Unsteady flow effects due to fluctuating wind pressures in natural ventilation design—mean flow rates

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Abstract

The paper presents the results of a study into the effects of unsteady wind pressures on the mean flow rates in certain types of purpose-designed naturally ventilated buildings. The study used nondimensional parameters and the results should therefore cover a wide range of conditions and should be of general application. It is concluded that unsteady effects are restricted to a relatively narrow band of conditions. These conditions have been quantified in terms of nondimensional parameters. In particular a simple procedure for calculating the mean flow rates when the unsteady effects are large has been derived. In its simplest form the procedure does not require knowledge of the instantaneous wind pressures. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

1.1. Aims of work

The work described forms part of a larger project to determine unsteady flow effects in relation to natural ventilation design. The re-emergence of natural ventilation as a serious option for non-domestic buildings has led to renewed interest in design procedures e.g. [1]. This is partly due to commercial necessity, but also because the use of purpose-provided openings of known geometry at known positions makes relatively sophisticated procedures more viable [2]. However as far as is known, all current design procedures employed in practice ignore unsteady flow effects. One major reason for this is the greatly increased amount of information needed to take full account of unsteady effects. At present such data is scarce, mainly because it is difficult and costly to obtain.

Thus, the overall aim of the project is to determine, in a general way, the conditions for which unsteady effects can be ignored. There are two aspects to this problem, namely the effects on time-averaged flow rates and the effects on flow patterns (e.g. the direction of flow through passive stacks). The present paper is concerned only with the first aspect i.e. mean (time-averaged) flow rates. One aim is to define a general way, by means of nondimensional parameters, the conditions under which unsteady wind effects may need to be considered when calculating mean flow rates. The second aim is to provide a simple means of accounting for these effects when desirable.

Although the study is of general application, it does have limitations which need to be recognised. The study is concerned primarily with two-opening cases, but as explained later these do have practical significance. The single-opening case is considered, but this is more of academic rather than practical interest. A further limitation is that the unsteady flows are assumed to be generated purely by unsteady surface wind pressures i.e. the effects of unsteadiness in the external velocity field around the openings are neglected. This is partly because the treatment of such effects is extremely difficult and partly because they will generally be less important than the effects of
pressure fluctuations. Unsteadiness arising from temperature fluctuations is neglected, on the basis that bulk temperature fluctuations occur at very low frequencies (much lower than wind pressure fluctuations) and they can therefore be treated by conventional steady flow theory. Finally the study is for uniform internal temperature, but there would be no particular difficulty in extending it to non-uniform temperature.

1.2. Previous work

The literature contains quite a few papers on unsteady flows in relation to ventilation and to internal pressures (from the viewpoint of structural forces generated by the wind). Useful summaries can be found in [3–5]. Various approaches to predicting unsteady effects have been made and they can be roughly divided into two categories i.e. those which operate in the frequency domain and those which operate in the time domain. Examples of frequency-domain methods can be found in [4,6]. They have the advantage that solutions can be obtained quickly and in an elegant way from a knowledge of the frequency spectra and the correlations of the external pressures. However these methods make assumptions to linearise the flow equations so that the spectra can be used in this way. A further assumption may also be made that the mean flow rates and mean internal pressure can be obtained from a conventional steady-state solution. Both of these assumptions become less tenable when fluctuating flow rates are large in relation to mean flow rates. Time-domain methods [7,8] rely on knowledge of the variation of external pressures with time (time histories) and the histories need to be recorded simultaneously so that they contain information about the correlations between pressures. There is no essential difference between the two methods in this respect, because time histories can in principle be obtained from spectral data, and vice versa, by means of Fourier transform techniques. However, no special assumptions about flow equations are required to obtain solutions in the time domain and for this reason a time-domain approach was adopted for the present work. This does not mean that the flow equations do not contain assumptions (they certainly do), only that fewer assumptions are needed. In fact, in the present work the effects of some of the assumptions are specifically investigated.
A problem faced by users of either approach is the dearth of simultaneous data on fluctuating external pressures. For the present work some real time histories have been used, but much of the investigation has been made with simplified pressure variations at discrete frequencies and phase shifts. This has the advantage that it reveals the effects of the various parameters in a simple way. Although it is true that such variations do not actually occur in practice it seems reasonable to assume that this approach will give a good indication of the importance of the various parameters. Some support for this assumption will be found from the results for real time histories.

2. Mathematical models

The following summary of mathematical models which are relevant here is based on the terminology used in [5]. Models can be distinguished by the assumptions which are made about the flow equation and the continuity (mass conservation) equation.

2.1. Flow equation assumptions

2.1.1. Pseudo-steady flow assumption

The time-averaged flow characteristic of an opening is the same as that for truly steady flow. The flow equation for a sharp-edged orifice in truly steady flow is

\[ q = C_r A \sqrt{\frac{2\Delta p}{\rho}} \]  

and this is assumed to be valid when the flow and pressure are replaced by their mean values and the discharge coefficient has the same constant value

\[ \tilde{q} = C_r A \sqrt{\frac{2|\Delta p|}{\rho}} \]  

This is the assumption usually made for design procedures.

2.1.2. Quasi-steady flow assumption

At each instant of time the flow behaves as if it were truly steady i.e.

\[ q(t) = C_r A \sqrt{\frac{2|\Delta p(t)|}{\rho}} S_p \]  

or

\[ q^2(t) S_p = 2C_r^2 A^2 \Delta p(t) \rho \]  

2.1.3. Quasi-steady/temporal inertia assumption

This is an extension of the quasi-steady model to include an inertia term. For the present study Eq. (3) is replaced by

\[ q^2(t) S_p A \frac{\Delta q(t)}{dt} = 2C_r^2 A^2 \frac{\Delta p(t)}{\rho} \]  

The origin and justification for this equation are given in Appendix A. The term \( l_e \) denotes the so-called effective length of the opening. It is intended to take account of the inertia of the mass of air which is accelerated under unsteady conditions. As far as is known \( l_e \) can not be obtained theoretically from a knowledge of the geometry of the opening and has to be determined empirically. Sharp-edged openings have a non-zero value despite the absence of any depth to such openings. The question of the specification of \( l_e \) in terms of the geometry of the opening is discussed in more detail in Section 5 below.

2.2. Continuity equation assumptions

The instantaneous form of the continuity equation for the air within the envelope is

\[ \frac{d\rho_1}{dt} = \rho_1 q_1(t) + \rho_2 q_2(t) \]  

For the conditions of interest here the difference between the internal and external densities can usually be neglected in the continuity equation (i.e. the Boussinesq approximation is valid). This is clearly the case when the ventilation arises purely from wind effects i.e. the temperature difference between inside and outside is negligible. When the temperature difference is large the unsteady effects of wind become small and the conditions are therefore not relevant. Thus, Eq. (5) can be replaced by

\[ \frac{1}{\rho_1} \frac{d\rho_1}{dt} = q_1(t) + q_2(t) \]  

The fluctuations of \( \rho_1 \) about the mean value are small, since they are due to pressure variations, so that any correlation they might have with flow rates is negligible. The time averaged form of Eq. (6) is therefore

\[ \bar{q}_1 + \bar{q}_2 = 0 \]  

This equation is in a form which can be directly used with the pseudo-steady flow equation to form the pseudo-steady model. It is this model which is invariably adopted for practical design procedures. Note that Eq. (7) states that the compressibility of the air has no effect on the mean flow rates.

For quasi-steady flow models an instantaneous form of the continuity equation is needed. The neglect of compressibility gives the incompressible form
From the above, the following hierarchy of mathematical models is obtained.

