# DETERMINATION OF VENTILATION RATES BY $CO_2$ MONITORING: ASSESSMENT OF INACCURACIES

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## ABSTRACT

The present paper presents an analysis of the inaccuracy of ventilation rates determined by monitoring of person-generated $CO_2$ in indoor air. The analysis includes consideration of the inaccuracy associated with the $CO_2$ concentration measurement and the uncertainty of the estimated $CO_2$ generation by humans. In addition to the traditional method of steady-state measurements, a method based on monitoring of the decay of person-generated $CO_2$ has been investigated. The study is carried out on a theoretical basis by analysis of the equations describing the ventilation process.

In an ideal situation, it would be possible to determine the ventilation rate, by a steady-state measurement, with an uncertainty of about ±10-15%. However, depending on the uncertainty of the $CO_2$ generation rate and if the indoor $CO_2$ concentration is low, the uncertainty of the ventilation rate obtained can easily exceed 20-30%, even if an accurate gas analyser is used.

By measurements of the decay of person-generated $CO_2$, lower inaccuracies can be obtained. The uncertainty of such a measurement depends strongly on the uncertainty of the background concentration (i.e. the outdoor concentration) in relation to the initial indoor concentration. Examples are given of situations where the uncertainty of the measured air change rate can be as low as ±3%. One prerequisite for this is that the initial concentration is substantially higher than the background concentration, and that systematic errors are eliminated.



## KEYWORDS

Tracer gas, Concentration decay, Measuring techniques, Model experiments.


## INTRODUCTION

Several indoor factors influence the prevalence of discomfort, allergies and other types of health related problems. The knowledge about these factors is still limited, but there are strong indications that there is a correlation between low outdoor airflow rates and problems with respect to discomfort and health. Thus, it is important to have access to methods for checking outdoor airflow rates. Two important features of such methods are that they should be both reliable and easy to use.

The concentration of $CO_2$ is frequently measured with the objective of indicating the indoor air quality or to check the outdoor airflow rates in buildings [Persily 1996; ASTM 1996; Reindl 1997]. Furthermore, $CO_2$ monitoring has been successfully used in connection with systems for demand controlled ventilation [Strindehag and Persson 1989]. Instruments for $CO_2$ measurements are often easy to use but, nevertheless, the applicability of the method is often disputed. Many of the difficulties with the interpretation of the results could be avoided by primarily regarding the method as a tracer gas method, for determination or estimation of ventilation rates. It is also vital that the user has sufficient knowledge, not only about the operation of the instruments, but also about the building and ventilation system where measurements are planned to take place.

The present paper contains an estimation of the uncertainties associated with two principally different $CO_2$ methods. Firstly, the quite frequently used method of steady-state $CO_2$ measurements is analysed, and secondly, a method based on analysis of the $CO_2$ concentration decay that can be observed after a room is left unoccupied after a period of occupancy [Ekberg and Strindehag 1996].

The objective of the presentation is to demonstrate some prerequisites for accurate determination of ventilation rates by analysis of $CO_2$-concentrations measured indoors and outdoors. The analyses presented are based on theoretical model data.

## METHODS

The theoretical analyses presented in the paper are principally based on equation (1), which is a balance equation for $CO_2$ in the room or building studied.

$$\dot{V} \cdot C_S + S = \dot{V} \cdot C_E + V \frac{dC_I}{dt} \qquad (1)$$

*where:* $\dot{V}$ *= airflow rate (l/s)*
$S$*= strength of indoor sources (l/s)*
$C_S$*= concentration in supply air (volume by volume)*
$C_E$*= concentration in exhaust air (volume by volume)*
$C_I$*= concentration in indoor air (volume by volume)*

If equation (1) is integrated between time $t_a$ and $t_b$, an expression for the total amount of the tracer gas that has been added to the indoor air is derived, see equation (2).

$$\int_{t_a}^{t_b} \dot{V} \cdot C_S \, dt + \int_{t_a}^{t_b} S \, dt = \int_{t_a}^{t_b} \dot{V} \cdot C_E \, dt + V \cdot \left[ C_{I(t_b)} - C_{I(t_a)} \right] \qquad (2)$$

