# VENTILATION AND STRATIFICATION IN NATURALLY VEN SPACES DRIVEN BY HEATED INTERNAL VERTICAL SURFACES

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## ABSTRACT

This paper reports on research into the fundamental fluid mechanics mechanisms that lead to thermal stratification in a naturally ventilated room containing sources of heat, or cooling. This aspect of natural ventilation has an important influence on both air exchange rates and thermal comfort in a naturally ventilated space. Particular attention is paid to the situation where the major source of heat is a vertical surface, such as a wall heated by solar insolation, for example. A theoretical model is presented that allows prediction of the ventilation flow stratification in such a room for the first time. Numerical results from this model and some preliminary experimental validation tests are presented. An example of how this theoretical model might be used by a designer for design of a naturally ventilated space subject to major heat gains at vertical surfaces is discussed.

## 1 INTRODUCTION

Natural ventilation of rooms inside buildings may be driven by wind and/or buoyancy forces. This paper focusses on the situation where a room contains vertical surfaces, such as walls or windows, that act as distributed heat sources and which lead to a buoyancy driven natural ventilation flow through vents at the top and bottom of the space.

A summary of research in the field of Natural Ventilation has been compiled by Linden (1999) and it is clear that significant progress has been made in recent years in our understanding of the fundamental flow mechanisms that are involved in buoyancy driven natural ventilation flows. The steady state case of natural displacement ventilation of a room containing a single, point source of buoyancy was first analysed by Thomas *et al.* (1963) and later in more detail by Linden *et al.* (1990). The same situation with multiple sources of heat (or cooling) of differing strengths was reported by Cooper and Linden (1996) and Linden and Cooper (1996). Hunt (1998) has analysed the transient development of the ventilation flow and thermal stratification in a room containing a single, point source of buoyancy. He developed an approximate analytical method to predict the height of the ambient layer of fresh air in the lower part of the room as a function of time.

However, in many buildings significant sources of thermal energy cannot be realistically treated as point sources. Walls and windows are often cooler or hotter than the mean room temperature and thus natural convection boundary layers form adjacent to such building elements. These boundary layers generate thermal stratification and a natural ventilation flow through the room.

This problem has received less attention in the past than the case of displacement natural ventilation and thermal stratification arising from point sources of heat. In part, this may be due to the fact that many aspects of fully turbulent natural convection boundary layers on vertical surfaces have yet to be determined. In this paper we take a simplified view of the turbulent boundary layer itself and model it as a plume arising from a source of buoyancy where the buoyancy is distributed uniformly over the surface. This approach is justified if the Rayleigh number is very large. In the present study we are concerned with only one thermal exchange, the interaction of the natural convection boundary layer with the air in the space and it is assumed that all other room surfaces are adiabatic. Of course in a real building there will be many additional thermal exchanges both convective and radiative between wall surfaces and between these surfaces and the room air. However, part of the motivation for pursuing this fundamental research is not only to shed light on the mechanisms that determine the ventilation and stratification in a room but also to provide numerical and experimental data for validation of CFD codes.

## 2 THEORETICAL MODEL

### 2.1 Sealed Room

The most fundamental geometry of the problem at hand is shown in Figure 1 where a sealed building space with no ventilation contains a wall which acts as a uniformly distributed source of heat of strength $q''$ (W/m$^2$). When the heat source is switched on a natural convection boundary layer forms on that wall. This is essentially a "filling box" problem where we wish to determine the temperature in the room as a function of both time, $t$, and height above the floor, $z$. The filling box containing a point source of heat (or buoyancy) has been analysed by many researchers, notably Baines and Turner (1968) and Worster and Huppert (1983).

