NATURAL VENTILATION INDUCED BY COMBINED WIND AND THERMAL FORCES IN A TWO-ZONE BUILDING

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ABSTRACT

When buoyancy forces, wind forces and envelope heat losses interact in a naturally ventilated building, the behaviour of the flow rate as a function of these parameters can be quite complex. This paper derives the equations for the flow rate in a two-zone building where one zone is above the other, and where each zone has a high and a low opening. When the wind force opposes the buoyancy force, several solutions for the flow rate can be found for a given set of parameters, and a wide variety of behaviours for the flow rate is possible, some of which are not possible for a one-zone building.

1 INTRODUCTION

When natural ventilation is induced fully or partly by thermal buoyancy forces, ventilation rates and indoor air temperatures are interdependent. Wind forces can either assist or oppose thermal buoyancy forces. It is well known that ventilation analysis becomes much simpler when one of the driving forces is dominant. The situation where two driving forces oppose each other and are of similar magnitude is difficult to analyse and quantify.

Li and Delsante (1999) recently derived an analytical solution for the ventilation flow rate and indoor air temperature in a single-zone building. The ventilation dynamics when both wind and thermal forces act together was studied in detail. Analytical solutions only exist when there are two effective ventilation openings, because of the non-linearity of the flow rate balance equation. It was found that the natural ventilation system is characterised by three air change rate parameters, α, β and γ, which represent respectively the effects of the thermal buoyancy force, the envelope heat loss and the wind force:

$$\alpha = \left(C_d A^*\right)^{\frac{2}{3}} (Bh)^{\frac{1}{3}}, \quad \beta = \frac{\sum U_j A_j}{3\rho c_p}, \text{ and } \gamma = \frac{1}{\sqrt{3}}\left(C_d A^*\right)\sqrt{2\Delta P_w},$$

where

$$A^* = \frac{A_t A_b}{\sqrt{A_t^2 + A_b^2}}$$

and

$$B = \frac{Eg}{\rho c_p T_o}.$$

Non-dimensional graphs were presented for calculating ventilation flow rates and air temperatures, and for sizing ventilation openings. An important finding in Li and Delsante [1] was that the simple natural ventilation system considered can exhibit hysteresis. That is, when the wind opposes the buoyancy flow and the indoor heat gain lies between a certain range of values, there are three possible flow rates – two downward flows and one upward flow. One of the downward flow states was shown to be unstable; which of the other two states the system is in depends on whether the heat gain value was reached by increasing it from a low value or decreasing it from a high value.

The purpose of this paper is to extend the single-zone analytical solutions to a two-zone building, with two effective openings in each zone. The approach used here is similar to that used in Li and Delsante (1999). The purpose of this paper is to provide preliminary answers to the following questions:

- Are there still three simple air change parameters that can be used to characterise the flow?
- Will the hysteresis behaviour found in the single-zone building also exist in the two-zone building?
- How do the wind and thermal buoyancy forces interact with the envelope losses to drive the natural ventilation flow rate?

Even though the building analysed here is very simple, it is useful to study the air flow in such a building using analytical solutions. These can be used to give insight into the main features of the air flow behaviour, and they can also be used to verify multi-zone airflow programs, as they do not have any experimental uncertainties.

In this paper, we assume that there is no thermal mass in the building. If thermal mass is included, no analytical solution exists. However, the model can still be applied to some practical buildings such as agricultural and industrial buildings which have relatively low thermal mass. When ventilation airflow rates are very large, the thermal mass may also be neglected. The effect of thermal mass is currently being studied and the results will be published elsewhere.

2 A TWO-ZONE BUILDING WITH WIND, THERMAL BUOYANCY AND HEAT CONDUCTION LOSS

Consider a simple two-zone building with two openings at different vertical levels in each zone, as shown in Figure 1. We assume that the height of the vertical openings is relatively small. There is an indoor source of heat, E_i, in each zone i. The wind force can assist or oppose the thermal buoyancy force; Fig. 1 shows the assisting wind case.

