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Velocity distribution in a room ventilated by displacement ventilation and wall-mounted air terminal devices

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Abstract

The article describes experiments with wall-mounted air terminal devices. The airflow from an air terminal device influences the occupants' thermal comfort and, therefore, it is important to develop an expression for this flow in the occupied zone. The velocity at the floor is influenced by the flow rate to the room, the temperature difference and the type of diffuser. The flow is stratified at Archimedes numbers larger than four. The article gives expressions for the velocity distribution close to the floor. It is shown that openings between obstacles placed directly on the floor generate a flow similar to the air movement in front of a diffuser, and expressions for the velocity distribution in that situation are also given in the article. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Displacement ventilation; Air terminal device; Velocity distribution; Stratified flow

1. Introduction

For many years ventilation systems with vertical displacement flow have been used in industrial areas with high thermal loads. The displacement flow systems have recently grown popular as comfort ventilation in rooms with thermal loads as, e.g., offices where the air is supplied directly into the occupied zone at low velocity from wall-mounted diffusers. The plumes from hot surfaces, equipment and persons entrain air and create a natural convection flow upwards in the room, see Fig. 1. The displacement flow system has the following characteristics which are different from the traditional mixing system.

• The system is energy efficient because it is possible to remove exhaust air from the room where the temperature is several degrees above the temperature in the occupied zone. This allows a higher air inlet temperature at the same load.

• The system can obtain an appropriate distribution of the pollution in the air. The vertical temperature gradient and the stratification imply that fresh air and polluted air are separated, and that the pollution can be found above the stratification height when the sources are combined heat and pollution sources.

• The vertical temperature gradient in a room ventilated by displacement ventilation may cause discomfort if the gradient exceeds a certain level. It is recommended to keep the gradient below 3 K/m.

The research described in this article is focused on the flow from wall-mounted low velocity air terminal devices. It is the aim of this work to obtain results which can simplify and improve the practical design procedure. In this connection it is important to examine the flow in front of an air terminal device and to investigate if this flow in practice can be treated unconnectedly with parameters as room geometry, heat source location and location of exhaust opening, etc. The design procedure is simplified if the flow depends on main parameters only as, e.g., type of diffuser, obstacles on the floor, flow rate and Archimedes number of the flow and, furthermore, if the influence of the width and the length of the horizontal section is insignificant. The expectation of this simplification is indicated by the dotted line in Fig. 1. An equivalent situation is known in mixing ventilation where the flow from air terminal devices can be described relatively independent of the recirculating flow in the room. It is also important to obtain a quantitative description of the flow along the

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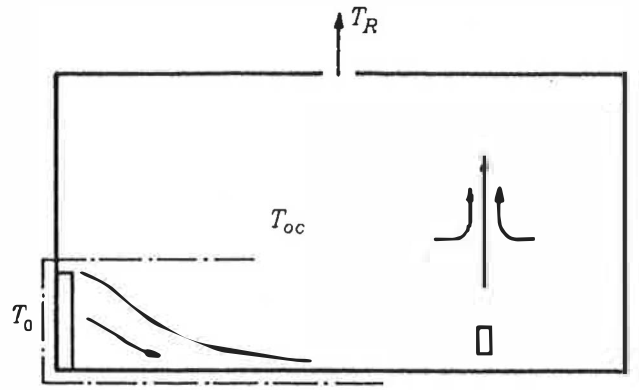

Fig. 1. Room with low-level diffuser, heat source and displacement flow.

floor. In a room with buoyancy driven ventilation, this flow is the air movement which influences the occupants' comfort, and a description of this air movement will therefore make it possible to obtain a detailed picture of the thermal comfort of the room.

2. Wall-mounted low velocity diffuser

Fig. 2 shows the wall-mounted low velocity diffusers which are tested and discussed in this article. The diffusers are of different designs and they cover flow rates of 50–300 m^3/h, except diffuser type F which is designed for a flow rate of 500–1400 m^3/h.

Diffuser type A has a supply velocity profile which is very constant over the entire supply area, while diffuser type B has a supply velocity with a large variation over the supply area both in speed and in direction (see Ref. [1]).

Diffuser type D can be adjusted to two different modes. It can work either without induction D_1 or with an internal induction unit D_2, which increases the flow rate at the diffuser surface with a factor of 2.5 compared with the supply flow q_o. The supply temperature T_o will be increased accordingly. The diffuser with the induction unit is especially used for displacement ventilation in systems generally designed for mixing ventilation (low flow rate and high temperature difference). The diffuser generates a semiradial flow at the supply surface.

