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Solutions to a mixed boundary problem for soil gas flow

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Abstract

The principal soil gases of current concern to building are radon and landfill gas. The flow of these is generally thought to be dominated by viscous flow under a pressure gradient. This paper presents results for such pressure-driven flow of gas for problems relating to a building with a bare soil floor, for example below a suspended timber floor. The solutions address this problem in two dimensions as a mixed boundary problem. In the following papers this initial solution is then mapped conformally onto a more complex geometry, and the results are compared to another analytical result and an experiment. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

In order to understand how soil gases enter buildings, the driving forces for this flow have to be understood. There are two processes for soil gas movement, namely diffusion and pressure-driven viscous flow. It is in general necessary to consider both, but [1] has shown that under most conditions diffusion alone is not enough to achieve the radon levels found in houses and that radon entry due to advection is expected to dominate.

If there is a pressure difference then soil gas will be transported by it, with the flow directed from high to low pressure. There are several possible causes of this pressure difference, all of which can be significant. In a normal British house, the pressure inside is slightly less than that outside, because of the combined effect of the wind and temperature differences between outside and in. The effect of these two mechanisms is to give an indoor pressure at floor level which is nearly always slightly less than that outside. Using a ventilation model, for example BREVENT [2], or by direct measurement, it is found that this pressure difference is usually less than 5 Pa, and typically only 1 Pa. However, it is still able to generate a significant flow rate. This natural driven flow is the subject of this paper.

There are other mechanisms for driving the flow of which three deserve consideration. One is the presence of a mechanical ventilation system in the house, which can enhance or reduce the natural pressure-driven flow. The second is a permanent driving force due to the production of soil gas in the soil. This applies to landfill gas when a building is built on top of a landfill site, or near to an unprotected site. This is beyond the scope of the present work. The third process is caused by changes in atmospheric pressure, or other time-dependent effects. This has been discussed elsewhere [3].

2. Theory and literature review

The key equation used is Darcy's Law here written as

$$Q = -\frac{k}{\mu}\cdot A \cdot \nabla P \tag{1}$$

where Q is the flow rate ($m^3\ s^{-1}$); k is the permeability of the soil (m^2); μ is the viscosity of the fluid flowing (Pa s^{-1}); A is the area of flow (m^2); P is the excess pressure of the fluid compared to ambient (Pa); ∇ is the gradient operator, written as d/dx in one dimension (m^{-1}); x is the length over which flow occurs (m).

It can be combined with the continuity equation, where v is the velocity of flow ($m\ s^{-1}$), defined as

$$\nabla \cdot v = 0 \tag{2}$$

to give Laplace's Equation [4]

$$\nabla^2 P = 0 = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}. \tag{3}$$

This equation describes the pressure field within a region of soil. If it can be solved, then the flow rate can be found

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from Darcy's Law. It is worth noting that there is some variation in the way that the terms permeability and Darcy's Law are used in different sciences. In particular, in studies of liquid flows in porous media a different definition of permeability is often used, with units of m s^{-1}. Hence it is important to check which form of the equation is being used; most work on soil gas has used the form given here.

Hence Laplace's Equation is used to predict the pressure distribution in soil wherever Darcy's Law for gas flow is valid. This is a fortunate result, since there has been much work done on solving Laplace's Equation in a wide range of geometries, as it appears in many different areas of science.

Work at Lawrence Berkeley Laboratory has shown that soil gas flow is not always described by a linear equation, [5]. However this generally happens where there is a sub-slab ventilation system being used (or radon sump in the UK), and gas flow velocities become high, of the order of 0.1 m s^{-1} or more. The problems considered here have velocities of order 10^{-4} m s^{-1}, so Darcy's Law remains valid.

There have been many studies of the movement of radon due to pressure-driven flow, and these are too numerous to mention here. However, work by Landman and Delsante [6] is the closest to that described here, and the two sets of results are compared in a later paper. They worked on radon and also on heat flow, but as the equations are the same as those used for flow described by Darcy's law, the results can be transferred to gas flow.

3. Analytic solution to pressure field for a mixed boundary-value problem

A mixed boundary-value problem is one in which the type of boundary condition changes from fixed pressure to fixed flow rate (or pressure gradient). In particular, in moving from open air to a solid wall, we move from a pressure boundary to a no-flow boundary. This can be solved numerically, although the detail of the result near the change-over point can be difficult.