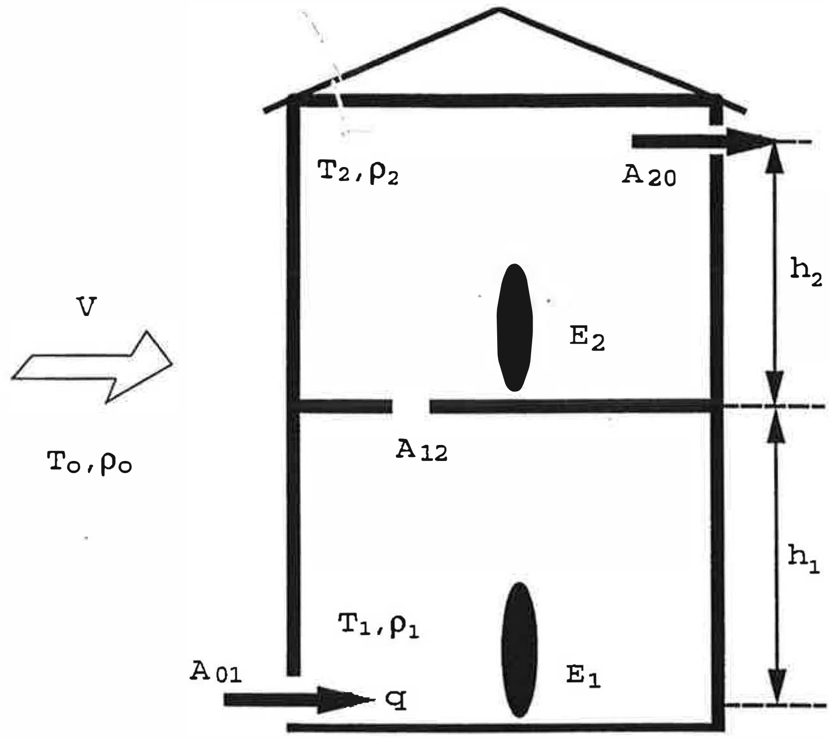

Figure 1. Geometry and notation for the two-zone building.

We assume that the indoor air is fully mixed, i.e. the air temperature is uniform. This assumption is generally not valid for flows dominated by thermal buoyancy forces. A non-uniform temperature model has also been developed and the results will be presented elsewhere. Wind turbulence effects are not included. Additionally, we assume that the partitions between the zones are perfectly insulated, i.e. heat loss only take place through the external walls, roof and floor.

We consider two cases: assisting wind (i.e. the wind assists the buoyancy flow); and opposing wind (i.e. the wind opposes the buoyancy flow). With an opposing wind, the flow can be either downward (the wind force is stronger than the buoyancy force) or upward (the buoyancy force is stronger).

2.1 Assisting winds

The flow rate can be calculated as (see Li and Delsante 1999 for details):

$$q = C_d A^* \sqrt{2gh_1 \frac{T_1 - T_0}{T_0} + 2gh_2 \frac{T_2 - T_0}{T_0} + 2\Delta P_w}\,, \tag{1}$$

where

$$A^* = 1 \Big/ \sqrt{\frac{1}{A_{01}^2} + \frac{1}{A_{12}^2} + \frac{1}{A_{20}^2}}$$

A heat balance on each zone gives:

$$\rho c_p q(T_1 - T_0) + \sum_{j=1}^{N_1} U_{1j} A_{1j}(T_1 - T_0) = E_1$$

$$\rho c_p q(T_2 - T_1) + \sum_{j=1}^{N_2} U_{2j} A_{2j}(T_2 - T_0) = E_2 \qquad (2)$$

Substituting (1) into (2) gives, after some manipulation,

$$q^4 + 3(\beta_1 + \beta_2)q^3 + \left(9\beta_1\beta_2 - 3\gamma^2\right)q^2 - \left(2\alpha_U{}^3 + 9(\beta_1 + \beta_2)\gamma^2\right)q$$

$$-6\beta_1\alpha_2^3 - 6\beta_2\alpha_1^3 - 27\beta_1\beta_2\gamma^2 = 0, \qquad (3)$$

where

$$\alpha_U = \left(C_d A^*\right)^{\frac{2}{3}}\left[B_1(h_1 + h_2) + B_2 h_2\right]^{\frac{1}{3}},$$

$$\alpha_i = \left(C_d A^*\right)^{\frac{2}{3}}\left(B_i h_i\right)^{\frac{1}{3}},$$

$$\alpha_{ij} = \left(C_d A^*\right)^{\frac{2}{3}}\left(B_i h_j\right)^{\frac{1}{3}},$$

$$\beta_i = \frac{\sum_j U_{ij} A_{ij}}{3\rho c_p} \qquad (4)$$

Although the roots of the quartic equation (3) for the flow can be obtained explicitly, their form is too long and complex to give any insight into the key features of the flow behaviour. However, it is possible to deduce from (3) the following:

- $q \geq \sqrt{3}\gamma$
- q increases monotonically as the heat source strengths are increased.

