INTERACTION OF RADIATION ABSORBED BY MOISTURE IN AIR WITH OTHER FORMS OF HEAT TRANSFER IN AN ENCLOSURE

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ABSTRACT

The impact of the radiation absorbed by room air moisture on heat transfer and air temperature distribution was investigated. Both analytical and CFD approaches were used. For large spaces such as atria, industrial workshops, hotel lobbies, and aircraft hangers, the neglect of radiation absorbed by the moisture within the air volume can lead to significant errors.

INTRODUCTION

Almost all efforts to predict velocity and temperature distributions within building interiors have neglected radiant interaction between the walls and the air. It is known that water vapor has several infrared absorption bands where the gas absorbs and emits radiant energy. Brandli et al (1996), carried out a detailed line by line integration in the infrared and concluded that within the narrow convective boundary layer near the walls radiation heat transfer has a secondary influence. However, they did not consider radiant heat transfer between the walls and the entire volume of the moist air.

In this paper we will investigate radiant transfer between walls and the entire air volume of large interior spaces.

THEORY OF RADIANT HEAT TRANSFER IN A ROOM

Consider the simple case of two large parallel walls where the spacing between the walls is less than the height or width of the walls - some large atria spaces approach this geometry. Further, if the air is well mixed within the space as a first assumption the air temperature can be considered uniform. The uniform temperature case will serve as an illustration of the important physical conditions of radiation interaction over the air volume.

The walls of the space will be considered black bodies, most building materials approach black body behavior in the infrared wavelengths. The two bounding walls will be taken as isothermal with the same temperature, T_w, which is different from the air and water vapor temperature, T_g.

Consider first only the radiant energy emitted by the walls and absorbed by the gas. Black body energy emitted by the walls in a certain direction s and certain wavelengths λ will be absorbed as it propagates through the air/water vapor moisture,

$$d i_\lambda = -i_\lambda k_\lambda \, ds \qquad (1)$$

where k_λ the monochromatic absorption coefficient of the gas mixture, is a function of the water vapor concentration. It varies substantially with λ since the water vapor can only absorb radiation in certain wavelength ranges. Integrating this expression over the length with the requirement that i_λ is equal to $i_{w\lambda}$ at s equal to 0, gives,

$$i_\lambda = i_{w\lambda} e^{-k_\lambda s} \qquad (2)$$

If the wall is a black body, the variation of $i_{w\lambda}$ with wavelength and temperature is given by Planck's expression for black body

$$i_{w\lambda} = \frac{eb_\lambda}{\pi} = \frac{2C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)} \tag{3}$$

The total intensity at location s in direction θ is found by integrating i_λ over all wavelengths containing appreciable energy

$$i(s) = \int_0^\infty i_\lambda d_\lambda = \int_0^\infty \frac{eb_\lambda}{\pi} (e^{-k_\lambda s}) d\lambda \tag{4}$$

The ratio of i(s) to the intensity leaving the wall is the directional transmissivity of the gas mixture at T_g for a black body source at T_w.

$$\tau'(s) = i(s) / i_w \tag{5}$$

The amount of radiation absorbed in the distance, s is given by the directional absorptivity.

$$\alpha' = 1 - i(s) / i_w = \frac{1}{i_w} \int_0^\infty i_{w\lambda} [1 - e^{-k_\lambda s}] d_\lambda \tag{6}$$

The total radiant energy flux emitted by the wall which passes through the imaginary plane at x is composed of energy emitted in all directions in the hemisphere facing the right hand side of the plane

$$q_w(x) = \int_{2\pi} i_{(s)} \cos\theta d\omega = \int_{2\pi} \propto'(s) i_w \cos\theta d\omega$$

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} i(s) \cos\theta \sin\theta d\theta d\phi \tag{7}$$

where q_w (x) represents the radiant energy flux emitted by the wall which passes through the plane at a distance x from the wall and q_w (x=0)-q_w(x) represents the amount of radiation emitted from the wall which is absorbed in a layer of thickness x.