Note that the indoor concentration, $C_{I(t)}$, represents the average concentration in indoor air at time $t$. In order to solve the integrals containing the concentrations $C_S$ and $C_E$ in equation (2), it is necessary to have simultaneous observations of both the airflow rate and the concentrations. Such data are rarely accessible in practise, and it is often necessary to make the assumption that the airflow rate is constant during the time period between $t_a$ and $t_b$. Equation (3), which is derived from equation (2) under the assumption that the air flow rate is constant, can be used for calculation of the airflow rate.

$$\dot{V} = \frac{\overline{S} - \frac{V \cdot (C_{I(t_b)} - C_{I(t_a)})}{t_b - t_a}}{\overline{C}_E - \overline{C}_S} \qquad (3)$$

*where:* $\overline{S}$ *= average source strength (l/s)*
$\overline{C}_E$ *= average exhaust air concentration (volume by volume)*
$\overline{C}_S$ *= average supply air concentration (volume by volume)*

Equation (3) is valid without respect to whether steady-state conditions are reached or not and also without respect to the mixing conditions. If steady-state conditions prevail, the expression reduces to equation (4).

$$\dot{V} = \frac{\overline{S}}{\overline{C}_E - \overline{C}_S} \qquad (4)$$

The maximum uncertainty of the airflow rate determined by steady-state $CO_2$ monitoring can be estimated by a differential analysis of each of the terms included in equation (4). An estimate of the total uncertainty of the airflow rate will then be obtained as the sum of the contributions from the uncertainty of the $CO_2$ generation rate, the measured exhaust air concentration and the measured supply air concentration. The

expression for the maximum uncertainty is shown i equation (5).

$$\Delta\dot{V} = \Delta S \cdot \left|\frac{1}{C_E - C_S}\right| + \Delta C_E \cdot \left|\frac{S}{(C_E - C_S)^2}\right| + \Delta C_S \cdot \left|\frac{S}{(C_E - C_S)^2}\right| \quad (5)$$

Equation (6) is the analytical solution to the balance equation (1). If the generation of $CO_2$ in the room is zero and $C_I = C_{I(0)}$ at t=0, equation (6) is valid under the assumption of complete mixing. It can be used in a regression analysis of the decay of the concentration of $CO_2$, in order to assess the air change rate in a room, $n = \dot{V}/V$.

$$C_{I(t)} = C_S - (C_S - C_{I(0)}) \cdot e^{-\frac{t}{T}} \quad (6)$$

*where:* $T = 1/n = V/\dot{V}$ *= nominal time constant of the ventilation system*
$C_{I(0)} = C_I$ *at time t=0*

The inaccuracy of air change rates obtained by concentration decay measurements was studied by a sensitivity analysis based on equation (6).

## RESULTS

Figures 1 and 2 are based on calculations using equation (5), which means that they show the maximum uncertainty of the airflow rate obtained by steady-state $CO_2$ measurements. It is presupposed that the exhaust air concentration represents the average indoor concentration.

Figure 1 shows how the uncertainty of the airflow rate is influenced by the uncertainty of the $CO_2$ concentration and the concentration level at which the measurement is carried out.

From the figure it is clear that the airflow rate will be determined with a higher degree of uncertainty the lower the indoor concentration. The calculations are carried out under the assumption that the $CO_2$ generation rate is determined with an uncertainty of ±10%. Consequently the uncertainty of the airflow rate will be at least ±10%.

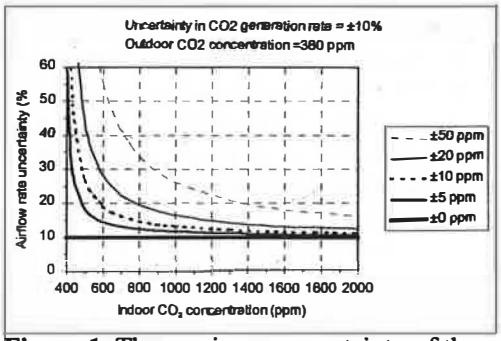


**Figure 1.** The maximum uncertainty of the airflow rate vs. the indoor $CO_2$ concentration calculated for various uncertainties of the concentration measurement.

Figure 2 shows another example of how the uncertainty of the obtained airflow rate is influenced by the uncertainty of the concentration measurement. The calculations are carried out with the indoor $CO_2$ concentration set to 1000 ppm, at a $CO_2$ generation rate of 20 l/h,p. The five lines in the figure represent various levels of the uncertainty of the $CO_2$ generation rate (between ±0 and ±8 l/h,p).