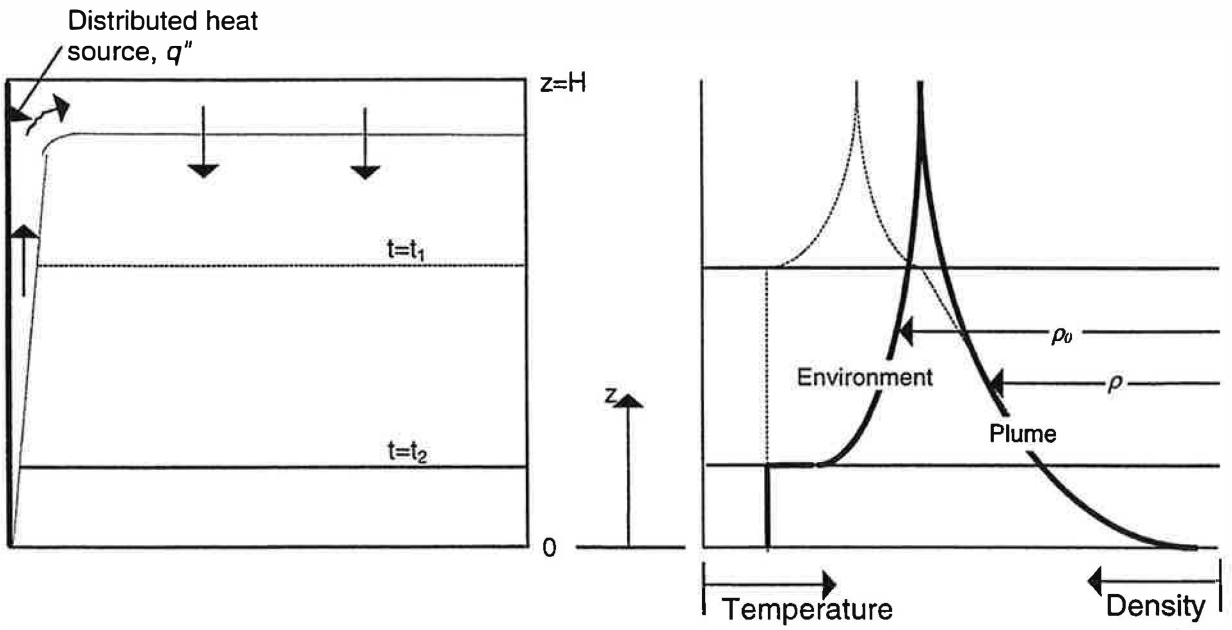


**Figure 1. Schematic of: a) development of a plume from a plane, vertical, distributed source of buoyancy on one wall of a sealed room showing position of the first density front at times $t_1$ and $t_2$; b) density profiles in the plume and the ambient fluid (not to scale), after Baines and Turner (1968).**

When the heat source on the wall is switched on it is assumed that a turbulent natural convection boundary layer forms adjacent to the wall. The flow in the boundary layer *per se* is complex. Indeed there is still a great deal we do not know about the behaviour of high Rayleigh number natural convection boundary layers despite the fact that they play a very important part in the thermal behaviour of buildings and other structures. (Note: a Rayleigh number of order $10^{11}$ is typical of a large space in a building with a heated wall of height H=5m and temperature difference of 10°C between wall and the air). However, at high boundary layer Rayleigh number one might reasonably assume that effects of molecular viscosity are small compared to those of turbulent mixing and that the "entrainment assumption" applies. The entrainment assumption is used frequently in the analysis of buoyant plumes and models the observation that the horizontal velocity of fluid entrained into the plume is directly proportional to the mean vertical velocity of the plume. If these assumptions are applicable to a boundary layer then it may be modelled using techniques similar to that used by Morton *et al.* (1956) in analysing the flow in a plume above a point source of buoyancy.

A similarity solution for the flow in a turbulent boundary layer next a wall in a *uniform ambient* has been developed (Cooper, 1999). In this case it can be shown that the volumetric flow rate per unit width, $Q(z)$, in the boundary layer is given by:

$$Q = \frac{3}{4}\left(\frac{4}{5}\right)^{1/3} \alpha^{2/3} F_0^{1/3} z^{4/3} \tag{1}$$

where $\alpha$ is the entrainment constant (assuming top-hat velocity and temperature profiles across the plume) and $F_0$ is the buoyancy flux per unit area of the heat source. This buoyancy flux, which has units of $m^2s^{-3}$, may be expressed in terms of the heat flux, $q''$, and the following properties of the air: coefficient of thermal expansion, $\beta$, specific heat capacity, $c_p$, gas constant, $R$, pressure, $p$, and density, $\rho$, as follows.

$$F_0 = \frac{g\beta}{\rho c_p} q'' = \frac{gR}{c_p p} q'' \tag{2}$$

At normal room temperatures this equates to $F_0 \approx 2.8 \times 10^{-5} q''$.

The governing differential equations for conservation of volume and heat in both the boundary layer and the environment may derived (Cooper, 1999) by adopting an approach similar to that of Baines and Turner (1968). When the heat source at the wall is switched on the boundary layer then lays down a buoyant layer at the top of the room. The height of the "first front" of this buoyant layer, $z_0$, may be shown to be given by:

$$z_0 = \left[1 + \frac{1}{4}\left(\frac{4}{5}\right)^{1/3} \alpha^{2/3} \left(\frac{L}{A}\right) H^{1/3} F_0^{1/3} t\right]^{-3}, \tag{3}$$

where $H$ is the height of the room.

