PREDICTION OF NATURAL CONVECTION FROM AN ARRAY OF HORIZONTAL LINE HEAT SOURCES IN A LARGE SPACE

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ABSTRACT

The buoyant plume characteristics of heat sources and their relation to geometric factors are of fundamental importance to the effectiveness of the displacement ventilation. The interactions in buoyant plumes from an array of horizontal line heat sources are investigated systematically with Mach-Zehnder interferometer.

Based on the discussion about convective flow patterns of a single lime heat source, the characteristic velocity and temperature to character the accumulating buoyancy effects of an array of horizontal line heat sources are proposed. The accumulating buoyancy effect Grasholff number is suggested and defined. A combined natural and "forced" convection model is proposed for predicting the natural convection from horizontal line heat sources. The measurements for natural convection are performed. The agreement between the analytical results and experimental data is satisfactory.

KEYWORDS

Natural convection, Plumes, Convective heat transfer

Heat sources can be divided into four types according to their basic shapes in air-conditioned rooms. They are point heat source, line heat source, surface heat source and bulk heat source. The effectiveness of displacement ventilation (including vertical temperature distribution, air-exchanging efficiency and air life span) has close relationship with the geometric shapes and dimensions of heat sources. Nowadays. except for point heat sources and single line heat sources. we know little about line array heat sources and above-floor heat sources. which are often used in air-conditioning and refrigerating engineering. Studying the characteristics of natural convective heat transfer about an array of line heat source has great industrial importance in predicting temperature distributions of the rooms.

ANALYSIS FOR INTERFEROGRAMS OF PLUMES FROM HORIZONTAL LINE HEAT ARRAYS

The graphs of plume flow of wire arrays are shown in Fig 1, Fig 3 to Fig 6. From the figures, s/d (s、 d is the space between adjacent wires and diameter of the wire. respectively) has intense influence to heat transfer performance of wire arrays. As for the heat transfer feature of wire arrays, the reason may be as follows: when intervals between adjacent wires are quite small, each wire is overlapped by their heat boundary layer. and much similarly in the same isothermal line (see Fig 1), fluid temperatures between heat wires are same on the whole, mutual heat transfer seldom happens, which forms stagnant air zone and thus decreases effective area of heat transfer. Inside the stagnant air zone. fluid temperature is relatively high because it is heated in double directions, but flow intensity is relatively weak, the model of heat transfer is mainly heat conduction, which worsen the heat transfer among wires. Cylinder (wire)heat release is achieved by the heat convection of plumes rising from sides and heat conduction of stagnant air zone. From Fig 2, the model of heat transfer of meddle wire 2 consists of the heat conduction of *ab* and *cd* in the stagnant zone and heat convection of *ad* and *bc* in buoyant plumes. Suppose the rate of heat conduction is q_1, rate of

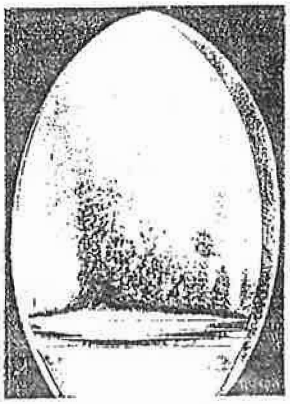


Fig 1. Interferogram of the plumes from wire arrays at small spaces s/d=8.3 d=0.3mm Q=9.95w/m

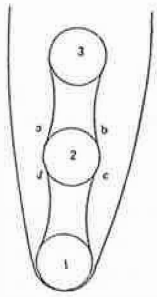


Fig 2. Natural convection heat transfer from horizontal wire arrays

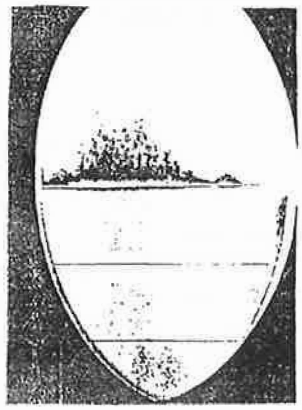
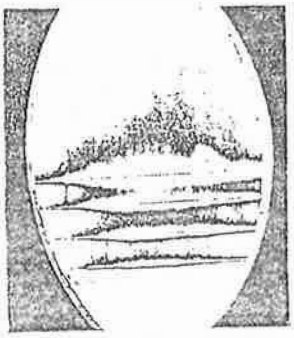
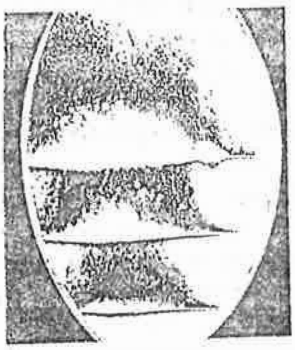


