DESIGN OF NATURAL VENTILATION BY THERMAL BUOYANCY WITH TEMPERATURE STRATIFICATION

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ABSTRACT

A set of formulae for natural ventilation by thermal buoyancy is derived for a room with two openings and with a linear temperature stratification. The formulae are based on the fundamental flow equations, and they cover air velocities, temperature differences and ventilation rates in relation to opening areas, opening position, net heat input, building geometry, and temperature stratification.

The temperature stratification can simply be taken into account by introducing a stratification factor ϵ and by using the mean difference between indoor and outdoor temperatures.

The validity of the formulae is discussed as well as specific topics concerning design and control.

The use of the formulae is illustrated on an atrium.

KEYWORDS:

Natural ventilation, thermal buoyancy, temperature stratification.

INTRODUCTION

Formulae for natural ventilation by thermal buoyancy are usually derived assuming uniform indoor temperature. However, temperature stratification always occurs in practice.

Only a few references re available, where temperature stratification is taken into account. Hemeon (1963) assumes that the pressure difference at an opening is proportional to the squareroot of the temperature difference at that opening. Kreichelt et al. (1976) assume that the difference between the indoor temperatures at opening and neutral plane level is the determining factor.

The aim of the theoretical work described in this paper has been to develop a reliable tool for designing and analyzing natural ventilation by thermal buoyancy, where the temperature stratification is taken into account.

STRATIFICATION

Temperature stratification depends on how heat is supplied to the room and how fresh air is mixed with room air. Typical stratifications are shown in Figure 1:

- Curve A: heat is supplied evenly and close to floor level.
- Curve B (uniform): a theoretical situation approximately obtainable when heat is supplied evenly in the room and when fresh air is well mixed with room air.
- Curve C (linear): heat is supplied evenly in a room.
- Curve D: heat is supplied from the ceiling or the upper part of the walls.
- Curve E: heat is supplied from the upper part of the room or from a concentrated heat source in the room.

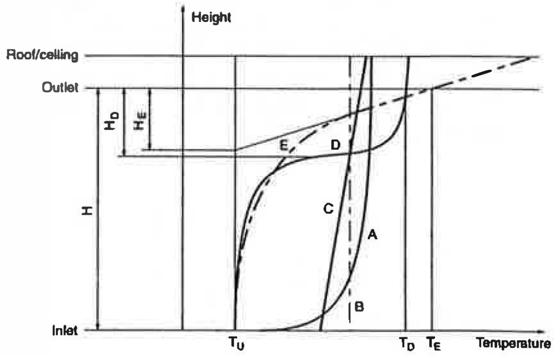

Figure 1 Vertical temperature distributions in heated rooms.

The linear stratification is the easiest to deal with. Besides, me surements show, that the linear stratification is frequently a good approximation in practice.

EQUATIONS

A simple case of two openings in a room as shown in Figure 2 is considered in this paper. The air flow through the openings will take the shape of jets, where the air pressure in the socalled *vena contracta* is equal to the surrounding pressure.

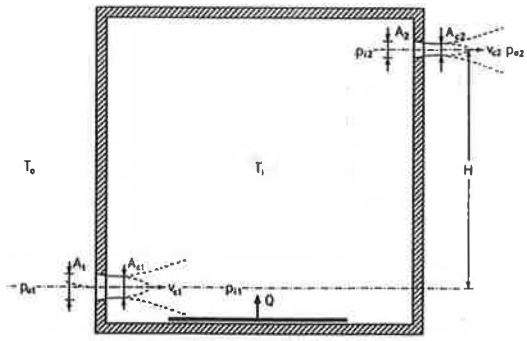

Figure 2 Natural ventilation through two openings by thermal buoyancy.

The fundamental flow equations can be used on a control volume enclosed by the room surfaces, the two vena contracta sections and the two jet surfaces between opening and vena contracta.

It should be mentioned beforehand that the solution results in pressures and pressure differences as shown roughly in Figure 3. One gets an indoor negative pressure at the bottom opening (inlet) and a positive one at the top opening (outlet). In between, one has the socalled neutral plane where indoor and outdoor pressures are equal.