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The above models form the basis of the present study and are identified in the text by the acronyms. Most attention is given to the quasi-steady temporal inertia model, QT, because this is the most rigorous. It includes compressibility. In principle an incompressible form should also be included for completeness, but this was not considered worthwhile, because there is little to justify this form of model (the major increase in complexity arises from the inclusion of inertia rather than compressibility).

3. Present study

The basic approach in the presentation of results is to consider solutions of the various models in turn, beginning with the PS model and progressing to the QT model, which is the most rigorous of the models listed above. Solutions are described initially for wind pressure fluctuations which occur at discrete frequencies, rather than the more realistic case of pressure fluctuations which cover a range of frequencies. Discrete frequencies have the advantage that they allow the effects of the various parameters to be more readily identified. They are of course less time-consuming in their calculation, but this is not a major problem (computer run times for time histories were less than 20 min). A particular point to note is that the equations are solved in nondimensional form, so that the results have general application.

3.1. Building types

Although the case of two openings may seem rather simple, it is in fact relevant to several practical cases (see Fig. 1). Details are given in Section 8, but in summary there are five cases as follows.

Cases 1 to 3: large spaces with two dominant openings, one or both of which may be a chimney. Good examples of this case are the lecture theatres at De Montfort University in the UK [1]. The so-called ‘wind catcher’ systems are other examples, when a single system is used for the space.

Cases 4 and 5: rooms or floors which are isolated from other parts of the building and which can be treated separately as far as ventilation is concerned. There are two cases here, namely ‘single-sided’ and ‘crossflow’ ventilation.

4. Equations

The derivation of the following equations and the assumptions and approximations on which they rely are given in Appendix A. The following nondimensional variables are used.

The nondimensional flow rates \( q_1[t] \) and \( q_2[t] \) are obtained by dividing \( q_1[t] \) by \( C_{A_1} U A_1 \) and \( C_{A_2} U A_2 \), respectively, to give

\[
q_1'[t] = \frac{q_1[t]}{C_{A_1} U A_1}, \quad q_2'[t] = \frac{q_2[t]}{C_{A_2} U A_2}
\]  

The relative strength of the buoyancy and wind forces is measured by the Archimedes number, defined as

\[
Ar = \frac{\Delta \rho g H}{\rho U^2}
\]

(\( \Delta \rho \) is worth noting that \( Ar \) is defined differently to that used in [5] when dealing with envelope flows.)

Time is nondimensionalised as follows

\[
t' = t/\tau
\]

where

\[
\tau = \frac{H}{U}
\]
Case 1. Single space with two sharp-edged openings

Case 2. Single space with one sharp-edged opening and one long opening

Case 3. Single space with two long openings

Case 4. Isolated room with single-sided openings

Case 5. Isolated space with openings on different walls

Fig. 1. Cases of practical interest

\[
D = \frac{2C_A A_1 H}{V^2} \frac{Hc^2}{2U^2}, \quad B_1 = A_s z_1, \quad B_2 = A_s z_2,
\]

(13)

\[
E = \frac{C_{A2} A_2}{C_{A1} A_1}
\]

and

\[
F_1 = \frac{H}{l_{e1}}, \quad F_2 = \frac{H}{l_e}
\]

(14)

Pressure coefficients arising from the wind (i.e. the unsteady pressure components) are defined by

\[
C_{p1}(t) = \frac{p_{w1}(t) - p_{wo}}{0.5 \rho U^2}
\]

and

\[
C_{p2}(t) = \frac{p_{w2}(t) - p_{wo}}{0.5 \rho U^2}
\]

(15)

and

\[
C_{p}(t) = \frac{p_{wo}(t) - p_{wo}}{0.5 \rho U^2}
\]

(16)

where \( p_{wo} \) is a steady reference pressure and \( p_{wo}(t) \) is defined by Eq. (A12).
In the above definitions \( H \) and \( U \) are reference quantities which can be arbitrarily specified, although it is more useful for comparative purposes if they are chosen in an appropriate way.

### 4.1. Pseudo-steady model

The equations to be solved are simply (A38)

\[
\overline{q}_1' = \overline{q}_2' = \sqrt{A r \left( \frac{z_2}{H} - \frac{z_1}{H} \right) + \left( \frac{C_{p1} - C_{p2}}{2} \right)}
\]

Since the term under the square root is an important one and frequently occurs, it is given its own symbol \( W \) i.e.

\[
W = A r \left( \frac{z_2}{H} - \frac{z_1}{H} \right) + \left( \frac{C_{p1} - C_{p2}}{2} \right)
\]

\[
= B_2 - B_1 + \Delta C_p/2
\]

### 4.2. Quasi-steady incompressible model

The instantaneous flow rates are given in terms of the instantaneous pressure differences [see Eqs. (A36) and (A37)] for the special case with \( E = 1 \)

\[
q_1'(t) = \frac{q_1(A_1 U)}{\overline{q}_1' A_1 U} = \sqrt{A r \left( \frac{z_2}{H} - \frac{z_1}{H} \right) + \left( \frac{C_{p1}[t] - C_{p2}[t]}{2} \right)} S_{pl}
\]

\[
q_2'(t) = \frac{q_2(t)}{\overline{q}_2' A_2 U} = \sqrt{A r \left( \frac{z_2}{H} - \frac{z_1}{H} \right) + \left( \frac{C_{p1}[t] - C_{p2}[t]}{2} \right)} S_{p2}
\]

The mean flow rates are defined by

\[
\overline{q}_1' = \frac{1}{\psi r} \int_0^{\psi r} q_1' dt'
\]

which gives

\[
\overline{q}_1' = \frac{1}{\psi r} \int_0^{\psi r} \sqrt{B_2 - B_1 + \Delta C_p(t)/2} S_{pl} dt'
\]

### 4.3. Quasi-steady compressible model

The differential equation for the internal pressure is

\[
\frac{dC_{p1}}{dt'} = D_1 \sqrt{-C_{p1}[t] - 2B_1 + C_{p1}[t]} S_{p1} + E \sqrt{-C_{p1}[t] - 2B_2 + C_{p2}[t]} S_{p2}
\]

The nondimensional flow rates are obtained from the solution for \( C_{p1} \) using (A32) and (A33)

\[
q_1'[t] = \sqrt{-C_{p1}[t] - 2B_1 + C_{p1}[t]} S_{p1}
\]

\[
q_2'[t] = \sqrt{-C_{p1}[t] - 2B_2 + C_{p2}[t]} S_{p2}
\]

### 4.4. Quasi-steady temporal inertia model

The three simultaneous differential equations are (A24), (25) and (23)

\[
\frac{dC_{p1}}{dt'} = \frac{F_i}{2} (-C_{p1} - 2B_1 + C_{p1} - q_1'[t] S_{p1})
\]

\[
\frac{dC_{p2}}{dt'} = \frac{F_i}{2} (-C_{p1} - 2B_2 + C_{p2} - q_2'[t] S_{p2})
\]

\[
\frac{dC_{p1}}{dt'} = D(q_1' + E q_1')
\]

### 5. Method of solution

#### 5.1. Specification of wind pressures

With the exception of the PS model, it is necessary to know the variation of wind pressures as a function of time. For most calculations the wind pressures are specified as a sinusoidal fluctuation of the pressure coefficient about a mean value

\[
C_p[t] = C_{p0} + C_p'[t]
\]

where

\[
C_{p0} = \frac{p_{w0} - p_{w0}}{p U^2}
\]

and

\[
C_p'[t] = C_{p0} \sin[2\pi(\omega t + \phi)]
\]

where the frequency and phase are denoted by \( \omega \) [Hz] and \( \phi \) [cycles], respectively.

