UPWARD FLOWS IN A MULTI-ZONE BUILDING WITH SUBFLOOR PLENUMS AND SOLAR CHIMNEYS

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ABSTRACT

Solar chimneys are often used to extract air from a building by thermal stacks, while subfloor plenums are used to passively cool air before it is supplied to a building. This paper examines the overall flow pattern in buildings with both solar chimneys and subfloor plenums. For a multi-zone flow system in which each zone has only two effective openings, an analytical solution is derived. A sufficient condition for upward flows to occur is derived from the analytical solution.

A two-room building with two solar chimneys and two subfloor plenums at two different heights, and an internal large opening is also considered. No analytical solutions exist for this configuration. A multi-zone natural ventilation program considering bi-directional flows is used to solve the problem numerically. Much more complex flow patterns are identified and analysed.

KEYWORDS

Natural ventilation, air flow pattern, modelling, solar chimneys.

INTRODUCTION

Solar chimneys provide effective natural ventilation, while subfloor plenums are often used as a passive cooling technique. A combination of the two mechanisms offers an ideal energy-efficient design option for some commercial and residential buildings.

Li and Delsante (1997) analysed a proposed building where such a combination was incorporated into the design. The building was to be built on a sloping site. It was designed to be cooled in summer by natural ventilation. Because of the site geometry, the building is quite complex, with floor levels changing stepwise as the building progresses up the site, and with the roof changing level as well, although generally not at the same location as the floor steps. Plenums are located underneath the occupied spaces. Solar chimneys are distributed fairly evenly over the roof. Simulated results showed an interesting *U*-tube flow pattern through the solar chimneys. Because the openings between the main office zones are relatively large and there is no flow resistance between them, it is easier for air to enter the building through lower chimneys, flow upward through the main zones, and then leave the building through the higher chimneys. This is an undesirable ventilation flow pattern. Further investigation shows that in other situations air can also enter the building through higher chimneys and leave through lower chimneys.

This earlier work has inspired us to examine some much simpler flow systems. The question is under what conditions is there an upward flow in buildings with subfloor plenums and solar chimneys. It should be mentioned that in this paper only a no-wind situation is considered.

ANALYTICAL SOLUTIONS

Consider a simple one-zone system with two openings (see Figure 1a). The system has a uniform indoor air temperature. The solution is well known. The derivation procedures for both single-zone and multi-zone situations are similar. Thus, a preferred derivation process for the single-zone problem is included for demonstration purposes.

The bottom opening height is taken as the datum level. The indoor and outdoor pressure at any height are:

$$p_i(z) = p_{i0} - \int_0^z \rho_i g dz$$

$$p_o(z) = p_{o0} - \int_0^z \rho_o g dz \qquad (1)$$

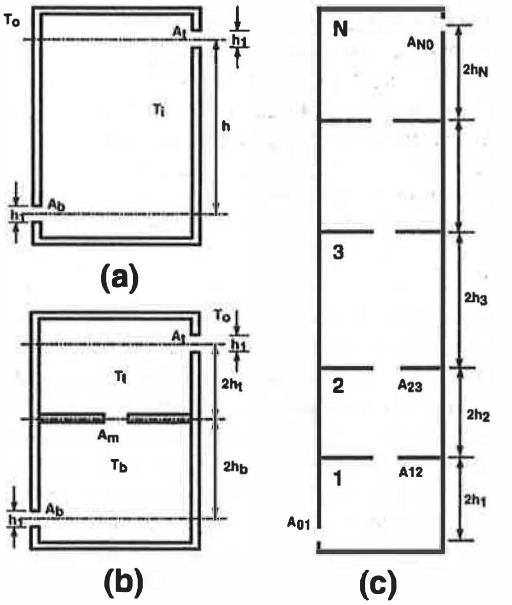

Figure 1 Geometry of three simple flow systems: (a) one-zone building, (b) two-zone building and (c) multi-zone buiding.

The outdoor and indoor pressure difference at any height is:

$$\Delta p_{tot}(z) = -\int_0^z (\rho_o - \rho_i) g dz - (p_{i0} - p_{o0}) \quad (2)$$

Define $p_i = p_{i0} - p_{o0}$ as the internal pressure.