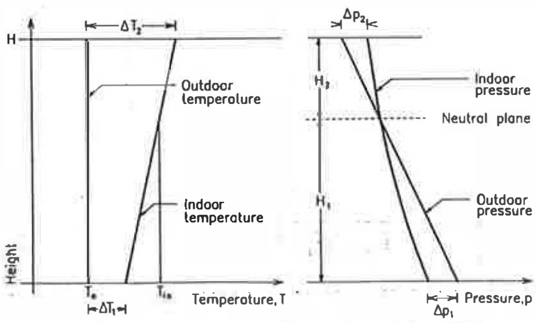

Figure 3 Pressure conditions by linear indoor temperature stratification

The fundamental flow equations are the equations for mass balance, energy conservation and vertical momentum. For the mass balance and the heat conservation one gets:

$$\varrho_o A_{c1} v_{c1} = \varrho_{i2} A_{c2} v_{c2} \tag{1}$$

and:

$$Q_s = c_p \varrho_{i2} A_{c2} v_{c2} \Delta T_2 = c_p \varrho_o A_{c1} v_{c1} \Delta T_2 = c_p \varrho_o V \Delta T_2 \tag{2}$$

where:

- ϱ_o, ϱ_{i2} = air densities in the flows through bottom and top openings
- A_{c1}, A_{c2} = areas of vena contracta for bottom and top openings
- v_{c1}, v_{c2} = air velocities in the two vena contractas
- Q_s = surplus heat or net heat input (i.e. heat load subtracted from the heat transmission loss)
- ΔT_2 = temperature difference between indoor and outdoor at top opening
- V = ventilation rate

The vertical momentum equation results in the following indoor pressure distribution (Andersen 1995):

$$p_i = p_{in} - g \varrho_{in} y + 0{,}5\, g\, b\, y^2 \tag{3}$$

with:

$$b = \varrho_{in} a / T_{io} \tag{4}$$

and where:

- p_{in} = indoor pressure at neutral plane level
- g = gravity acceleration
- ϱ_{in} = indoor air temperature at neutral plane level
- y = vertical distance from neutral plane, positive upward
- a = indoor temperature gradient

Together with the linear outdoor pressure distribution one gets the following differences between outdoor and indoor pressures at inlet and outlet, *Tabel 1. Formulae for natural ventilation by thermal buoyancy in room with two openings and with temperature stratification.*

	Formulae based on the mean temperatur difference ΔT_m	Formulae based on the modified surplus heat $Q_{s\epsilon}$ (Q_s/ϵ)
Inlet conditions pressure difference, Δp_1 (Pa)	$\rho_u \, \Delta T_m \, g \, H_1 / T_{im}$	
air velocity, v_{c1} (m/s)	$\left[\frac{2 \, \Delta T_m \, g \, H_1}{\psi_1 \, T_{im}}\right]^{1/2}$	$0.038 \left[\frac{Q_{s\epsilon} \, H_1}{C_{d1} \, A_1}\right]^{1/3} \left[\frac{1}{\psi_1}\right]^{1/2}$
Outlet conditions pressure difference, Δp_2 (Pa)	$\rho_i \, \Delta T_m \, g \, H_2 / T_u$	
air velocity, v_{c2} (m/s)	$\left[\frac{2 \cdot \Delta T_m \, g \, H_2}{\psi_2 \, T_o}\right]^{1/2}$	$0.039 \left[\frac{Q_{s\epsilon} \, H_2}{C_{d2} \, A_2}\right]^{1/3} \left[\frac{1}{\psi_2}\right]^{1/2}$
Temperature difference, ΔT_m (K or °C)		$7.3 \cdot 10^{-5} \, T_{im} \left[\frac{Q_{s\epsilon}}{C_{d1} \, A_1}\right]^{2/3} \left[\frac{1}{H_1}\right]^{1/3}$
Ventilation rate, V (m^3/s)	$C_{d1} \, A_1 \left[\frac{2 \, \Delta T_m \, g \, H_1}{T_{im}}\right]^{1/2}$	$0.038 \, (Q_{s\epsilon} \, H_1)^{1/3} \, (C_{d1} \, A_1)^{2/3}$
Inlet area A_1 (m^2) 1)		$6.2 \cdot 10^{-7} \, \frac{Q_{s\epsilon}}{C_{d1}} \left[\frac{1}{H_1}\right]^{1/2} \left[\frac{T_{im}}{\Delta T_m}\right]^{3/2}$ $140 \, \frac{V^{3/2}}{(Q_{s\epsilon} \, H_1)^{1/2} \, C_{d1}}$ 2)

1) The outlet area is calculated from the area ratio A_1/A_2 used when determining the neutral plane position.
2) V is a required ventilation rate

$$\Delta p_1 = \Delta \varrho_n \, g \, H_1 - b \, g \, H_1^2/2 \qquad (5)$$

$$\Delta p_2 = \Delta \varrho_n \, g \, H_2 - b \, g \, H_2^2/2 \qquad (6)$$

where:

