TIME-DEPENDENT DISPLACEMENT VENTILATION CAUSED BY VARIATIONS IN INTERNAL HEAT GAINS: APPLICATION TO A LECTURE THEATRE

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ABSTRACT

We examine transient displacement flows in naturally ventilated spaces that are subject to an increase in internal heat gains as in, for example, an empty lecture theatre which is then occupied by an audience. Heat gains create a layer of warm air at the ceiling which initially increases in depth and temperature, and descends towards the occupied regions. A theoretical model is developed to predict the time-dependent movement of the interface that separates the warm upper and cool lower layers of air, and comparisons are made with the results of laboratory experiments. The time scale for the flow to reach a steady state is found to depend upon the height and cross-sectional area of the space, the opening areas and the strength of the heat gains. We estimate that this time scale is of the order of an hour for a typical lecture theatre. An interesting feature of these flows is that the interface descends *below* or 'overshoots' the steady-state interface position during the transient period, thus potentially exposing the occupants to the higher air temperatures and pollutant levels of the upper layer. We predict that the initial transient ventilation flow rates are significantly less than those established in the steady state. As a consequence, ventilation designs based upon steady-state flow conditions may fail to provide adequate ventilation if the time scale for the development of the flow is comparable with the time the space is occupied.

KEYWORDS

Convection flows, displacement ventilation, modelling, temperature gradients.

INTRODUCTION

Valuable insight into the parameters which control stack-driven flows in naturally ventilated enclosures has been gained through the development of simple theoretical models and the use of small-scale analogue laboratory experiments in water tanks. A variety of fundamental stack-driven flows have been investigated using these techniques. These include transient draining flows, such as those established in enclosures initially containing air at a temperature different to the ambient, and steady-state flows established in enclosures containing continuous sources of buoyancy. A review of this work is presented by Linden (1996) and recent developments, see Hunt and Linden (1997,1998), have extended these ideas to include ventilation by the combined effects of stack and wind.

The theoretical models of Linden, Lane-Serff and Smeed (1990) are based on plume theory and predict the steady-state stratification profiles and ventilation flow rates for displacement flows driven by sources of buoyancy. These models allow decisions to be made at the design stage regarding, for example, the opening areas required to provide a prescribed number of air changes per hour over the range of heat gains expected. One aspect of these stack-driven flows which has until now been neglected is the time-dependent behaviour of the flow leading to the steady state. For example, when an audience enters an empty lecture theatre there is a sudden increase in internal heat gains which begins to drive warm air through upper openings and draw cooler ambient air into the space through low-level openings. Initially, there will be a transient period in which the thermal stratification and air flow rates vary until a steady-state flow is established. Under certain conditions the transient flow may evolve over a time scale comparable with the time the space is occupied and comfort conditions during the transients could be far worse than those established, and presumably designed for, in the steady state. Thus, the time scale for the flow to reach the steady state is an important quantity to consider, as are the temperatures and air flow rates in the space during the transient period. If, for example, the time scale is short compared with the time the space is occupied, say, during a lecture, then any discomfort experienced by the occupants, due say to inadequate air flow rates or high temperatures, will not be prolonged providing, of course, the steady-state conditions satisfy those for comfort. If, however, the transient flow occurs on a time scale comparable with or greater than the occupied time a knowledge of the transient behaviour is vital so that openings may be sized correctly to ensure that adequate ventilation will be maintained during these periods.

In this paper we investigate the time-dependent displacement ventilation of a naturally ventilated space subject to a sudden increase in internal heat gains. A simple theoretical model is developed in order to predict the thermal stratification, ventilation flow rates and typical time scale for the transient motion. Comparisons are made with the results of laboratory experiments. The theory is then extended and applied to the case of a lecture theatre with tiered seating and the effect the sloped seating has on the ventilation is investigated.

METHODS

Time-dependent displacement flows in naturally ventilated enclosures driven by an abrupt increase in internal heat gains were investigated through simple theoretical analysis and small-scale laboratory experiments. A brief description of a theoretical model which predicts the transient flow generated by a heat source of strength B in an enclosure of height H and cross-sectional area S is now presented.