Where in each direction,

$$s = \frac{x}{\cos\theta} \tag{8}$$

The ratio of (σT_w^4 - q_w (x)) to the radiant energy emitted by the wall, at x=0, is defined as the hemispherical absorbivity of the gas to a black body source at T_w.

$$\alpha(x) = 1 - \frac{q_w(x)}{q_w(x=0)} = 1 - \frac{q_w(x)}{\sigma T_w^4} \tag{9}$$

The radiant energy from the wall absorbed in a plane section of gas of thickness)x can be expressed in terms of the absorptivities as,

$$\Delta q_w = q_w(\text{absorbed in } 0 \text{ to } x + dx) - q_w$$
$$(\text{absorbed in } 0 \text{ to } x) = \sigma T_w^4 [\propto(x + \Delta x) - \propto(x)] \tag{10}$$

or in the limit of small thickness,

$$\frac{dq_w}{dx} = \sigma T_w^4 \int_{2\pi} \frac{d \propto'(s)}{dx} \cos\theta d\omega \tag{11}$$

Note that $\propto$(x) involves an integration over wavelengths, where k_λ and eb_λ varies, and over orientation θ,φ where s varies for a given x.

In a similar fashion the radiant energy emitted by a layer of isothermal gases can be considered. Since all of the gas within the room is at uniform temperature, T_g , $i_{b\lambda}$(Tg) in the equation of transport is a constant,

$$\frac{di_\lambda}{ds} = -k_\lambda i + k_\lambda i_{b\lambda}(T_g) \tag{12}$$

It can be solved to yield,

$$i_\lambda = i_{b\lambda}(T_g)[i - e^{-k_\lambda s}] \tag{13}$$

taking the condition that the intensity of emitted radiation for a small thickness layer, s) approaches zero. Integrating over all the wavelengths,

$$i=\int_0^{\infty} i_{\lambda}\,d\lambda=\int_0^{\infty} i_{b\lambda}(T_g)[1-e^{-k_{\lambda}s}]\,d\lambda \quad (14)$$

The directional emissivity is the ratio of intensity to the intensity of a black body at Tg,

$$\varepsilon'=\frac{i(s)}{i_b(s,T_g)} \quad (15)$$

Note in the limiting case where the black body wall and gas temperature approach each other Ξ and ε approach each other.

The amount of energy flux emitted by one surface of an isothermal gas layer of thickness x is

$$q_{emitted}(x)=\int_{2\pi} i\cos\theta\,d\omega=\varepsilon(x)\sigma T_g^4 \quad (16)$$

The amount of radiant energy emitted by a gas layer of t ickness)x to a bounding wall is the difference between that emitted from the surface of a gas layer of thickness, x+)x over that emitted by an isothermal gas layer of thickness x is given by the difference,

$$dq(gas\ at\ x\rightarrow x+dx\ to\ wall)=\sigma T_g^4[\varepsilon_g(x+dx)-\varepsilon(x))]$$

or

$$\frac{d\,q_{(g\ to\ w)}}{dx}=\sigma T_g^4\frac{d\,\varepsilon_g(x)}{dx} \quad (17)$$

This is similar to the expression for the radiation emitted from the wall to the gas layer at x, Eq. 11.

Since all of the gas is at a uniform temperature T_g there must be zero net radiant flux between one gas layer and any other gas layer. Thus, the net radiant flux between any gas layer and one bounding wall is simply the difference between Eqs. 17 and 11. When the wall and gas tempe atures approach each other,

$$\frac{dq_{Net}(w\ to\ g)}{dx}\approx\frac{d\ \varepsilon_g(x)}{dx}[\sigma T_w^4-\sigma T_g^4] \quad (18)$$

This expression requires the evaluation of ε_g from experimental data, or models of the gas radiation. For purposes of these calculations we will use a simple closed form expression for the total gas emissivity. Cess and Liam presented a simple expression for water vapor which at 1 atm total pressure and a temperature of 300°K becomes,

$$\varepsilon_{H_2O}=a_o[1-\exp(-a_1\sqrt{P_{H_2O}LP_{air}})] \quad (19)$$

where a_0 is 0.683 and a_1, is $1.17 m^{-1/2}\ atm^{-1}$ at 300°K. For our case, P_{air} is 1 atm. Cess and Liam show this representation gives close agreement with Hottel's measurement of total emissivity for between 0.03 and 3 m-atm.