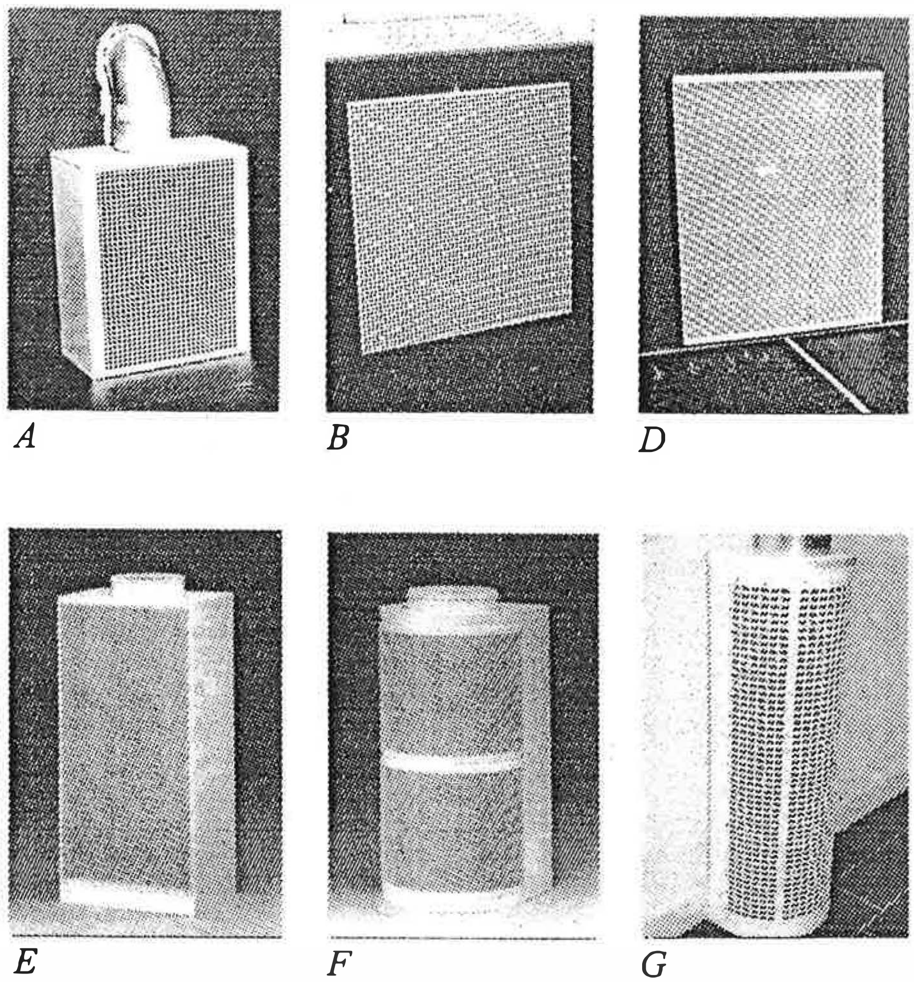

Fig. 2. Six different wall-mounted low velocity diffusers for displacement ventilation.

Table 1
Area a_f and height h of the six different low velocity diffusers

Diffuser	A	B	D₁	E	F	G
a_f (m²)	0.159	0.306	0.437	0.267	1.293	0.188
h (m)	0.48	0.58	0.73	1.00	1.42	0.56

Diffuser type E is a conventional diffuser for displacement ventilation without any devices for the generation of radial flow at the supply surface.

The experiments with diffuser F are mainly carried out to test the influence of the diffuser size. The diffuser is designed for a flow q_o of 500–1400 m³/h, but it is tested in the range of 100–200 m³/h. The flow from the diffuser is radial.

Diffuser type G generates a radial flow at the supply surface. The velocity distribution is varying over the surface from 70 to 140% of the nominal face velocity.

The flow from the diffusers is either given by the flow rate q_o or by a face velocity u_f calculated from

$$u_f = \frac{q_o}{a_f} \tag{1}$$

where a_f is the surface area of the perforated part of the diffuser. u_f is easy to calculate, but it is different from the supply velocity measured in the openings of the diffuser.

The height h of the different diffusers may be an important parameter because the cold flow is influenced by vertical acceleration due to gravity. The height and the area of the diffuser are given in Table 1.

The Archimedes number Ar for a flow is given by

$$\mathrm{Ar} = \frac{\beta g h (T_{oc} - T_o)}{u_f^2} \tag{2}$$

where β, g and $(T_{oc} - T_o)$ are volume expansion coefficient, gravitational acceleration and temperature difference between the temperature at a height of 1.1 m and the supply temperature, respectively. T_{oc} is measured in the middle of the room, but it is rather independent of the location due to the temperature stratification in the room.