The nondimensional frequency is defined by

\[
\omega' \equiv \omega \tau
\]

and since \( t' = t/\tau \)

\[
C_p'[t] = C_{p0} \sin[2\pi(\omega' t' + \phi)]
\]
which can be written as
\[ C_{pl}'(t) = C_{pd} \sin[2\pi\omega'(t' + \phi')] \]  
(32)

where
\[ \phi' = \phi/\omega' \]  
(33)

The other calculations use real time-histories (see Section 7).

5.2. Pseudo-steady model

For the case when \( z_2 > z_1 \) and \( C_{z1}A_1 = C_{z2}A_2 \) there is a simple algebraic solution (see Eq. 2.5.8 in [5] where \( H \) is defined as the difference between \( z_1 \) and \( z_2 \))

\[ Q_{ps} = \sqrt{W} \quad \text{and} \quad \bar{q}_{p1} = 0 \]  
(34)

where it is assumed \( C_{pl1} > C_{p2} \).

5.3. Quasi-steady incompressible

The QI case can be solved by numerical integration of the right-hand side of Eqs. (19) and (20) when the \( C_{pl} \) are known as functions of time. This is a relatively simple procedure. The difficulty of course lies in obtaining simultaneous records of \( C_{pl1} \) and \( C_{pl2} \).

For the case where the pressure coefficients are sinusoidal with equal amplitudes and frequencies, the time-averages over a period take the functional form

\[ \frac{\bar{q}}{\sqrt{C_{pl}}} = f \left( \frac{W}{C_{pd}} \phi' \right) \]  
(35)

and using (34) this can be written as

\[ \frac{\bar{q}}{\sqrt{C_{pl}}} = f \left( \frac{Q_{ps}^2}{C_{pd}} \phi' \right) \]  
(36)

There is thus a direct relationship between the pseudo-steady flow rates and the corresponding quasi-steady incompressible rates. The former actually correspond to the case with \( \phi' = 0 \).

5.4. Quasi-steady QC and QT models

The QC and QT models require initial values for the variables at time \( t' = 0 \). The initial values have an influence on the subsequent values of \( q_{s1}' \) and \( C_{pl}' \) and since the initial values are chosen more or less arbitrarily, it is desirable to restrict attention to values of \( t' > \) beyond which the influence of the initial values is negligible. This has been achieved by carrying out the calculation in two consecutive parts. The first part is long enough for initial transients to have died away and the values obtained at the end of this time are used for the initial values of the second part. All solutions have been obtained numerically using programs written with MATLAB software [12]. For most cases the standard solver for differential equations was used (i.e. 'ode45'), but in a few cases a stiff solver ('ode15 s') was employed to eliminate high frequency noise.

5.5. Choice of \( l_e \) and \( C_z \) in QT model

5.5.1. Sharp-edged openings

For a single sharp-edged opening in an otherwise sealed envelope, it is possible to derive theoretically an expression for the effective length \( l_e \) for the case of acoustic phenomena (Helmholtz resonator) in terms of the diameter of a circular opening i.e.

\[ l_e = 0.85d \]  
(37)

This relationship has been applied to ventilation flows through sharp-edged openings [7], but it is an assumption, because the main terms in the equations of motion are not the same as those for acoustic flows [5]. However as will be seen below an equation of the form of (37) does seem justifiable, at least when combined with the assumption of a constant \( C_z \). Bearing in mind that the use of Eq. (4) for describing the bulk flow through an opening is itself an approximation, it is probably best to view the choice of \( C_z \) and \( l_e \) as linked.

For the following calculations, \( C_z \) has been taken as 0.6 which corresponds to the steady flow value through a two-dimensional slit and this is consistent with the assumption of a constant streamline pattern. Specifying \( l_e \) is rather more difficult and it has been done by making use of the experimental data of Modera [11]. Modera tested single openings subjected to a nominally sinusoidal oscillating pressure differential. For such tests, Eqs. (26) to (28) reduce to

\[ \frac{dq'}{dt'} = \frac{F}{2} (C_{pd} \sin 2\pi\omega' t' - q_{ps}'^2 S_d) \]  
(38)

The relevant length scale is \( d \) i.e. \( F = d l_e \). There is no actual reference velocity in the boundary conditions, but one can be formed from the amplitude of the driving pressure difference \( \Delta p_d \)

\[ U = \sqrt{\frac{2\Delta p_d}{\rho}} \]  
(39)

The functional relationship

\[ q' = f \left( \frac{H}{2l_e}, \omega', C_{pd}, l_e' \right) \]  
(40)

follows from (38), where the function \( f \) relates to a given shape of opening.
Substitution for $H$ and $U$ gives

$$q' = f \left[ \frac{d}{2l_e}, \omega', t' \right]$$  \hspace{1cm} (41)

since $C_{pd}$ becomes a constant equal to 1.0. This reflects the fact that the number of boundary conditions is reduced by the absence of a reference wind speed.

Moderne presents results in the form of the phase lag between the pressure signal and the velocity at the centre of the opening. There is some approximation in this, because the fluctuating velocity is not a pure sinusoid. For the present comparison the zero crossing of the fluctuating flow rate has been used to define the phase lag i.e. the time between the points at which the pressure difference and the flow rate are equal to zero. Denoting this time by $t_{lag}$ it can be seen from (41) that the nondimensional time lag is a function only of $\omega'$ and $d/l_e$. For a given shape of opening. This follows because the function $f$ is periodic and $t'$ is eliminated when integrating over a period. By solving (38) with $C_{pd} = 1$ for different values of $F$ it is possible to determine which value of $F$ gives the best agreement with Moderne's data. The results are shown in Fig. 2 both as $t_{lag}$ and as the tangent of the phase angle.

It is clear that

$$F = \frac{d}{l_e} = 1.0$$ \hspace{1cm} (42)

gives close agreement. The results for $F = 1.96$ are shown for comparison. These results correspond to $l_e/d = 0.51$, which with $C_z = 0.6$ corresponds to $l_{eo}$. $d = 0.85$ i.e. the relationship according to Eq. (37). It can be seen that with $C_z = 0.6$ Eq. (42) is more appropriate. This result is useful in two ways. First, the value of $d/l_e$ allows the calculations to be related to practical situations (see Section 8). Second, it indicates that (38) and hence the QT model with $C_z = 0.6$ and $d/l_e = 1.0$ can describe the flow through sharp-edged slits reasonably well. There is a problem here, and that is the experimental data lies at values of $\omega' < 0.1$, whereas it will be seen that larger values (at least up to $\omega' = 1$) also need to be considered. In the absence of more extensive data it has to be assumed that the QT model remains valid at these values of $\omega'$.

It should also be noted that Holmes concluded that a value of 0.15 for $C_z$ gave better predictions of internal pressure fluctuations. This casts some doubt on the chosen value of 0.6. However Holmes chose Eq. (37) to specify $l_e$ and this is not necessarily justifiable, since the concept of flow separation (which actually determines $C_z$) is not appropriate for acoustic phenomena. Moderne concluded from his experiments that a slightly larger value than 0.6 was more appropriate. Bearing in mind that Moderne's tests are much more relevant to the present work and the fact that the chosen value should also satisfy steady conditions, it is believed that retaining the value of 0.6 is justifiable.

