitot-tube arrays, using differential pressure transmitters with ranges of 0-1 in.w.g. (0-249 Pa) and 0-0.25 in.w.g. (0-62.25 Pa) for the supply and return ducts, respectively. Each transmitter was assumed to have an accuracy of 1% of full scale. Errors due to nonuniform flow profiles were ignored, as were those introduced when there is any exhaust flow. Even by neglecting these errors, which would be unavoidable in a real system, volumetric fan tracking is unable to maintain minimum outside airflow rates for operating conditions typically found in VAV systems.

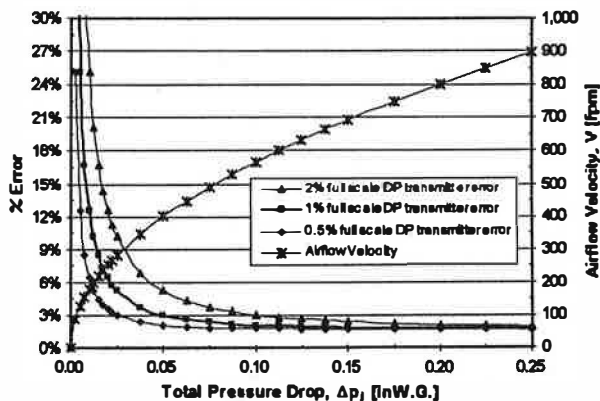


Figure 12 Predicted errors of plenum pressure control.

Plenum Pressure Control

The outside air intake rate is related to the pressure drop across a fixed orifice by Equation 8. When the outside air damper is left in a fixed position, the value of the loss coefficient for the outside air duct remains constant. For the purpose of this analysis, a value of 5 was assumed for the local loss coefficient, C . Predicted errors for the plenum pressure control technique at standard atmospheric conditions are shown in Figure 12. Errors are shown for three different differential pressure transmitters. All three transmitters were assumed to have a range of 0-0.25 in. w.g. (0-62.25 Pa), but the accuracy of each transmitter varied from 0.5% of full scale to 2% of full scale. Results of the analysis show that with only a minimal pressure drop, accurate control of the outside air intake rate can be achieved.

Measurement and Control of Outside Airflow Rate

The next three control strategies—measurement and control in an economizer duct, a dedicated outside air duct, and a dedicated duct with an injection fan—require the measurement of the airflow rate and the positioning of dampers to maintain the minimum outside airflow rate. Actuators used to position the damper have an error affiliated with them, and the dampers also have errors associated with their linkages and hysteresis effects. With the use of PID control, however, these errors associated with dampers become precision errors and, hence, the average of them is zero. The only remaining source of error is that of the outside airflow measurement.

For control strategies that utilize the direct measurement of the outside airflow rate, predicted errors are the same as those shown in Figure 7. In systems where the outside airflow rate is calculated using the CO_2 concentration balance technique, predicted errors are the same as those illustrated in Figure 9.

When a dedicated outside air duct is available, any desired degree of accuracy may be obtained by sizing the dedicated air duct to obtain the necessary flow rate. The only requirement is that the face velocity of the airflow at the outside louver should be less than 400 fpm (2.03 m/s) to minimize the possibility of water penetration (ASHRAE 1997b). This analysis also applies when an injection fan is installed in the dedicated outside air duct. However, the use of an injection fan may require longer lengths of duct to establish uniform airflow profiles necessary for accurate measurement using an averaging pitot-tube array or electronic thermal anemometer.

CONCLUSIONS

Accurate measurement and control of outside air intake rates in VAV systems are possible when careful attention is paid to proper installation and operation of system equipment. In systems where uniform airflow profiles exist, the use of an averaging pitot-tube array or an electronic thermal anemometer, depending upon the expected velocities, for the direct measurement of outside airflow rates allows for direct control of minimum outside air intake rates. When these conditions are not met, the installation of a separate, dedicated minimum outside air duct, or the use of the concentration balance airflow measurement technique, provides adequate alternatives. However, calculating the outside airflow rate using a temperature balance will not provide accurate results for all building operating conditions. Plenum pressure control in systems where measurement of the outside airflow rate is not possible should provide adequate control of minimum outside air intake rates. The traditional CAV control strategy of a fixed minimum outside air damper position and the more robust volumetric fan tracking technique are not capable of accurately controlling outside airflow rates in VAV systems.

ACKNOWLEDGMENTS

The authors would like to thank ASHRAE, the sponsor of the research project RP-980, and ASHRAE Technical Committee 1.4, which initiated the project. Particular acknowledgement is due to the members of the project monitoring subcommittee, S. Bushby, J. Kettler, F. Mayhew, and S. Taylor.

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