Fig 3. Interferogram of the plume from the single wire

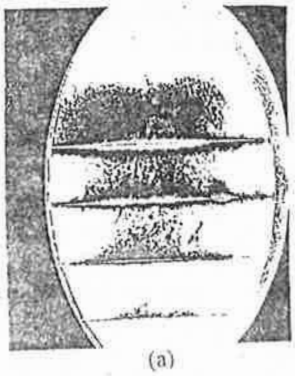


(a)



(b)

Fig 4. Interferograms of the plume flow against the space between adjacent wires
(a) s/d=16.7 d=0.6mm Q=14.9w/m
(b) s/d=41.7 d=0.6mm Q=14.9w/m



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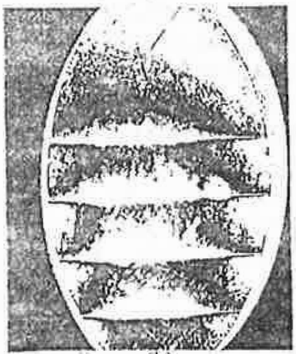
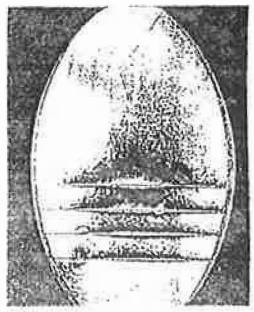


(b)

Fig 5. Interferograms of the plume flow against the rate of heat flow
(a) s/d=66.7 d=0.3mm Q=4.42w/m
(b) s/d=66.7 d=0.3mm Q=10.26w/m



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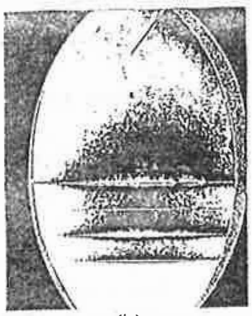

(b)

Fig 6. Comparison of the interferograms of the plume flow with 3th wire heated or not
(a) s/d=33.3 d=0.3mm Q=9.95w/m All wires are heated
(b) s/d=33.3 d=0.3mm Q=9.95w/m 3th wire is not heated

heat convection is q_2, then total rate of heat flow of cylinder 2 is q:

$$\iint_{ab+cd} -\lambda(\frac{\partial t}{\partial n})dA_1 = q_1$$

$$\iint_{ad+bc} h(t_w - t_f)dA_2 = q_2$$

$$\iint_{ab+cd} -\lambda(\frac{\partial t}{\partial n})dA_1 + \iint_{ad+bc} h(t_w - t_f)dA_2 = q_1 + q_2 = q$$

In the expressions, $\partial t/\partial n$ is the temperature variation in the normal direction of cylinder surface, dA_1, dA_2, are areal infinitesimal element of *ab*, *cd* and *ad*, *bc*, respectively.

As s/d increases, the convection effect from rising plumes to upper heat wires become stronger, convective heat transfer zones *ad*, *bc* enlarge, yet stagnant zones gradually reduce. The explanation is confirmed by visualized graphs (Fig 1, Fig 4, Fig 5) got from flow field visualization technique.

When s/d further increases and exceeds the critical value, the development and acceleration of rising plumes make the upper wires similarly be in forced flow completely ; heat transfer takes place by means of convection. However, this "forced flow" is a comprehensive effect of rising plumes from lower heat wires. The rising plumes from wire arrays are illustrated in Fig 5. under the different rate of heat flow with large s/d.

As the rate of heat flow increases, plumes are strengthened. We have also observed that plume flow intensity of each wire successively increases from bottom to top.