Although Cess and Liam suggest that the simple expression should be valid for ε above 0.1, a comparison by the present authors to the values given by Edward and Matavosion (1984, 1987), given in Brewster (1992), shows that at 400K this expression also gives good results at much lower emmissivities, Fig 1.

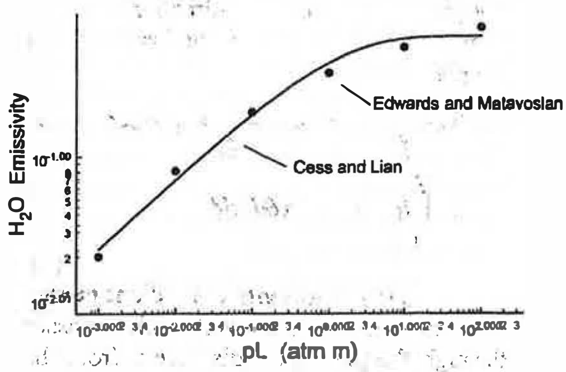


Fig. 1: Water vapor emissivity at room temperature.

The emissivity presented in Eq.19 represents the directional emissivity for a length L In our case, we are considering radiation between plane slabs of gas and a plane wall. Here, the path length from the gas layer to the wall varies with the orientation of the ray of radiation considered, see Eqs. 7 and 8. In the limit of optically thin gas, a simple mean length can be used to evaluate the hemispherical emissivity instead of explicitly accounting for the emissivity variation at each angular inclination. The mean length, in the optically thin limit, is twice the normal distance, x, between the wall and the layer in question. That is, in Eq.18 for the emissivity we will use Cess and Liam's approximation, Eq. 19 with L equal to 2x. The required change of emissivity with the distance becomes,

$$\frac{d\,\varepsilon_g}{dx} = +\left(\frac{a_0 a_1}{2x}\sqrt{P_{H_20}2x}\right) exp\,(-a_1\sqrt{P_{H_20}2x}) \tag{20}$$

Using Eq. 20, the net radiant flux of the gas between the black wall and a gas layer per unit volume, can be evaluated using Eq. 18. The results for air-water vapor at 20°C and 50% relative humidity (P_{H20} Of .0117 bars) and a wall temperature of 25 are shown on Fig. 2. Also shown are the results by Brändli et al (1996), using a line by line integration of 5950 lines in the water vapor frequency spectrum, with each line assumed to have a Lorentz line shape. Their results obtained in the center of a three dimensional room agree closely with the present results down to distances less than 1cm from the wall.

Although the radiant flux per unit volume is highest in the very thin layers directly adjacent to the wall it is mistaken to conclude that the majority of radiant interaction between the wall and the water vapor occurs in this region. We must integrate the radiant flux per unit volume over the entire gas volume to determine where the majority of the interaction occurs. In addition, in the actual case the air and water vapor temperature will not be uniform throughout, rather the gas temperature will approach the wall temperature in the boundary layer substantially reducing the net radiant flux between the gas and the wall in the wall region.

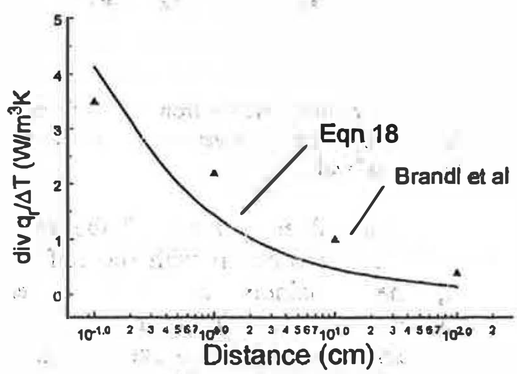


Fig 2: Radiant energy exchange, wall to gas, per unit volume of gas.