As ρ_o and ρ_i are constant along the height, at the bottom and top openings:

$$\Delta p_{tot}(0) = -p_i$$
$$\Delta p_{tot}(h) = -(\rho_o - \rho_i)gh - p_i \quad (3)$$

Assume inflow through the bottom opening and outflow through the top opening:

$$\Delta p_b = \Delta p_{tot}(0)$$
$$\Delta p_t = -\Delta p_{tot}(h) \quad (4)$$

The flow rates through two openings are:

$$q_b = C_d A_b \sqrt{\frac{-2p_i}{\rho}}$$
$$q_t = C_d A_t \sqrt{\frac{2[(\rho_o - \rho_i)gh + p_i]}{\rho}} \quad (5)$$

For incompressible flows, $q_b = q_t$, so that:

$$p_i = -\frac{A_t^2}{A_t^2 + A_b^2}(\rho_o - \rho_i)gh \quad (6)$$

Thus:

$$q_t = q_b = C_d \sqrt{\frac{2A_t^2 A_b^2}{A_t^2 + A_b^2}\left(\frac{\rho_o - \rho_i}{\rho}\right)gh} \quad (7)$$

If $T_i > T_o$, then we have an upward flow. This relationship will be confirmed even in some two-zone and multi-zone situations.It should be mentioned the flow direction is independent of the size of the ventilation openings. Strictly speaking the ventilation flow rate will affect the indoor air temperature which will control the overall flow direction, and the ventilation openings govern the ventilation flow rate. With a specified indoor air temperature, the flow direction is simply determined by the buoyancy pressure.

The expression for the pressure difference across each opening is:

$$\Delta p_b = \frac{A_t^2}{A_t^2 + A_b^2}(\rho_o - \rho_i)gh$$

$$\Delta p_t = \frac{A_b^2}{A_t^2 + A_b^2}(\rho_o - \rho_i)gh \quad (8)$$

For a two-zone system (Figure 1b), it is assumed the temperature in the bottom zone is T_b and in the top zone T_t. We take the bottom zone mid-height as the datum level. We have:

$$\Delta p_b = (\rho_o - \rho_b)gh_b - p_b$$
$$\Delta p_t = (\rho_o - \rho_t)gh_t + p_t + \rho_o g(h_t + h_b) \quad (9)$$

The pressure at floor level in the top zone is:

$$p_{tf} = p_{t0} + \rho_t g h_t \quad (10)$$

The pressure at ceiling level in the bottom zone is:

$$p_{bc} = p_{b0} - \rho_b g h_b \quad (11)$$

Thus, the pressure difference across the opening between the two zones is:

$$\Delta p_m = p_{bc} - p_{tf} = -g(\rho_b h_b + \rho_t h_t) + p_b - p_t \quad (12)$$

The flow rates are:

$$q_b = C_d A_b \sqrt{\frac{2\Delta p_b}{\rho}}$$

$$q_t = C_d A_t \sqrt{\frac{2\Delta p_t}{\rho}}$$

$$q_m = C_d A_m \sqrt{\frac{2\Delta p_m}{\rho}} \quad (13)$$

With incompressible flows, $q_b = q_t = q_m$, so that:

$$p_b = (\rho_o - \rho_b)gh_b - \frac{2A_t^2 A_m^2}{A_{tmb}^2}[(\rho_o - \rho_b)gh_b + (\rho_o - \rho_t)gh_t]$$

$$p_t = -(\rho_o - \rho_b)gh_t - \rho_o g(h_t + h_b) + \frac{2A_b^2 A_m^2}{A_{tmb}^2}[(\rho_o - \rho_t)gh_t + (\rho_o - \rho_b)gh_b] \quad (14)$$

where

$$A_{tmb}^2 = A_m^2 A_b^2 + A_m^2 A_t^2 + A_t^2 A_b^2 \quad (15)$$

The pressure difference across each opening is:

$$\Delta p_m = \frac{2A_b^2 A_t^2}{A_{tmb}^2}[(\rho_o - \rho_b)gh_b + (\rho_o - \rho_t)gh_t]$$

$$\Delta p_b = \frac{2A_m^2 A_t^2}{A_{tmb}^2}[(\rho_o - \rho_b)gh_b + (\rho_o - \rho_t)gh_t]$$

$$\Delta p_t = \frac{2A_b^2 A_m^2}{A_{tmb}^2}[(\rho_o - \rho_b)gh_b + (\rho_o - \rho_t)gh_t] \quad (16)$$

That is, if $\rho_o g(h_t + h_b) > \rho_b g h_b + \rho_t g h_t$, there is always an upward flow. The physical meaning of this condition is simple. The total weight of the indoor air columnis less than that of the outdoor air column. Again, this condition is independent of the size of ventilation openings, if the indoor air temperatures are maintained. To apply the above formula to solar chimney applications, one can consider the top zone as the solar chimney. It should be mentioned that the flow resistance in the solar chimney due to wall friction is neglected. Even if the zone b temperature is lower than the outdoor air temperature, if the above-mentioned condition is satisfied there is still an upward flow.

