# IMPROVEMENT OF A PLUME VOLUME FUX CALCULATION METHOD 

Z. Popiołek, Z. Trzeciakiewicz, S. Mietzwiński,<br>Department of Heating, Ventilation and Dust Removal Technology of the Silesian Technical University in Gliwice, POLAND


#### Abstract

The paper presents the results of the research on application of the equation describing the increase in the air volume flux in buoyant plume above a point heat source to calculate plumes in rooms with displacement ventilation: $$
V=k_{v} Q^{1 / 3}\left(z+z_{v}\right)^{5 / 3}
$$

The tests carried out in test room have given information about practical defining of the distance from the origin, assuming entrainment coefficient values and possibilities of assuming equal widths of temperature and velocity profiles in order to determine the origin distance. They have also informed whether it is possible to use the model of plume above a point heat source in an enclosure where displacement ventilation is applied. The tests were preceeded by an analysys of buoyant plumes, on the basis of available literature data.


## KEYWORDS

displacement ventilation, buoyant plumes, air distribution

## INTRODUCTION

The model describing the increase in volume flux of a plume above a point heat source in an unbounded space:

$$
\begin{equation*}
V=k_{v} Q^{1 / 3}\left(z+z_{v}\right)^{5 / 3} \tag{1}
\end{equation*}
$$

poses a problem of the choice of proper values of $k_{v}$ coefficient, which describes the air entrainment by the plume, and of so-called origin distance $z_{v}$. The above equation (1) makes it possible to calculate, in a relatively simple way, the plume volume flux and could be applied in displacement ventilation designing.

The test of various free plumes, carried out so far, have proved that $k_{v}$ coefficient values vary between 0.0040 and 0.0082 . The values of $z_{v}$ also vary within a wide range of parameters and are calculated by means of different equations, relating these values with parameters such as heat source diameter or the value of velocity profile width measured in the plume. In practice, the discrepancies in assuming $z_{v}$ and $k_{v}$ values r.ay result in differences of the order of $200-300 \%$.

The aim of the research, the results of which are presented in the paper was to define values of $k_{v}$ and $z_{v}$ more precisely in order to increase of the accuracy of volume flux determination in plumes. The research covered plumes in rooms where displacement ventilation was applied. The tests were proceeded by an analysis of available literature data regarding both free plumes and plumes occuring in enclosures with displacement ventilation as well as without ventilation. The answers to the following questions were sought:

- how to define the origin distance in practice?,
- how to relate the plume visualisation with the plume width?
- can temperature profile width be assumed instead of velocity profile width in order to determine the origin distance?
- can the model of plume above a point heat source be used to determine the volume flux in plumes in enclosures with displacement ventilation and what is the accuracy?

The tests employed visualisation where laser was used as the light source and the plume was observed in the plane formed by a laser beam. The use of an eight-channel anemometer made it possible to verify experimentally, on the way of measurements, the value of the ratio of temperature and velocity profile widths in the plume which is often assumed equal one in the literature available.

## VOLUME FLUX IN PLUMES

The volume flux in plumes can be determined from a relatively simple relation resulting from the model of plume above a point heat source (PPHS) i.e. a holding true in the selfpreserving zone of plume developing in an unbounded space, in environment without thermal stratification:

$$
\begin{equation*}
V=\left[\frac{3 \pi^{2} \beta g}{2 c_{p} \rho} \frac{m+p}{m^{2} p}\right]^{1 / 3} Q^{1 / 3} z_{b}^{5 / 3}=k_{\nu} Q^{1 / 3} z_{b}^{5 / 3} \tag{2}
\end{equation*}
$$