DISTRIBUTION OF RADIANT ENERGY

The amount of radiant energy absorbed from the black walls by a gas layer of thickness x adjacent to the walls can be obtained simply as the hemispherical total absorptivity of the gas thickness x. With a black wall at a temperature close to the gas, i.e. T_w-T_g is small compared to the absolute temperature, the absorptivity is closely approximated by the gas emissivity evaluated at a temperature between the wall and gas temperature.

Fig. 3 shows the results for gas at 50 e of 20^{0}C using Cess's approximation for γ_{H20}. In this case, the results are given as the ratio of radiant energy from one wall absorbed by the gas within a distance x of the wall to the radiant energy absorbed over the full width of the room. Note that

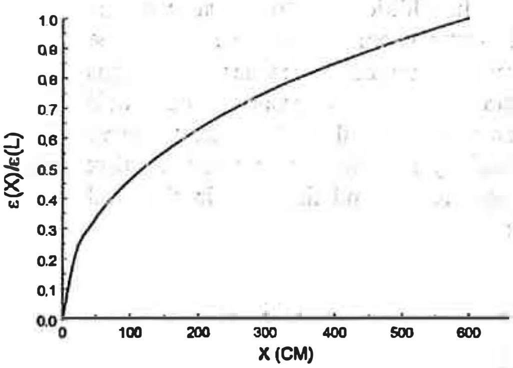


Fig. 3: Net radiant energy from wall which is absorbed by the air-water vapor within distance x of wall.

only about fifteen percent of the total energy absorption occurs with 10cm of the wall, the remainder occurs at larger distances for the wall. In a typical measurement of natural convection in a room, Olson et al (1990), the thermal boundary layer on a vertical wall is approximately 5 to 10 cm thick. At 100 cm away from the wall, about 46 percent of the total absorption occurs. When we consider the radiation energy emitted by two parallel walls at the same temperature, only 15 percent of the total radiant energy absorbed from the two walls occurs within

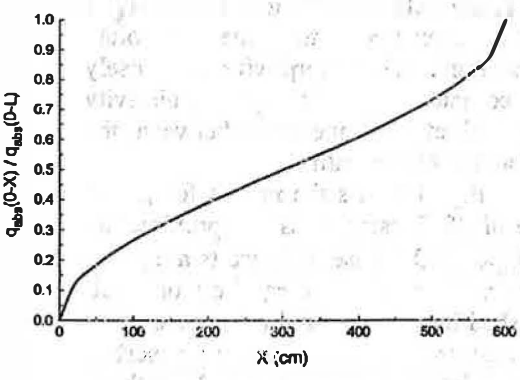


Fig. 4: Radiant energy absorbed in gas layer of thickness x from two parallel walls.

10cm of either wall, Fig. 4. About 40 percent occurs within 1m of each wall when the walls are separated by a distance of 6m. When the distance between the two walls is increased to 15m, a smaller portion of the total absorption , about 11%, occurs within 10 cm of each wall and 35 percent occurs within 1m of the wall. In all of these cases the percent of the total radiant energy incident in a wall which comes from a given gas layer thickness is the same as the percent absorbed from the wall if the gas is at a uniform temperature close to the wall temperature.