The above solution procedure can be easily applied to a three-zone system (Figure 1c shows an N-zone system), where the bottom zone 1 may be considered as a subfloor plenum. The solutions are:

$$\Delta p_{01} = \frac{2A_{all}^2}{A_{01}^2}\sum_{i=1}^{3}(\rho_0 - \rho_i)gh_i$$

$$\Delta p_{12} = \frac{2A_{all}^2}{A_{12}^2}\sum_{i=1}^{3}(\rho_0 - \rho_i)gh_i$$

$$\Delta p_{23} = \frac{2A_{all}^2}{A_{23}^2}\sum_{i=1}^{3}(\rho_0 - \rho_i)gh_i$$

$$\Delta p_{30} = \frac{2A_{all}^2}{A_{30}^2}\sum_{i=1}^{3}(\rho_0 - \rho_i)gh_i \quad (17)$$

where

$$A_{all}^2 = \left(\frac{1}{A_{01}^2} + \frac{1}{A_{12}^2} + \frac{1}{A_{23}^2} + \frac{1}{A_{30}^2}\right)^{-1} \quad (18)$$

and A_{ii+1} is the area of the opening between zone i, and zone $i + 1$ and $2h_i$ is the vertical distance between two openings in zone i. These solutions are in fact very obvious. The flow system is simply three "air columns" connected in series. Each air column introduces a pressure difference between outdoors and the air column. For a steady state problem, the overall pressure difference determines the flow direction. In buildings with subfloor plenums and solar chimneys, the air in the subfloor plenum and in the occupied zones is expected to be cooler than the outdoor air during the day. The thermal stack introduced into the solar chimneys should be sufficiently large to extract the air out of the building.

For example, consider a single-zone office building 3 m high with a 2 m high subfloor plenum. We assume the air temperatures are 15 and 22°C in the subfloor plenum and the office respectively. The air temperature in the solar chimney is 35°C and the outside air is 25°C. Based on the above condition, the solar chimney has to be at least 3.2 m high in order to maintain an upward flow: note however, that a solar chimney height only slightly greater than 3.2 m will only provide an upward ventilation flow with an almost zero ventilation flow rate. Similarly, the analytical solution can also be used to determine the required height and air temperature of a solar chimney in order to achieve a specified ventilation flow rate.

These solutions can be easily extended to N-zone flow systems:

$$\Delta p_{01} = \frac{2A_{all}^2}{A_{01}^2} \sum_{i=1}^{N} (\rho_0 - \rho_i) g h_i$$

$$\Delta p_{12} = \frac{2A_{all}^2}{A_{12}^2} \sum_{i=1}^{N} (\rho_0 - \rho_i) g h_i$$

$$\cdots$$

$$\Delta p_{n0} = \frac{2A_{all}^2}{A_{n0}^2} \sum_{i=1}^{N} (\rho_0 - \rho_i) g h_i \qquad (19)$$

where

$$A_{all}^2 = \left(\frac{1}{A_{01}^2} + \frac{1}{A_{12}^2} + \ldots + \frac{1}{A_{n0}^2} \right)^{-1} \qquad (20)$$

Similar to a two-zone problem, if $\rho_o g \sum_{i=1}^{N} h_i > g \sum_{i=1}^{N} \rho_i h_i$, there is an upward flow. Analytically, this is the sufficient condition for maintaining upward flows. The basic assumptions behind this sufficient condition include that no bi-directional flows exist through all openings and the air temperature in each zone is uniform. In realistic situations, the above derived condition for maintaining upward flows may be considered as a necessary condition. It should be mentioned that any flow friction in a solar chimney can only reduce the flow rate, but not change the flow direction.

