Heat Transfer under Thermally Stratified Conditions at Horizontal Surfaces in Buildings

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ABSTRACT

In room air flow, thermally unstably stratified conditions very near the wall can occur on horizontal surfaces as heated floors and cooled ceilings. In this case, turbulence production due to buoyancy can not be neglected. Using dimensional analysis, appropriate scales are found to build a parameter describing the strength of thermal stratification. It is the Richardson number $Ri^+_{q\tau}$, containing momentum and heat fluxes at the wall. Furthermore dimensional analysis shows, that the velocity and temperature profiles in the inertial sublayer are not logarithmic as in pure forced convection. The meteorological correlations of Businger and Dyer, derived for a similar problem in the surface layer near the ground, are adapted here to room air flow for estimating the difference between the profiles of pure forced and stratified profiles. In room air flow the layer near the wall can be even more stratified than the experiments found in meteorology. For this stability range, numerical calculations are used to predict the profiles, tacking also account of low-Reynolds-Number and buoyancy effects near the wall. The profiles show, that the logarithmic law of the wall needs modifications for the calculation of thermally unstably stratified situations at horizontal surfaces in rooms.

KEYWORDS

Boundary layer, Convective heat transfer, Cooled ceiling, Modelling, Temperature gradient

INTRODUCTION

In room air flow, many different flow phenomena are interacting. Even when these phenomena are considered separately with well defined boundary conditions, as for example natural convection on a heated vertical wall, their numerical calculation has got its difficulties. This means, that for improving a calculation model, leading to an inaccurate flow prediction of a real room, one has to know at least the shortcomings of the calculation of each of the interacting phenomena.

One such phenomena is the convection on a horizontal surface under thermally unstably stratified conditions. This kind of flow can be observed for example on heated floors or cooled ceilings, both used to control the indoor climate, but also in the case of hot smoke on a cold ceiling or air over a floor, heated by solar radiation.

To omit the numerical resolution of the region with the high gradients of the dependent variables between the wall and the inertial sublayer, usually the well known logarithmic laws of pure forced convection are used, which consider only turbulent transport due to shear production. In the case of thermal stratification, turbulence is also produced by buoyancy due to the large turbulent heat flux, and the dependent variables deviate from the laws of pure forced convection. The strength of thermal stratification is described locally by the Flux Richardson number, the ratio of the turbulence production due to buoyancy to the one due to shear. Using dimensional analysis, it is shown, which characteristic scales of the flow near the wall can form an appropriate Richardson number, being a parameter for the discrepancy between the logarithmic

For convenience $\Theta^+ = \frac{T^*(x^*,y^*)-T^*_w(x^*)}{T^*_\tau(x^*)}$ is introduced.

The integration constants can be derived from the velocity and temperature profiles in the wall layer:

$$C^+ = \int_0^1 \frac{du^+}{dy^+} dy^+ + \lim_{y^+\to\infty} \int_1^{y^+} \left(\frac{du^+}{dy^+} - \frac{1}{\kappa y^+} \right) dy^+ \quad (19)$$

$$C_H^+ = \int_0^1 \frac{d\Theta^+}{dy^+} dy^+ + \lim_{y^+\to\infty} \int_1^{y^+} \left(\frac{d\Theta^+}{dy^+} - \frac{1}{\kappa_H y^+} \right) dy^+ \quad (20)$$

In the case of thermal stratification, first a function for Φ and one for Φ_H has to be found, which are valid for all stability parameters $\zeta = \frac{y^*}{L^*_{MO}}$. This is only possible, if ζ is the appropriate scaled dimensionless wall distance. As explained in the next section, meteorologists found such functions for the so called surface layer, where constant turbulent fluxes of momentum and heat are assumed, as in the inertial sublayer. The u^*- and T^*-distributions result then by integration of (15b) and (16b).

With that, one can describe all velocity and temperature profiles up to the inertial sublayer by two curves $u^+(\zeta)$ and $\theta^+(\zeta)$. Furthermore it shows, that using the nondimensional wall distance y^+, leading in the case of pure forced convection to two curves $u^+(y^+)$ and $\theta^+(y^+)$, results in families of curves $u^+(y^+, PA)$ and $\theta^+(y^+, PA)$ in the case of thermal stratification. PA is a parameter, describing the thermal stratification of a particular curve.

To compare a stratified situation with pure forced convection quantitatively, we have to find a parameter and need to know how and with which characteristic scales to build this parameter.