COMPARISON TO NATURAL CONVECTION

The effective radiative heat transfer coefficient between a black wall and isothermal gas layer of thickness L is given as,

$$h_r = \frac{\varepsilon_g(L)\sigma(T_w^4 - T_g^4)}{T_w - T_g} \qquad (21)$$

At 50 percent relative humidity h_r varies between 0.5 and 1.5 w/m^2 ^{0}K when the distance between walls varies between 1 and 10 m, respectively. For natural convection on vertical walls in rooms, Olson et al (1990), found that results varied between about 100 and 150% of the results for turbulent natural convection on an isolated wall obtained by Siebers et al (1985).

$$Nu = 0.05\ Ra^{1/3} \qquad (22)$$

which yields value of h between 1 and 1.5 W/m^{2}C for a ΔT of 10°C. For gas layers of 3m or thicker, radiative transfer between the wall and the gas is approximately equivalent to natural convection. Note, for gas layers of the order of 10cm or thinner, radiative transfer is an order of magnitude smaller than natural convection. In the natural convection boundary layer, radiative transfer is a second order effect. However, in large building spaces radiative transfer between the walls and the entire air volume is equivalent to or exceeds natural convection.

COMBINED FLOW AND RADIANT HEAT TRANSFER

A fundamental calculation of heat transfer and air movement in a room requires the simultaneous solution of Navier-Stokes equations and radiant heat transfer equations. The inter-dependence and interaction of these equations make the solution extremely complex. The present investigation solves the time-averaged Navier-Stokes equations by using the k-γ model, Launder and Spalding (1974). The flow equations can be expressed in a single form:

$$\frac{\partial \rho\phi}{\partial t} + div(\rho\bar{U}\phi - \Gamma_{\phi,eff}\ grad\ \phi) = S_\phi \quad (23)$$

For steady-state conditions without scattering, the equation of radiant energy transfer is given by Eq. 12.

The present investigation uses the flux model of Lockwood and Spalding (1971), which allow the exact integro-differential radiation transfer equations to be reduced to a system of approximate partial differential equations in six principal coordinate directions. The resultant flux distribution equations on a small volume of the medium (room air) are:

$$\frac{d}{dx_i}F_{x_i}^+ = -k\,F_{x_i}^+ + k_a eb \quad (24)$$

and

$$\frac{d}{dx_i}F_{x_i}^- = k_a F_{x_i}^- - k_a eb \quad (25)$$

where $F_{x_i}^+$ and $F_{x_i}^-$ are the radiation fluxes in the positive and negative coordinate direction x_i, i.e.

$$F_{x_i}^+ = \int_{\cos\theta>0} \cos\theta\, I\, d\Omega \quad (26)$$

and E is emissive power. The above flux equations are rearranged to yield the composite radiation fluxes in x_i direction:

$$\frac{d}{dx_i}\left(\frac{1}{k_a}\frac{dR_{x_i}}{dx_i}\right) = k_a(R_{x_i} - eb) \quad (27)$$

where $R_{x_i} = (F_{x_i}^+ - F_{x_i}^-)/2$

The contribution of radiative heat transfer to the energy equation is a source term involving the divergence of the radiative heat-flux vector, Qr_i, as follows:

$$S_{rad} = -\frac{dqr_i}{dx_i} \quad (28)$$

Once the composite radiation fluxes are determined, the net radiative heat fluxes can be evaluated from:

$$qr_i = -\frac{2}{k_a}\frac{dR_{x_i}}{dx_i} \quad (29)$$

Thus, the contribution of the radiation fluxes to the energy source term is:

$$S_H = 2k_a(R_x + R_y + R_z - 3eb) \quad (30)$$

The radiant energy boundary conditions take the form:

$$\frac{d}{dx_i}\left(\frac{1}{k_a}\frac{dR_{x_i}}{dx_i}\right) + \frac{\varepsilon_w}{2-\varepsilon_w}\frac{e_w}{\Delta x_i} - \frac{\varepsilon_w}{2-\varepsilon_w}\frac{R_{x_i}}{\Delta x_i} = 0 \quad (31)$$

While the composite-flux model has the advantage of simplicity and computational economy, it does suffer from a number of limitations, such as:

(1) radiation is transmitted in coordinate direction only;

(2) no interlinkages arise between the radiation fluxes in the respective coordinate directions.