The fluid mechanics of the situation shown in Figure 1 may be generalised by using the following *non-dimensional* variables for height above the floor, $\zeta$, time, $\tau$, and buoyancy of the ambient fluid in the room, $\delta$.

$$\zeta = zH^{-1} \tag{4a}$$

$$\tau = \alpha^{2/3} H^{1/3} \left(\frac{L}{A}\right) F_0^{1/3} t \tag{4b}$$

$$\delta = \alpha^{2/3} H^{1/3} F_0^{-2/3} \Delta_0 \tag{4c}$$

Note that the dimensional buoyancy of the stratified ambient fluid, $\Delta_0$, (with units of m/s$^2$) is related to the absolute temperature of the ambient air, $T_0$, and the initial temperature of the air in the room, $T_1$, through the following relationship $\Delta_0 = g(\rho_0 - \rho_1)/\rho_1 = -g(1 - T_1/T_0)$.

Using a numerical method similar to that employed by Germeles (1975) the governing differential equations for flow and temperature in the plume and the ambient have been solved. The predicted thermal stratification in the sealed room is shown in Figure 2 in terms of non-dimensional buoyancy of the air in the room as a function of non-dimensional height and time.

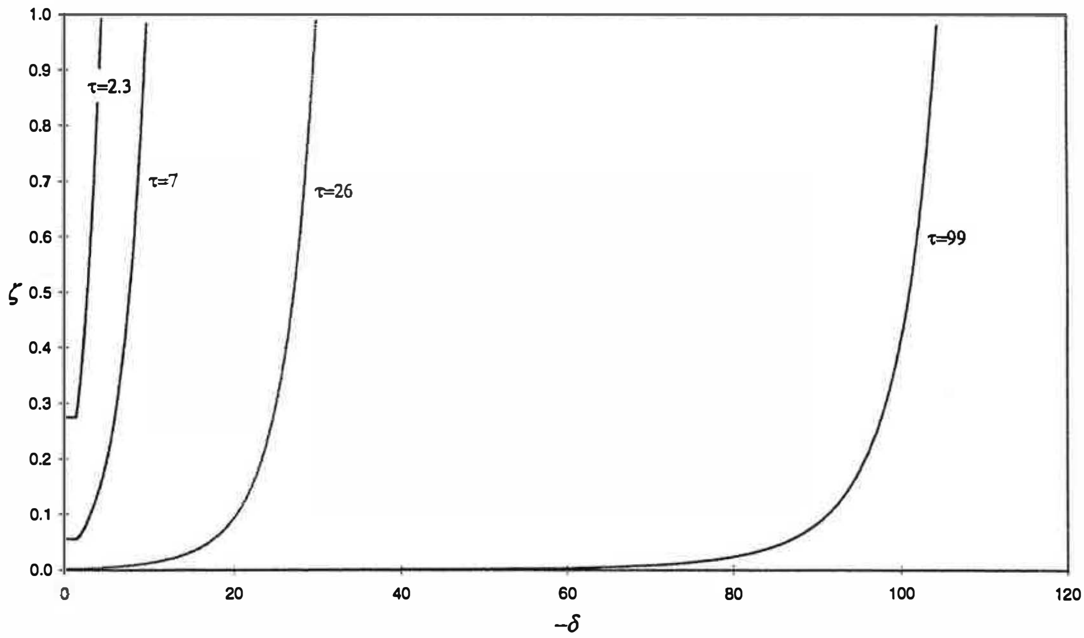


**Figure 2 Development of the non-dimensional density field in a filling box containing a vertical, plane distributed source of buoyancy as a function of non-dimensional height, $\zeta = z/H$, and time, $\tau$.**

Figure 2 clearly shows the descent of the first front with increasing time and also the way in which non-dimensional buoyancy, $\delta$, of the air in the room (equivalent to temperature) will continue to increase indefinitely with time since the walls of the sealed room are assumed to be adiabatic.

## 2.1 Ventilated Room

The case of a room with vents located in the ceiling and floor is shown schematically in Figure 3. Here the temperature in the room will not increase indefinitely but will tend to a steady state condition where the heat added to the room through the distributed heat source on the wall is exactly balanced by the energy lost from the upper vent. There will exist a layer of air in the bottom of the room that is drawn in from outside by the displacement natural ventilation process.