Thus the dependence of q on the heat source strengths is straightforward and the assisting winds case will not be discussed further here.

2.2 Opposing winds

The flow direction for assisting winds is always upward. When the wind force opposes the thermal buoyancy force, the flow can be either upward or downward, depending on the relative strengths of the forces. The flow rate is given by

$$q = C_d A^* \sqrt{\left|2gh_1 \frac{T_1 - T_0}{T_0} + 2gh_2 \frac{T_2 - T_0}{T_0} - 2\Delta P_w\right|} \qquad (5)$$

For opposing winds, care must be taken with the heat balance equation. If we have upward flow, then (2) still applies, but if we have downward flow, the heat balance equations become

$$\rho c_p q(T_1 - T_2) + \sum_{j=1}^{N_1} U_{1j} A_{1j}(T_1 - T_0) = E_1$$

$$\rho c_p q(T_2 - T_0) + \sum_{j=1}^{N_2} U_{2j} A_{2j}(T_2 - T_0) = E_2 \tag{6}$$

For upward flows, substituting (2) into (5) gives, after some manipulation,

$$q^4 + 3(\beta_1 + \beta_2)q^3 + (9\beta_1\beta_2 + 3\gamma^2)q^2 - (2\alpha_U{}^3 - 9(\beta_1 + \beta_2)\gamma^2)q$$

$$-6\beta_1\alpha_2^3 - 6\beta_2\alpha_1^3 + 27\beta_1\beta_2\gamma^2 = 0. \tag{7}$$

For downward flows, substituting (6) into (5) gives

$$q^4 + 3(\beta_1 + \beta_2)q^3 + (9\beta_1\beta_2 - 3\gamma^2)q^2 + (2\alpha_D{}^3 - 9(\beta_1 + \beta_2)\gamma^2)q$$

$$+6\beta_1\alpha_2^3 + 6\beta_2\alpha_1^3 - 27\beta_1\beta_2\gamma^2 = 0, \tag{8}$$

where

$$\alpha_D = (C_d A^*)^{\frac{2}{3}}[B_1 h_1 + B_2(h_1 + h_2)]^{\frac{1}{3}}$$

As a check, we can recover the equations for a one-zone building as follows. For upward flows, we must set $E_1 = 0$ (so that $T_1 = T_0$) to obtain the single-zone limit. Equation (7) becomes

$$(q + 3\beta_1)(q^3 + 3\beta_2 q^2 + 3\gamma^2 q + 9\beta_2\gamma^2 - 2\alpha_U^3) = 0$$

Since $q + 3\beta_1$ is non-zero, the other bracketed term must be zero, and this is indeed the equation for the single-zone case obtained by Li and Delsante (1999). For downward flows we must set $E_2 = 0$. Equation (8) then factorises in a similar way.

Again, while (7) and (8) can be solved analytically or numerically, it is preferable to first obtain some general insight into the behaviour of the flow rate as a function of the governing parameters. For the single-zone case, this was conveniently done by solving the flow equation for α and analysing the behaviour of α as a function of q. For the two-zone case this approach can still be applied, but with some additional complexity because there is no longer a single α parameter. However, we can proceed as follows. Let us write

$\beta_i = g_i\beta$, $\beta = \beta_1 + \beta_2$, $\alpha_i^3 = f_i\alpha_D^3$, $B_2 = \delta B_1$, and $h_2 = \varepsilon h_1$.