3. Flow from a wall-mounted diffuser

The flow from three different wall-mounted diffusers is shown in Fig. 3. The maximum velocity u_x close to the floor is given as a function of the distance x to the diffuser. The cold air from supply diffuser A has a high initial acceleration due to the buoyancy effect, and a velocity of 0.34 m/s is obtained at a distance of 0.8 m from the diffuser. Type B has a larger diffusion of the supply flow and the gravity will only increase the velocity to 0.23 m/s. The diffuser type G shows an even lower velocity level, although the flow to the room is almost the same in all three situations. Fig. 3 indicates that the

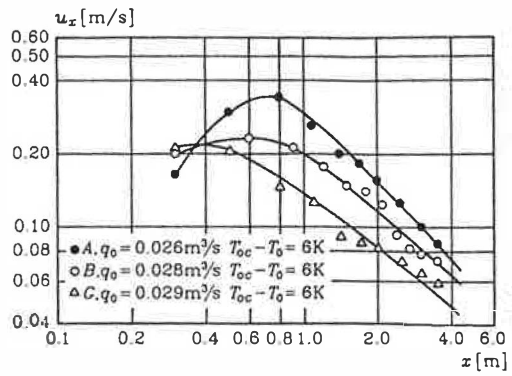

Fig. 3. Maximum velocity close to the floor vs. distance x.

maximum velocity in the symmetry plane is proportional to $1/x^n$, where the exponent n is close to 1.0, as pointed out by Nielsen et al. [2]. It is also obvious from Fig. 3 that different diffuser designs generate a different velocity level at the same flow rate and heat load.

The velocity at the floor is not only influenced by the flow rate to the room and the type of diffuser. Fig. 4 shows that the Archimedes number is an important parameter. A 3 K increase in temperature difference will, for example, increase the velocity from 0.10 m/s to 0.12 m/s at a distance of 2 m from the diffuser. The figure shows that the gravity accelerates the flow close to the diffuser and gives a high initial velocity level at large Archimedes numbers. This effect is very important for the flow in rooms with displacement ventilation and the outcome can be surprising. The velocity level in a room may for example be uninfluenced, although the flow rate is reduced because the heat load in the room requires a reduction of the supply temperature and consequently an increase in the relative velocity level u_x/u_f (see Ref. [3]).

Some of the tested diffusers discussed in this article generate a velocity decay of $1/x^n$ where n is slightly different from 1.0. Fig. 5 shows an example where the measurements correspond to $n = 1$, only for $x > 2.0$ m. The presence of a virtual origin located at some distance x_o behind the diffuser can explain the velocity decay

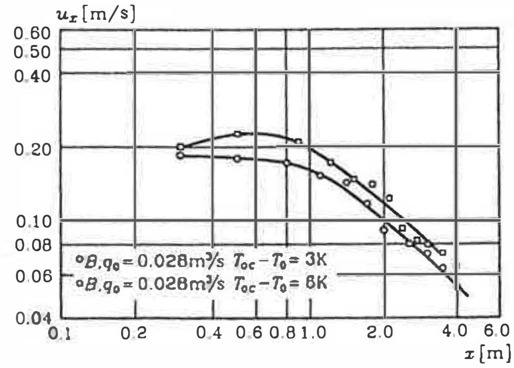

Fig. 4. Velocity decay along the floor at different Archimedes numbers [3].

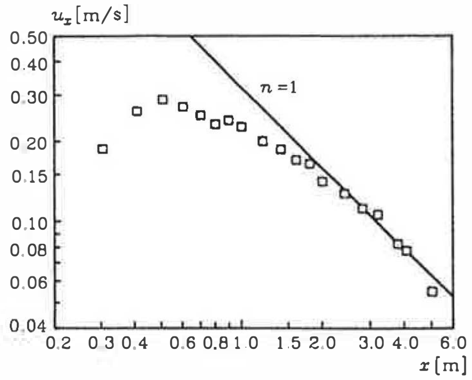

Fig. 5. Velocity decay in the flow from a wall-mounted air terminal device type D_I. $q_0 = 0.028$ m^3/s and Ar = 45.8.

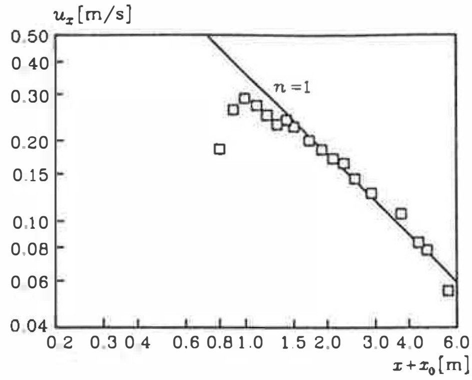

Fig. 7. Velocity decay in the flow from diffuser D_I vs. $(x + x_0) \cdot q_0 = 0.028$ m^3/s, Ar = 45.8 and $x_0 = 0.5$ m.

shown in Fig. 5. Fig. 6 gives the flow direction measured by smoke close to the floor. The conditions for the experiment are almost the same as the conditions in Fig. 5. The figure shows both that the flow close to the symmetry line has a location of the virtual origin corresponding to $x_0 \sim 0.5$ m, although the measurements are rather scattered, and that the general flow is radial, even rather close to the side walls.