The appropriate parameter can be derived by factorizing the stability parameter ζ. Because we want to use y^+ as the wall distance, it makes sense to write

$$\zeta = \frac{y^*}{L^*_{MO}} = y^+ Ri^+_{q\tau} \kappa \quad (21)$$

$$Ri^+_{q\tau} = \frac{T^*_\tau/(\nu^*/u^*_\tau) g^* \beta}{u^{*2}_\tau/(\nu^*/u^*_\tau)^2} \quad (22)$$

$Ri^+_{q\tau}$ is the proper parameter, describing the strength of thermal stratification of a certain situation. From the convective heat flux and the friction velocity, e.g. on a cooled ceiling or heated floor, one can calculate $Ri^+_{q\tau}$ and say if the situation is thermally stratified.

How much the values of a stratified situation deviate from the laws of pure forced convection for certain values $Ri^+_{q\tau}$ is investigated in the following.

CORRELATIONS FROM METEOROLOGY

In meteorology, correlations for $\Phi(\zeta)$ and $\Phi_H(\zeta)$ have been found in field measurements and wind tunnel experiments. All explanations here are given on the example of velocity, whereas the procedure for temperature is analogous. The correlations of Businger (1972) and Dyer (1974) for $\zeta < 0$ (unstable) are most often used:

$$\Phi(\zeta) = \kappa^{-1}(1 - 15\,\zeta)^{-1/4} \quad (23)$$

Integration of (15b) with (23), with the aerodynamic roughness length y^*_0 and $\zeta_0 = y^*_0/L^*_{MO}$, leads to:

$$\frac{\overline{u^*}}{u^*_\tau} = \frac{1}{\kappa}\left[\ln\left(\frac{y^*}{y^*_0}\right) - \Psi_M(\zeta) + \Psi_M(\zeta_0)\right] \quad (24)$$

For simplicity the function $\Psi_M(\zeta)$ is not written here. For forced convection with $\zeta = 0$, it is zero and (24) yields the logarithmic law of the wall. This is more obvious from (25), which is (24) written in scales used in room air flow and with $\zeta = y^+ Ri^+_{q\tau}\kappa$ and $\zeta_0 = y_0^+ Ri^+_{q\tau}\kappa$:

$$u^+ = \frac{1}{\kappa}\ln(y^+) \underbrace{-\frac{1}{\kappa}\ln(y_0^+)}_{C^+} \quad (25)$$
$$-\frac{1}{\kappa}(\Psi_M(\zeta) - \Psi_M(\zeta_0))$$

It is important to notice that in the logarithmic law of pure forced convection over a smooth plate, C^+ is derived by integration of the velocity profile from the wall to the inertial sublayer, as shown in (19). This means, that C^+ can be influenced by buoyancy, when the plate is heated, because turbulence and with this the velocity between the wall and the inertial sublayer are affected by buoyancy! We could write $C^+(Ri^+_{q\tau})$.

In meteorology on the other hand, the velocity is taken zero at $y^+ = y_0^+$ over the ground. Knowing the aerodynamic roughness height y_0^+, one can calculate the constant C^+ from $-\ln y_0^+ = C^+$. y_0^+ and with that C^+ are assumed to be mainly functions of geometry. In the case of a smooth ground, we can choose from pure forced convection $C^+ = 5.2 = -\frac{1}{\kappa}\ln(y_0^+)$ and get $y_0^+ = 0.119$.

Using these values, (25) is plotted in Figure 1 for different values of $Ri^+_{q\tau}$.

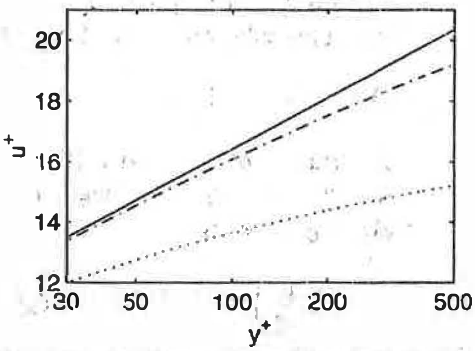

Figure 1: Businger-Dyer Correlation with $Ri^+_{q\tau} = 0$ (—), $Ri^+_{q\tau} = -0.001$ (– · –) and $Ri^+_{q\tau} = -0.026$ (· · ·)

$Ri^+_{q\tau} = -0.0011$ is the maximum absolute value considered in the famous experiment of Arya (1975) and it is found to be the upper boundary of the $Ri^+_{q\tau}$ range occurring in meteorology. Its u^+- value at $y^+ = 30$ corresponds about to the one of pure forced convection at this point and the curve also corresponds to measurements made in meteorological experiments. This means, that the assumption of a constant C^+ is correct for the typical $Ri^+_{q\tau}$ range of meteorology, where the region between the wall and the inertial sublayer is only very slightly influenced by buoyancy.