5.5.2. Long openings

For openings with appreciable depth $l_e$ has been taken as equal to the depth, $L$, plus the effective length of the inlet and outlet (i.e. $L + d$). This assumes that

![Fig. 2. Comparison of Eq. (38) with data from Moderne [11]. Upper part of figure shows results for $t_{lag}$ and lower part the results for $\tan \phi$. The curves show the predictions of Eq. (38) for $l_e/d = 1.0$ and 1.176. Experimental points are shown by • and x.](image-url)
the air within the opening behaves as a simple plug flow and for the present purposes this seems to be justified by the results in [8]. It should also be noted that the definition of \( L \) includes \( C_r \) (see Appendix) so that by adopting the above simple form for the total \( L \), the effective length is \( C_r L \). For the purposes of the present exercise, where the interest lies in the practical range of values of \( F \) rather than specific cases, this is not a major issue. When dealing with the instantaneous flows however, it will need to be examined more carefully (this is the subject of a separate paper).

6. Results for discrete frequencies

When dealing with mean flow rates it is necessary to distinguish between the entry of fresh air and the exit of air. In the following the mean rates at which fresh air enters through openings 1 and 2 are denoted by \( q_F1 \) and \( q_F2 \) respectively. The total fresh air entry is denoted by \( Q_F \) where

\[
Q_F = \bar{q}_{F1} + \bar{q}_{F2} \tag{43}
\]

and

\[
Q_F' = \frac{Q_F}{C_{21}A_1U} = \bar{q}_{F1} + \bar{E}d_{l/2} \tag{44}
\]

It should be noted that \( Q_F' \) is defined using \( A_1 \), not the total area.

For large spaces where opening 2 is situated at high level relative to opening 1, fresh air entry through opening 2 may not contribute effectively to the ventilation of the space and it may be neglected. In the following therefore both \( \bar{q}_{F1} \) and \( \bar{Q}_F \) are considered.

6.1. Pseudo-steady

The results for the pseudo-steady model are shown by the curve for \( \phi' = 0 \) in Fig. 3(a) and (b). Both sides of Eq. (34) have been divided by \( \sqrt{C_{pd}} \).

6.2. Quasi-steady incompressible

For the case where \( E = 1 \) the total mean fresh air entry rate is given by

\[
Q_F' = \bar{q}_{F1}' = \bar{q}_{F2}' \tag{45}
\]

In general \( Q_F' \neq \bar{q}_{F1}' \), but \( Q_i' = q_i' / A \) when \( C_{pi} \) is greater than \( C_{pd} \) at all times and for the particular case where \( q_i' > 0.5 \), \( A_r = 0 \) and \( \Delta C_{ri} = 0 \), \( Q_i' = \bar{q}_{F1}' = \bar{q}_{F2}' \).

Fig. 3(a) and (b) show the variation of \( Q_i' / \sqrt{C_{pd}} \) and \( \bar{q}_{F1}' / \sqrt{C_{pd}} \) with \( \phi' \) for a range of values of \( W/C_{pd} \) with the corresponding values of \( Q_{ps} / \sqrt{C_{pd}} \). As noted above the PS solution corresponds to \( \phi' = 0 \), so that from this one plot the difference between the two solutions arising from the phase difference can be seen immediately for any combination of wind and buoyancy.

However it is more useful to plot the results in the form \( Q_F' / \sqrt{\sigma_{\Delta P}} \) and \( q_F' / \sqrt{\sigma_{\Delta P}} \) against \( W \sigma_{\Delta P} \), where \( \sigma_{\Delta P} \) denotes the standard deviation of the instantaneous pressure difference

\[
\Delta C_p[t] = C_{pi}[t] - C_{p2}[t] \tag{46}
\]

In this form the results collapse onto single curves which are independent of phase angle. As shown in Fig. 4. The basic reason for this is that \( \Delta C_p[t] \) is a sine wave, as explained in Appendix B. For the general case this will not be true, but it will be seen below that the two curves can be applied generally with acceptable accuracy and Fig. 4 is therefore important and is discussed further in Section 8.

As expected, Figs. 3 and 4 show that the higher the ratio between the mean and fluctuating pressures, the lower are the effects of the fluctuations. For \( W \) \( \sigma_{\Delta P < 0.8 \text{ the effect of } \phi' \text{ on the mean flow rate } Q_F' \text{ is less than } 10\% \text{ and this is likely to be an acceptable error in the context of design.} \)

6.3. Quasi-steady compressible

For this case two extra parameters \( \omega' \) and \( D \) are introduced, namely the frequency parameter of the wind pressure fluctuations and the compressibility parameter of the volume of air enclosed by the space. The interest lies in the values of these parameters for which the effects of compressibility are significant and this can best be seen by plotting results against \( \omega' \) for given values of \( D \), as shown in Fig. 5(a) and (b). (Although this plot has a similar form to a spectral plot, it should not be confused with one—the results are for discrete frequencies.) The value of \( W/C_{pd} \) is 0.1667.

The first point to note from Fig. 5(a) is that the effects of compressibility are negligible for \( D = 100 \). With \( D = 1 \) and low values of \( \phi' \), the differences between QC and Ql results can greatly exceed 20% when \( \omega' \) is greater than about 0.5. At frequencies below 0.5, the effects of compressibility can be neglected for all values of \( D \) and \( \phi' \). The situation for \( q_{F1}' \) is similar but less pronounced, and the effects are apparent down to values of \( \omega' \) of about 0.1.

It can be seen that for \( Q_F' \) the effects are greatest with \( \phi'=0 \). This corresponds closely to the case of a single opening, for which the air entry is due purely to compressibility (and for which \( W/C_{pd} = 0 \)).

Although the effects of compressibility can be large, they occur at relatively high frequencies where one can expect the effects of inertia to be most pronounced.
6.4. *Quasi-steady temporal inertia*

Two extra parameters, $F_1$ and $F_2$, related to the effective lengths of the two openings are introduced in the QT model. The smaller the value of $F$, the greater is the inertia.

It is convenient to retain the same form of plot as above and Fig. 6(a) and (b) show the equivalent results to Fig. 5(a) and (b) for $\phi = 0.125$ ($W/C_{pd} = 0.1667$ for both sets of results). For comparison the QC results for $D = 1$ and $D = 1000$ are shown in Fig. 6(a).

It can be seen that inertial effects significantly alter...
the results, but the largest effects are restricted to the high frequencies. With \( F = 100 \) (relatively low inertia) and \( D = 1 \) (relatively high compressibility) the effects are only apparent for \( \omega' > 1 \), where the increased flow rates at high frequencies due to compressibility are much reduced. When the inertia is relatively high (\( F = 10 \)) there is a clear tendency to reduce the mean flow rate and this is noticeable at much lower values of \( \omega' \), down to 0.01. Very similar results were obtained for \( \phi' = 0.25 \). On balance therefore, it seems that the more complicated QT model is generally required, except for cases where compressibility is high and inertia is low (low \( D \), high \( F \)).

Real pressure fluctuations will of course cover a wide range of frequencies, often extending to very low values of \( \omega' \). The average effects of compressibility and inertia will therefore tend to be smaller than those indicated for a particular frequency in Fig. 6.

6.5. Single opening in a sealed room

Results for a single opening are obtained by setting \( E = 0 \) in Eqs. (23) and (28). Values of \( \bar{q}_{f1} \) as a function of \( \omega' \) are shown in Fig. 7(a) and (b). Three sets of results were obtained with the QT model (with \( F = 10 \) and \( D = 1 \), 10 or 100) and three sets with the QC model using the same values of \( D \), all with \( C_{pd} = 0.3 \).