$\Delta\varrho_n$	=	difference between outdoor and indoor air density at neutral plane level
H_1, H_2	=	distances from neutral plane to centre of inlet and outlet

Further, the modified Bernouilli equation can be used on the conditions at inlet and outlet resulting in the following equations:

$$\Delta p_1/\varrho_o = \tfrac{1}{2}(1 + \zeta_1)\, v_{c1}^2 = \tfrac{1}{2}\, \psi_1 \, v_{c1}^2 \qquad (7)$$

$$\Delta p_2/\varrho_{i2} = \tfrac{1}{2}(1 + \zeta_2)\, v_{c2}^2 = \tfrac{1}{2}\, \psi_2 \, v_{c2}^2 \qquad (8)$$

where:

Δp_1, Δp_2	=	pressure differences at inlet and outlet
ζ_1, ζ_2	=	resistance coefficients for inlet and outlet
ψ_1, ψ_2	=	"flow" coefficients for inlet and outlet

SOLUTIONS

By solving the Equations 1 to 8, the position of the neutral plane can be determined and afterwards, solutions can be derived based on either the temperature difference or the surplus heat.

Neutral plane position

The air velocities in the vena contractas can be found from equations 7 and 8 together with equation 5 and 6. By inserting the velocities into equation 1 one gets after simplifications and assuming $\varrho_{in}/\varrho_o \sim 1$ and $\psi_1/\psi_2 \sim 1$:

$$A_{c1}^2 \, (\Delta\varrho_n \, g \, H_1 - b \, g \, H_1^2/2) =$$

$$A_{c2}^2 \, (\Delta\varrho_n \, g \, H_2 + b \, g \, H_2^2/2) \qquad (9)$$

where $H_2 = H - H_1$ and besides, $\Delta\varrho_n$ is dependent on H_1. After some manipulations, one gets an equation of second degree for H_1 as a function of $\Delta T_2/\Delta T_1$ (ΔT_1 being the temperature difference at inlet) and with A_1/A_2 or A_{c1}/A_{c2} as parameter. Figure 4 shows the solution for different values of A_1/A_2. As it can be seen, the neutral plane is above the position valid for uniform indoor temperature. For $\Delta T_2/\Delta T_1$ moving towards infinity (or ΔT_1 moving towards zero) one gets H_1/H = 0,71, and 0,45, and 0,32 for A_1/A_2 = 1/1, and 2/1, and 3/1, respectively.

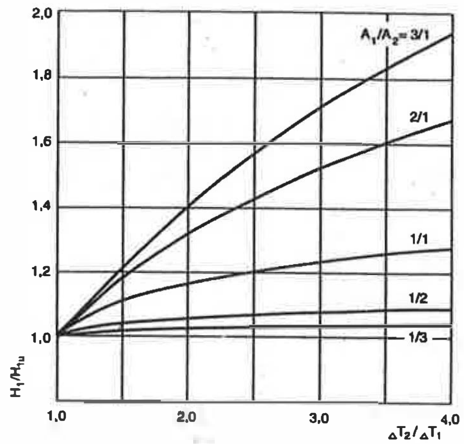

Figure 4 The relationship between the distance H_1 and the temperature difference rate $\Delta T_2/\Delta T_1$ with the area ratio as parameter

Solution based on temperature difference.
By introducing the following relationship between density and temperature:

$$\Delta\varrho = \varrho_o \, \Delta T/T_i = \varrho_i \, \Delta T/T_o \qquad (10)$$

one gets a solution based on the temperature differences.

The air velocities can be determind with a good approximation by:

$$v_{c1} = (2 \, \Delta T_m \, g \, H_{1u}/(\psi_1 \, T_{im}))^{1/2} \qquad (11a)$$

$$v_{c2} = (2 \, \Delta T_m \, g \, H_{2u}/(\psi_2 \, T_o))^{1/2} \qquad (11b)$$

with:

$$\Delta T_m = (\Delta T_1 + \Delta T_2)/2 \qquad (12)$$

$$H_{1u} = \frac{H}{1 + (T_{im}/T_u)\,(C_{a1}/C_{a2})^2\,(A_1/A_2)^2} \sim \frac{H}{1 + (A_1/A_2)^2} \qquad (13a)$$

$$H_{2u} = H - H_{1u} \qquad (13b)$$

and where:

ΔT_1, ΔT_2 = difference between indoor and outdoor temperature at inlet and outlet

T_{im} = mean indoor value of temperature between inlet and outlet

H_{1u}, H_{2u} = distance from the neutral plane position to the centre of inlet and outlet by uniform indoor temperature

The equations 11a and 11b are identical to the expressions valid for uniform indoor temperature when using a temperature difference of $\Delta T = \Delta T_m$. It can be shown that the "stratified" velocity is equal to the "uniform" one with an error smaller than 1%. This can be explained by the fact that the neutral plane moves upward and the temperature difference at the inlet decreases at the same time, resulting in a pressure difference at the inlet, almost equal to the one, one gets by uniform temperature. Similarly, one gets almost equal pressure differences at the outlet.

The ventilation rate is determined by:

$$V = A_{c1} V_{c1}$$
$$= C_{d1} A_1 (2\Delta T_m g H_{1u}/T_{im})^{½} \qquad (14)$$

with:

$$C_{d1} = (A_{c1}/A_1)/\psi_1^{½}$$

and where:

C_{d1} = discharge coefficient for inlet
A_1 = opening area of inlet

The solution is summarized in Table 1.

Solution based on surplus heat

In Equation 2, the ventilation rate can be eliminated by using Equation 14. One gets:

$$Q_s = c_p \rho_o C_{d1} A_1 \left[\frac{2 g \Delta T_m H_{1u}}{T_{im}}\right]^{½} \Delta T_2$$

This equation can be solved with regard to ΔT_m by introducing

$$\Delta T_2 = (\Delta T_2/\Delta T_m) \Delta T_m = \epsilon \Delta T_m$$

resulting in:

$$\Delta T_m = \left[\frac{Q_s/\epsilon}{c_p \rho_o C_{d1} A_1}\right]^{2/3} \left[\frac{T_{im}}{2g H_{1u}}\right]^{1/3}$$

By eliminating ρ_o, introducing $Q_{s\epsilon} = Q_s/\epsilon$, and inserting the values of the constants involved, one gets:

$$\Delta T_m = 7.1 \cdot 10^{-5} T_{im} \left[\frac{Q_{s\epsilon}}{C_{d1} A_1}\right]^{2/3} \left[\frac{1}{H_{1u}}\right]^{1/3}$$

By using this expression, one gets air velocities, ventilation rate and opening areas in dependance of the surplus heat Q_s and the stratification factor ϵ as seen in Table 1.

REGION OF VALIDITY

The set of formulas in Table 1 implies linear temperature stratification and uniform air velocity through the openings. In practice, the stratifications can be curved in different ways as shown in Figure 1. Besides, the air velocities through vertical openings will only be approximately uniform, being more and more parabolic as the neutral plane comes nearer to one of the openings.

Stratification variations

The stratification situations represented by the curves A, B and C in Figure 1 can be considered by direct use of the formulas in Table 1.

The situation represented by curve D can be considered by replacing the opening height H by the height H_D of the high temperature region and by using the temperature T_D as indoor temperature.

The situation represented by curve E can be considered by replacing the opening height H by the height H_E and assuming a temperature stratification determined by the top temperature T_E and a temperature at the inlet equal to the outdoor temperature T_o.

Bidirectional Flow

By increasing the inlet opening area A_1 compared to the outlet area A_2, the neutral plane moves downward towards the inlet as shown in Figure 5. At the same time the velocity distribution changes from almost uniform to parabolic.

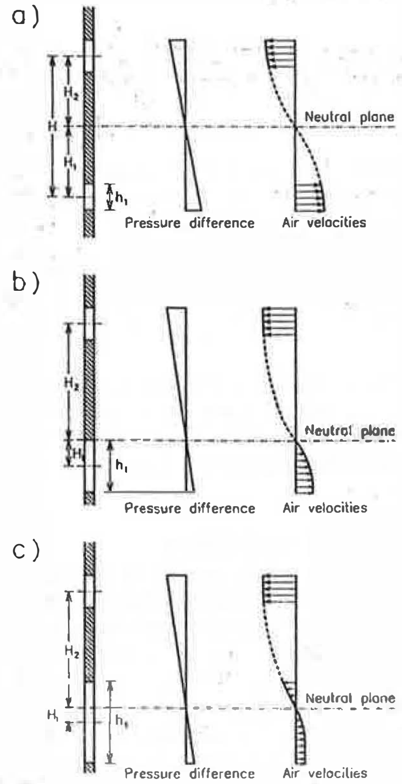

Figure 5 The position of the neutral plane by different area ratios with corresponding pressure differences and air velocities.