For simplicity, we will assume that internal heat gains may be represented as a single point source of buoyancy on the floor, and that heat transfers between the building fabric and the air in the interior are negligibly small. The thermal plume entrains ambient fluid as it rises from the source and creates a layer of buoyant fluid at the ceiling which is separated from the ambient layer by an interface. The rate of change in the depth h of the ambient layer is controlled by the difference between the volume flow rate $Q_P = cB^{1/3}h^{5/3}$ (see Morton, Taylor and Turner 1956) at which buoyant fluid is supplied to the upper layer via the rising thermal plume, and the volume flow rate $Q_o = A^*\sqrt{g'(H-h)}$ (see Linden et al. 1990) at which buoyant fluid drains through the upper openings. By conservation of volume, this may be expressed as

$$\frac{d}{dt}\left[(H-h)S\right] = Q_P - Q_o \,. \tag{1}$$

The constant c (≈ 0.143) is related to entrainment into the plume, and for an ideal gas $g' = g\Delta T/T$, where ΔT denotes the temperature step across the interface, T denotes the absolute temperature of the ambient and g the acceleration due to gravity. The areas of the top and bottom openings, a_t and a_b, respectively, are combined and expressed as an 'effective' area A^*, where

$$A^* = a_t a_b \left(\frac{2C_e C_d}{C_e a_b^{\,2} + C_d a_t^{\,2}} \right)^{1/2} , \tag{2}$$

and C_e and C_d are the coefficients of expansion and discharge, respectively. Taking a heat balance for the upper layer we obtain

$$\frac{d}{dt}\left[(H-h)Sg'\right] = B - Q_o g' \,. \tag{3}$$

Temperatures are non-dimensionalised with respect to the temperature in the plume at the top of the space, thus $G' = g'/B^{2/3}H^{-5/3}$, and we denote $\xi = h/H$. Introducing the non-dimensional quantities into (1) and (3) and assuming the cross-sectional area S is independent of height we obtain

$$\frac{d}{d\tau}(1-\xi)=c\xi^{5/3}-\frac{A^*}{H^2}\sqrt{G'(1-\xi)} \qquad (4)$$

and

$$\frac{d}{d\tau}\left[(1-\xi)G'\right]=1-\frac{A^*}{H^2}\sqrt{G'^3(1-\xi)}\,, \qquad (5)$$

where the non-dimensional time scale

$$\tau=t/(S/B^{1/3}H^{2/3})\,. \qquad (6)$$

Thus, the theory predicts that the time scale for the flow to reach a steady state may be increased by increasing the cross-sectional area S or by decreasing the strength B of the heat source or the height H.

Our primary concern is the flow which results from an increase in internal heat gains and we shall focus our attention on the *initial* transients, i.e. those which occur when the internal heat gains in a space at ambient temperature are increased from zero to strength B at time $t=0$. The initial temperature of the upper layer is taken to be the temperature of the plume when it collides with the ceiling. These initial conditions, namely, $h=H$ and $g'=B^{2/3}/cH^{5/3}$ at $t=0$, may be expressed in the non-dimensional form

$$\xi=1 \text{ and } G'=1/c \text{ at } \tau=0\,. \qquad (7)$$

The time-varying depth of the layer at ambient temperature and the temperature step across the interface is modelled by the solution of (4) and (5) subject to the initial conditions (7).

The transient development of the stratification profile within an enclosure following a sudden increase in internal heat gains was investigated through a series of laboratory experiments in which fresh water and saline solutions were used to create density differences and thereby simulate the stack effect. A detailed description of this technique is given by Baker and Linden (1991). The experiments were conducted in a clear Perspex box (cross-sectional area 29.5 cm x 15 cm and height 25 cm) which represented a room or single-spaced building. A number of circular holes at high and low-levels in the side walls of the box served as ventilation openings. The opening area was varied between experiments by adding or removing plastic bungs from the holes. The box was immersed in a large tank containing fresh water. Heat gains were modelled by supplying dense salt solution to the box via a nozzle located at the centre of the top face. The salt solution descended through the surrounding fresh water forming a turbulent plume which created a layer of dense salt solution on the floor of the box and drove a flow through the openings. The density and volume flow rate at which brine was supplied to the plume was measured and the buoyancy flux or strength of the source determined. To visualise the flow, coloured food dye was added to the salt solution. Using an optical technique and digital image analysis the concentration of dye in the dense saline layer laid down by the plume was related to the known initial dye concentration at the source and the density of the layer thereby inferred. A description of this technique is given by Holford (1994). To avoid confusion the results of the experiments are now described in the context of a thermal plume rising upwards as is the case from a heat source in a building.