For the *i*th heat wire, it is subjected to synthetical influences of plume flow field of (*i*-1)heat wires below. In Fig 6(a), natural convection interferograms of heat arrays is shown with a wire not heated. Contrasting with Fig 6(b), we can see that under the same rate of heat flow rate and s/d. the disappearance of plume field of the 3th wire itself would make the other wires' heat transfer and flow field become stronger instead. Inhibitory action of heat convection between heat wires in the stagnant zone is confirmed again by the fact. The plume field interferogram from single horizontal heat wire is shown in Fig 3. Contrasting from Fig 1 to Fig 6. we can observe that wire arrays' natural convection plume fields obviously differ from that of single wire's flow field. In other words, calculation of wire arrays' heat exchange is different from that of single wire and is more complicated.

BUOYANT PLUME OF A SINGLE HORIZONTAL LINE HEAT SOURCE

Morgan (1967) once made comprehensive comments on natural convective heat transfer from a cylinder heat source, diameters of cylinders ranging from 10^{-3}mm to 113.5mm. $10^{-10}<Gr_d\ Pr<10^{12}$,where Grasholff number is based on the diameter of cylinder. However, Morgan only gave us the natural convection heat transfer coefficient of the cylinder. not discussing the characteristics of the plume field. Fujii (1963,1973,1982). Forstrom (1967) and Gebhart (1970) once did many experimental and theoretical studies on the buoyant plume field of single horizontal heat wire respectively. Through theoretical analysis and experiment. Fujii studied the plume velocity and temperature distribution from single horizontal heat wire. He pointed out that the plume field of heat wire can be approximately disposed as boundary layer flow.

As simplification, the following basic assumptions are made:

(1) The buoyant plume field of each wire conforms to boundary layer flow and also steady laminar flow. The buoyant effect of a certain point is the accumulating effects of each plume below;

(2) Satisfied with Boussinesq hypothesis;

(3) Newton fluid, neglecting viscous dissipation;

(4) No-slip condition near solid wall.

In considering the heat transfer of the line heat source (heat wire array)with constant heat flux, the definition of Grasholff number is made as follows

$$Gr_d = \frac{g\beta d^3}{\nu^2} \cdot \frac{Q}{\rho C_p \nu} \tag{1}$$

$$Gr_x = \frac{g\beta x^3}{\nu^2} \cdot \frac{Q}{\rho C_p \nu} \tag{2}$$

After introducing Boussinesq assumptions, the boundary layer equations of the plume field are

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial(T - T_\infty)}{\partial x} + \upsilon\frac{\partial(T - T_\infty)}{\partial y} = a\frac{\partial^2(T - T_\infty)}{\partial y^2} \tag{5}$$

boundary conditions

$$\left.\begin{array}{l} y = 0;\quad \upsilon = 0,\quad \partial u/\partial y = 0,\quad \partial(T - T_\infty)/\partial y = 0 \\ y = \infty;\quad u = 0,\quad T = T_\infty \end{array}\right\} \tag{6}$$

Convert dimension coordinates into dimensionless coordinates. Introducing similar variable

$$\zeta = Gr_x^{1/5}\frac{y}{x} \tag{7}$$

stream function $\Psi = \nu Gr_x^{1/5} f(\zeta)$ (8)

$$T - T_\infty = \theta Gr_x^{1/5} \cdot \varphi(\zeta), \quad \theta = Q/(\rho C_p \nu) \tag{9}$$

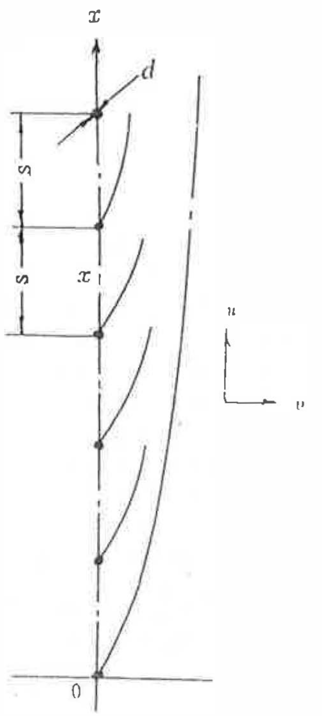

Fig 7. Natural convection flow arising from heated wire-arrays

Vertical component of velocity in the boundary layer of single horizontal heat wire:

$$\frac{ux}{\nu} = Gr_x^{2/5} f'(\zeta) \tag{10}$$

horizontal component of velocity

$$\frac{\upsilon x}{\nu} = -\zeta\left(\frac{3}{5}f - \frac{2}{5}\zeta f'\right) \tag{11}$$

temperature distribution

$$T - T_\infty = Gr_x^{1/5}\theta\varphi(\zeta) \tag{12}$$

Where Q is rate of heat flow of unit wire; T, T_∞ stands for temperature of buoyant plume and environment respectively ρ, ν, and C_p are physical parameters. β is expansion coefficient of the fluid. With the exception of β, all of the other parameters are defined on the basis of the average temperature of the boundary layer ($T_0 + T_\infty$)/2.