It is fairly easy to further extend the above analytical solutions to situations where air temperature in each zone is not uniform. Each zone is divided into a number of small parallel subzones, say N_i small subzones with relative height $2dh_i$, where air temperature in each subzone can be assumed to be uniform. The pressure difference can be calculated individually for each small subzones. The sufficient condition for a $N \times N_i$-zone problem can be written as:

$$2g \sum_{i=1}^{N \times N_i} (\rho_0 - \rho_i) dh_i$$

$$= 2g \sum_{i=1}^{N} (\rho_0 h_i - \sum_{j=1}^{N_i} \rho_i dh_i) > 0 \qquad (21)$$

When N_i approaches infinity, we obtain:

$$2g \sum_{i=1}^{N} (\rho_o h_i - \int_0^{h_i} \rho_i dh_i) > 0 \qquad (22)$$

It can be easily shown that if the ventilation openings are much smaller than the floor areas of all zones, the pressure difference across each ventilation opening can be calculated as follows:

$$\Delta p_{01} = \frac{2gA_{all}^2}{A_{01}^2} \sum_{i=1}^{N} (\rho_o h_i - \int_0^{h_i} \rho_i dh_i)$$

$$\Delta p_{12} = \frac{2gA_{all}^2}{A_{12}^2} \sum_{i=1}^{N} (\rho_o h_i - \int_0^{h_i} \rho_i dh_i)$$

$$\Delta p_{n0} = \frac{2gA_{all}^2}{A_{n0}^2} \sum_{i=1}^{N} (\rho_o h_i - \int_0^{h_i} \rho_i dh_i) \qquad (23)$$

where

$$A_{all}^2 = \left(\frac{1}{A_{01}^2} + \frac{1}{A_{12}^2} + \ldots + \frac{1}{A_{n0}^2} \right)^{-1} \qquad (24)$$

NUMERICAL SOLUTIONS

The details of the numerical method used in our multi-zone natural ventilation model MIX2.0 were described by Li *et al.* (1998). The basic assumption behind MIX2.0 is that each zone is fully mixed. For the purpose of this paper, the method can be evaluated against the analytical solutions presented earlier, i.e. equations (7) and (13).

Evaluations

For simplicity, we introduce the notation $W\times(h_1, h_2)$ to represent a ventilation opening of width W with vertical coordinates from h_1 to h_2.

The single-zone building is considered first. The first test is of a building with two openings with equal areas. The two openings are 10 m × (0 m, 1 m) and 10 m × (9 m, 10 m). Thus, the relative height h is 9 m. The predicted ventilation flow rates as a function of temperature difference are shown in Figure 2. The analytical solution based on equation (7) and the numerical solution of MIX2.0 are compared. Two other different opening heights are chosen in the MIX2.0 runs: 5 m × (0 m, 2 m) and 5 m × (9 m, 11 m), and 2.5 m × (0 m, 4 m) and 2.5 m × (9 m, 13 m). The vertical opening locations are adjusted in order to have the same h for all three MIX2.0 runs. All the results agree with each other very well. The results show that for a building with two equal-area openings separated vertically, the analytical solution (7) can be used for predicting ventilation flow rates. The question is, can this simple analytical solution be applied for a building with two unequal-area openings.

Figure 3 shows the results of a test with two unequal openings. The two openings are 5 m × (0 m, 4 m) and 2.5 m × (9 m, 10 m). The bottom opening is seven times larger than the top opening. This combination may be found in some industrial buildings. The results of the simple analytical solution and MIX2.0 are compared. The simple theory underpredicts the overall ventilation flow rate by about 50%. This is because bi-directional flows occur in the bottom opening. The neutral height is within the bottom opening. It is interesting that the simple theory still successfully predicts the outflow through the top opening.

Now consider a two-zone building (Figure 1b). Each zone has a height of 8.05 m. There are three openings, the bottom opening is 100 m × (0 m, 0.1 m), the middle opening 10 m^2 and the top opening 100 m × (7.95 m, 8.05 m). The purpose of validation here is to test the multi-zone pressure calculation aspect of MIX2.0, but not the bi-directional flows. Thus, both vertical openings are considered to be a slot. The exact solution and the MIX2.0 prediction are compared in Figure 4. Again, excellent agreement is obtained.

The above test results provide some confidence that MIX2.0 can be applied to

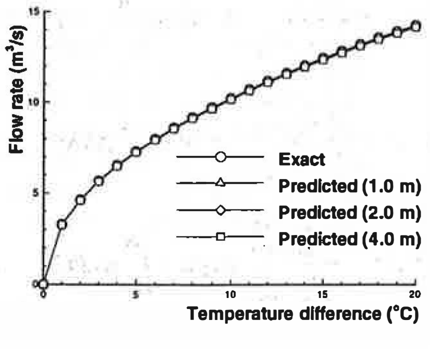

Figure 2 Comparison of exact solutions with predicted results for a building (Figure 1a) with two equal-area openings; opening heights of 1, 2 and 4 m are shown.