The QT results exhibit resonant peaks as expected for Eqs. (26) and (28). The theoretical resonant frequency, \( \omega_r \) [Hz] is given by, see for example [7],

\[
\omega_r = \frac{1}{2\pi} \sqrt{\frac{A_0^2}{k_0 V}}
\]

which in nondimensional form is

\[
\omega_r' = \frac{1}{2\pi} \sqrt{\frac{DF}{2}}
\]

The theoretical values corresponding to the values for \( D \) and \( F \) used in Fig. 7(a) are 0.356, 1.13 and 3.56.

Under certain conditions the flow rate can become independent of area \( A \) (see Eq. 3.9.6 in [5]) i.e.

\[
q(t) = \frac{1}{2\pi} \sqrt{\frac{A_0^2}{k_0 V}} \frac{df}{dr}
\]

which in nondimensional form is

\[
q'(t) = \frac{1}{2\pi} \sqrt{\frac{DF}{2}} \frac{dC_{pd}}{dr}
\]

For a sinusoidal variation of the pressure coefficient this reduces to
Fig. 5. Quasi-steady compressible results compared with quasi-steady results for \( W_{C,0.167} \). (a) Plot of \( Q_2 \) obtained from the QC model divided by \( Q_1 \) obtained with the QI model against \( W' \) for various values of \( \phi' \) and \( D \). (b) Plot of \( \bar{Q}_1 \) obtained from the QC model divided by \( \bar{Q}_1 \) obtained with the QI model against \( \phi' \) for various values of \( \phi' \) and \( D \).
Fig. 6. Quasi-steady temporal inertia results compared with quasi-steady compressible results for $W'_j/C_p = 0.167$. (a) Plot of $Q_f$ obtained from the QT model divided by $Q_f$ obtained with the QI model against $\omega'$ for various values of $F$ and $D$ with $\phi' = 0.125$. (b) Plot of $\overline{q}_{\text{QT}}$ obtained from the QT model divided by $\overline{q}_{\text{QI}}$ obtained with the QI model against $\omega'$ for various values of $F$ and $D$ with $\phi' = 0.125$. 
Fig. 7. QT and QC results for single opening for $C_{pa} = 0.3$. (a) Plot of $Q_c$ obtained from the QT model against $\alpha'$ for various values of $F$ and $D$.
(b) Plot of $Q_F$ obtained from the QC model against $\alpha'$ for various values of $D$. 
Table 1
Mean and standard deviations of surface pressure coefficients for time history records

<table>
<thead>
<tr>
<th>Record</th>
<th>( \bar{c}_p )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.459</td>
<td>0.228</td>
</tr>
<tr>
<td>2</td>
<td>-0.224</td>
<td>0.079</td>
</tr>
<tr>
<td>3</td>
<td>-0.763</td>
<td>0.253</td>
</tr>
<tr>
<td>4</td>
<td>-0.450</td>
<td>0.319</td>
</tr>
<tr>
<td>5</td>
<td>-0.463</td>
<td>0.246</td>
</tr>
</tbody>
</table>

Eq. (51) gives a reasonable description of the behaviour at low values of \( \omega' \), whereas at high \( \omega' \) the effects of inertia are dominant.

For the single opening case, it is particularly important to note that the results are only of value when the length scales of the turbulence close to the opening are large in relation to the opening. If this is not true, the flow rates can be expected to be significantly larger (see Section 3.9.2 of [5]).

7. Results for time histories

7.1. Pressure distributions

Simultaneous measurements of surface pressures on a test building with a pitched roof are reported in [9]. A small selection of this data has been obtained [10] and used for the present investigation. There are five time histories (referred to here as records 1 to 5), three being over one time period and two over another. Each record consists of 18,000 data points (relative to a steady reference pressure) recorded at a sampling frequency of 5 Hz. The pressures have been converted to coefficients using the mean wind speed (7.6 m/s for records 1, 2 and 3 and 10.0 m/s for records 4 and 5). The mean and standard deviation of the pressure coefficients are given in Table 1 and Fig. 8 shows the power density spectra of records 1 to 3, evaluated using Matlab software [12]. It can be seen that at this wind speed most of the turbulence energy lies at frequencies well below 1 Hz. In addition it should be noted that some of the energy apparent at the high frequencies will be due to aliasing.

From Table 1 it can be seen that the mean pressure difference between any two of the first three records is large. The values of \( W \) are high and therefore the corresponding effects of unsteadiness are small. In view of this, calculations have been made by applying a bias to the mean values to obtain smaller values of \( W \). This means that these time histories are somewhat artificial.

However the fluctuating components are still representative of what will be encountered in practice and are suitable for the present investigation where there is no interest in predicting ventilation rates for the particular building. Basically the time histories are a source of signals which have characteristics (spectra, standard deviations) which are representative of real signals. The correlation between the signals is perhaps least representative. There are no such problems with records 4 and 5.

For the calculations the nondimensional frequency has been obtained using \( H = 2.533 \) m i.e. \( \omega' = \omega/3 \) for records 1 to 3. The value was chosen because it is consistent with the value of \((z_2-z_1)\) which might occur for a building with the dimensions of the test structure (see Section 8).

For the calculations with the QT model, only 8192 points were used. This was partly to reduce the computation time, but the time span is still 27 min which is appropriate to ventilation phenomena. (The number 8192 is a power of 2 which is convenient for spectral analysis.)

7.2. Quasi-steady incompressible model

The results for values for \( \Delta c_p \) of 0, 0.1 and 0.3 are shown in Fig. 9 with the curves from Fig. 4 for comparison and it can be seen that they lie close to the basic curves for the simple sinusoidal fluctuations.

7.3. Quasi-steady compressible model

Calculations were not carried out with the QC model, since the QT model includes compressibility.

7.4. Quasi-steady temporal inertia model

The results obtained with the QT model are also shown in Fig. 9. They exhibit more spread than the QI results, but they are not wildly different. The differences reflect the inclusion of both compressibility and inertia effects. In the context of design it would seem that the greater accuracy achievable with the QT model may not be worth the extra computational effort required (for mean flow rates) compared to the QI model, at least for the pressure histories chosen. This is an important point and will be returned to later.

8. Application of results to cases of interest

8.1. Ranges of values of parameters

In the following some examples are given of the values of the various parameters which are likely to be
encountered in practice with the five cases of interest. It is worth noting that parameter $F$ depends only on the reference height and the size of the opening, $B$ depends on the Archimedes number and the height of the opening and $D$ depends on the volume of the space, the size of the opening and the wind speed.

It is convenient to take the reference height as the difference between $z_2$ and $z_1$. It is then unlikely that a design for natural ventilation will have a value of $H$ equal to zero, because it is the general rule to maximise the effect of buoyancy. The value of $H$ will normally lie somewhere between $0.5H_b$ and $H_b$ where $H_b$
denotes the height of the space or room. The exception to this is Case 3 where \((z_2 - z_1)\) is the distance between the external outlet and the internal inlet.

The values of \(A_1\) and \(A_2\) have been estimated using a normal design procedure for summer and winter \((\Delta T = 3\) and \(20^\circ C\), respectively with \(U = 0\)). For Cases 1 to 3 where the openings are assumed to be square or circular, \(l_c\) has simply been taken as the square-root of \(A\). For the openings with appreciable depth in Cases 2 and 3, the axial length (depth) of the opening has then been added. With Cases 4 and 5, the openings are assumed to be in windows with a width of 1 m, from which \(d\) can be calculated from the design area for use in (42).

8.1.1. Case 1. Single large space with two sharp-edged openings

This case is perhaps the most hypothetical of the five, but it can be considered as a simple example of a building with a large upper opening in an atrium.