METHOD TO DETERMINE THE CHARACTERISTIC VELOCITY AND TEMPERATURE OF THE PLUME FIELD FROM AN ARRAY OF HORIZONTAL LINE HEAT SOURUES

Previous studies of Li (1997) show that flow field around each of the line arrays can be regarded as the combination of its own flow field and that produced by the other heat sources below. That is to say, heat transfer of the heat wire is composed of natural convection itself and the plume convection, the latter of which represents the accumulating buoyant effect of the other heated wires below. Before changing plume (*Gr* number) into "forced" convection (*Re* number), We will determine the characteristic velocity and temperature of "forced" convection. Applying the definition of accumulating buoyant effectiveness *Gr* number into Eq. (10) yields

$$u = \frac{v}{x} - Gr_x^{1/3} f'(\zeta), \qquad \overline{Gr}_x = \frac{g\beta}{v^2} \cdot \frac{iQ}{\rho C_p v} \cdot \overline{x}^{-3}$$

and $$\overline{x} = s[\frac{i-1}{2}]^{2/3} \cdot i^{1/3}$$

Where s is the space between adjacent wires, i is the number of wires. The velocity distribution on the center line (y=0) is

$$u_{max} = \frac{v}{x} \overline{Gr}_x^{1/3} f'(0)$$

With the consideration of the existance of other wires and mixing of each plume, the characteristic velocity u_c of "forced" convection is defined as the mean velocity over the boundary layer

$$u_c = \frac{1}{y_c} \int_0^{y_c} u dy \tag{13}$$

Where y_c is the thickness of boundary layer determined by

$$f'(\zeta_c) = 0.01 f'(0) \tag{14}$$

and $y_c = Gr_x^{-1/4} \cdot x\zeta_c$ (15)

That is to say, the thickness of boundary layer y_c is the radial distance when the velocity reaches 1% of the center velocity, u_{max} is the maximum velocity.

Further expressed as

$$u_c = \frac{f(\zeta_c)}{\zeta_c f'(0)} u_{max} \tag{16}$$

Through numerical calculation on single line heat wire, the value of $f'(0)$, ζ_c and f(ζ_c) is obtained, which are presented in Table 1.

In the same way, the characteristic temperature of the plume field is defined. Dimensionless thickness of heat boundary layer ζ_{ct}

$$\varphi(\zeta_{ct}) = 0.01\varphi(0) \tag{17}$$

The characteristic temperature of the oncoming flow may be expressed as

$$T_c - T_\infty = \frac{(T_{max} - T_\infty)}{\varphi(0)} \cdot \frac{1}{\zeta_{ct}} \int_0^{\zeta_{ct}} \varphi(\zeta) d\zeta \tag{18}$$

$$T_{max} - T_\infty = \overline{Gr}_x^{-1} \theta\varphi(0) \tag{19}$$

Where $\varphi(0)$, ζ_{ct} and $\int_0^{\zeta_{ct}} \varphi(\zeta) d\zeta$ can be obtained through numerical calculation.

Thus, Re number of "forced "convection, which characters the plume field. is

$$Re_f = \frac{d}{x} \cdot \overline{Gr}_x^{1/4} \cdot \frac{f(\zeta_c)}{\zeta_c} \tag{20}$$

Table 1. Numerical calculation results of coefficients concerned

items	$f'(0)$	$\varphi(0)$	ζ_c	$f(\zeta_c)$	$\int_0^{\zeta_{ct}} \varphi(\zeta) d\zeta$
value	0.808	0.373	8	2.055	0.832

COMBINED NATURAL AND FORCED CONVECTION MODEL FOR PREDICTING NATURAL CONVECTIVE HEAT TRANSFER OF AN ARRAY OF LINE HEAT SOURCES

As mentioned above, the natural convective heat transfer of each wire (for example, the ith wire) consists of two parts:

(1) Natural convective heat transfer from itself;

(2)"Forced" convection created by the accumulating buoyant effects from heated wires below

Hatton (1970) once used the equal-effective Reynolds number to solve the combined

Natural and forced convection heat transfer of single horizontal cylinder.