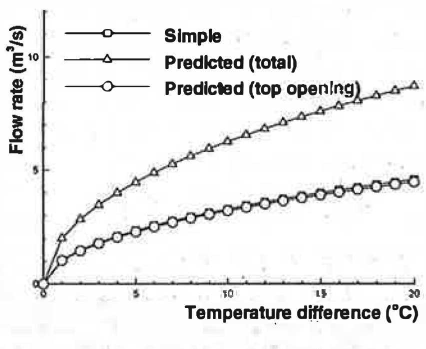

Figure 3 Comparison of exact solutions with predicted results for a building (Figure 1a) with two unequal-area openings.

more complex situations to investigate flow patterns in multi-zone buildings.

Two-room *U*-tube flows

To further investigate the *U*-tube flow pattern demonstrated in Li and Delsante (1997), a building with two offices is considered. The two offices are connected via a large opening. Each office room is designed with a solar chimney and a subfloor plenum. The solar chimneys are at two different heights, as are the two subfloor plenums. It is obvious that each office with its subfloor plenum and solar chimney is the simple three-zone system considered earlier. However, the large opening between the two offices provides an additional flow path within the building. The flow system becomes much more complex and no analytical solution exists.

For simplicity, all ventilation openings are considered to have a discharge coefficient of 0.6. A typical summer condition is considered with an outdoor air temperature of 30°C. The plenum air temperature is assumed to be 20°C and the solar chimney air temperature is assumed to be 40°C. Office air temperatures ranging from 25 to 35°C are considered to study the effect of thermal buoyancy on the flow pattern through the ventilation openings. The temperature range considered here is for demonstration purpose only, not necessarily suggesting that the natural ventilation introduces an indoor air temperature as high as 35°C.

We first simulate a situation with the standard ventilation parameters shown in Figure 5 (the first set of opening sizes; all openings are equal). The results are shown in Figure 6. A dominant upward flow pattern is found for office air temperatures ranging from 25 to 35°C. This is a desirable air flow pattern. When the office air temperature is 30°C there is no interchange air flow between the two office zones. The achieved ventilation capacity is the same for both solar chimneys. However, with an office air temperature lower than 30°C, there is a net air flow from the high office to the low office and, accordingly, the ventilation capacity is higher through the low chimney than through the high one. The situation is just the opposite when the office air temperature is higher than 30°C.

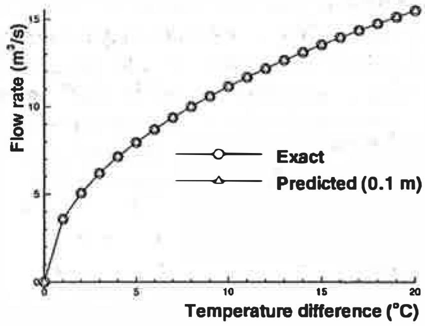

Figure 4 Comparison of exact solutions with predicted results for a two-zone building (Figure 1b).

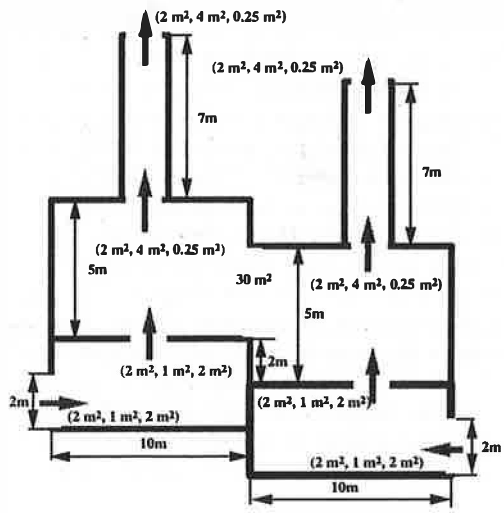

Figure 5 A hypothetical building with two offices, two plenums and two solar chimneys. The large grey arrows indicate the desirable air flow pattern. Three sets of total free opening areas are shown: A_1, A_2, A_3, where A_1 shows the first set of openings, A_2 the second set, and A_3 the third set of openings.

The results obtained with the second set of opening sizes are shown in Figure 7. Openings connecting offices and chimneys, and chimneys and outdoors are larger than openings connecting offices and plenums, and plenums and outdoors. There is always an upward flow through plenums. Compared to the result for the first set of opening sizes, a new type of air flow pattern occurs in the building.

- When the office air temperature is less than 27°C the air enters the building through the higher office ceiling, passes through the internal office opening and leaves the building through the lower office ceiling opening.