8.1.2. Case 2. Single large space with one sharp-edged opening and one long opening

Good examples of this case are the lecture theatres in the Queen's building at De Montfort University in the UK. Also covered by this case are certain configurations of 'windcatchers'. The large distance between the inlet and outlet implies that the correlation between the pressures will be relatively low.

8.1.3. Case 3. Single space with two long openings in close proximity

This case corresponds to a certain type of 'windcatcher' whereby the inlet and outlet ducts form a single unit. The proximity of the inlet and outlet implies that differences between the phase and frequencies of the pressure fluctuations at the two points will be relatively small i.e. high correlation.

8.1.4. Case 4. Isolated room with two single-sided sharp-edged openings

Single-sided ventilation of office rooms is a common strategy with natural ventilation.

8.1.5. Case 5. Isolated room with a sharp-edged opening on two different walls

This case corresponds to crossflow ventilation strategy and is probably less common than Case 4.

Table 2 shows the basic dimensions chosen for each case and Table 3 shows the corresponding values of the parameters.

Thus, with the above definition of \(H\), the values of \(D\) range from about 0.5 to \(200 \times 10^3\) and the values of \(F\) from about 0.5 to 100. The values of \((B_1 - B_2)\), which
equals \( \Delta P \) with this choice of \( H \), range from 0.0015 to 10.

However if the ratio between the mean pressures and the fluctuating pressures, \( W/\sigma_{\Delta P} \), is greater than about 1.0, it is not necessary to be concerned about unsteady effects. This is almost certainly the case when \( (B_1-B_2) \) exceeds 1.0 irrespective of the mean pressure difference. It can be seen therefore that there is no need for concern for a large part of the winter conditions.

8.2. Pressure fluctuation parameters \( \omega' \) and \( \phi' \)

It is clear from Table 3 that low values of both \( D \) and \( F \) can occur such that significant effects of compressibility and inertia (compared to the PS and QI models) can arise, but only at high values of \( \omega' \). The frequency parameter is therefore crucial in determining whether these effects are significant. For most of the time histories used here very little energy resides at \( \omega' > 1 \) but for one of the records about 30% of the energy lies above that value. Even for this record the mean flow rates lie close to the QI results in Fig. 9.

To investigate further the effect of \( \omega' \), for one of the calculations with the QT model the values of \( \omega' \) have been increased by a factor of 10. For this calculation the effects of compressibility and inertia were maximised by setting \( D = 1 \) and \( F = 10 \). Table 5 shows the results compared with the original values. Although there is a discernible effect, it is only about 5% and the results are quite close to the others in Fig. 9.

As well as the energy spectrum, the correlation between the two surface pressures is important. If both pressures tend to increase and decrease at the same time (i.e. they are positively correlated), this will tend to reduce the fluctuations of the instantaneous pressure difference. In the extreme case (where the two surface fluctuations are identical), the fluctuations in their difference is equal to zero (this corresponds to \( \phi = 0 \) for the discrete sine waves). If the two surface pressures are negatively correlated, the standard deviation of \( \Delta P \) will tend to be large. On the other hand, if the two pressures have zero correlation their standard deviations would be related to \( \sigma_{\Delta P} \) by a simple relation (Eq. 52).

In general some correlation can be expected and the values of \( \sigma_1, \sigma_2 \) and \( \sigma_{\Delta P} \) given in Table 4 indicate that in practice it can be positive or negative. Each surface pressure record can be approximated by a Fourier series consisting of a large number of cosine waves at discrete frequencies and with a specific amplitude and phase. At a given frequency, the contribution of the two surface pressures to the pressure difference will be determined by the amplitudes of the two pressures and their phases. However, the effects of compressibility and inertia are only important at the higher frequencies and \( \omega' \) is therefore a more important parameter than \( \phi \).
Table 4
Comparison between actual and estimated values of $\sigma_{\Delta C_p}$

<table>
<thead>
<tr>
<th>Data sets used for pressure difference</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>Actual $\sigma_{\Delta C_p}$</th>
<th>$\sigma_{\Delta C_p}$ from (52)</th>
<th>$\sigma_{\Delta C_p}$ from (53)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 2</td>
<td>0.228</td>
<td>0.079</td>
<td>0.255</td>
<td>0.242</td>
<td>0.308</td>
</tr>
<tr>
<td>1 and 3</td>
<td>0.228</td>
<td>0.253</td>
<td>0.401</td>
<td>0.341</td>
<td>0.481</td>
</tr>
<tr>
<td>2 and 3</td>
<td>0.079</td>
<td>0.253</td>
<td>0.226</td>
<td>0.265</td>
<td>0.333</td>
</tr>
<tr>
<td>4 and 3</td>
<td>0.319</td>
<td>0.246</td>
<td>0.348</td>
<td>0.403</td>
<td>0.565</td>
</tr>
</tbody>
</table>

8.3. Simple method for estimating unsteady effects on mean flow rates

To be able to estimate the importance of unsteady wind effects requires some knowledge of the pressure fluctuations due to the wind. At the present time simultaneous records of the instantaneous surface pressures $C_{pi}(t)$ and $C_{pi}(t)$ are relatively rare in the literature and unlikely to be available to the designer. Values for the standard deviations, $\sigma_1$ and $\sigma_2$, are more common, because they do not require simultaneous measurement. If not, typical values can be used. This opens the way to estimating the importance of wind effects using Fig. 9.

The crucial parameter is $W\sigma_{\Delta C_p}$. The value of $W$ can be calculated using Eq. (18), which requires values for the mean pressure coefficients for which use can be made of tabulated data. If the standard deviations of the pressure fluctuations are known, $\sigma_{\Delta C_p}$ can be estimated from equations such as

$$\sigma_{\Delta C_p} = \sqrt{\sigma_1^2 + \sigma_2^2}$$  \hspace{1cm} (52)

or

$$\sigma_{\Delta C_p} = \sigma_1 + \sigma_2$$  \hspace{1cm} (53)

Both equations are approximate. The former equation assumes no correlation between the pressures and the latter equation will lead to a larger value than the former. In this connection it is worth comparing the values of $\sigma_{\Delta C_p}$ obtained from the equations with the actual values from the time histories, see Table 5.

The values of $\sigma_1$ and $\sigma_2$ given in [9] are virtually all less than 1.0. However the building was in an exposed position and the values are not necessarily representative of other environments.

Table 5
Effect of increasing $\omega'$ by factor of 10 on mean flow rates

<table>
<thead>
<tr>
<th>Nondimensional frequency</th>
<th>$Q_Q' / \sigma_{M^{1/2}}$</th>
<th>$Q_Q' / \sigma_{M^{1/2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>0.5105</td>
<td>0.2733</td>
</tr>
<tr>
<td>Increased by 10</td>
<td>0.4900</td>
<td>0.2668</td>
</tr>
</tbody>
</table>

8.4. Comments on multiple openings

It has been seen that with two openings, the mean flow rates are not greatly influenced by unsteady wind effects when the parameter $W\sigma_{\pi_d}$ exceeds unity. For buildings with multiple openings this observation is likely to become less valid as the number of openings increases. Probably the most relevant practical example is a high-rise building with a large outlet opening at the top of an atrium. This corresponds to Case 1 with the lower opening replaced by smaller openings on each floor level. Design examples for such a building are given in [1,2]. The mean pressure differences across the outlet opening and the small openings at the higher levels will tend to be smaller than when there are only two openings. The reason for this is that the neutral level for the buoyancy pressures is designed to be just below the level of the outlet opening. When there are only two openings, the neutral level will tend to lie midway between the two openings.