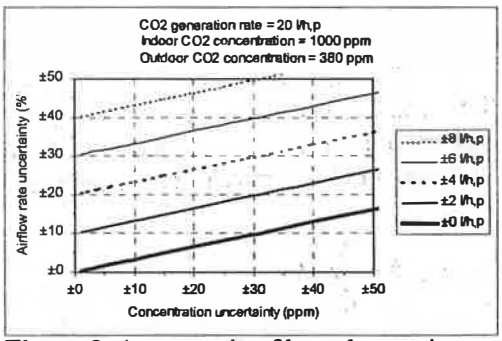


**Figure 2.** An example of how the maximum uncertainty of the obtained airflow rate is influenced by the uncertainty of the concentration measurement and the uncertainty of the $CO_2$ generation rate.

When interpreting figures 1 and 2 it should be noted that the values given of the uncertainty of the concentration measurement refers to the uncertainty of each concentration measurement. This causes the same effect as if the uncertainty of the indoor-outdoor concentration difference was twice the uncertainty of one single concentration measurement. This is a "worst case scenario" and may be applicable if the indoor and outdoor concentrations are measured using two different instruments, between which the correlation has not been established.

Figure 3 shows the result of a sensitivity analysis of equation (6) used together with a regression analysis in order to obtain the time constant of the ventilation system, T. The figure shows the error of the time constant obtained by the regression analysis as a function of the measurement period (i.e. the length of the period over which the regression analysis is carried out). The measurement period is expressed as multiples of the time constant, T. The five curves represent calculations carried out at different levels of errors of the background concentration, $C_S$. Please note that the effect of random errors of the concentration measurement are not considered in the calculations behind figure 3. Random errors will be treated in an subsequent section of the paper.

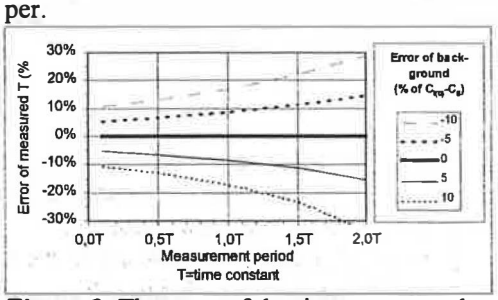


**Figure 3.** The error of the time constant obtained by regression of the $CO_2$ concentration decay as a function of the measurement period (i.e. the length of the period over which the regression analysis is carried out) and the error of the background concentration.

The following example is given in order to explain figure 3: Assume that the initial indoor concentration (at time t=0) is accurately measured to 900 ppm, and that the background concentration is assumed to be 350 ppm. If the actual concentration is 400 ppm, the background is underestimated by 50 ppm, which is -10% of the initial indoor-outdoor concentration difference ($C_{I(0)}$-$C_S$=500 ppm). The figure shows that if the regression analysis is carried out over a time period that equals the time constant for the ventilation, the analysis will result in an overestimation of the time constant by about 17%. The figure also shows that the deviation from the true time constant increases with increasing measurement period.

It is important to note that figure 3 is valid if the background concentration is determined with a systematic error while the indoor concentration is correctly measured, which can be the case if: 1) the indoor and outdoor concentrations are measured with different instruments, or 2) the outdoor concentration is not measured, but chosen based on a "best guess". However, errors of the calibration (gain and offset) of the instrument will not influence the obtained time constant, as long as the indoor and outdoor concentrations are measured using the same instrument.

Figure 4 and 5 show how random errors of the concentration measurement influence the time constant obtained by the concentration decay method. Figure 4 is based on equation (6) using the following input data: T=60 min; $C_S$=380 ppm; $C_{I(0)}$=1400 ppm. The bold curve shows the room concentration calculated using equation (6) and the upper of the two thin curves is obtained by adding a random error corresponding to a standard deviation of ±5% to the values shown by the bold curve. The lower thin curve represents the background concentration of 380 ppm with a random error of ±5% added (again expressed as the standard deviation of the concentration). The data in the

figure are calculated and plotted using a time interval of 2 minutes, i.e. the data simulate $CO_2$ concentrations monitored with a sampling interval of 2 minutes.

We know that the true value of the time constant is 60 minutes, as this is the value inserted to equation (6). However, a regression analysis including the first 120 minutes of the data set shown in figure 4 give an apparent time constant of 59 minutes. If the regression analysis is limited to the first 60 minutes the result will be a time constant of 54 minutes. As already mentioned, the true value of the time constant is 60 minutes.