Here the problem is tackled analytically. It cannot be evaluated easily because the solution consists of integrals which have to be found numerically. However it has been possible to calculate the flow rate into the 'house' in a fairly simple way, and this result can be useful.

No assumption is made about the pressure change across the wall of the house. The solution can be obtained fairly easily if this pressure drop is assumed to be linear, using a Laplace Transform technique. Here the only assumption is that there is no flow vertically into the base of a wall. Although walls built with no footings are no longer allowed within building regulations, there were many houses built without foundations in the past. Hence the problem has some validity in its own right. In addition it can be extended to a more general case with the technique discussed in the following paper.

3.1. The bare soil house

The boundary conditions come from considering the problem described in Fig. 1.

This house is then modelled by assuming some behaviour on the surface, defined at $y = 0$. The conditions on $y = 0$ are shown in Fig. 2. This represents the soil surface, $P = 0$; no flow within solid walls, $\partial p/\partial y = 0$; and a fixed pressure inside a house, $P = P_0$. This gives a representation of the position in a house built with a timber floor with no concrete oversite, and the test huts which were built on the Building Research Establishment (BRE) radon pit.

In the method used the pressure problem is then represented using a complex variable form. Let $w(z)$ be function of the complex variable $z = x + iy$. Then $w(z)$ can be written

$$w(z) = U(x,y) - i \cdot V(x,y) \tag{4}$$

The use of a function of a complex variable is useful because of the fact that any function of a complex variable automatically satisfies Laplace's Equation. This means that the problem here is to find a solution which matches the boundary conditions. The Cauchy–Riemann Equations state that

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y} \tag{5}$$

and

$$\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x} \tag{6}$$

Using these and the boundary conditions for the pressure field problem in Fig. 2 allows us to define the boundary conditions for the related problem (i.e. finding $w(z)$) as in Fig. 3.

Here B is a constant to be determined. The boundary of the w-region is called L. A part of the boundary where

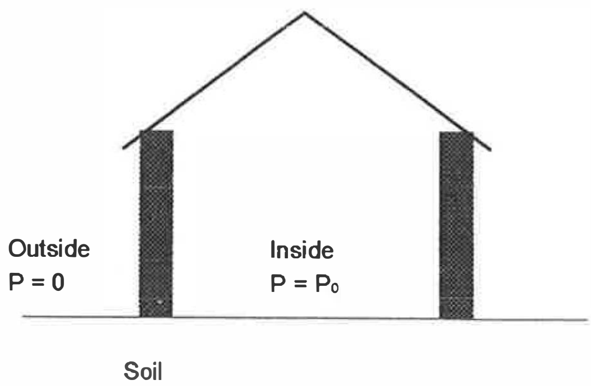

Fig. 1. Schematic diagram of 'house'.

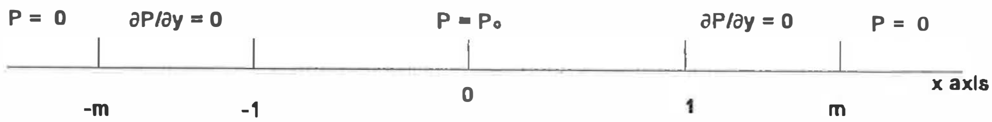

Fig. 2. The pressure field problem.

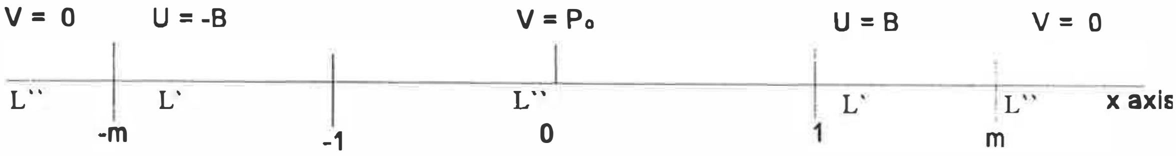

Fig. 3. Boundary conditions for the w problem.

the real part is defined is labelled L', while those where the imaginary part are defined are labelled L''. Note that since V is an even function of x, from the Cauchy–Reimann Eqns (5) and (6), U must be odd, hence the choice of $\pm B$ for the L'' regions. Finding the constant B is a main part of the solution to the problem. The constant B is also important in finding the flow rate into the house.

So, if we can solve for w we may then take the imaginary part, which will give the pressure field $V(x,y)$. The U-part of the solution can provide information about the pressure gradients.