RESULTS

The position of the interface against time is shown in figure 1a for a typical experiment; the corresponding buoyancy profiles are depicted in figure 1b. Initially, the volume flow rate in the plume is large compared with the volume flow driven through the openings by the newly formed, shallow and weakly buoyant, upper layer. Consequently, the initial descent of the interface is rapid (figure 1a). As the buoyant upper layer rapidly increases in depth it is supplied at a reduced flow rate, yet with increasingly buoyant fluid, from the plume. The volume flow rate through the upper openings therefore increases and slows the descent of the interface. The upper layer depth is observed to descend below or 'overshoot' the steady-state layer depth and then asymptote to the steady-state value.

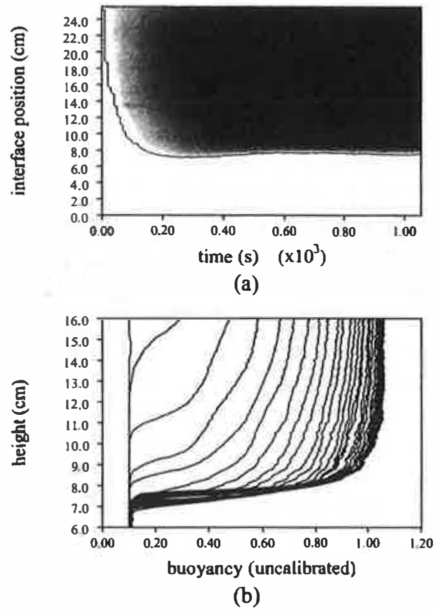

Figure 1. Experimental results a) interface position *vs*. time (s). The shaded region indicates the buoyant layer and the intensity of the shading is indicative of temperature: darker regions correspond to higher temperatures. b) Corresponding buoyancy profiles for $6\ \mathrm{cm} \le h \le 16\ \mathrm{cm}$ shown at 30 s intervals. $B = 1.24\mathrm{x}10^{-6}\ \mathrm{m^4 s^{-3}}$. After correcting for the virtual origin $A^*/H^2 = 4.96\mathrm{x}10^{-2}$.

To understand why the interface overshoots, we note that in the steady state the upper layer is of a uniform temperature T_{ss}, which is equal to the temperature in the plume at the height h_{ss} of the interface, and the volume flow in the plume at this height is identical with the volume flow driven through the openings by the buoyant layer, see Linden et al. (1990). When the interface initially descends to $h = h_{ss}$ the plume begins to supply fluid at temperature T_{ss} to the upper layer. However, at this time the upper layer is not homogeneous and is on average cooler than T_{ss} and therefore the volume flow in the plume still exceeds the volume flow driven through the upper openings by the buoyant layer. Therefore, the interface descends towards the plume source and the upper layer is supplied with fluid at a temperature greater than T_{ss}. At some later time the volume flow driven through the openings will equal the volume flow in the plume at the height of the interface, although as the supply temperature exceeds T_{ss} the interface continues to ascend to its steady position.

The buoyancy profiles, shown superimposed on one another in figure 1b, were recorded at 30 s intervals during the course of the experiment and are seen to evolve from an initially uniform profile to a sharp two-layer stratification. The overshooting phenomena is clearly shown by the overlapping profiles.

Theoretical predictions of interface height $\xi = h/H$ against time τ obtained by numerical solution of (4) and (5) are depicted in figure 2a for a range of dimensionless opening areas A^*/H^2. The values of A^*/H^2 considered are those required to produce steady-state interface heights at $\xi = 0.3, 0.4, 0.5$ and 0.6.