The essence of equal-effective Reynolds method is to convert *Gr* number of natural convention into *Re* number, during the course, *Nu* number maintains the same. Equal-effective *Re* number is the sum of the oncoming flow *Re* number and converted *Re* number.

In analyzing the convective heat transfer of the *i*th wire, its equal-effective *Re* number includes two parts:

(1)The "forced" convection flow, which indicates the accumulating buoyant effect of the other (*i*-1)heat wires below. The characteristic velocity is u_c; the characteristic temperature is T_c; the *Re*, Reynolds number of the flow field, can be obtained from the equation (20)

(2)Gr_d number, which indicates the natural convective buoyant effect of the *i*th wire itself, can be converted into Re_n.

The convective heat transfer equation recommended by most literatures is given by Morgan (1975).

$$Nu_n = C_n (Gr_d \ \mathrm{Pr})^p \tag{21}$$

According to the definition of converted Reynolds number,

$$Nu_n = C_f (\mathrm{Re}_n)^q \tag{22}$$

then
$$\mathrm{Re}_n = [C_n / C_f (Gr_d \ \mathrm{Pr})^p]^{1/q} \tag{23}$$

Where the coefficients C_n, C_f and index *p*, *q* are given in literature (Morgn, 1975).

When the oncoming flow has the same direction as the buoyant convection of the present heat wire,

$$\mathrm{Re}_{eff} = \mathrm{Re}_n + \mathrm{Re}_f \tag{24}$$

The heat transfer calculation of the *i*th wire is generalized as following

$$Nu_i = C_f \left\{ [C_n / C_f (Gr_d \cdot \mathrm{Pr})^p]^{1/q} + \mathrm{Re}_f \right\}^q \tag{25}$$

The mean Nusselt number of wire arrays is

$$Nu = \frac{1}{n} \sum_{i=1}^{n} Nu_i \tag{26}$$

Where, *n* is the number of wires, the *Nu* number is based on the reference temperature $0.5(T_w + T_c)$.

EXPERIMENTAL RESULTS AND DISCUSSION OF NATURAL CONVECTIVE HEAT TRANSFER FROM HEATED WIRE ARRAYS

Experimental equipment for measuring the natural convection heat transfer from heated wire arrays has been built. The diagram of the experimental installation is indicated in Fig 8.

Line heat source arrays are constituted of heated wires tightened in an epoxy frame. The interior size is 300mm×300mm; the outer size is 360mm×360mm.The diameters of Ni-Cr wires are 0.3mm and 0.6mm.The experimental installation mainly consists of heat power measurement system and temperature measurement system. Using standard resistor and V-A (Joule heating) method, we can measure the heat power of the wires. Standard resistor and heat wires are connected into electric circuit in series, through measuring the voltage of standard resistor and Ni-Cr wire, we can get the rate of heat flow of the wire. The temperature of heat wire can be measured by thermocouples. To reduce the additional disturbance of thermocouples to the plume flow, each thermocouple is welded on the center part of heat wire arrays in an oblique way from top to bottom. As indicated earlier, the effect of radiant heat exchange is of considerable importance in the results presented here. The approximate method developed to compute the heat of exchange via radiation between adjacent cylinders is discussed fully by Marsters (1972). All error factors considered, the maximum relative error of heat transfer coefficient measurement is ±14.5%.

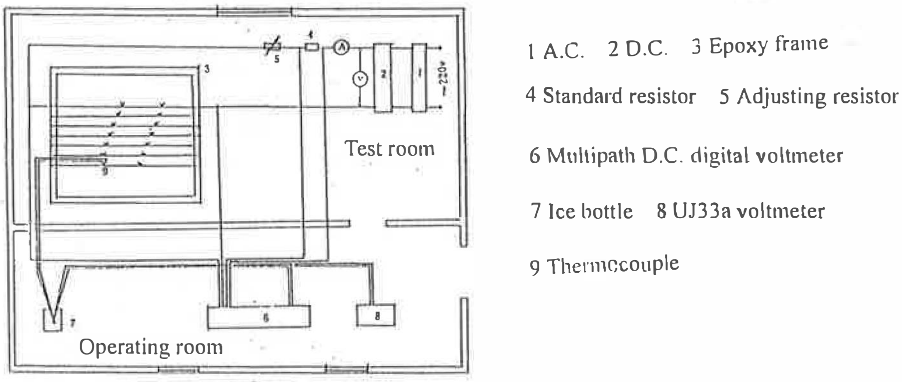

Fig 8 Flowchart of wire-array experiment system

Figure 9 shows us the contrast between the predicted and the experimental results of natural connective heat transfer from wire arrays. In the range of $10^{-2} \leqslant Gr_d \cdot Pr \leqslant 10$,the analytical expression of heat transfer coefficient is