Then $\alpha_U^3 = f_U\alpha_D^3$, where $f_U = f_1(1 + \delta\varepsilon + \varepsilon)$. We also have $g_1 + g_2 = 1$. Then (7) and (8) can both be written in terms of α_D as follows:

Upward flow:

$$q^4 + 3\beta q^3 + (9g_1g_2\beta^2 + 3\gamma^2)q^2 - (2f_U\alpha_D{}^3 + 9\beta\gamma^2)q$$

$$-6\beta\alpha_D^3(f_2g_1 + f_1g_2) + 27g_1g_2\beta^2\gamma^2 = 0. \tag{9}$$

Downward flow:

$$q^4 + 3\beta q^3 + \left(9g_1g_2\beta^2 - 3\gamma^2\right)q^2 - \left(2\alpha_D{}^3 - 9\beta\gamma^2\right)q$$

$$+6\beta\alpha_D^3\left(f_2g_1 + f_1g_2\right) - 27g_1g_2\beta^2\gamma^2 = 0. \tag{10}$$

From (9) and (10) we can easily see that the flow is zero when

$$2\alpha_D^3 = \frac{9\beta g_1g_2\gamma^2}{f_1g_2 + f_2g_1}. \tag{11}$$

We are interested in the slope of q at $q = 0$, i.e. where the buoyancy force is exactly balanced by the wind force. From (9) and (10) and using (11) we obtain:

Upward flow:

$$\frac{\partial q}{\partial \alpha_D}\Big|_{q=0} = \frac{2\alpha_D^2\left(f_1g_2 + f_2g_1\right)^2}{\gamma^2\left(f_1g_2 + f_2g_1 - f_Ug_1g_2\right)}. \tag{12}$$

Downward flow:

$$\frac{\partial q}{\partial \alpha_D}\Big|_{q=0} = \frac{2\alpha_D^2\left(f_1g_2 + f_2g_1\right)^2}{\gamma^2\left(f_1g_2 + f_2g_1 - g_1g_2\right)}. \tag{13}$$

Note that the corresponding expression for the single-zone case is simply

$$\frac{\partial q}{\partial \alpha}\Big|_{q=0} = \frac{2\alpha^2}{\gamma^2},$$

for either upward or downward flow. Thus whereas in the single-zone case the slope of q at q = 0 is always positive, and is the same for upward and downward flow, for the two-zone case (12) and (13) indicate a more complex behaviour at this point. Firstly, unless f_U = 1, the slopes are different at $q = 0$, and can differ in sign as well as magnitude; secondly, the slopes can be positive, zero or negative, depending on the relative sizes of the parameters f_i, g_i, δ and ε.

Figures 2a-2d show schematically some possible solution behaviours for the upward and downward flows as a function of the buoyancy force parameter α_D (for clarity, the flow rate for downward flow is shown as being negative). Fig. 2a shows the simplest case: the slope at q = 0 is positive for upward flow and negative for downward flow and the flows are monotonic. Thus for any value of α_D there is a unique solution for the flow. Note that there is no analogue for this behaviour in the single-zone case, since there slope is always positive at q = 0. Fig. 2b shows similar hysteresis behaviour to that found for the single-zone case: for certain values of α_D there are three possible solutions – one upward flow (S1), and two downward flows (S2 and S3). Fig. 2c also shows three possible solutions for certain values of α_D, but with two upward flows and one downward flow. Finally, Fig. 2d shows five possible solutions – two upward and three downward. This set of behaviours is clearly not exhaustive.