The volume flow rate through the upper and lower vents is determined by the balance between the total buoyancy of the hot layer of air in the room and the pressure loss due to friction of the air flow through the vents. This may be expressed as (Linden *et al.*, 1990):

$$Q_{vent} = A^*\left(\int_0^H \Delta_0 dz\right)^{1/2} = A^*\left(\int_{z_0}^H \Delta_0 dz\right)^{1/2}. \qquad (5)$$

The "effective vent area", $A^*$, is defined as in Cooper and Linden (1996)

$$A^* = \frac{c_d a_t a_b}{\left(\frac{1}{2}\left(\frac{c_d^2}{c}a_t^2 + a_b^2\right)\right)^{1/2}}, \qquad (6)$$

where $c$ is the pressure loss coefficient associated with the inflow through a sharp edged opening and $c_d$ is a discharge coefficient to account for the vena contracta arising at the downstream side of the sharp edged upper vents.

In terms of the dimensionless variables (4) this ventilation flow rate (per unit width of the heated wall) is therefore

$$q_{vent} = \frac{A^*}{\alpha HL}\left(\int_0^1 \delta\, d\zeta\right)^{1/2}, \qquad (7)$$

where $L$ is the horizontal width of the heated wall. The term $A^*/\alpha HL$ is a non-dimensional vent area and is similar to $A^*/H^2$ for the case of a ventilated filling box containing a point source of buoyancy (Linden *et al.*, 1990). Thus, $A^*/\alpha HL$ is a geometrical parameter that influences the steady state (or asymptotic) stratification within the box and is in fact the only parameter that determines the *shape* of the thermal stratification in the box (the heat flux , $q''$, and dimensions of the vents and room determine the magnitude of the stratification).

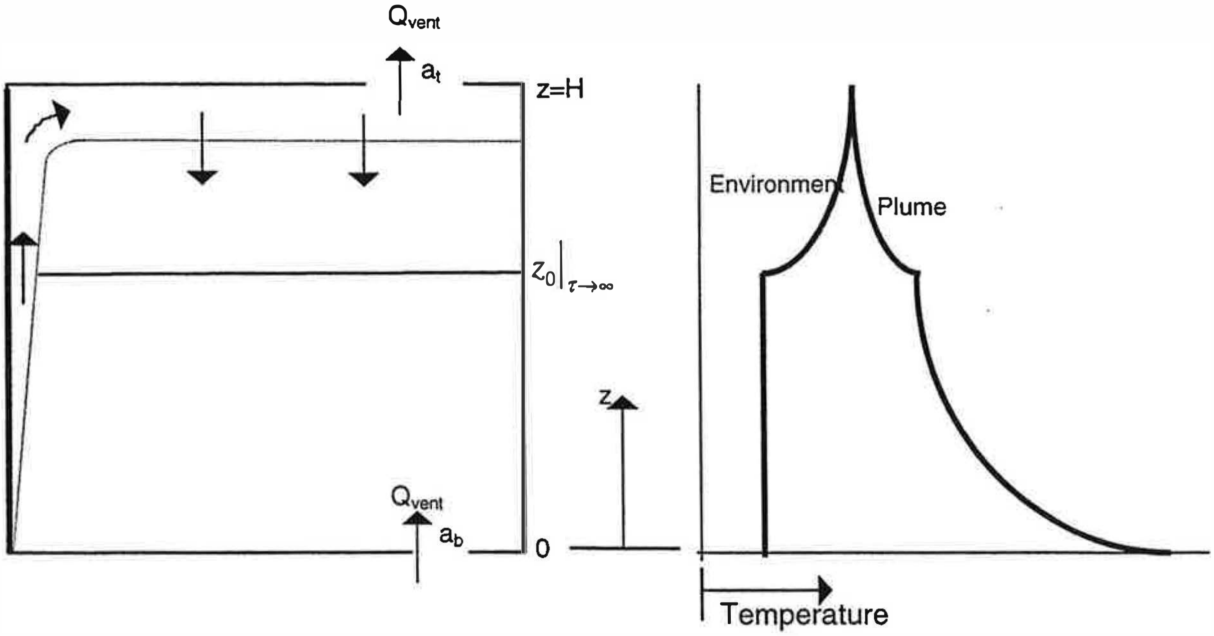


**Figure 3. Schematic of: a) development of a boundary layer from a vertically distributed source of buoyancy in a box ventilated by upper and lower vents of areas $a_t$ and $a_b$, respectively; b) density profiles in the plume and ambient (not to scale).**

Unlike the case of the sealed room it is theoretically possible for the boundary layer to have a density equal to the air in the room at a given height, $z$. If this occurs the boundary layer must cease rising and form a horizontal intrusion into the room as shown in Figure 4. Note that this situation cannot occur in the ventilated room containing a *point source* of heat.