Fig. 7 shows the earlier given velocities u_x vs. $x + x_0$ where $x_0 = 0.5$ m. It can be seen that the velocity decay is close to $1/(x + x_0)$ for $x > 1.5$ m, which indicates that the measurement of a virtual origin will improve the presentation of the results close to the diffuser. Many measurements show, however, that most of the diffusers have virtual origins which are located very close to the surface of the diffusers which leads to a small x_0. The influence of a small x_0 is only important close to the diffuser.

Velocity measurements show that the flow in the vicinity of the floor can be characterized by a normalized velocity profile identical to the universal profile used for the description of a wall jet flow (see Refs. [4,5]). Fig. 8 shows a number of measured profiles normalized with respect to the maximum velocity u_x in the profiles and thickness δ of the profiles. The length scale δ is defined as the distance from the floor to the height where the velocity has a level which is half of the maximum velocity close to the floor, $u_x/2$. The curve in Fig. 8 shows a universal wall jet profile $u/u_x = f(\eta)$, where $\eta = y/\delta$ (see Ref. [6]).

Fig. 9 shows the development of δ for three different Archimedes numbers. It can be seen that the height of the flow region is much smaller than the height of the diffuser, even at a distance of 0.5 m from the diffuser. The cold air from the diffuser accelerates towards the floor due to gravity and it behaves as a stratified flow in its further progress along the floor. The length scale δ has a rather constant level compared with the development of δ in a wall jet, where it is proportional to x as indicated by the dotted line in Fig. 9. The figure shows that the length scale will decrease at increasing Archimedes number.

The entrainment of air into the flow, or the turbulent mixing process, will be low when a vertical temperature gradient is present because the gravity will work against both an upward movement of heavy fluid and a downward movement of light fluid. This is shown in hydraulics by, for example, Turner [7], and in displacement ventilation by

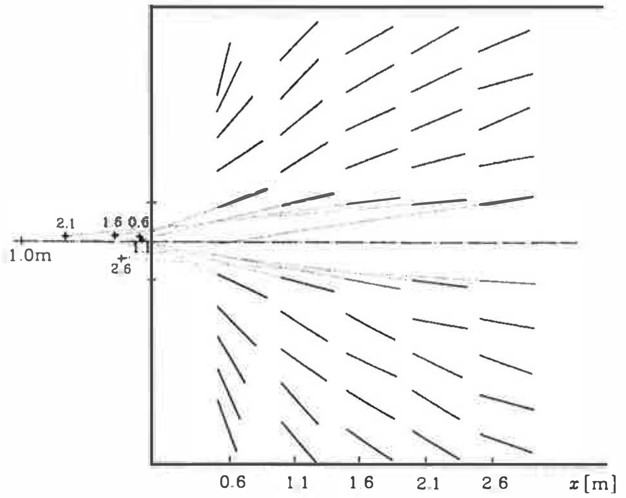

Fig. 6. Flow directions at the floor. Diffuser type $D_I \cdot q_0 = 0.028$ m^3/s and Ar = 47.

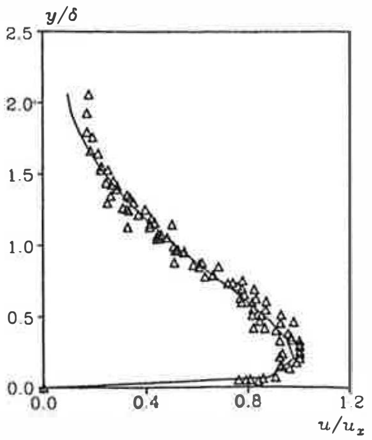

Fig. 8. Measured velocities compared with a universal wall jet profile. Ar = 1.4.

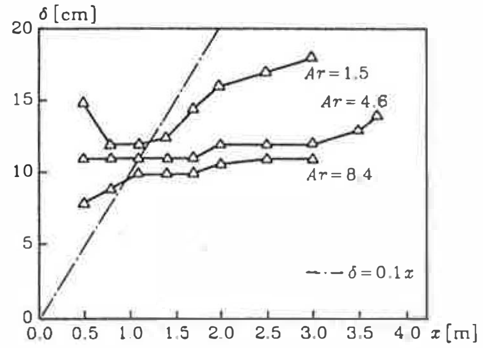

Fig. 9. Length scale δ in the flow vs. distance from the diffuser. Diffuser type G.