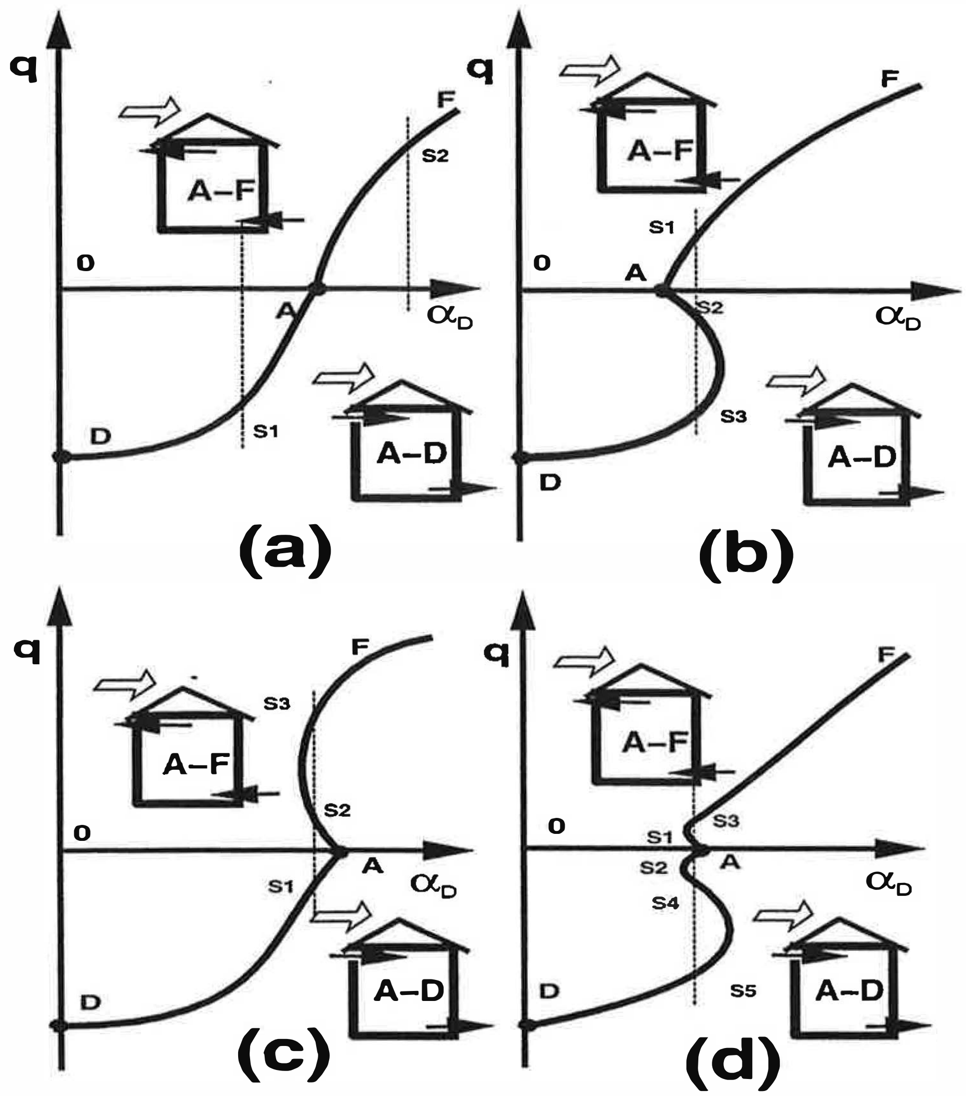

Figure 2. Some possible behaviours of the flow rate q for opposing winds as a function of the heat gain parameter α_D. Positive q indicates upward flow; negative q indicates downward flow.

It is not difficult to find values of the parameters f_i, g_i, δ and ε to illustrate the various behaviours. For example, if we take $f_1 = f_2 = 0.4$, $g_1 = g_2 = 0.5$, $\delta = 0.4$ and $\varepsilon = 1.5$, then from (12) and (13) we see that both the downward and upward slopes are positive at $q = 0$. Fig. 3 shows numerical solutions of the flow rate for these parameters for both assisting and opposing winds as a function of α_D/γ for β/γ ranging from 0.1 to 0.9 (note that the flow rate and the buoyancy force parameter have both been scaled by the wind parameter γ). For this range of β/γ the solution is of the form shown in Fig. 2b. Alternatively, if $f_1 = f_2 = 0.1$, $g_1 = g_2 = 0.5$, $\delta = 20.0$ and $\varepsilon = 0.2$, then both downward and upward slopes are negative at $q = 0$. Fig. 4 shows numerical solutions of the flow rate for these parameters. It is interesting to note that for small values of β/γ the solution is of the form shown in Fig. 2d, but as β/γ increases the form becomes that of Fig. 2c.