It is outside the scope of this paper to comment further on multiple openings, except to note that a general treatment of such cases will be relatively difficult due to the need to know the simultaneous surface wind pressures at each opening. However it may be possible to derive an approximate procedure, similar to that using Fig. 9, where the instantaneous pressures are not required.

9. Conclusions

1. Comparisons with published experimental data indicate that the quasi-steady temporal inertia model [Eqs. (26)-(28)] with $C_z=0.6$ and $l_c/d = 1.0$ gives a good description of the unsteady volume flow through sharp-edged openings for values of $\omega'$ up to 0.1.

The nondimensional study described has led to the following conclusions which are of a general nature and of wide application, subject to the proviso that experimental verification of the model at higher values of $\omega'$ is desirable. There is also a need to establish more definite relationships between $l_c$ and the geometry of long openings. It should also be re-stated that the conclusions concern mean flow
rates in spaces with only two well-defined openings.

2. Unsteady flow effects can be ignored if the parameter $W/\sigma_{DCP}$ exceeds 1.

3. For lower values of $W/\sigma_{DCP}$ the mean fresh air flow rates can be adequately estimated, at least for design purposes, by using the nondimensional curves given in Fig. 9. If necessary the procedure can be carried out without specific knowledge of the instantaneous pressures. This is a very significant benefit, although it is accompanied by increased uncertainty in the results.

4. There is little benefit to be gained by including the effects of compressibility and inertia if a large proportion of the energy in the wind pressure fluctuations lies at nondimensional frequencies below about 0.5.

5. As a general rule, compressibility and inertia effects should not be treated in isolation.

Acknowledgements

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Appendix A. Equations for the models

In the following the equations for the QT model are first derived, from which the other models are obtained by applying the relevant assumptions.

A.1. Quasi-steady temporal inertia model

![Diagram](image)

A.1.1. Flow equation

The starting point is to relate the instantaneous flow rate, $q(t)$, through an opening to the instantaneous pressure difference, $\Delta p(t)$, across the opening, where $\Delta p(t)$ is determined from pressures at points which are not influenced by the flow through the opening.

With steady conditions an accurate description of the flow through a two-dimensional slit can be obtained by applying free streamline theory which leads to the result $C_x = 0.6$. The solution can be expressed in terms of a velocity potential $\Phi$, which is a function of $x$ and $y$.

The unsteady bulk flow through a sharp-edged opening is derived by making the quasi-steady assumption that the streamline pattern remains the same as that for steady flow. The function $\Phi$ corresponds to a particular pattern of streamlines and the pattern is unchanged when $\Phi$ is multiplied throughout by a factor $C$. By making $C$ a function of time, the quasi-steady flow can be expressed as

$$\Phi(x,y,t) = C(t)\Phi(x,y)$$

where the unsteady flow rate is related to the steady flow rate $q_s$ by $q(t) = q_s C(t)$. Application of the unsteady form of the Bernoulli equation between a point (1) on the inlet side where the air speed, $w_1$, is negligible and a point (2) in the jet on the outlet side where the speed is $w_2$ gives (see [5])

$$\frac{p_1(t) - p_2(t)}{\rho} = C^{-1}(t)\frac{w_1^2}{2} + (\Phi_1 - \Phi_2)\frac{\partial C}{\partial t}$$

where

$$\Phi_1 - \Phi_2 = \int_1^2 w \, ds$$

At some distance from the outlet the speed at any point in the jet is given closely by $w_2(t) = q(t)/C_x A$ so the above equation can be written

$$\frac{p_1(t) - p_2(t)}{\rho} = C^{-1}(t)\frac{q(t)^2}{2C_x^2 A^2} + \int_1^2 w \, ds \frac{\partial C}{\partial t}$$

The problem is to express the term $\int_1^2 w \, ds$ in a form which can be evaluated. This is done empirically by defining an effective length $l_e$

$$\frac{q_s}{C_x A} l_e = \int_1^2 w \, ds$$

Substituting in (A4) gives

$$\left(\frac{\rho q(t)^2}{2C_x^2 A^2}\right) S_4 + \rho \frac{l_e}{C_x A} \frac{dq}{dt} = \Delta p(t)$$

It should be noted that the discharge coefficient has
been included in the definition of \( l_d \) so that the definition differs from that in Eq. (3.2.67) of [5]. This has been done for consistency, since it leads to a slightly simpler form of nondimensional equation i.e.

\[
q^2 S_q + 2 \frac{l_d g}{H} \frac{dq}{dt} = \frac{\Delta p(t)}{0.5 \rho U^2}
\]  

(A7)

For the present purposes \( l_d H \) is determined empirically by comparing the predictions of Eq. (A7) with the measurements of phase lag reported by Modera [11], see Section 5 in the main text.

A.1.2. Pressure difference across openings

When the external density distribution is unchanged by the presence of wind, the wind pressures can be simply added to the hydrostatic pressure. Thus the external surface pressure at height \( z \) is given by

\[
P_{E}(z) = P_{E0} - \rho_g g z + p_w(z)
\]  

(A8)

For the internal pressure the internal air motion is neglected, because the velocities are very small. The hydrostatic equation can therefore be applied i.e.

\[
\frac{\partial P_1}{\partial z} = -\rho_1 g z
\]  

(A9)

The internal surface pressure at height \( z \) is thus given by

\[
P_1(z) = P_{10}(z) - \rho_1(z) g z
\]  

(A10)

Now, the pressure variations are very small (a few Pa), so the changes in \( \rho_1(z) \) are very small e.g. a 2 Pa change in pressure gives a change in density of about 0.002%. This may be significant in the continuity equation where changes in density are factored by the volume \( V \), but it is of no significance in the internal pressure equation and it is therefore allowable to replace \( \rho_1(z) \) by the mean value of \( \rho_1 \) in Eq. (A10) to give

\[
P_1(t) = P_{10}(t) - \bar{\rho}_1 g z
\]  

(A11)

It is also convenient to put

\[
P_{10}(t) \equiv P_{10}(t) - P_{E0}
\]  

(A12)

so that

\[
P_1(t) = P_{10} + P_{10}(t) - \bar{\rho}_1 g z
\]  

(A13)

The pressure difference at height \( z \) is therefore

\[
\Delta \rho(z) = P_E(z) - P_1(z) = p_w(z) - (\rho_E - \bar{\rho}_1) g z - P_{10}(t)
\]  

(A14)

A.1.3. Internal conditions and continuity equation

Conservation of mass applied to the air contained within the envelope leads to the following instantaneous form of the continuity equation

\[
\frac{1}{\rho_1} \frac{d\rho_1}{dt} = q_1(t) + q_2(t)
\]  

(A15)

With unsteady flow the density of the air inside the room varies and influences the continuity equation. It is therefore necessary to describe the variation of density by some equation which relates it to the flow. This is done by means of an equation of state which describes the bulk behaviour of the air enclosed by the envelope. The bulk behaviour can be assumed to follow a polytropic process as follows

\[
\frac{dP_{w}(t)}{P_{10}(t)} = K
\]  

(A16)

where \( K \) is a constant. Here, the small spatial variations with height of pressure and density are neglected. The exponent \( n \) is equal to 1 for a constant temperature process and equal to \( \gamma \) for an isentropic process. (Note: It is assumed that the dimensions of the space are small enough and/or the speed of sound is high enough for \( P_{10}(t) \) to be spatially uniform at all times.)