In this equation the distance from the origin (virtual point heat source) should include the distance from the top of the heat source, $z_{b}=z+z_{v}$. The equation should be therefore used in practice as:

$$
\begin{equation*}
V=k_{v} Q^{1 / 3}\left(z+z_{v}\right)^{5 / 3} \tag{3}
\end{equation*}
$$

where:
$z$-distance between the top of the heat source and the cross-section considered
$z_{v}$ - distance between the top of the heat source and the origin
$m, p$-distribution and velocity coefficients in PPHS
It is easy enough to determine the plume enthalpy flux $Q$, from the heat source balance. The enthalpy evaluation error, if there is one, does not effect the volume flux value significantly since the enthalpy exponent in Eq. 3 is $1 / 3$. The other two parameters on which the calculated $V$ value depends, have been assumed so far in an arbitrary way. The following relations for calculation of the origin distance depending on the heat source width, $D$, can be found in the literature available:

$$
\begin{equation*}
z_{\nu}=D ; \quad z_{\nu}=1.7 D ; \quad z_{\nu}=2 D ; \quad z_{\nu}=2.5 D^{1.14} \quad z_{\nu}=3 D ; \tag{4}
\end{equation*}
$$

In practical calculations the above discrepancies may cause results different even by 200 $-300 \%$. The available values of the other parameter in Eq. 3 are also different. Table 1 shows the values of $k_{v}$ coefficient suggested by different authors.

Table 1 Comparison of $k_{v}$ coefficient values

| No | Author | $\mathrm{k}_{\mathrm{v}}$ |
| :---: | :--- | :---: |
| 1 | Schmidt | 0.0074 |
| 2 | Rouse | 0.0047 |
| 3 | Abramowicz | 0.0082 |
| 4 | Shepielev | 0.0055 |
| 5 | George | 0.0063 |
| 6 | Nagakome | 0.0057 |
| 7 | Popiotek'87 | 0.0080 |
| 8 | Kofoed'91 | 0.0040 |

The table shows that the $k_{v}$ values which can be found in the literature may be almost twice different. If inaccuracies in the origin defining and in $k_{\nu}$ coefficient determination are taken into account together, the calculation results may differ even several times. The results of the analysis of the volume flux increase are affected essentially by the proper defining of the plume origin position with the use of the method suggested in [1,2], where plumes
developing in the environment, without thermal stratification are analysed. It may be proved that the origin position depends on Archimedes number value at the boundary cross-section of the mean flow self-preserving zone. When analysing the volume flux increase it is assumed that $z_{v}$ value is determined on the basis of the measured profile width values. If the measurement shows that the velocity profile width is $R_{w 1}$ at the height $z_{l}$ above the heat source, the origin distance ought to be calculated from the relation:

$$
\begin{equation*}
z_{v}=7.75 R_{w l}-z_{l} \tag{5}
\end{equation*}
$$

## ANLYSIS OF THE AVAILABLE RESULTS OF PLUME MEASUREMENT

On the basis of the available experiment results the ways of volume flux calculation employing the PPHS model were verified. The analysis covered more than 40 plumes presented in Table 2. In real plumes, enthalpy excess is often reduced considerably when the height above the heat source increases, whereas PPHS model refers to plumes in the environment where no stratification is observed and assumes a constant enthalpy excess value. In the plume analysis it was assumed that the enthalpy flux value was equal to the value measured at the beginning of the self similarity of mean motion zone. The analysis results are shown in Fig. 1

Table 2 Data on the plumes in which the volume flux was analysed

| Heat source | Entalphy flux $Q_{0}$ | Number of measured cross-section | Author | Marker on Fig. 1 |
| :---: | :---: | :---: | :---: | :---: |
| - | W | - | - | - |
| human body | 10-30 | 8 | Stanisław Mierzwiński | black circle |
| sphere, $\mathrm{d}=35 \mathrm{~mm}$ | 15 | 3 | Zbigniew <br> Popiołek | rhombus |
| sphere $=700 \mathrm{~mm}$ | 17 | 3 |  |  |
| cylinder, $\mathrm{d}=15 \mathrm{~mm}, \mathrm{~h}=120 \mathrm{~mm}$ | 25 | 3 |  |  |
| plate d=500mm | 380 | 9 |  |  |
| cylinder, $\mathrm{d}=55 \mathrm{~mm}, \mathrm{~h}=150 \mathrm{~mm}$ | 80-350 | 106 | Peter Kofoed | square |
| plate d=356mm | 20-250 | 125 |  |  |
| cylinder, $\mathrm{d}=320 \mathrm{~mm}, \mathrm{~h}=1200 \mathrm{~mm}$ |  |  |  |  |
| radiator, $\mathrm{h}=600 \mathrm{~mm}, \mathrm{l}=600 \mathrm{~mm}$ | $\sim 100$ | 4 | Irma Welling | triangle top oriented up |
| squared plate, $300 * 300 \mathrm{~mm}$ | 270-600 | 10 |  |  |