Now [7] gives the solution to this type of problem as,

$$w(z) = \frac{1}{\pi i}\sqrt{\left(\frac{(z-a_1)(z-a_2)}{(z-b_1)(z-b_2)}\right)}\int_L \sqrt{\left(\frac{(t-b_1)(t-b_2)}{(t-a_1)(t-a_2)}\right)} \cdot \frac{h(t)\,\mathrm{d}t}{t-z} + C\sqrt{\left(\frac{(z-a_1)(z-a_2)}{(z-b_1)(z-b_2)}\right)}. \quad (7)$$

where C is an arbitrary real constant to be determined; $h(t)$ is given by the condition on w on the boundary L (i.e. $y = 0$); the points a_j are where a boundary of type L'' changes to one of type L'; the points b_j have the opposite change, L' to L''.

For this problem the a_1, a_2 have values $-m$ and $+1$, and the b_1, b_2 terms -1 and $+m$. Inserting these in Eqn (7) defines the solution to the problem. However the value of $h(t)$ has be inserted into the expression, and this results in five different terms to the result, because there are five different regions to be considered. However, for two of these, it is zero so they make no contribution.

The function $h(t)$ is defined at follows:

$t < -m \quad h(t) = 0,$
$-m < t < -1 \quad h(t) = -B,$
$-1 < t < +1 \quad h(t) = i.P_0,$
$+1 < t < +m \quad h(t) = +B,$
$t > +m \quad h(t) = 0.$

The other problem to consider at this stage is the argument of expressions containing t within the root sign in the integral. As the value to t changes the expression in the root changes sign, and hence the argument of the root of it needs to be considered separately. The arguments are defined in Fig. 4.

Using these allows the correct calculation of the root terms in Eqn (7). That the solution defined above meets the boundary condition can be checked by calculating the values on the boundary by examining the terms in

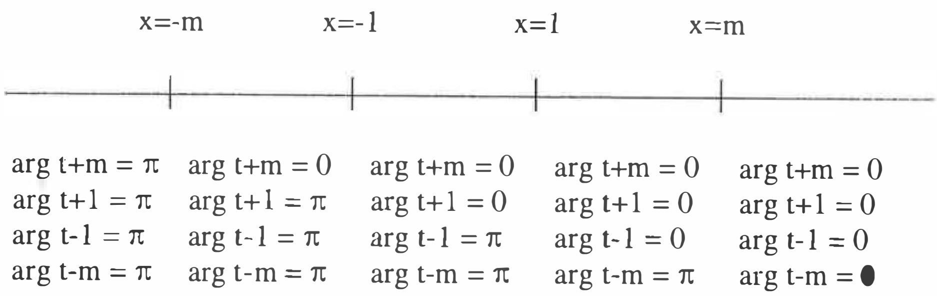

Fig. 4. Arguments of expressions in t.

Eqn (7). Some care needs to be taken in treating the principal values of integrals where the integrands diverge at some point on the boundary. The solution does match the condition, but the confirmation of this is too long to reproduce here.

3.2. Finding the constants B and C

To evaluate the constants B and C we use the fact that the expressions for w must be finite. Concentrating on the x-axis, where the boundary is defined, we can re-write Eqn (7) in terms of x instead of z, and substituting for the a and b terms, as

$$w(x) = \frac{1}{\pi i}\sqrt{\left(\frac{(x+m)(x-1)}{(x+1)(x-m)}\right)}\left[\int_{-m}^{+m}\sqrt{\left(\frac{(t+1)(t-m)}{(t+m)(t-1)}\right)}\cdot\frac{h(t)\,\mathrm{d}t}{t-x}+iC\pi\right]. \quad (8)$$

From Eqn (8) at $x = m$, or at $x = -1$, the outer factor goes to infinity, so the inner terms must also total zero or w will diverge. Considering this equation at $x = m$, and at $x = -1$, and subtracting one from the other gives an expression for B. After some rearrangement it is given by

$$B = -P_0\cdot\frac{\displaystyle\int_0^1\frac{\mathrm{d}t}{\sqrt{(1-t^2)(m^2-t^2)}}}{\displaystyle\int_1^m\frac{\mathrm{d}t}{\sqrt{(t^2-1)(m^2-t^2)}}}. \quad (9)$$

Using this expression leads to the simple result that

$$C = 0 \quad (10)$$

The result for B is defined by two elliptical integrals. These are given in standard tables, or can be evaluated numerically. Some values for B for different values of the parameter m are given below. For values of m near to 1, or large values of m, an approximate result can be found for B.