$$Nu_r = 0.804(Gr_d^{0.334} + 0.25\overline{Gr}_x^{0.4}\frac{d}{x})^{0.384} \tag{27}$$

or

$$Nu_r / Nu_o = (1 + 0.25\overline{Gr}_x^{0.067})^{0.384} \tag{28}$$

Where Nu_o is the Nusselt number from single wire under the same operating conditions

$$Nu_o = 0.804Gr_d^{0.129} \tag{29}$$

The equations (27) and (28)are verified by experiment in the range $50 \leqslant s/d \leqslant 100$, $10℃ \leqslant \Delta t_w \leqslant 50℃$ and $2\times10^4 \leqslant \overline{Gr}_x \leqslant 10^7$.

From figure 9. the predicted results are higher than the experimental results by 10% roughly. The possible reasons are as following:

1.The friction dissipation on the surface of wires makes buoyant potential energy partly convert into the kinetic energy of the fluid. Thus analytical velocities are higher than the real ones;

2.The energy loss caused by the mixing of each plume is neglected:

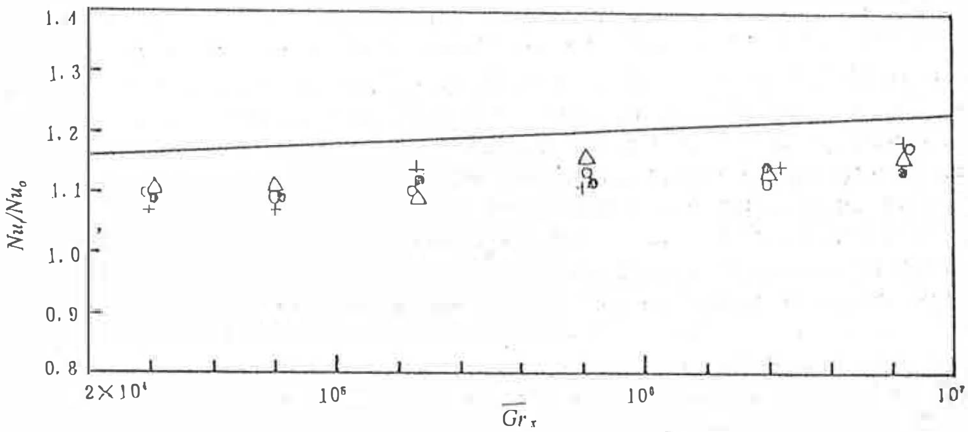

Figure 9. Comparisons between the predicted and the experimental results of natural convective heat transfer from wire arrays

CONCLUSION

With the help of Mach-Zehnder interferometer, author has studied the mechanisms of interactions in buoyant plumes from an array of horizontal line heat sources.

In a summary, author concludes that the physical characteristics of natural convection heat transfer from horizontal line heat arrays can be distinguished into two zones:

(1)Stagnant zone at the small intervals. In this zone, the heat transfer is composed of heat conduction and heat convection. As the intervals between adjacent wires increase, the effect of heat convection is of greater importance.

(2)Intensified heat transfer zone. In this zone, heat transfer proceeds in the way of convection; we can regard the buoyant plumes as a kind of boundary layer flow.

The characteristic velocity and temperature to describe the accumulating buoyant effects from heated line arrays are analyzed. The combined natural and "forced" convection model to predict the natural convection of wire arrays is proposed, and the characteristics of natural convection heat transfer is also studied by experiments. The agreement between the predated and experimental results is satisfactory.

The significance of the analytical model is that the method is given for predicting the natural heat transfer of heated elements with complex structures.

ACKNOWLEDGEMENT

This research is sponsored by the Fundamental Research Foundation from the Department of Metallurgy Industry and the National Natural science Foundation of China.

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