Jacobsen and Nielsen [5] (see Appendix A for a further discussion).

The assumption of low entrainment, constant length scale and the knowledge from measurements that the flow is radial will be used in the following equations. Fig. 10 shows a small section $\Delta\theta$ of this flow which has a virtual origin located at a distance of x_o from the diffuser. The flow $q_{\Delta\theta}$ within the section $\Delta\theta$ is given by

$$q_{\Delta\theta} = \Delta\theta(x + x_o)\int_o^{\infty} u\,\mathrm{d}y \tag{3}$$

where $\Delta\theta\ (x + x_o)$ is the width of the section and u is the velocity at the height y. The velocity profile can be given by the normalized profile $f(\eta)$

$$q_{\Delta\theta} = \Delta\theta(x + x_o)\delta u_x \int_o^{\infty} f(\eta)\,\mathrm{d}\eta \tag{4}$$

$q_{\Delta\theta}$ will be constant and independent of the distance $x + x_o$ for a fully stratified flow with constant δ and low entrainment.

The flow rate $q_{\Delta\theta}$ will be proportional to q_o, initial entrainment close to the diffuser and to $\Delta\theta/\theta_o$, where θ_o is the angular width of the radial flow close to the diffuser (see Fig. 10). The stratified flow will obtain the radial pattern partly due to the gravity effect and partly due to the construction of the diffuser

$$q_{\Delta\theta} = \frac{\Delta\theta}{\theta_o} q_o e b_m \tag{5}$$

e is a factor which represents the initial increase in flow rate due to entrainment in the accelerating flow close to the opening, and b_m is a factor which adjusts the flow in the direction $\theta = 0$ to the flow profile generated by the diffuser. e and b_m may both be a function of the Archimedes number.

Eqs. (4) and (5) can be written as

$$\frac{u_x}{q_o} = K_{Dr}\frac{1}{x + x_o} \tag{6}$$

where

$$K_{Dr} = \frac{e b_m}{\theta_o \delta \int_o^{\infty} f(\eta)\,\mathrm{d}\eta} \tag{7}$$

K_{Dr} is independent of the distance $x + x_o$, but it is a function of the Archimedes number as well as an individual function of different types of air terminal devices. Both x and u_x are defined in the symmetry plane of the flow.

The development of Eq. (6) implies a high Archimedes number, but the structure is also valid for cases where the number is very small. In that case, the flow will be part of a potential core or part of a three-dimensional or a radial wall jet. The velocity will in most cases be proportional to $1/(x + x_o)$, and Eq. (6) will therefore be able to predict the velocity u_x when the K_{Dr}-value is adjusted to the situation.

A large number of experiments have been made to establish K_{Dr} as a function of the Archimedes number, and the results are shown in Fig. 11. It is obvious that K_{Dr} may be very different for different products as it varies from 5 to 13 m^{-1} at high Archimedes numbers. The figure shows also that K_{Dr} increases with increasing Archimedes number. This is due to the fact that gravity will accelerate the vertical flow close to the opening and generate a stratified air movement in a relatively thin layer along the floor where the obtained velocity level will be retained. This effect is also shown in Fig. 4.

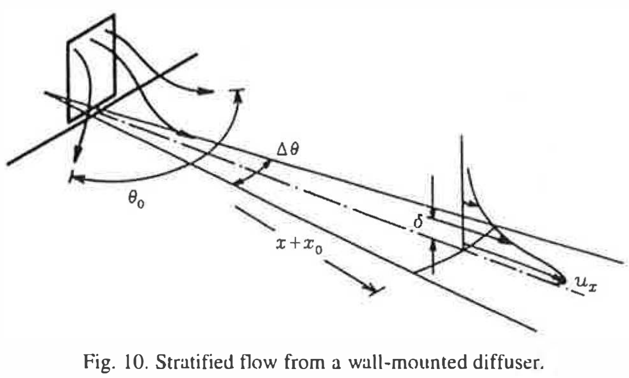

Fig. 10. Stratified flow from a wall-mounted diffuser.

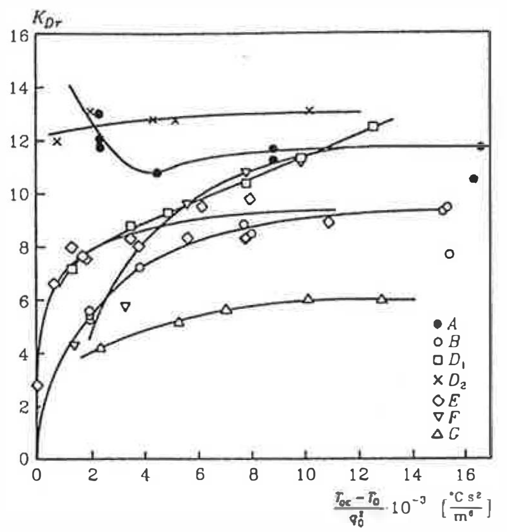

Fig. 11. K_{Dr} vs. temperature difference and supply flow rate for seven different wall-mounted air terminal devices. $x_o = 0.0$.