When the neutral plane passes below the upper edge of the inlet, air starts to flow outward through the opening between the neutral plane and the upper edge.

By uniform indoor temperature, the set of formulas can be used with an error smaller than 6% as long as the neutral plane position is neither below the upper edge of the inlet nor above the lower edge of the outlet (Andersen 1995).

By linear stratification the same criterion for validity can be used. When the neutral plane is close to the inlet, the error for using the set of formulae is smaller than for uniform temperature as the stratification makes the neutral plane move upward. When the neutral plane is close to the outlet the error is only sligtly higher than by uniform temprature.

Opening orientation

Air velocities can be considered as completety uniform in horizontal openings, see Figure 6. But generally it is not very important whether the openings are oriented horizontally, vertically or sloped as long as the openings are relatively small.

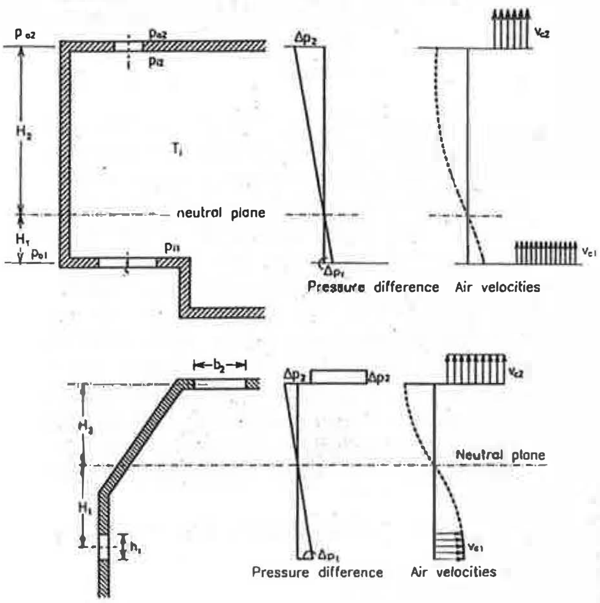

Figure 6 Pressure difference distribution and air velocities by horizontally and vertically placed openings.

DESIGN AND CONTROL CONSIDERATIONS

When designing natural ventilation, a typical task is to determine the opening areas ensuring that a certain temperature difference between indoor and outdoor is not exceeded on a hot calm summerday.

A fi st step is to calculate the neutral plane position which implies a knowledge of the area ratio A_1/A_2. This ratio should be chosen so that the total opening area $A_1 + A_2$ does not become bigger than necessary.

Optimum opening areas

A theoretical analysis shows that by uniform temperature, the largest ventilation rate is obtained when the total opening area is shared equally between inlet and outlet. The analysis also shows that the ventilation rate is reduced less than

10% as long as the area ratios are within $1/2 < A_1/A_2 < 2/1$ (Andersen 1995).

This result can also be used when the temperature stratification is considered, as the ventilation rate is only slightly dependent on the stratification.

Ventilation rate and area ratios.

For a fixed outlet area, the ventilation rate is not proportional to the variation of the inlet area. For uniform indoor temperature a doubling of the inlet area only increases the ventilation rate by 26% when the indoor temperature is kept constant (by extra heat supply), and the increase is reduced to 17% when the heat supply is kept constant. By a six fold increase of inlet area, the increase in ventilation rate is about 39% and 25%, respectively (Andersen 1995).

This result is valid approximately when considering temperature stratification, as the ventilation rate is only slightly influenced by the stra ification.

Air velocities and area ratios

The air velocities in the openings are dependent on the position of the neutral plane as shown in Equations 11a and 11b and thereby on the area ratio A_1/A_2, cf Equations 13a and 13b. The air velocity in the inlet can thus be decreased by increasing the area ratio and be increased by decreasing the area ratio.

EXAMPLE. ATRIUM

Figure 7 shows the cross section of an atrium which is 30 m long, 6 m wide and 15 m high. To the right, the atrium is limited by walls to a store area and to the left, it is open to public areas. The atrium is supposed to be ventilated by natural ventilation with one big inlet and with outlets in the glass roof. It is required that the indoor temperature on the level of the public areas should not exceed 30 °C on a calm summerday with an outdoor temperature of 27 °C.

The heat load from the sunshine is approximately 16kW assuming that 30% of the sun heat on the indoor surfaces is returned to the indoor air.