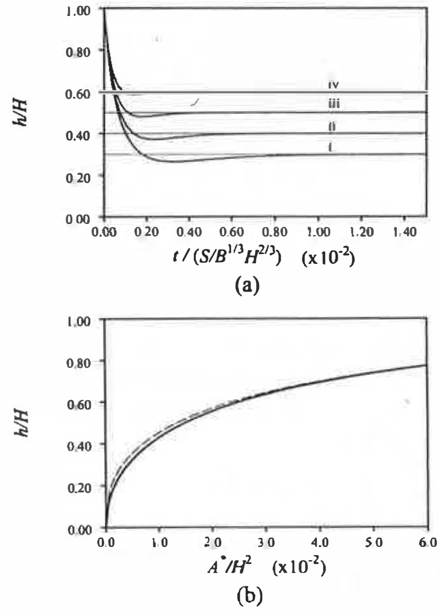

Figure 2. Predictions of a) h/H *vs*. τ for $A^*/H^2 = \{3.18\mathrm{x}10^{-3}$(i), $7.05\mathrm{x}10^{-3}$(ii), $1.35\mathrm{x}10^{-2}$ (iii), $2.38\mathrm{x}10^{-2}$(iv)$\}$. Steady-state layer depths are shown by the horizontal lines. b) Minimum ambient layer depth (solid line) and steady-state layer depth (dashed line) *vs*. A^*/H^2.

The predicted movement of the interface is similar to that observed with the buoyant layer initially increasing rapidly in thickness and overshooting the steady-state layer depth. At larger times the interface asymptotes to the steady state. The maximum overshoot distance, i.e. the difference between the maximum and the steady-state layer depths, see figure 2b, increases as the opening area A^*/H^2 is decreased. The ratio of the volume flow rate supplied to the upper layer (via the plume) to the volume flow rate draining from this layer increases as the opening area decreases thereby increasing the overshoot distance. Furthermore, the time taken for the flow to reach the steady state also increases as A^*/H^2 is decreased; this dependence of the time to reach steady state with A^*/H^2 is also evident from figure 3 which shows the time-variation in the temperature step across the interface (figure 3a) and the ventilation flow rate (figure 3b).

DISCUSSION

In many lecture theatres the seating rows are tiered above the stage used by the speaker and typically the cross-sectional area of the theatre increases with height. A simplified vertical cross-section through a generic lecture theatre is shown in figure 4. Following an analysis similar to that presented earlier in this paper, the equations governing the development of the stratification profiles within the theatre may be shown to take the form:

$$\left.\begin{aligned} \lambda_2\lambda_3\frac{d}{d\tau}\left[(1-\xi)\right] &= c\xi^{5/3}-\frac{A^*}{H^2}\sqrt{G'(1-\xi)} \\ \lambda_2\lambda_3\frac{d}{d\tau}\left[(1-\xi)G'\right] &= 1-\frac{A^*}{H^2}\sqrt{G'^3(1-\xi)} \end{aligned}\right\} \quad (8)$$

for $h^*/H \le \xi \le 1$, and

$$\left.\begin{aligned} J(\xi)\frac{d\xi}{d\tau} &= c\xi^{5/3}-\frac{A^*}{H^2}\sqrt{G'(1-\xi)} \\ J(\xi)G'\frac{d\xi}{d\tau}+K(\xi)\frac{dG'}{d\tau} &= 1-\frac{A^*}{H^2}\sqrt{G'^3(1-\xi)} \end{aligned}\right\} \quad (9)$$

for $0 \le \xi < h^*/H$, where

$$0 \le \theta \le \tan^{-1}(1/(\lambda_3-\lambda_1)),$$

$$h^* = H(\lambda_3-\lambda_1)\tan\theta,$$

$$J(\xi)=\left(\frac{h^*/H-\xi}{\tan\theta}\right)\lambda_2-\lambda_2\lambda_3,$$

$$K(\xi)=\lambda_2\lambda_3(1-\xi)-\frac{\lambda_2\left(h^*/H-\xi\right)^2}{2\tan\theta},$$

and now

$$\tau = t/(H^{4/3}/B^{1/3}). \quad (10)$$

Note that in (6) the cross-sectional area of the space is of the order of H^2, and hence, the time scales (6) and (10) are identical to within a multiplicative constant.

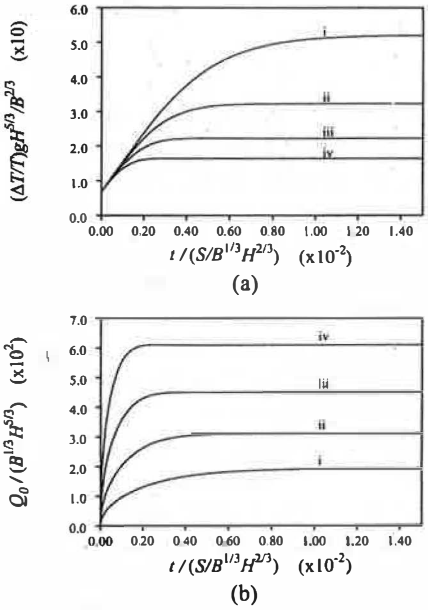

Figure 3. Predictions of a) temperature step $(\Delta T/T)gH^{5/3}/B^{2/3}$ across the interface *vs.* time τ and b) the ventilation flow rate $Q_0/(B^{1/3}H^{5/3})$ *vs.* time τ. $A^*/H^2 = \{3.18\times10^{-3}$(i), 7.05×10^{-3}(ii), 1.35×10^{-2} (iii), 2.38×10^{-2}(iv)$\}$.