In room air flow a value of e.g. $Ri^+_{q\tau} = -0.026$ is also possible. In this range an experiment of Mizushina (1982) could be found, where significant deviation form the forced convection profile in the wall layer could be observed. It may be that the curve for $Ri^+_{q\tau} = -0.026$ in Figure 1 predicts the correct gradient at $y^+ = 30$ but its start value is wrongly predicted by (25), because the effects of buoyancy on turbulence and velocity in the range $y^+ < 30$ has not been considered in C^+ and buoyancy can there not be considered accurately by $\psi(\zeta)$, including $\Phi(\zeta)$, which is valid only in the inertial sublayer. Because of the lack of measurements in this stability range, numerical methods are presented in the next section.

NUMERICAL ANALYSIS

From (6) to (8) or (10) and (11) respectively, it is obvious, that in the wall layer all equations reduce to ordinary differential equations (ODE) for high *Re*-Numbers. If the logarithmic law of the wall is not appropriate for a certain problem, one can build and solve a system of ODEs, including all the terms that have not been considered in the derivation of the logarithmic law. E.g. for the calculation of mixed convection on a vertical wall, a buoyancy term can be included in (6), which represents in this case the vertical momentum equation. For the present investigation of the thermally stratified convection on horizontal surfaces, buoyancy effects on τ_t^+ and q_t^+ are considered.

Combining this method with a three dimensional fluid solver, the ODEs can be solved numerically between the wall and the inertial sublayer, using the usual boundary values at the wall and the values of the last iteration of the fluid solver in the inertial sublayer. The fluxes resulting from the ODEs in the inertial sublayer are then the boundary conditions for the next iteration of the fluid solver. The solution of all the ODEs do not have to be kept in memory, leading to an applicable method for the calculation of room air flow!

In this article we are interested only in the distribution of $u^+(y^+)$, depending on the parameter Ri_{qr}^+. It suffices to integrate the first order ODEs (10) and (11) from the wall, with τ_t^+ and q_t^+ considering buoyancy and near wall low Reynolds-number effects.

τ_t^+ has been calculated using the Van Driest Formula for the mixing length:

$$l^+ = \kappa y^+[1 - \exp^{-y^+/26}] \qquad (26)$$

Buoyancy forces enhance the turbulent transport and with this the eddy viscosity and eddy diffusivity. We used the stability dependence of Mizushina (1982)

$$f_{BV} = \frac{\nu_{tB}^+}{\nu_t^+} = (1 - 25Ri_l)^{1/3} \qquad (27)$$

with the local Richardson number

$$Ri_l = Ri_{qr}^+ \frac{\partial \theta^+}{\partial y^+} \left(\frac{\partial u^+}{\partial y^+}\right)^{-2} \qquad (28)$$

to calculate the eddy viscosity ν_{tB}^+, influenced by buoyancy, from ν_t^+ of forced convection.

Mizushina (1982) mentioned that this correlation is the same near the wall and far apart from the wall. This is not valid for the value of the buoyancy-influenced turbulent Prandtl number:

$$Pr_{tB} = \nu_{tB}/a_{tB} \qquad (29)$$

with the buoyancy affected eddy diffusivity a_{tB}. We used the correlation proposed by Gibson and Launder (1978), shown in Figure 2.

Townsend (1972) found the averaged Pr_{tB} across the layer to be in the range of 0.6 and 0.85.

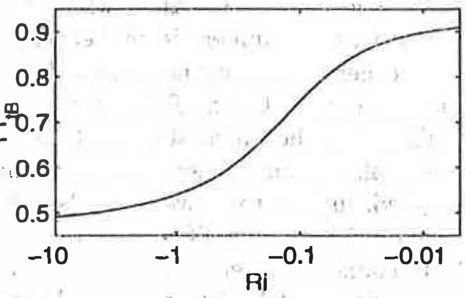

Figure 2: $Pr_{tB}(Ri_l)$ near the wall, following Gibson/Launder.