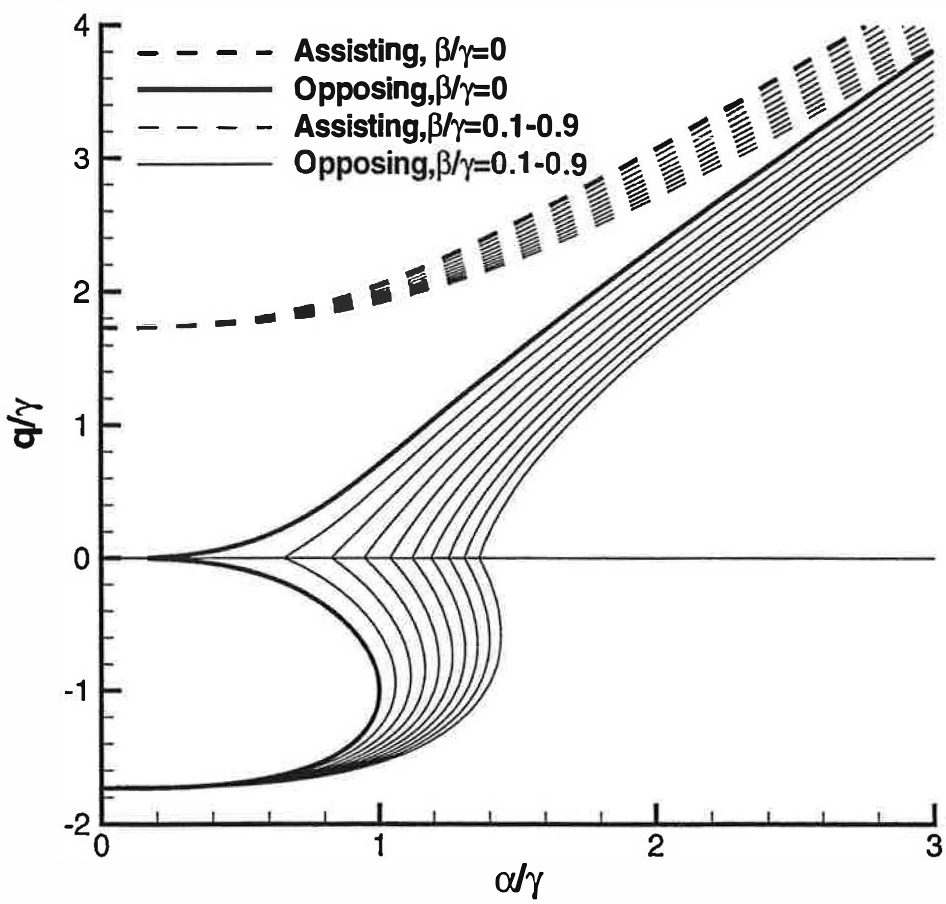

Figure 3. Scaled flow rate as a function of the scaled heat gain parameter α_D and heat loss parameter β for both assisting and opposing winds. For opposing winds the parameters for the two zones have been chosen to give a solution behaviour corresponding to Fig. 2b.