Diffuser A shows an increase in velocity at small Archimedes numbers which in this situation can be explained by the decrease in the radial flow (θ_o).

Eq. (6) can only be used at some distance from the diffuser as it appears in Figs. 3, 4 and 7. This distance is 1.0 to 1.5 m for most of the diffusers.

It is typical that all diffusers, except diffuser F, are designed for rooms with a size comparable to the size of the test room. It might be concluded that the diffusers are tested under conditions and with dimensions close to the conditions they are meant to cover in practice, and that the velocity level given by the variable K_{Dr}, in Fig. 11, is therefore typical of a practical application.

Fig. 11 shows a large variation in the level of K_{Dr} for the individual products. The influence of different parameters is given in Eq. (7). It is possible to make statements on the design of a diffuser from the influence of these parameters. The following values:

$$\int_o^\infty f(\eta)\mathrm{d}\eta \sim 1.1 \quad \delta \sim 0.1\,\mathrm{m} \quad \theta_o = \pi$$

$$e \sim 2.5 \text{ and } b_m \sim 1$$

will give a K_{Dr}-value of ~ 7. Many of the early designs of diffusers have a radial distribution of the flow with a relatively high level in the symmetry plane as, e.g., a b_m-value of 1.5. This will, in the above-mentioned example, give a K-value of 11, in good agreement with the measurements in Fig. 11. A further increase in the velocity level will be obtained by a design where θ_o is smaller than π, which also is typical of an early diffuser design.

New designs will have a lower flow in the direction into the room perpendicular to the wall and a higher flow parallel with the diffuser wall. This can for example be expressed by a b_m-value of ~ 0.85 giving a K-value of ~ 6. This level is typical of the diffuser type G, and this diffuser does in fact have a restricted flow in the direction $\theta = 0$ compared with the flow in other directions.

Velocity distribution in rooms ventilated by displacement ventilation is also discussed by Mathisen [4], Sandberg and Holmberg [8], Sandberg and Mattsson [9] and by Skistad [15].

It is known from stratified flow in hydraulics that obstacles located downstream may influence the length scale δ of the flow (see Ref. [7]). Most of the measurements are made in test rooms of equal size, so it is difficult to determine the influence from the end wall and the side walls, but practical experience from the ventilation industry indicates that room dimensions are of minor importance. Measurements in two different rooms with the lengths of 4.2 m and 6.0 m do not show any influence from room dimensions (see Ref. [12]).

The design procedure for displacement ventilation often deals with the velocity in front of a wall-mounted diffuser by expressing the distance ℓ_n from the diffuser to an area where the velocity has decreased to 0.2 m/s. The variation of the length ℓ_n from the diffuser to the 0.2 m/s velocity contour can be found from Eq. (6) to be

$$\ell_n = 5 q_o K_{Dr} \tag{8}$$

The length ℓ_n is given for many commercial diffusers and, therefore, it is possible to find the velocity distribution in front of the diffuser from the equation

$$u_x = 0.2 \frac{\ell_n}{x} \tag{9}$$

based on the assumption that $u_x \sim 1/x$ as shown in Eq. (6).

4. Flow between obstacles

The flow in the vicinity of the floor may be influenced by furniture and other obstacles in the occupied zone. The maximum velocity in the flow is located rather close to the floor (between 1 to 5 cm above the floor), and a large part of the air movement will therefore be in this region. Conventional furniture will only have a small influence on the air movement, while obstacles placed directly on the floor will block the flow. An opening between this type of obstacles will work as a new supply opening because the flow in the room is stratified. Fig. 12 shows the flow between two obstacles where the cold air is supplied in the left side of the room and the heat sources are located in the right side of the room.

Experiments have shown that the flow from an opening between obstacles can be described as a semiradial flow as the air movement from a wall-mounted supply opening. The velocity decay can be given by the equation

$$\frac{u_x}{q_{ob}} = K_{ob} \frac{1}{x} \tag{10}$$

u_x is maximum velocity at distance x from the opening, and q_{ob} is the excess air supplied to the upstream side. u_x is measured in the symmetry plane.

Fig. 13 shows the measurements of K_{ob} in Eq. (10). The structure of the equation and the distribution of K_{ob}-values are equivalent to the structure of Eq. (6) and the distribution of K_{Dr}-values. The temperature difference $T_{oc} - T_{ob}$ is the difference between the temperature measured at a height of 1.1 m in front of the opening and the lowest temperature in the opening between the obstacles.