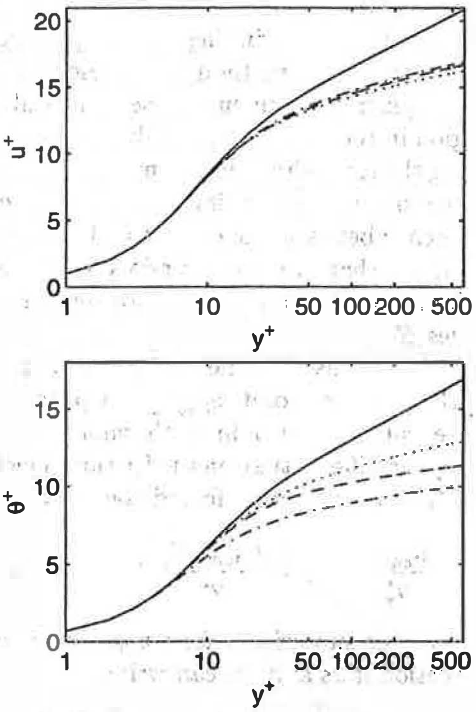

Figure 3 and 4: u^+ and θ^+ distribution for forced convection (—) and for $Ri_{qr}^+ = -0.026$, using Pr_{tB} of Gibson/Launder (- - -), $Pr_{tB} = 0.9$ (· · ·) and $Pr_{tB} = 0.5$ (— · —) respectively

Calculations of u^+ and θ^+ are shown in Figure 3 and 4 for $Ri^+_{q\tau} = -0.026$, using the correlation of Gibson and Launder and its extreme values $Pr_{tB} = 0.9$ and $Pr_{tB} = 0.5$, respectively, throughout the layer.

The u^+ and θ^+ values with Pr_{tB} of Gibson and Launder lie in between the two others. Let us use these data for further explanations. To improve the reliability of the calculations, at the moment also other models are being investigated, such as the low-Reynolds number model with the amplification functions of Murakami, Kato et. al. (1996) or four-equations models that are tested against DNS Data of thermally stratified channel flow. These investigations are not the subject of this article.

DISCUSSION:

At the beginning of the preceding section a new method was presented for the general treatment of the near wall region in room air flow. But before improving the logarithmic law of pure forced convection, we better first estimate the difference between the correct and predicted fluxes, that are the boundary conditions for numerical calculations, for certain values $Ri^+_{q\tau}$.

Let us assume, a given velocity u^*_{known} at a point y^*_{known}. Up to now we calculate also in a thermally unstably stratified situation the friction velocity $u^*_{\tau f}$ from the law of forced convection:

$$\frac{u^*_{known}}{u^*_{\tau f}} = f\left(\frac{y^*_{known}u^*_{\tau f}}{\nu^*}\right) \tag{30}$$

with the subscript $_f$ for using forced convection laws and we can write:

$y^*_{known} = \frac{y^+_f \nu^*}{u^*_{\tau f}}$ and $u^*_{known} = f(y^+_f)u^*_{\tau f}$.

Inserting these known values into the law for thermally unstably stratified conditions (subscript $_B$):

$$\frac{u^*_{known}}{u^*_{\tau B}} = f_B\left(\frac{y^*_{known}u^*_{\tau B}}{\nu^*}\right) \tag{31}$$

leads to an equation for the ratio r_u between the wrongly predicted friction velocity $u^*_{\tau f}$ and the correct value $u^*_{\tau B}$:

$$f(y^+_f)\frac{u^*_{\tau f}}{u^*_{\tau B}} = f_B\left(y^+_f\frac{u^*_{\tau B}}{u^*_{\tau f}}\right) \tag{32}$$

$$f(y^+_f)r_u = f_B\left(\frac{y^+_f}{r_u}\right) \tag{33}$$

For a given $Ri^+_{q\tau}$, one can calculate for every y^+_f a corresponding r_u! In a similar procedure, $r_\theta = T^*_{\tau f}/T^*_{\tau B}$ can be calculated with the exception, that on the right side of (34) r_u is used instead of r_θ:

$$f_\theta(y^+_f)r_\theta = f_{\theta B}\left(\frac{y^+_f}{r_u}\right) \tag{34}$$

The distributions $r_u(y^+)$ and $r_\theta(y^+)$ resulting from a high stratification of $Ri^+_{q\tau} = -0.026$, are shown in Figure 5. They are calculated on the basis of the calculations shown in Figure 3 and 4.