CASE STUDY

The case study was performed for two different size of rooms (6 m x 3 m x 3 m and 18 m x 9 m x 9 m). The ceiling temperature of the rooms is 30 °C, window temperature 5 °C, and wall and floor temperature 20 °C. The investigation was for two relative humidities (ϕ): 0% and 50%. The corresponding absorption coefficient used in the computations is 0.122 for the small room and 0.197 for the large room.

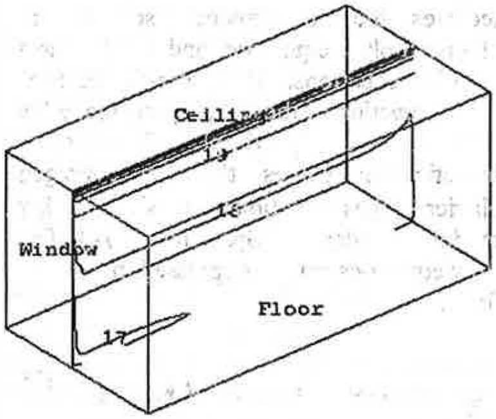
Table 1 shows the convective heat fluxes calculated for different room size and relative humidity. The higher fluxes calculated with 50% relative humidity are due to the radiative contribution. When a room is small, the radiative contribution is small and is negligible. However, the radiative contribution for a large room is very significant as shown in the table. From the results, we can further calculate the convective heat transfer coefficients as shown in Table 2. The coefficients are small for natural convection. However, the wall function used to calculate the convective heat transfer near a wall is grid dependent and can contribute some errors.

Table 1. Convective heat fluxes calculated (W)

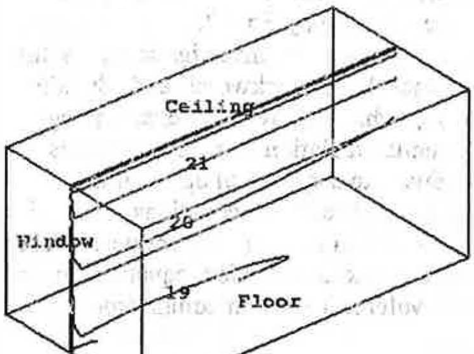
Room size and relative humidity	Window	Ceiling
small, 0%	-334	97
small, 50%	-350	110
large, 0%	-2,185	371
large, 50%	-2,803	1,105

Table 2. Convective heat transfer coefficients calculated (W/m^2K)

Room size and relative humidity	Window	Ceiling
small, 0%	2.65	0.49
small, 50%	2.68	0.58
large, 0%	2.07	0.19
large, 50%	2.31	0.68



$\phi = 0\%$



$\phi = 50\%$

Fig. 5: Temperature distribution in the large room.

The values of R_x, R_y, R_z and E depend on the room location. For the larger room with 50% relative humidity, this leads to about two degrees higher air temperature, as shown in Fig. 5. The impact of the radiative contribution on air temperature for the small room is less than one degree.

CONCLUSIONS

Since radiation absorption and emission by a gas is dependent on the path length, very little radiant transfer occurs with a typical convective boundary layer adjacent to the wall. However, the radiation is absorbed and emitted by the total gas volume within the interior space.

For large interior spaces with humid air, typically 50 percent relative humidity, radiation heat transfer between the walls and the air volume has an important influence on the air temperature and the total heat flux at the walls.

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SYMBOLS

e_b Emissive power
F Hemispherical radiant flux
h_r Effective radiant heat transfer
i Intensity
k Absorption coefficient
L Total wall spacing
Nu Nusselt Number
p Partial pressure
q Heat transfer rate
R Defined in equation 27
Ra Raleigh Number
s Path length
S Source term
T Temperature
U Velocity
$\propto$ Absortivity
Γ Diffusion coefficient
ε Emissivity
θ,φ Spherical Coordinator
ρ Denisty
σ Stefan-Boltzman Constant
π Transmissivity

Superscripts
( )' Directional Property

Subscripts
λ monochromatic
w wall
g gas