Table 1
Values of B/P_0 for different values of m

m	B/P_0
1.1	1.4101
1.2	1.20
1.5	0.95
2.0	0.7817

3.3. Complete solution

Hence combining the previous results the solution is

$$w(z) = \frac{1}{\pi i}\sqrt{\left(\frac{(z+m)(z-1)}{(z+1)(z-m)}\right)}\cdot\left[\int_{-m}^{-1}\sqrt{\left(\frac{(-1-t)(m-t)}{(t+m)(1-t)}\right)}\frac{-B\,\mathrm{d}t}{t-z}+\int_{-1}^{+1}\sqrt{\left(\frac{(t+1)(m-t)}{(t+m)(1-t)}\right)}\frac{P_0\,\mathrm{d}t}{t-z}+\int_{+1}^{+m}\sqrt{\left(\frac{(t+1)(m-t)}{(t+m)(t-1)}\right)}\frac{B\,\mathrm{d}t}{t-z}\right]. \quad (11)$$

3.4. Using the result

In order to evaluate the result from Eqn (11) and the definition of B a computer program was needed to calculate the integrals numerically. The author wrote his own program for this, but believes that the mathematical software available would be able to calculate these integrals.

This method was used to produce the plots shown in Figs 5–7. These give a result which meets the boundary conditions, and appear reasonable.

In Figs 5 and 6 the parameter m had the value of 2. This is an unrealistic situation, since it implies walls half the width of the space they enclose. However it shows the boundary conditions better than an example with smaller m.

Figure 5 shows a pressure contour plot of the whole region. The flow of gas would be perpendicular to the pressure contours at all points, with the rate proportional to the pressure gradient or separation of the lines. In this plot the main pressure boundary conditions are seen to be met correctly.

Figure 6 is an expanded view of the right hand wall of Fig. 5. This shows the pressure contours meeting the $y = 0$ line perpendicular to it. Hence, because no flow occurs along pressure contours, no flow occurs across $y = 0$ between $x = 1$ and $x = 2$, as required by the boundary conditions. The fact that the pressure drop between $x = 1$ and $x = 2$ is non-linear can be seen clearly in Fig. 6.

Figure 7 shows the situation when $m = 1.1$. This means the walls are one-tenth of the width of the room, which is more realistic than the previous result. The pressure contours are squeezed closer together at the walls, which implies faster flow in those regions.

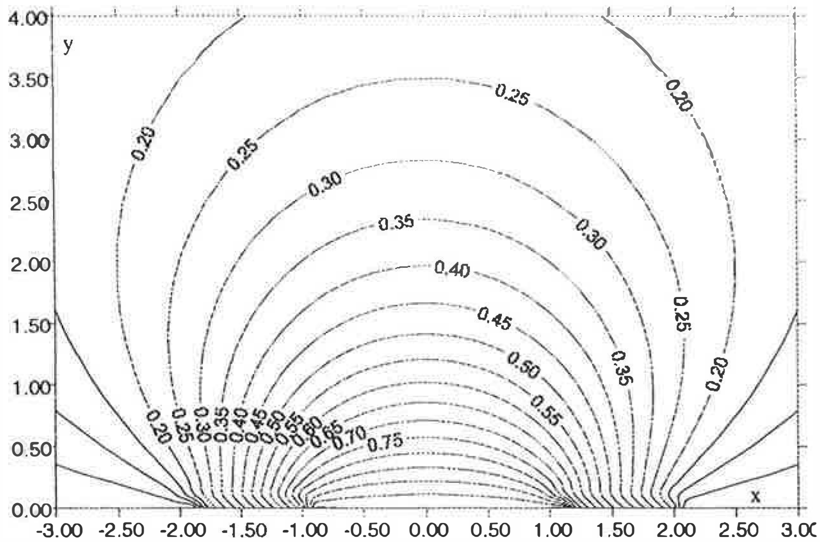

Fig. 5. Pressure contours when $m = 2$.