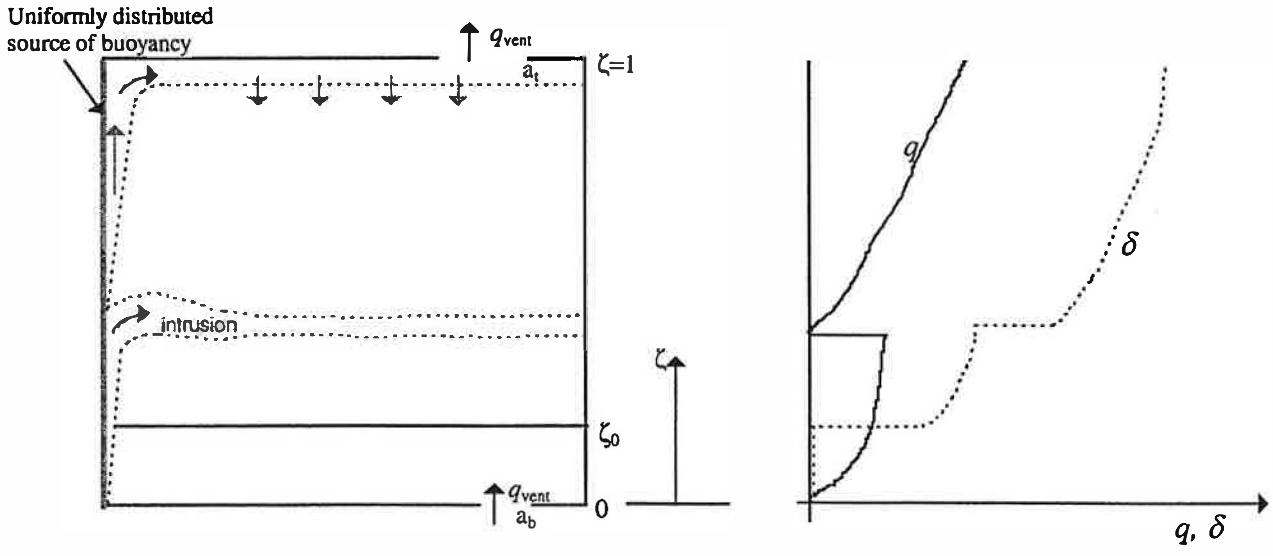


**Figure 4. Schematic of one possible scenario in a ventilated box containing a heated wall with a small effective vent area, *A**. The boundary layer in this case rises to a point where it is neutrally buoyant with respect to the local ambient and hence is forced to form a horizontal intrusion. A new boundary layer is modelled as forming above the intrusion.**

The numerical method developed to model the sealed room was modified to take account of the ventilation flow through the top and bottom vents. The numerical results for a number of non-dimensional vent areas indicate that horizontal intrusions will form. Figure 5 gives an example of the predicted development of the density field in the room as a function of time for one particular value of $A^*/\alpha HL$=0.0419.

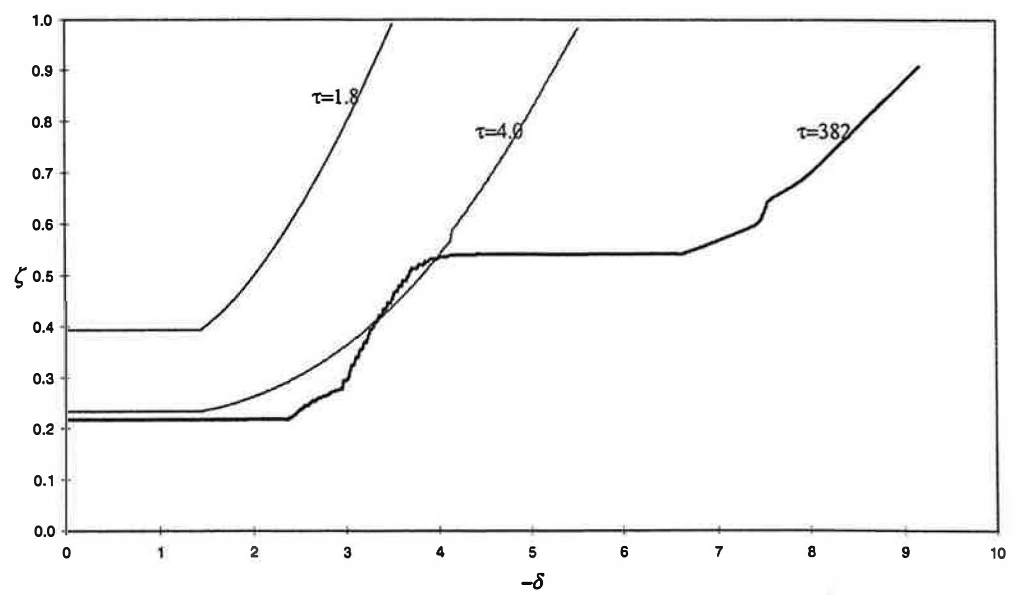

**Figure 5 Numerical simulation of the transient development of the ambient stratification within a naturally ventilated room containing a plane, vertically distributed source of heat with a non-dimensional vent area of A*/(αHL)= 0.0419 as a function of non-dimensional height, $\zeta$, and non-dimensional time, $\tau$.**