Thus

\[
\frac{dP_{w}(t)}{dt} = nC_{p_1}\frac{\partial \rho_1}{\partial z} + 1 \frac{d\rho_1}{dt}
\]  

(A17)

with

\[
\frac{d\rho_1}{dt} = \frac{1}{nC_{p_1}} \frac{dP_{w}}{dt}
\]  

(A18)

so that

\[
\frac{1}{\rho_1} \frac{d\rho_1}{dt} = \frac{V}{nP_{10}(t)} \frac{dP_{w}}{dt}
\]  

(A19)

Since

\[
P_{10}(t) \equiv \bar{P}_{10} \quad \text{and} \quad \frac{dP_{10}}{dt} = \frac{d\rho_{10}}{dt}
\]

the continuity equation can be written as

\[
\frac{V}{n} \frac{1}{\bar{P}_{10}} \frac{d\rho_{10}}{dt} = q_1(t) + q_2(t)
\]  

(A20)

A.1.4. Equations for QT model

The equations for the case of two openings are

\[
\frac{V}{n} \frac{1}{\bar{P}_{10}} \frac{d\rho_{10}}{dt} = q_1(t) + q_2(t)
\]  

(A20)
It is convenient to express the equations in a nondimensional form, because solutions then become more general. Using the definitions given in Eqs. (9)–(12) in the main text the equations become

\[
\frac{V}{n} \frac{1}{P_{10}} \frac{dP_{10}}{dt} = q_1(t) + q_2(t) \quad \text{(A23)}
\]

\[
q_1^2 S_{q1} + \frac{2l_{q1}}{H} \frac{dq_1}{dt} = -C_{p1} - 2Ar^2 + C_{p1} \quad \text{(A24)}
\]

\[
q_2^2 S_{q2} + \frac{2l_{q2}}{H} \frac{dq_2}{dt} = -C_{p1} - 2Ar^2 + C_{p2} \quad \text{(A25)}
\]

where use in (A23) has been made of the expression for the speed of sound, \(c\).

\[
c^2 = \frac{P_{10}}{\gamma \rho} \quad \text{(A26)}
\]

and \(\gamma\) is the ratio of specific heats.

A.2. Quasi-steady compressible, QC model

The relevant equations are

\[
\Delta p(t) = p_0(t) - \Delta pgz - p_0(t) \quad \text{(A27)}
\]

\[
\frac{V}{n} \frac{1}{P_{10}} \frac{dP_{10}}{dt} = q_1(t) + q_2(t) \quad \text{(A28)}
\]

and the flow equation

\[
q(t) = C_\xi \sqrt{\left(\frac{2\Delta p(t)}{\rho}\right)} S_p \quad \text{(A29)}
\]

Substituting for \(\Delta p(t)\) in (A29) and for \(q(t)\) in (A28) leads to the following differential equation for \(P_{10}(t)\) for the case of two openings

\[
\frac{V}{n} \frac{1}{P_{10}} \frac{dP_{10}}{dt} = \frac{C_{z1} A_1}{\sqrt{\left[-p_1(t) - \Delta pgz + p_{10}(t)\right]}} S_{P_{10}} \quad \text{(A30)}
\]

Eq. (A30) is a differential equation for \(P_{10}(t)\) which can in principle be solved for given initial values of \(P_{10}(t)\) and \(t\). The flow rates \(q_1(t)\) and \(q_2(t)\) can then be evaluated.

The nondimensional forms of the equations are

\[
V \frac{1}{n} \frac{\gamma U^2}{\Delta H} \frac{dC_{pi}}{dt} = q_1(t) + q_2(t) \quad \text{(A23)}
\]

\[
q_1^2 S_{q1} + \frac{2l_{q1}}{H} \frac{dq_1}{dt} = -C_{p1} - 2Ar^2 + C_{p1} \quad \text{(A24)}
\]

\[
q_2^2 S_{q2} + \frac{2l_{q2}}{H} \frac{dq_2}{dt} = -C_{p1} - 2Ar^2 + C_{p2} \quad \text{(A25)}
\]

and

\[
q_1(t) = \frac{q_1(t)}{C_{z1} A_1} U \quad \text{(A32)}
\]

\[
q_2(t) = \frac{q_2(t)}{C_{z2} A_2} U \quad \text{(A33)}
\]

A.3. Quasi-steady incompressible, QI model

For this model Eq. (A31) reduces to

\[
-C_{P1}(t) - 2Ar^2 + C_{p1}(t) \quad \text{(A34)}
\]

so that

\[
C_{P1}(t)(1 + E^2) \quad \text{(A35)}
\]

Substitution into (A32) and (A33) for the special case
with \( E = 1 \) gives

\[
q_f[t] = \frac{q_f[t]}{C_{\text{a}1} \cdot A_1 \cdot U}
\]

\[
= \left[ \frac{\pi}{\Delta f} \left( \frac{\Xi_2}{H} - \frac{\Xi_1}{H} \right) + \left( \frac{C_{\text{pl}}[t] - C_{\text{p}2}[t]}{2} \right) \right] \cdot S_{\text{pl}}
\]  

(A36)

\[
q_g[t] = \frac{q_g[t]}{C_{\text{a}2} \cdot A_2 \cdot U}
\]

\[
= \left[ \frac{\pi}{\Delta f} \left( \frac{\Xi_2}{H} - \frac{\Xi_1}{H} \right) + \left( \frac{C_{\text{pl}}[t] - C_{\text{p}2}[t]}{2} \right) \right] \cdot S_{\text{pl}}
\]  

(A37)

A.4. Pseudo-steady, PS model

For the pseudo-steady model, the instantaneous values in the above equations are replaced by their mean values to give (for the case with \( E = 1 \))

\[
\bar{q}_f = \bar{q}_g = \left[ \frac{\pi}{\Delta f} \left( \frac{\Xi_2}{H} - \frac{\Xi_1}{H} \right) + \left( \frac{C_{\text{pl}}[t] - C_{\text{p}2}[t]}{2} \right) \right] \cdot S_{\text{pl}}
\]  

(A38)

Appendix B. Results for QI model

The instantaneous pressure difference coefficient can be expressed as

\[
\Delta C_{p}[t] = k f[t]
\]  

(B1)

where \( k \) is a constant and \( f[t] \) denotes some function of time. The mean flow rate through the opening is then given by

\[
\bar{q} = \frac{1}{T} \int_{t_0}^{T} \sqrt{W + k f[t]} \, dt
\]  

(B2)

For a given \( f[t] \) and integration period \( T \), the standard deviation of \( \Delta C_{p}[t] \) can be written as \( k F_T \) if \( F_T \) is a constant. Thus one can write

\[
\frac{\bar{q}}{\sqrt{\Delta C_{p}}} = \frac{1}{T} \int_{t_0}^{T} \sqrt{\frac{W}{\sigma_{\Delta C_{p}}} + \frac{k f[M]}{k F_T}} \, dt
\]  

(B3)

Since integrating between fixed limits 0 and \( T \) fixes both \( \sigma_{\Delta C_{p}} \) and the above integral it can be seen that, for a given \( f[t] \) and \( T \),

\[
\frac{\bar{q}}{\sqrt{\Delta C_{p}}} = f \left( \frac{W}{\sigma_{\Delta C_{p}}} \right)
\]  

(B4)

where \( f \) denotes another function.

For the particular case where \( \Delta C_{p}[t] \) is formed from two sine waves with phase difference \( \phi \),

\[
\Delta C_{p}[t] = k \cos(2\pi sint + \phi)
\]

where

\[
k = \sqrt{2C_{pl}\sqrt{1 - \cos \phi}}
\]

It can be seen from Eq. (B3) that the values of \( k \) cancel, so that Eq. (B4) is independent of \( \phi \).

References


[10] Preiser T. 1998: Internal communication. Dept. of Civil Engineering, University of Nottingham, Nottingham, UK.