Fig. 1 Air volume flux in buoyant plumes (symbols described in Table 2.)

The data presented in Fig. 1 show that the experiment results of the volume flux increase in buoyant plumes may be generally described by Eq. 3, with the accuracy $\pm 20 \%$, after having assumed $k_{v}=0.006$. The general description is possible if the distance from the origin is determined on the basis of the plume width measured at the beginning of the self similarity of mean motion zone. The origin position cannot be defined only on the basis of dimensions of the heat source and its thermal power.

## DETERMINATION OF THE PLUME ORIGIN DISTANCE BY MEANS OF FLOW VISUALISATION

Since it is sufficient to know only the widths of the plume parameter profiles in order to determine the origin position, flow visualisation was employed in the tests. The tests consisted in the flow in a free buoyant plume recording and simultaneous temperature measurement. A VHS camera and PC computer were used for the recording and the flow pattern analysis, respectively. Fig. 2 shows a single shot of the recorded flow picture.

The temperature distributions were measured with a multichannel thermometer assuming the averaging time 180 s. Ten shots, recorded every 20 s were selected for analysis. The plume diameter and origin distance were defined from the flow picture at the height 1.2 m . The temperature distribution and the plume diameter are compared in Fig. 3.


Fig. 2 A single shot of the recorded picture of flow in a buoyant plume


Fig. 3 Comparison of temperature excess distributions in a buoyant plume and the plume diameter observed during flow visualisation

The measurements have proved that the average plume diameter $D_{a v}$ corresponds to about two widths of the temperature profile, $R_{t}$. The point most distant from the plume edge observed defines the maximum diameter of the plume $D_{\max }$ corresponding to about three widths of the temperature profile. Thus, when the plume diameter $D_{\max }$ is determined at the height $z_{l}$ above the top of the heat source, the origin distance may be expressed by the following relation:

$$
\begin{equation*}
z_{v}=2.6 D_{\text {max }}-z_{l} \tag{6}
\end{equation*}
$$

Most of the plumes analysed developed in the environment where thermal stratification was observed. It seems, however, that the method of volume flux determination above a point heat source may be used for plumes in environment both with or without thennal stratification. In order to check whether it is possible to use the method to calculate the latter in rooms with displacement ventilation some other tests were carried out.

## TESTS OF AIR VOLUME FLUX IN BUOYANT PLUME IN A ROOM WITH DISPLACEMENT VENTILATION

The tests were carried out in a room of cubature $27 \mathrm{~m}^{3}$ and height 3 m . Different heat sources were simulated by a buoyant plume generator and the enthalpy flux of the plumes was 150-300W. The air was supplied to the room through two quasilaminar diffusers placed just above the floor and removed through an outlet in the ceiling. The ventilating flow rate in the room during the tests was $50 \div 150 \mathrm{~m}^{3} / \mathrm{h}\left(1.85 \div 3.7 \mathrm{~h}^{-1}\right)$. The measurement stand diagram is shown in Fig. 4.