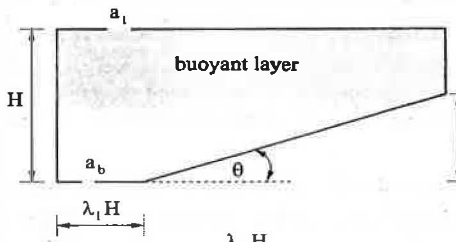

Figure 4. Vertical cross-section through a typical lecture theatre with tiered seating inclined at an angle θ. The length of the speaker's platform is $\lambda_1 H$, the width (into the page) of the theatre is $\lambda_2 H$ and its length is $\lambda_3 H$, where λ_1, λ_2 and λ_3 are constants.

As an illustrative example we estimate the maximum overshoot distance, the upper layer temperatures and the time taken to reach steady state for a lecture theatre in the Chemistry department of the University of Cambridge, UK. The approximate height of the theatre $H = 8.5$ m, the overall length $\lambda_3 H = 25$ m, the speaker's stage length $\lambda_1 H = 5$ m, width $\lambda_2 H = 16.5$ m, and the inclination of tiered seating is $\theta = 17°$. For a typical audience of 100 people and assuming a power output E of 100 watts per person we may express the total heat gains as a buoyancy flux, namely

$$B = g\beta E/\rho c_p = 0.275\ \mathrm{m^4 s^{-3}}.$$

The physical properties of the ambient air at 15°C, namely, the coefficient of thermal expansion, the specific heat capacity and the density, are taken to be $\beta = 3.48\mathrm{x}10^{-3}\,°\mathrm{C}^{-1}$, $C_P = 1012\ \mathrm{Jkg^{-1}°C^{-1}}$ and $\rho = 1.225\ \mathrm{kg\,m^{-3}}$, respectively. For illustrative purposes we take $A^*/H^2 = 3.18\mathrm{x}10^{-3}$ which leads to a steady-state interface height at $\xi = 0.3$. Predictions of h, ΔT and Q_0 in the theatre at the time of the maximum overshoot ($\tau \approx 125$) and in the steady state ($\tau \approx 462$) are shown in Table 1. The 'steady-state' conditions shown refer to those established when the ambient layer depth is 99 % of the actual steady-state layer depth.

	Maximum overshoot	Steady state
time (hours)	0.93	3.42
h (m)	2.16	2.55
ΔT (°C)	9.9	18.3
Q_0 ($\mathrm{m^3s^{-1}}$)	0.336	0.443

Table 1. Comparison of stratification and air flow rates at the time of maximum overshoot and at steady state.

Approximately 56 minutes after the audience has entered the theatre the interface is at its lowest level, almost 0.4 m below the steady-state position. The upper layer is then ≈ 10°C above ambient. Bearing in mind the lecture theatre in question is usually occupied for an hour's lecture we note that the time of maximum overshoot coincides approximately with the time the space is occupied. Furthermore, for the duration the theatre is occupied the ventilation flow rate Q_0 increases from zero to just 75 % (approx.) of the steady-state flow rate.

The full time-dependent behaviour of the interface position and temperature step in the theatre is depicted in figure 5 which also shows predictions for tiered seating at angles of $\theta = \{0,\ \pi/32\ (\equiv 5.625°),\ \pi/16\ (\equiv 11.25°)$ and $\pi/8\ (\equiv 22.5°)\}$.