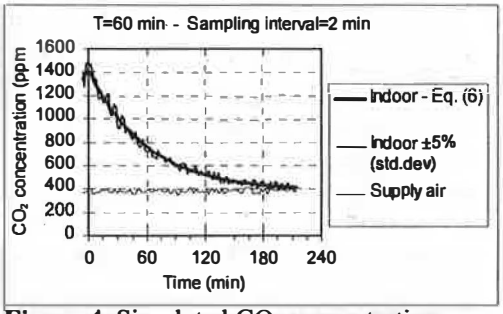


**Figure 4.** Simulated $CO_2$ concentrations vs. time. The bold curve represents the concentration calculated using equation (6) while the two thin curves are calculated by adding a random error corresponding to a standard deviation of ±5% to the concentration in the room and in the supply air, respectively.

Figure 5 shows the result of calculations similar to the example shown in figure 4. The calculations are carried out for three different levels of the precision of the concentration measurement, ±1%, ±2 and ±5%. The determination of the time constant by regression analysis is repeated 100 times for each set of input data. The only parameters varied between data sets are the length of the period over which the regression analysis is carried out (measurement period) and the concentration precision levels. The standard deviation of these 100 determinations of the time constant is then calculated, i.e. each data-point in the figure represents the standard deviation of the time constant, obtained after 100 regression analyses.

As expected, the uncertainty of the obtained time constant (due to random errors of the concentrations measured) increases with decreasing analysis period. The example shown in figure 5 indicate that, if the concentrations are measured with an uncertainty less than ±2%, and if the regression analysis includes at least the first 60 minutes of the concentration decay, the uncertainty of T, due to random errors of the concentration measurement, will be less than ±3%. If $C_{I(0)}$ is changed from 1400 ppm to 700 ppm, while all other parameters are kept unchanged, the uncertainty will be approximately twice as high.

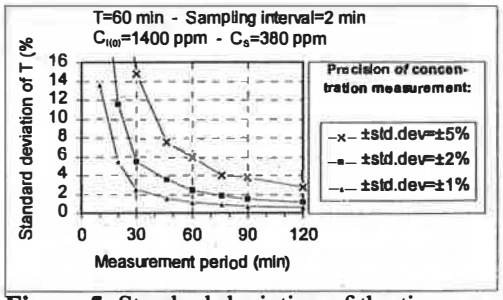


**Figure 5.** Standard deviation of the time constant, T, obtained by the concentration decay method, as a function of the length of the period over which the regression analysis is carried out, calculated for various levels of the precision of the concentration measurement (random errors). The precision of the concentration measurement is expressed as one standard deviation.

## DISCUSSION

Measurements of $CO_2$ concentrations indoors and outdoors can be a valuable tool for determining the outdoor airflow rates and air change rates. However, there are a number of factors that may lead to unacceptable measurement errors, but accurate results can often be obtained if the planning of the measurements include consideration of these possible sources of errors. It is a

prerequisite that the measurements are carried out by a person who:

- is familiar with the operation of the instruments being used,
- is aware of the limitations of the method, and
- has sufficient knowledge about the building and the ventilation system where measurements are to be carried out.

Assessment of the outdoor airflow rate under steady-state conditions requires accurate information about the total $CO_2$ production in the room, and that occupancy is constant during a period long enough to allow the concentration to reach equilibrium. The assumption of steady-state can often be checked in single rooms, but the uncertainty of the $CO_2$ generation rate may be difficult to estimate.

**Occupant-generated $CO_2$**

The human production of $CO_2$ and other bioeffluents have been found to vary approximately linearly with the level of physical activity [European Concerted Action 1992; ASHRAE 1989]. For adults the metabolic $CO_2$ production varies from about 10 l·h$^{-1}$ per person when sleeping up to about 170 l·h$^{-1}$ per person at high levels of physical activity. For persons at sedentary activities (1.0-1.2 met), such as office work, the $CO_2$ production is about 19 l·h$^{-1}$ per person [European Concerted Action 1992]. The corresponding figures according to ASHRAE (1989) are 18 l·h$^{-1}$ per person at a metabolic rate of 1.2 met. The $CO_2$ production may vary between individuals depending on, for example, body weight.