The measurement stand was equipped with a movable system for temperature measurement in the plume and a stationary system for temperature measurement in the plume surroundings. The values of the enthalpy flux in the initial plumes, the plume Archimedes numbers, starting temperature and ventilating air amount in the room, assumed in the tests, are shown in Table 3. The table 3 presents also $z_{v}$ values obtained for different plumes at different air flow rates in the room test. Having assumed rectangular velocity profiles, Archimedes number values were determined from the relation:

$$
\begin{equation*}
A r_{0}=\frac{\beta \cdot \Delta t \cdot d}{2^{5 / 2} \cdot w_{0}^{2}} \tag{7}
\end{equation*}
$$



Fig. 4 Measurement stand diagram

Table 3 Values of the origin distance, $z_{v}$, for the plumes tested at different ventilating air flow rates

| $\mathrm{V}_{\mathrm{N},} \mathrm{m}^{3} / \mathrm{h}$ |  | $\mathrm{Ar}_{0} / \mathrm{Q}$ |  |  |
| ---: | :---: | :---: | :---: | :---: |
|  | $0.05 / 150$ | $0.01 / 180$ | $0.008 / 370$ |  |
| 50 | $\left(1.9 \mathrm{~h}^{-1}\right)$ | $\mathrm{z}_{v}=0.03$ | $\mathrm{z}_{v}=-0.02$ |  |
| 100 | $\left(3.7 \mathrm{~h}^{-1}\right)$ | $\mathrm{z}_{v}=0.45$ | $\mathrm{z}_{v}=-0.05$ |  |

The data presented in Table 3 show that $z_{v}$ values vary and rise when the ventilating air flow rate increases. The range of the origin distance variability refers more to weak plumes of low initial enthalpy flux. For stronger plumes the effect of the air supply appears only at higher amounts of the ventilating air.

The value of $\lambda$ ratio expressing the ratio of temperature and velocity profile widths is about one. It confirms the assumption made so far that temperature and velocity profile widths are the same. When assuming $\lambda=1$ the error does not exceed $\pm 10 \%$. which is illustrated in Fig. 5. When analysing $\lambda$ ratio values it may be noticed that they rise when the distance from the heat source increases. At small heights, velocity profile widths, $R_{w}$ are greater than temperature profile widths, $R_{t}$. When the height increases, the relation is opposite.


Fig. 5 The temperature and velocity profiles widths ratio $\lambda$, versus height above the heat source.

Fig. 6 shows the results of air volume flux measurement in a plume. The results, similarly to the ones shown in Fig. 1, are all included in an area defined by two lines expressed by the relations: $V / Q^{1 / 3}=0.0048\left(z+z_{v}\right)^{5 / 3}$ and $V / Q^{1 / 3}=0.0072\left(z+z_{v}\right)^{5 / 3}$.

It proves that the value $k_{v}=0.006$, the most often assumed in calculations, is statistically correct and the error does not exceed $\pm 20 \%$. Apparently, the value of $k_{v}$ coefficient depends neither on the initial Archimedes number, $A r_{0}$ nor on the heat source power, $Q_{0}$. This is illustrated by Fig. 7. Only slight difference in $k_{\nu}$ values is observed and in practice it may be assumed that the value of $k_{v}$ is constant. However, a rise in $k_{v}$ is observed when the ventilating flow rate $V_{N}$ is increased, from $\mathrm{k}_{\mathrm{v}}=0.0059$ for $\mathrm{V}_{\mathrm{n}}=50 \mathrm{~m}^{3} / \mathrm{h}$ to $\mathrm{k}_{\mathrm{v}}=0.0066$ for $\mathrm{V}_{\mathrm{n}}=100 \mathrm{~m}^{3} / \mathrm{h}$. It may be explained by the fact that increase in the ventilating air flow rate, results in the turbulence increase in the plume surroundings due to the effect of the supply jets. Turbulent flow in the plume surroundings intensifies the entrainment process and it is reflected in $k_{v}$ coefficient value.