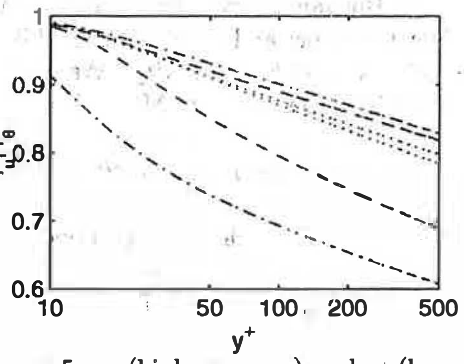

Figure 5: r_u (higher curves) and r_θ (lower curves) distributions for $Ri^+_{q\tau} = -0.026$, using Pr_{tB} of Gibson/Launder (- - -), $Pr_{tB} = 0.9$ ($\cdots$) and $Pr_{tB} = 0.5$ (— $\cdot$ —)

It is obvious, that the logarithmic law of the wall in a thermally unstable situation does not lead to accurate predicted friction velocities and friction temperatures. The predicted boundary fluxes, needed for the fluid solver, are even worse:

$$\overline{\tau^*_w} = \rho^* u^{*2}_{\tau f} = \rho^* u^{*2}_{\tau B} r^{*2}_u = \overline{\tau^*_{wB}} r^{*2}_u \tag{35}$$

$$\overline{q_w^*} = -\rho^* c_p^* u_{\tau f}^* T_{\tau f}^* = -\rho^* c_p^* u_{\tau B}^* r_u T_{\tau B}^* r_\theta = \overline{q_{wB}^*} r_\theta r_u \quad (36)$$

CONCLUSIONS

Using dimensional analysis, the parameter $Ri_{q\tau}^+$ can be found, determing the strength of thermal stratification in the wall layer. Experimental results found in meteorology are below the range of stratification occurring in some situations in room air flow, e.g. on heated floors and cooled ceilings. The velocity and temperature distribution in the case of high stratification can be numerically calculated, tacking account of low-Reynolds-Number and buoyancy effects near the wall. These models have to be investigated in more detail. Nevertheless they are accurate enough for estimating the ratio between the correct, thermally stratified values and the ones calculated using the forced convection law. Due to the high ratios, a modification of the logarithmic law of the wall is recommended. A possible solution method for horizontal surfaces, also covering the problem of mixed convection on vertical surfaces, has been presented.

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APPENDIX

In wall scales		*In outer scales*	
$x = \frac{x^*}{L_R^*}$,	$y = \frac{y^*}{L_R^*}$,	$y^+ = \frac{y^* u_\tau^*(x^*)}{\nu^*}$	Coord. along and normal to plate
$u = \frac{\overline{u^*(x^*,y^*)}}{U_\infty^*}$,	$v = \frac{\overline{v^*(x^*,y^*)}}{U_\infty^*}$,	$u^+ = \frac{\overline{u^*(x^*,y^*)}}{u_\tau^*(x^*)}$	Velocity along and normal to plate
$T = \frac{\overline{T^*(x^*,y^*)}}{T_\infty^*}$		$T^+ = \frac{\overline{T^*(x^*,y^*)}}{T_\tau^*(x^*)}$	Temperature
$k = \frac{k^*(x^*,y^*)}{U_\infty^{*2}}$		$k^+ = \frac{k^*(x^*,y^*)}{u_\tau^*(x^*)^2}$	Turb. kinetic energy
$\epsilon_u = \frac{\epsilon_u^*(x^*,y^*)L_R^*}{U_\infty^{*3}}$		$\epsilon_u^+ = \frac{\epsilon_u^*(x^*,y^*)\nu^*}{u_\tau^{*4}(x^*)}$	Dissipation rate
$\tau_t = \frac{\tau_t^*(x^*,y^*)}{\rho^* U_\infty^{*2}}$		$\tau_t^+ = \frac{\tau_t^*(x^*,y^*)}{\tau_w^*(x^*)}$	Turb. shear stress ($\tau_t^* = -\rho^* \overline{u^{*\prime} v^{*\prime}}$)
$q_t = \frac{q_t^*(x^*,y^*)}{\rho^* c_p^* U_\infty^* T_\infty^*}$		$q_t^+ = \frac{q_t^*(x^*,y^*)}{q_w^*(x^*)}$	Turb. heat flux ($q_t^* = \rho^* c_p^* \overline{v^{*\prime} T^{*\prime}}$)
$B = -\frac{\overline{v^{*\prime}(p^{*\prime} + q^{*2}/2)}}{U_\infty^3}$		$B^+ = -\frac{\overline{v^{*\prime}(p^{*\prime} + q^{*2}/2)}}{u_\tau(x^*)^3}$	Turb. flux of turb. kin. energy

Subscripts: t: turbulent, B: stratified, w: wall, f: forced convection, ∞: free stream, R: characteristic reference scale