It should be noted that the form of the stratification in the room at large values of non-dimensional time, $\tau$, includes two step changes in density (or temperature). The first occurs at the "first front" ie at $\zeta \sim 0.22$, the top of the layer in the bottom of the room fed by cool outside air. The second step change in temperature occurs at $\zeta \sim 0.5$. This result is in contrast to the theory presented by Linden *et al.* (1990) for vertically distributed sources in naturally ventilated spaces.

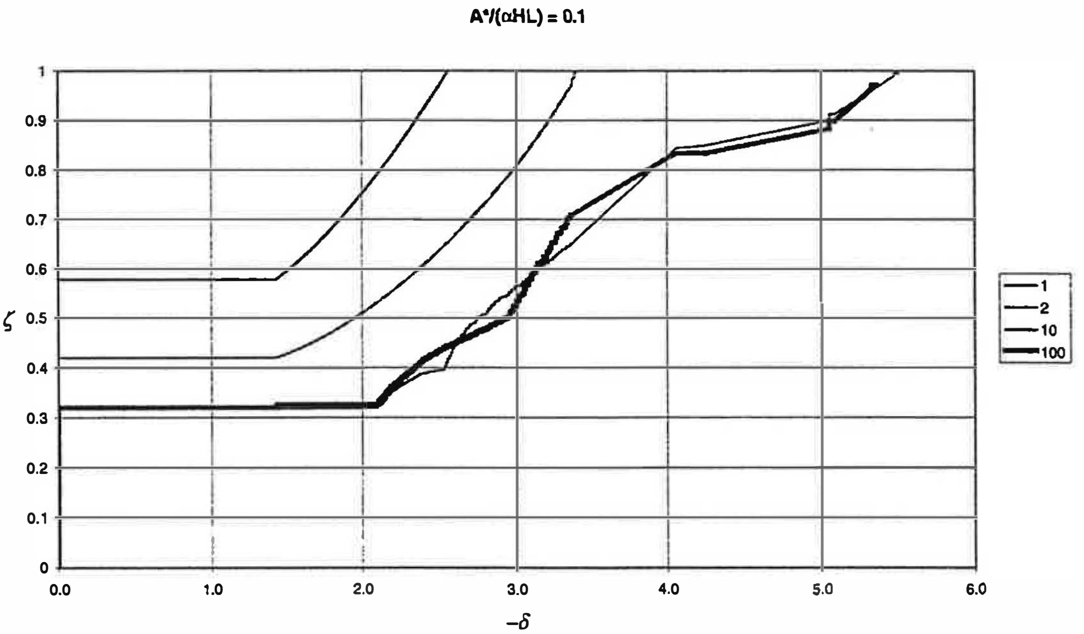


**Figure 6 Numerical simulation of the transient development of ambient stratification within a naturally ventilated room containing a plane, vertically distributed source of heat with a non-dimensional vent area of A*/(αHL)= 0.1 as a function of non-dimensional height, $\zeta$, and non-dimensional time, $\tau$.**

Figure 6 shows the predicted stratification in a room with a larger non-dimensional vent area $A^*/\alpha HL$=0.1. Here there appears to be no major step changes in density even though the simulation indicates that the boundary layer forms an intrusion at $\zeta = 0.51$.

## 3 EXPERIMENTAL METHOD AND RESULTS

A limited number of experiments have been carried out to validate some of the results from the theoretical modelling described above. These were carried out in the fluid mechanics laboratory at the Department of Applied Mathematics and Theoretical Physics, University of Cambridge, and at the University of Wollongong, Australia.

### 3.1 Determination of entrainment constant

The room shown in figures 1 and 3 was modelled experimentally using a water-filled enclosure made of acrylic sheet measuring 250mm in height internally and 300x200mm in horizontal cross-section. The "salt solution" method (as described by Linden *et al.*) was used as an analogue to the full-scale thermal situation whereby a solution of sodium chloride was injected through a vertical porous membrane to model the addition of a uniformly distributed buoyancy flux at a vertical surface. A light source was placed behind the acrylic enclosure and the intensity of light passing through the apparatus was recorded by means of a CCD video camera connected to both a video recorder and also directly to a frame grabber. The salt solution was dyed with a blue food dye and an image processing technique involving the use of the DigImage software (Dalziel, 1993) was used to convert the intensity of light passing through the enclosure to salt concentration. Thus, density was determined as a function of height above the floor of the enclosure and of time.