Fig. 6 Air volume flux increase in buoyant plumes with displacement ventilation


Fig. 7 Coefficient $k_{v}$ dependence on $\mathrm{Ar}_{0}$ and $\mathrm{Q}_{0}$

## CONCLUSIONS

1. It is possible to determine the increase in the air volume flux in buoyant plumes with the use of the model of plume over a point heat source not only for free plumes arising in environment where there is no themnal stratification but also in rooms with stratification. It refers to rooms with displacement ventilation in the range of parameters covered by the tests.
2. The assurnption of equal widths of the air temperature and velocity profiles in buoyant plumes $R_{w}=R_{t}$, when the origin distance $z_{v}$ is determined according to Eq. 5 , has been confirmed in the case of plumes developing in rooms with displacement ventilation, and the accuracy acquired is $\pm 10 \%$.
3. The entrainment coefficient value $k_{\nu}=0.006$, assumed so far, is statistically correct in the cases covered by the tests and the error does not exceed $\pm 20 \%$.
4. When the ventilating air flow rate is increased, $k_{v}$ value may rise due to the effect of supply jets.

## ACKNOWLEDGMENTS

The present paper includes the result of the research supported by Polish Committee for Scientific Research within the frame of Projects No 7 T07G 03911

## REFERENCES

1 Popiołek. Z. : "Problems of Testing and Mathematical Modelling of Plumes above Human Body and other Extensive Heat Sources", K.T.H. Stockholm, A4-Serien, No 54, Inst. Uppv. och Ventil., 1981
2 Popiołek, Z., Mierzwiński, S.: "Buoyant plume calculation by means of the integral method", K.T.H. Stockholm, A4-Serien, No 89, Inst. Uppvarmn. och Ventil., 1984

3 Schmidt, W.: [4] Rouse, H., Yih, C.S., Humpreys, H.W.: "Gravitational Convection from a Boundary Source", A Quarterly Journal of Geophysics, Tellus, Vol.4, 1952
4 Sutton, O.G.: "The dispersion of Hot Gases in the Atmosphere", J. Meteorology, vol.7, pp.307-312, 1950
5 Hatch, T.F., Barron-Oronzco, D.: "Air Flow in Free Convection over Heated Bodies", ASHAE Trans., vol.63, nr 1602, 1957
6 Baturin, W.D., Elterman, W.M.: "Aeracyja promyszlennych zdanij", Gos. Izd. Lit. po Strit. Mat., Moskov, 1963
7 Daws, L. F.: "Movement of Air Streams Indoors", J. Inst. Heat. Vent. Engrs, vol.37, pp.241153, 1970
8 Shepieliev, I.A.: "Aerodynamika wozdusznych potokow w pomieszczenii", Stroizdat. Moskov, 1978
9 Popiołek Z.: "Badanie i modelowanie strug konwekcyjnych z uwagi na ksztattowanie procesu wentylacji", Zeszyty Naukowe Politechniki Ślaskiej, Inżynieria Środowiska 28
10 Kofoed, P.: "Thermal plumes in ventilated rooms", Ph. D. Thesis, Aalborg Universitetsceneter. Aalborg, Denmark, 1991
11 Kofoed, P., and Nielsen, P.V.: "Thermal plumes in ventilated rooms- Measurements in stratified surroundings and analysis by use of an extrapolation method". Proc. Roomvent'92. Sess. B2. Aalborg, Denmark, 1992
12 Mierzwiński, S.: "Testing and modelling of thermal plumes in rooms". Proc. ISRACVE. University of Tokyo, 1992
13 George, W.K., Alpert, R.L., Tamanini, F.: "Turbulence measurement in an axisymmetrical buoyant plume", Int. J. Heat Mass Transfer, Vol.20, 1145-1154, 1977
14 Nakagome, H., Hirata,M.: "The structure of turbulent diffusion in an axisymmetric thermal plume", ICHMT, vol.1-2, Dubrownik, 1976
15 Abramowicz, G.N.: "Tieorija turbulentnych strui", Moskwa, 1960
16 Welling, I.: "An Investigation of the SVD-Method for Analyzing Convection Plumes", Lappeenranta Regional Institute of Occupational Health, Lappeenranta, 1993