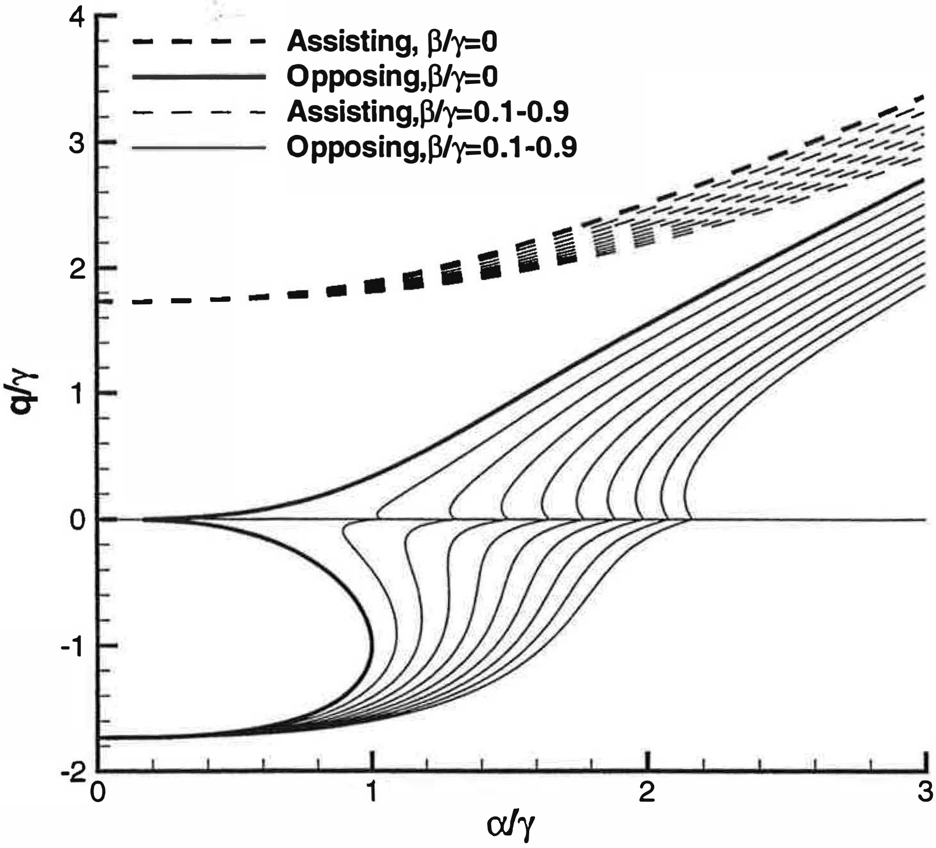

Figure 4. Scaled flow rate as a function of the scaled heat gain parameter α_D and heat loss parameter β for both assisting and opposing winds. For opposing winds the parameters for the two zones have been chosen to give a solution behaviour corresponding to Fig. 2d (for small β/γ) or Fig. 2c (for larger β/γ).

3 CONCLUSION

The initial study of the simple two-zone building presented in this paper has answered some of the questions posed in the introduction:

- Three air change parameters α, β and γ can still be defined to characterise the flow, but additional subsidiary parameters f_i, g_i, δ and ε must be introduced to account for the additional complexity.
- The hysteresis behaviour found in the single-zone building also exists in the two-zone building, and is indeed considerably more complex. Again we find that for opposing winds and a given set of heat gains, wind speeds and *U*-values, there appear to be several solutions for the flow.

The full spectrum of solution behaviours has not yet been fully analysed. Furthermore, the stability of the multiple solutions has also not yet been resolved. For the single-zone case Li and Delsante analysed the system dynamics and showed that solution S2 in Fig. 2b is unstable, leaving one stable upward flow solution and one stable downward flow solution. In the two-zone building there may be two or more stable solutions for a particular flow direction (e.g. S2 and S5 in Fig. 2d) However, unlike a single-zone building where the system dynamics can be easily analysed, a two-zone building presents a two-dimensional non-linear system. Further analysis of the system dynamics and identification of stable solutions will be carried out in the near future.

In summary, the derived flow equations allow us to understand the effect of the three air change parameters on ventilation flow rates in a two-zone building. The conclusion that multiple solutions can exist in a two-zone building has interesting implications for numerical modellers. For multi-zone airflow modelling where the basic governing equations are similar to the ones used in this paper, in particular when the airflow and thermal models are integrated, care should be taken in interpreting the results obtained when the buoyancy force and wind force are of similar magnitude.

4 REFERENCES

Li, Y. and Delsante, A. (1999) Natural Ventilation Induced by Combined Wind and Thermal Forces. Submitted to *Building and Environment.*

5 NOMENCLATURE

A_b	area of bottom ventilation opening in single-zone building
A_t	area of top ventilation opening in single-zone building
A_{ij}	area of opening between zone *i* and zone *j* (zone 0 = outdoors); or area of *j*-th wall in zone *i*
B	buoyancy flux
c_p	heat capacity of air
C_d	discharge coefficient
E_i	total heat power in zone *I*
f_i	α_i^3 / α_D^3
g	acceleration of gravity
g_i	β_i/β
h_i	height between two openings in zone *i*
N_i	number of external walls in zone *i*
ΔP_w	wind pressure
q	volumetric flow rate
T_i	air temperature in zone *i* (zone 0 = outdoors)
U_{ij}	*U*-value of *j*-th external wall in zone *i*

Greek symbols

α_U	buoyancy parameter for upward flow
α_D	buoyancy parameter for downward flow
α_i, α_{ij}	buoyancy parameters defined in (4)
β_i	heat loss parameter for zone *i*, defined in (4)
β	$\beta_1 + \beta_2$
γ	wind force parameter
δ	B_2/B_1
ε	h_2/h_1