To facilitate a rapid start to the influx of salt solution through the porous membrane the latter was fabricated as a vertical cylindrical of external diameter $d_0$= 15.5mm. The first front of the developing stratification was then tracked accurately from the digitised video images obtained from the image capture equipment.

Previous research on the volume flow in a turbulent natural convection boundary layer has been very limited. Thus, an important issue to be settled experimentally is the determination of the entrainment constant, $\alpha$, applicable to a natural convection boundary layer. This has been achieved by tracking the progress of the first front of the buoyant layer and comparing this to the theoretical model of equation (3). Figure 7 shows the results for the position of the first front height, $\zeta_0$, as a function of time together with the analytical prediction. A value for the turbulent entrainment constant, $\alpha$, of between 0.015 to 0.03 was found to give a reasonable fit to the experimental data (although there was significant scatter in the results). This magnitude of $\alpha$ is much smaller than that for a plume arising from a point source of buoyancy ($\alpha_{point\_source}$ ~ 0.1 assuming a "top-hat" plume velocity profile) which is to be expected since the effect of the vertical wall will be to suppress turbulence in the boundary layer in practice. However, examination of the data from previous research on the development of a thermal natural convection boundary layer with a constant heat flux boundary condition (Vliet and Liu, 1969) gives $\alpha$~ 0.03. This indicates that the entrainment constant found in the present experiments using the salt solution technique is of the same order as that in the thermal situation.

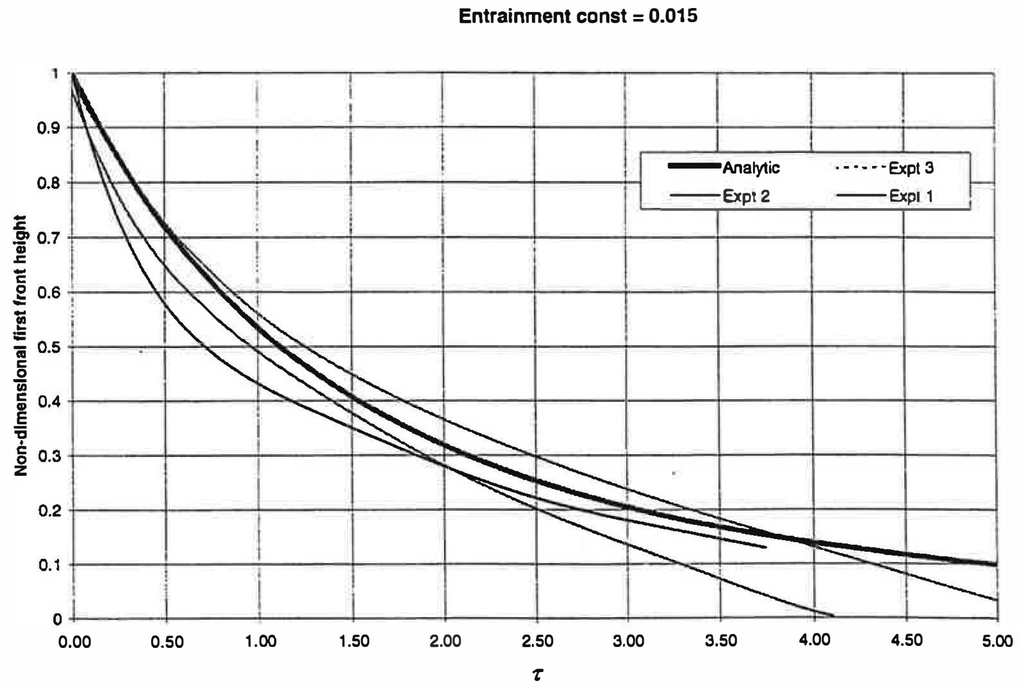


**Figure 7 Comparison of the experimental non-dimensional first front height, $\zeta_0$, and the analytical**

**prediction using equation (3). Data presented assuming the entrainment constant $\alpha = 0.015$.**

Validation of the theory modelling flow and stratification in the ventilated room shown in Figure 3 is currently underway. Some preliminary qualitative results have already been reported (Cooper *et al.*, 1998) and Figure 8 shows an example of this work where a shadowgraph technique was used to highlight the step changes in stratification the boundary layer forming an intrusion.