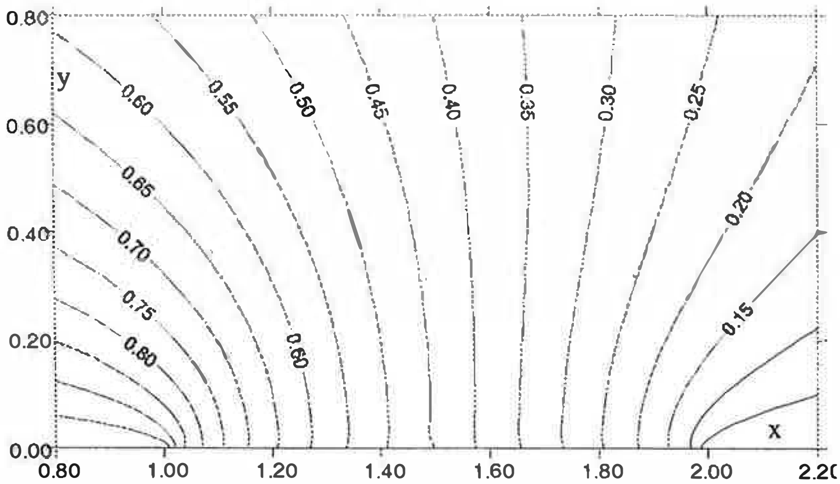

Fig. 6. Pressure contours when $m = 2$: enlargement of 'wall' region.

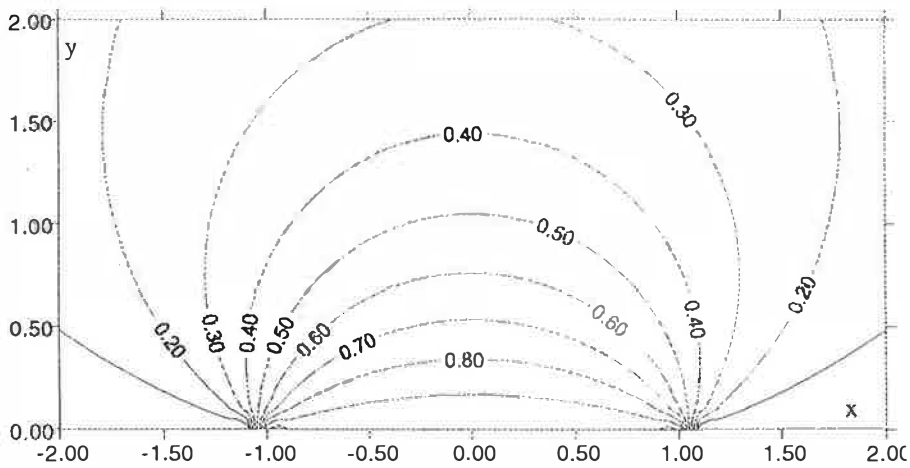

Fig. 7. Pressure contours when $m = 1.1$.

3.5. *Flow produced by the pressure distribution*

A key result is the flow rate into a house produced by a given pressure distribution. This is given, assuming linear i.e. Darcy flow, by the integral of the pressure gradient between the two walls. This gives the result for the flow, Q, as

$$Q = \int_{-1}^{+1} -\frac{k}{\mu}\cdot\left.\frac{\partial P}{\partial y}\right|_{y=0}\cdot \mathrm{d}x = \int_{-1}^{+1} -\frac{k}{\mu}\cdot\left.\frac{\partial V}{\partial y}\right|_{y=0}\cdot \mathrm{d}x. \quad (12)$$

From the Cauchy–Riemann Eqns (5) and (6), this is then equal to

$$Q = \int_{-1}^{+1} -\frac{k}{\mu}\cdot\left.\frac{\partial U}{\partial x}\right|_{y=0}\cdot \mathrm{d}x. \quad (13)$$

This is easily evaluated, since U is known at -1 and 1 as B and $-B$ respectively. Hence the flow rate is simply given by

$$Q = -\frac{2Bk}{\mu}. \quad (14)$$

Here the flow is given in $\mathrm{m}^3\ \mathrm{s}^{-1}\ \mathrm{m}^{-1}$ of wall. Using this and the known values of B found earlier we can predict the flow rate expected for different geometries.

4. Conclusions

This paper has presented the solution to a mixed boundary problem representing a house with bare soil below a suspended floor. The expression for the flow rate is quite simple, whereas the result for the pressure field in quite complicated. In the following paper this result is used with another technique, conformal mapping, to consider the more realistic case where the footings to the walls extend into the ground. In a third paper these results are compared to experiments and another theoretical result.

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