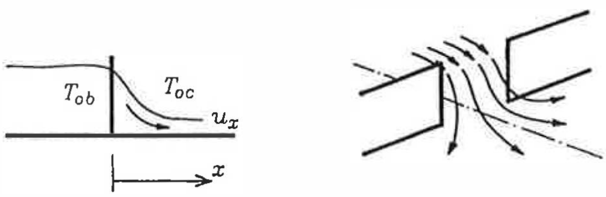

Fig. 12. Radial stratified flow between obstacles.

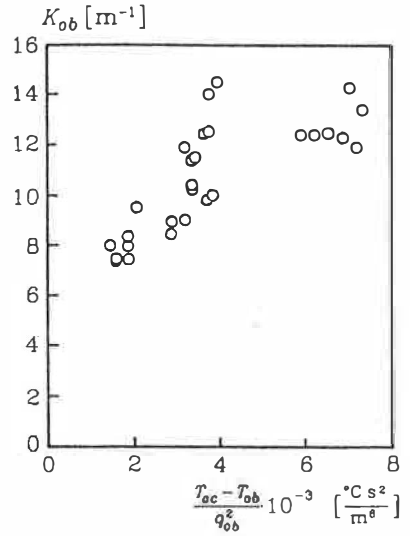

Fig. 13. K_{ob} vs. flow rate and temperature difference.

It is interesting to see that the level of the variable K_{ob} is only slightly larger than the level of K_{Dr}.

The width of the opening is varying from 0.1 to 1.5 m in the experiment. Measurements show that the importance of the width is less obvious, and results with different widths are given in Fig. 13.

5. Conclusions

Wall-mounted air terminal devices are often used in displacement ventilation. The flow from an air terminal device will accelerate in a vertical movement close to the opening due to the gravity effect when inlet air is colder than room air. Consequently, the airflow will move along the floor in a radial pattern and behave as a stratified flow. The airflow will influence the occupants' thermal comfort, and it is therefore important to develop an expression for the flow for design proposals. Measurements show that the velocity at the floor is not influenced by the flow rate to the room only. It is also influenced by the temperature difference—or by the Archimedes number—and the velocity level may vary for different types of diffusers.

The flow is stratified at large temperature differences. This is indicated by a constant height of the cold flow independent of the distance from the supply opening. It is shown that the radial flow has a virtual origin close to the front of the diffuser. The velocity level in the flow along the floor is inversely proportional to the distance from the diffuser. The velocity decay can be described individually for each type of diffuser by a single equation and a variable which is a function of the Archimedes number.

Openings between obstacles placed directly on the floor will generate a flow similar to the air movement in front of a diffuser. It is shown that the velocity distribution can be described by an equation system with the same structure as the system describing the stratified flow from wall-mounted diffusers.

Measurements of the entrainment coefficient of the radial flow along the floor show supercritical and subcritical areas of the flow, and a semiempirical theory of the stratified flow supports the measurements.

Appendix A. Stratified flow

The theory of stratified flow has been used to describe air movement with an entrainment close to zero and a constant length scale δ. The maximum velocity u_x in the radial flow close to the floor, given by Eqs. (6) and (10), is for example based on such a formulation. The theory can also be used for a more detailed description of the flow which takes account of a variable entrainment coefficient and a variable length scale.

Wilkinson and Wood [10] have shown how the flow entrains fluid with lower density close to the opening in a process similar to the flow in a wall jet. This process will be followed by a roller region and a density jump at a certain distance. The entrainment will disappear in the flow further downstream from the density jump. The flow in the entrainment region is called supercritical flow, and the flow in the area with diminishing entrainment (gravity current) is called subcritical flow. It has not been possible to identify all these details in the flow from a wall-mounted air terminal devices in displacement ventilation, but the decreasing entrainment coefficient as a function of distance (or as a function of local Archimedes number) is typical of a stratified flow in a room.

A local Archimedes number is defined by the expression

$$\mathrm{Ar}_x = \frac{\beta g \delta \Delta T_x}{u_x^2} \tag{A1}$$

where δ, u_x and $\Delta T_x = (T_{oc} - T_x)$ are all local reference values. T_x is the minimum temperature in the flow and it is located close to the maximum velocity u_x. The following expression can be used for an estimate of ΔT_x in case of high entrainment

$$\frac{T_{oc} - T_x}{T_{oc} - T_o} \sim \frac{u_x}{u_f} \tag{A2}$$

It is assumed that temperature variations, which are due to thermal radiation from ceiling to floor, can be included in this equation.