This flow pattern can be easily explained by the *N*-zone exact solution presented earlier. As the ceiling openings are considerably larger than the floor openings, the floor openings may be neglected. The flow network can then be simplified. Although the two office zones are horizontally connected and the air temperatures in the two zones are equal, there is no bi-directional flow through the opening connecting the two office zones. With these simplifications, the *N*-zone exact solution can be easily applied.

It can be shown that if the office temperature is less than the outdoor air temperature (30°C), the air will enter the building through the high chimney and leave the building through the low chimney. In Figure 7, additional flows through subfloor plenums prevent such a reverse flow pattern occurring when the office temperature is close to the outdoor air temperature. As the office temperature is further reduced to below 27°C the reverse flow occurs. It should be mentioned that the reverse flow phenomena presented here have not been experimentally verified.

The results obtained with the third set of openings are shown in Figure 8. The openings connecting offices and plenums, and plenums and outdoors are larger than the openings connecting offices and chimneys, and chimneys and outdoors. There is always an upward flow through both chimneys. Compared to the results for the first set of opening sizes, two new types of air flow patterns occurs in the building.

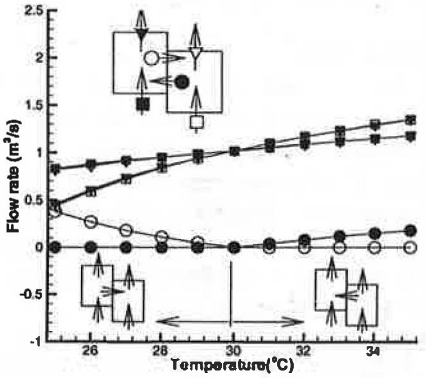

Figure 6 Air flow rates through ventilation openings as a function of air temperature in the two offices with the first set of openings in Figure 5.

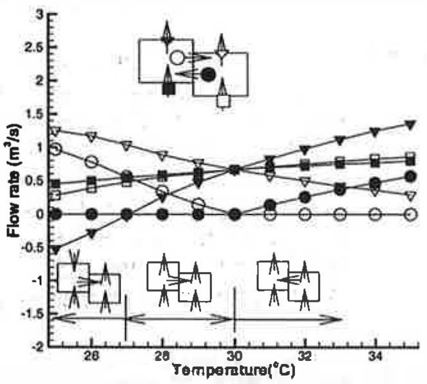

Figure 7 Air flow rates through ventilation openings as a function of air temperature in the two offices with the second set of openings in Figure 5.

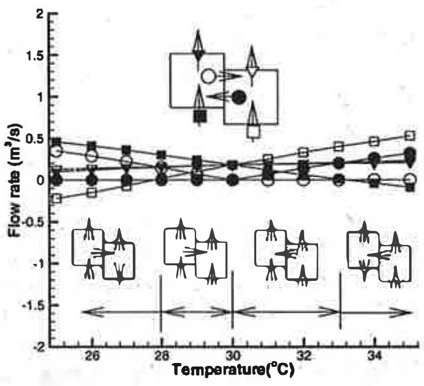

Figure 8 Air flow rates through ventilation openings as a function of air temperature in the two offices with the third set of openings in Figure 5.

- When the office air temperature is less than 28°C the air mainly enters the building through the higher office floor, passes through the internal office opening and leaves the building through the lower office floor opening.
- When the office air temperature is higher than 33°C the flow pattern is reversed.

Again, the *N*-zone analytical solution can be easily applied to explain these flow pattern.

In a multi-zone flow system in which each zone has only two effective openings, if the temperatures are maintained the flow direction is independent of the sizes of ventilation openings. However, the above numerical solutions show that for a two-room building with an internal opening, the flow direction depends strongly on the ventilation opening sizes.

CONCLUSIONS

An analytical solution exists for a multi-zone flow system in which each zone has only two effective openings. From this analytical solution, it can be shown that the sufficient condition for maintaining upward flows in a single room building with a solar chimney and a subfloor plenum is that $\sum_{i=1}^{3} (\rho_0 - \rho_i)gh_i > 0$. The sufficient condition does not depend on ventilation opening sizes. However, in a two-room building with solar chimneys and subfloor plenums and also inter-room air flows, the overall flow pattern is dependent of the size of ventilation openings. Additionally, it is also shown that for a single-zone building with two unequal openings, although the analytical solution underpredicts the overall ventilation flow rate, it does successfully predict the flow through the smaller opening.

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