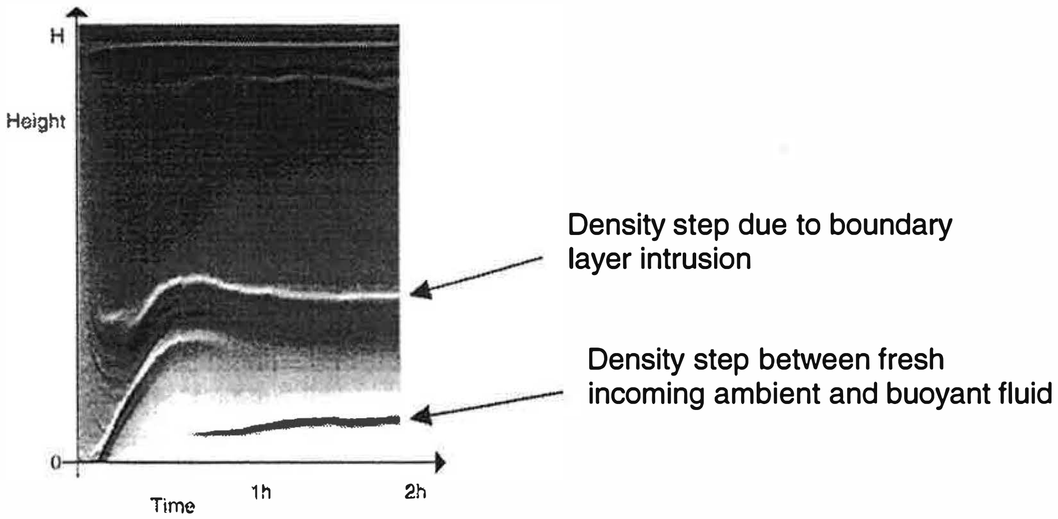


**Figure 8 Results from a shadowgraph technique showing the development of stratification in a ventilated enclosure with $A^*/(HL)$=0.00089. The white line at $z/H$ ~ 0.4 is a result of the boundary layer on the wall forming an intrusion.**

For a variety of reasons, quantitative experiments to validate the numerical results such as those of Figures 5 and 6 are difficult to carry out. However, our preliminary results indicate that quantitative agreement will be forthcoming. The maximum reduced density in the top of the ventilated enclosure with $A^*/\alpha HL$=0.1 is $\delta$ ~ -5.3 as shown in Figure 6. This matches that found experimentally by Cooper *et al.* (1998) to within 20% for $\alpha = 0.03$.

## 4 DISCUSSION

The theoretical model proposed and described in section 2 is clearly a considerable simplification of the full-scale situation shown in Figures 2 and 3. In particular, the behaviour of the boundary layer is not modelled in detail. In practice the entire boundary layer is unlikely to detach and form an intrusion as shown in Figure 4, for example. It is likely that the laminar sub-layer will not detach and this has been confirmed by our experiments.

However, the theoretical model does, for the first time, provide a means of making a first estimate of the maximum stratification in a natural, displacement-ventilated room containing a vertical heated surface. Data from graphs such as those shown in Figures 5 and 6 allow a designer to predict the temperature in such a room as a function of the heat flux input to the air at the wall, $q''$, the room dimensions and the area of the vents.

As an example of how this information might used in practice, consider the case of a room 5m in height with one wall 10m in width being uniformly heated such that a heat flux $q''$ = 100W/m$^2$ is transferred to the adjacent air (all other surfaces are adiabatic). The room measures 10x10m in plan. Let us also assume that the entrainment constant $\alpha = 0.03$ and that the effective vent area is rather small, $A^*$ = 0.15m$^2$. The buoyancy flux at the wall is therefore $F_0$ = 0.0028m$^2$s$^{-3}$ and the non-dimensional vent area is $A^*/\alpha HL$=0.1. From Figure 6 the non-dimensional density at the top of the room is therefore $\delta \sim -5.3$ and from equation (4) we can deduce the buoyancy of the air at the top of room to be $\Delta_0(1)=-0.64$. If the ambient temperature is 20°C then the temperature at the top of the room will be approximately 40.4°C.

## 5 CONCLUSION

A theoretical model of the transient development of ventilation flow and stratification in a naturally ventilated room containing a heated vertical surface has been developed. By assuming the boundary layer to be fully turbulent and that the effects of viscosity are therefore negligible it is then possible to model the boundary layer as a buoyant plume developing as a result of a vertically distributed source of buoyancy.

The governing differential equations governing the flow in both the boundary layer and in the room itself have been solved using a numerical technique for the transient cases of a sealed room and a ventilated room.

The results of this theoretical model provide a means of estimating the vertical temperature distribution in such a room. They also provide an explanation for the phenomenon observed in laboratory scale models of rooms whereby the natural convection boundary layer may detach from the heated wall and form one or more horizontal intrusions into the room for particular values of non-dimensional vent area.

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