The length scale δ will be proportional to x in the supercritical stage, and ΔT_x will be proportional to u_x (Eq. (A2)) and therefore proportional to $1/x$ (Eq. (6)). u_x^2 is proportional to $1/x^2$. The total effect is Ar_x being proportional to x^2 in the supercritical stage, and Ar_x will therefore increase with the distance x, which means that the flow moves towards a gravity current (density jump). It is also possible that the local Archimedes number will increase with the distance x in the subcritical stage due to decreasing velocity.

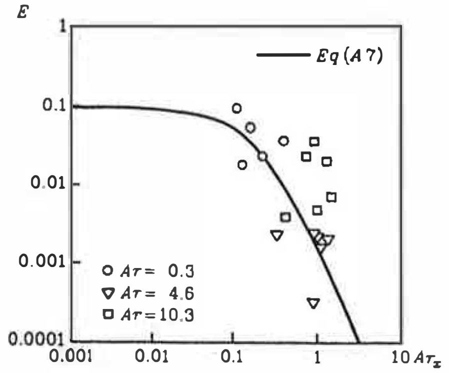

Fig. 14. Entrainment coefficient vs. local Archimedes number Ar_x for the diffuser type G [12].

The entrainment coefficient or the entrainment constant E is defined as the ratio between the vertical entrainment velocity u_e into the turbulent shear layer and the velocity scale of the layer u_x (see Morton et al. [11]).

$$E = u_e / u_x \tag{A3}$$

The determination of the velocity u_e is in practice based on measurement of the increase in the volume flow in the stratified layer at the floor.

The entrainment coefficient is measured at different positions in the flow and at different Archimedes number Ar. Fig. 14 from Ref. [12] shows that the stratified flow moves from a supercritical stage to a subcritical stage with an abrupt decrement of the entrainment coefficient in two of the experiments with high Archimedes number. Fig. 14 indicates that the transition from a supercritical stage to a subcritical stage takes place at a local Archimedes number Ar_x which is equal to 1.0.

It is possible to make some statements on the entrainment coefficient from the description in Eq. (3). The entrainment velocity u_e may be given as

$$u_e = \frac{1}{(x + x_o)\Delta\theta} \frac{\mathrm{d}q_{\Delta\theta}}{\mathrm{d}x} \tag{A4}$$

because the change in the flow $\mathrm{d}q_{\Delta\theta}$ is equal to the amount of the inflow $u_e(x + x_o)\,\Delta\theta\,\mathrm{d}x$.

It is assumed that $(x + x_o)u_x$ is constant and independent of x at all values of Ar_x, as confirmed by the measurements behind Eq. (6) because $q_o K_{Dr} = (x + x_o) u_x$. The expression $\mathrm{d}q_{\Delta\theta}/\mathrm{d}x$ is obtained from Eq. (4)

$$\frac{\mathrm{d}q_{\Delta\theta}}{\mathrm{d}x} = \Delta\theta(x + x_o)u_x \frac{\mathrm{d}\delta}{\mathrm{d}x} \int_o^\infty f(\eta)\mathrm{d}\eta \tag{A5}$$

Eqs. (A3), (A4) and (A5) give the following empirical expression for the entrainment coefficient

$$E = \frac{\mathrm{d}\delta}{\mathrm{d}x} \int_o^\infty f(\eta)\mathrm{d}\eta \tag{A6}$$

It is interesting to compare this equation with the measurements in Fig. 9. High Archimedes numbers and increasing distance x show a small $\mathrm{d}\delta/\mathrm{d}x$ corresponding to a small entrainment coefficient in good agreement with Fig. 14.

The dotted line in Fig. 9 for a wall jet type of flow corresponds to $\mathrm{d}\delta/\mathrm{d}x$ being equal to 0.1. In this situation Eq. (A6) gives $E \sim 0.1$, which is confirmed by the measurements in the supercritical stage in Fig. 14. Eq. (A6) will give a low entrainment coefficient in the subcritical stage when $\mathrm{d}\delta/\mathrm{d}x$ is small, which is confirmed by the measurements (Fannelöp [14] has pointed out that an entrainment equal to zero corresponds to a negative growth of δ. This effect is not contained in the empirical Eq. (A6)).

A work on turbulent buoyant jets in shallow fluid layers by Jirka [13] shows that the entrainment coefficient can be given by the following equation

$$E = E_o\left(1 - \frac{Ar_x}{\sqrt{Ar_x^2 + 0.063}}\right) \bigg/ (1 + Ar_x) \tag{A7}$$

where E_o is the entrainment constant for low Archimedes numbers (~ 0.1). It is implied that the local Archimedes number is identical to the overall Richardson number (see Ref. [12]). Fig. 14 shows that there is good agreement between measurement on the stratified flow in a room and the description given by Eq. (A7) for buoyant jets in shallow fluid layers.

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