The discharge coefficient is assumed to be C_d = 0.62 corresponding to sharp-edged openings.

The temperature stratification is assumed linear with an indoor temperature difference between top and bottom of 2K corresponding to a vertical gradient of 0.13 K/m.

The inlet opening shall be placed with the lower edge 2.2 m above floor level for reasons of comfort.

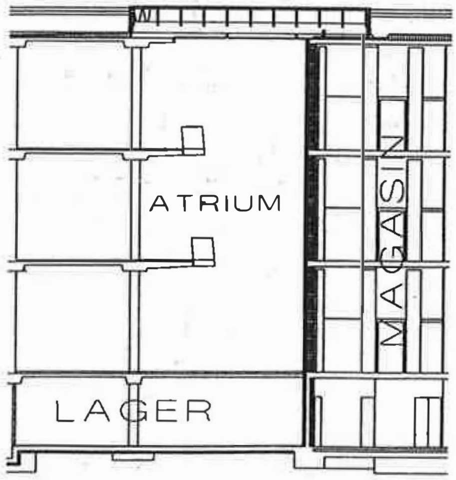

Figure 7 Cross section of atrium, 30 m long, 6 m wide and 15 m high.

Required opening areas

With a temperature difference of about 3K one gets a heat transmission loss of about 2 kW resulting in a surplus heat of Q_s = 16 - 2 = 14 kW.

With the requirement to the indoor temperature, one ge s a temperature difference at the top of ΔT_2 = 3.4 K, and a mean temperature difference of ΔT_m = 2.6 K, resulting in ϵ = 3.4/2.6 = 1.3.

Assuming an inlet height of 2 m one gets an opening distance H = 15 - 2.2 - 1.0 = 11.8 m, and with equally large opening areas one gets a neutral pla e position H_1 = H/2 = 5.9 m.

By using the inlet area formula in Table 1, one gets:

$$A_1 = 6.2\cdot 10^{-7}\,\frac{14000}{1.3\cdot 0.62}\left(\frac{1}{5.9}\right)^{1/2}\left(\frac{303}{2.6}\right)^{3/2}$$
$$= 5.6\ m^2$$

and one gets the outlet area $A_2 = A_1$ = 5.6 m^2.

For CFD calculations, the socalled efficient opening areas may be wanted as boundary values. They are determined by:

$$A_{1eff} = A_{2eff} = C_{d1}\,A_1 =$$

$0.62 \cdot 5.6 = 3.5\ m^2$

CONCLUSION

he set of formulae derived for natural ventilation by thermal buoyancy with temperature stratification extends the range of situations where natural ventilation can be analysed. The set is identical to the formulae valid for uniform indoor temperature when Q_s is replaced by $Q_{s\epsilon}$ and ΔT by ΔT_m.

The formulae are valid as long as the neutral plane does not intersect with the inlet or outlet, and the validity is independent of the opening orientation.

The optimum opening area ratio is $A_1/A_2 \sim 1/1$ and the ventilation rate is not particularly sensitive to deviations from this ratio as long as $1/2 < A_1/A_2 < 2/1$.

The air velocity and the flow direction in the opening can be varied by varying the opening area ratio.

If either inlet or outlet area is kept fixed, the possiblities for increasing the ventilation rate by increasing the other opening area are limited. Only by increasing both areas a significant increase in ventilation rate can be obtained.

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NOMENCLATURA

A = opening area (m^2)
C_d = discharge coefficient
H = vertical distance between centres of the tvo openings (m)
H_1 = vertical distance between neutral pressure plane and inlet (m)
H_2 = vertical distance between neutral pressure plane and outlet (m)
Q_s = surplus heat (W)
$Q_{s\epsilon}$ = surplus heat including the stratification factor ($=Q_s/\epsilon$)(W)
T = absolute temperature (K)
V = volume flow, ventilation capacity (m^3/s)

c_p = specific heat capacity of air (=1010 J/kg K)
g = gravity acceleration (= 9.82 m/s^2)
n = opening area ratio ($\sim A_1/A_2$)
p = pressure (Pa)
v = air velocity (m/s)

ϵ = stratification factor
ρ = air density (kg/m^3)
ζ = resistance coefficient
ψ = flow coefficient ($= 1 + \zeta$)
Δ = difference in pressure or temperature
ΔT_{12} = inside temperature difference between top and bottom (K)

Subscripts
c = contracted
d = discharge
i = indoor
m = nuan value
n = at neutral plane level
o = outdoor
u = uniform temperature
1 = inlet
2 = outlet