For children, the relationship between the $CO_2$ production and the metabolic rate is different compared to that for adults. In, for example, kindergartens an activity level of 2.7 met, corresponding to a $CO_2$ production of 18 l·h$^{-1}$ per person is a realistic assumption [European Concerted Action 1992]. Also in schools with children aged between 14 and 16 years, the $CO_2$ production is about equal to that for an adult at sedentary activity, 19 l·h$^{-1}$ per person. Since, as mentioned, the $CO_2$ production is depending both on personal characteristics and the metabolic rate, such figures can only be regarded as more or less rough estimations. Consequently, airflow rates determined by steady-state $CO_2$ measurements will suffer from uncertainties of the $CO_2$ generation, uncertainties that are often difficult to estimate.

**Outdoor $CO_2$ concentrations**

It is often stated in the literature that the background concentration of $CO_2$ in the atmosphere is about 350 ppm. However, it has been shown that various outdoor pollution sources, can significantly influence the $CO_2$ concentration outdoors. [Dols et al. 1992]. Ekberg and Strindehag (1996) presented outdoor $CO_2$ concentrations between 370 and 440 ppm measured continuously over a nine-day period. These data were collected 3 meters above ground level, outside an office building located close to a busy street in Göteborg, Sweden. Corresponding measurements of $CO_2$ were also carried out in a location not directly influenced by traffic or other sources of outdoor pollution. These measurements were made in the countryside about 25 km from the centre of Göteborg, and in this case the concentrations ranged from a minimum of 350 ppm up to a maximum of 370 ppm, with an average value of 360 ppm.

The present paper clearly shows that the quality of both steady-state measurements and concentration decay measurements for determination of airflow rates and air change rates are strongly depending on the accuracy of the background $CO_2$ concentration, which obviously may vary considerably both with time and between locations. In order to limit the uncertainties of ventilation rates and air change rates determined by $CO_2$ monitoring the outdoor or supply air concentration should be meas-

ured, preferably using the same instrument as used for the indoor concentration measurement.

By using the same instrument for the outdoor measurement and the indoor measurement, the calculated difference between indoor and the outdoor concentration will not be affected by an error of the zero-calibration of the gas analyser, and consequently, neither will the airflow rate obtained by a steady-state measurement. However, an error of the gain (i.e. the slope of the calibration curve) will influence the concentration difference, and, thus cause a systematic error of the airflow rate obtained, even if the same instrument is used for both the indoor and the outdoor $CO_2$ measurement.

**Concentration decay measurements**

It is shown that systematic errors of the background concentration may lead to considerable systematic errors of the time constant obtained by $CO_2$ concentration decay measurements. However, by using the same instrument for the indoor and the outdoor measurement such systematic errors can be eliminated, even if the calibration of the instrument used is incorrect with respect to both the zero-point and the slope of the calibration curve. This is true as long as the output signal is linearly increasing with the concentration measured.

When systematic errors have been eliminated, random errors due to noise of the instruments output signal remain to be considered. The example given in figure 5 indicate that it may be possible to determine the time constant for the ventilation system with an uncertainty less than ±3% provided that the precision of the concentration measurement is better than ±2% of the measured value. These figures are valid for measurements that are free from systematic errors. In the example shown the indoor $CO_2$ concentration was initially 1400 ppm. A lower initial indoor concentration would result in an increased uncertainty of the obtained air change rate.

## CONCLUSIONS

Guidelines for estimation of the $CO_2$ production by humans at various levels of activity are available. However, due to the difficulty of estimating the uncertainty of the $CO_2$ generation rate, the steady-state $CO_2$ method has primarily to be limited to rather rough estimations of outdoor airflow rates.

Ventilation rates and air change rates determined by steady-state $CO_2$ measurements and $CO_2$ concentration decay monitoring, respectively, are highly sensitive to errors of the background $CO_2$ concentration. Furthermore, it cannot be presupposed that the outdoor $CO_2$ concentration is constant with time, especially not in locations influenced by vehicle exhaust. Therefore, measurements in the outdoor air should be carried out continuously or at least before and after the indoor measurements. If more than one instrument is used in the same investigation the correlation between the concentrations measured by the different instruments should be established.

The air change rate can be accurately determined by analysis of the decay of the $CO_2$ concentration in a room. Again, one prerequisite for keeping the uncertainties at a low level is that the indoor and outdoor measurements are carried out with the same instrument or instruments that have been correlated with respect to their calibrations. It is also important that the initial indoor concentration is high compared to the outdoor concentration.

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