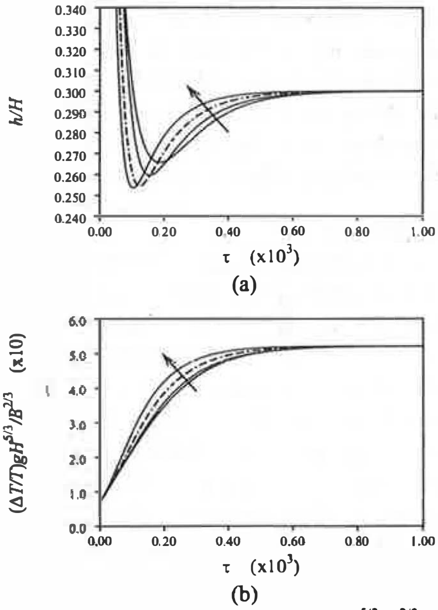

Figure 5. a) h/H *vs.* τ and b) $(\Delta T/T)gH^{5/3}/B^{2/3}$ *vs.* τ in a theatre with seating angles of $\theta = \{0,\ \pi/32\ (\equiv 5.625°),\ \pi/16\ (\equiv 11.25°),\ \pi/8\ (\equiv 22.5°)\}$. The dashed line represents the lecture theatre with $\theta = 17°$. The arrow indicates the direction of increasing θ.

The effect of the tiered seating is to create a space whose cross-sectional area increases with height. It is well known that the steady-state ventilation flow rate and stratification profiles are independent of the cross-sectional area of the space. This is confirmed by the present predictions as for large τ the curves, representing interface height and temperature step, figures 5a and 5b, respectively, collapse onto a common horizontal line for each value of θ considered. However, the initial transients may be strongly influenced by changes in cross-sectional area if the opening areas are such that the interface descends to the level of the inclined seating, as depicted in figure 4. For the shallowest slope angle considered, namely, $\theta = 5.625°$, the interface does not descend to the tiered seating level and the transient behaviour of the flow is identical to the case where the cross-sectional area is independent of height, i.e. $\theta = 0°$. Thus, if the zone above the tiered seating has uniform cross section and the interface remains in this zone during the transient period the air flow rates and stratification profiles will not be effected by obstacles, e.g. seating and desks, in the tiered zone and the above calculation applies. For larger slope angles we see that increasing θ has a three-fold effect on the transient behaviour of the interface position (figure 5a), namely, it increases the overshoot distance and decreases both the time taken for the flow to reach maximum overshoot depth and to reach steady state. Figure 5b indicates that increasing θ also causes the temperature of the descending layer to rise more rapidly. In other words, for a given opening area the interface descends further and more rapidly, and the upper layer temperature increases more rapidly, as the slope of the seating is increased. Note that while the interface lies above the level of the inclined seating the angle of the slope has no effect on the transient behaviour of the flow. Should a steep angle of slope be required in a lecture theatre it would be prudent to ensure that the ceiling height is sufficiently far above the tiered section so as to prevent the interface descending to the occupied levels. If this is not possible the length and/or width of the theatre should be chosen to ensure that the cross section at the ceiling is of sufficiently large area so that reductions in this area due to the angled seating are small. In addition, openings should be appropriately sized so as minimise the time taken to reach a steady state and to prevent the 'overshoot' effecting the audience.

To summarise, we have examined transient displacement flows which arise in naturally ventilated enclosures when there are increases in internal heat gains. Both simple box-like room geometries and typical lecture theatre geometries have been considered. Our results show that during the transient period the ventilation flow rates are significantly less than those established in the steady state. We estimate that for a lecture theatre the time taken for the flow to reach a steady state may be significantly greater than the length of time the space is occupied. As a consequence, ventilation designs based upon steady-state flow conditions may fail to provide sufficient driving force to adequately ventilate the space. Opening areas may therefore need to be increased or the ventilation wind-assisted in order to enhance the air flow rates and provide the necessary cooling during the transient period.

Although, the simple theoretical models developed provide a useful insight into the factors influencing the transient behaviour of stack-driven ventilation, the reader should be aware of the limitations imposed on the range of applicability of these models due to the assumptions made in their development. In the analysis we have assumed that heat gains may be represented as a point source of buoyancy on the floor of the space. However, the effect of an audience may be more closely represented by a distributed heat source and this is currently under investigation. Furthermore, we have assumed that the effect of the heat source is to create a horizontal layer of warm air at the ceiling and that all mixing is confined to within the plume. At the onset of the flow, however, the ceiling current produced by the plume will impinge with the side walls of the space and cause a localised overturning motion and additional mixing. This overturning motion is observed to be more pronounced as the ratio R of the height to the effective radius of the enclosure is increased. For $R < 1$ the effect of the overturning motion is small and the theoretical models presented here apply.

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