

Air-Change Effectiveness: Theory and Calculation Methods

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Abstract This paper reports the development of methods for calculating a ventilation performance metric that is a measure of the airflow pattern in a room or zone of a multi-zone ventilation system. Temporal mixing theory is used as the basis for these methods. The methods are applicable to all ventilated systems that can be modeled as a set of interconnected chambers. Relations between the ventilation performance metric defined in this paper and those defined previously are derived. The theoretical results of this paper are consistent with published experimental findings. They also illustrate that the conclusions in some experimental studies about the airflow patterns in working buildings may be incorrect. Re-analysis of previously published data illustrates how common features of mechanically ventilated buildings, such as recirculation of return air and multiple chambers, confound information about airflow patterns in tracer gas data. The calculation methods developed in this paper can be used to undo this confounding.

Key words Ventilation; Effectiveness; Efficiency; Age-of-air; Multizone; Recirculation.

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Introduction

Ventilation performance measures are primarily aimed at characterizing either indoor airflow patterns or pollutant removal behavior. Performance measures that characterize airflow patterns are commonly referred to as air-change effectiveness, while performance measures that characterize pollutant removal behavior are commonly referred to as pollutant removal effectiveness. Fisk et al. (1997) describe calculation methods for both of these parameters, field measurements of each, and relationships between them. Although airflow patterns, pollutant removal

behavior, and outdoor air delivery are closely related topics, this paper focuses exclusively on the characterization of airflow patterns and the calculation of air-change effectiveness.

Concerns about indoor airflow patterns are often expressed by engineers, facility managers, and architects with questions such as "what fraction of outdoor air is actually delivered to the breathing zone?" or "how much of the supply air short-circuits the zone?" One may also be interested in characterizing whether or not the airflow pattern in a displacement ventilation system is actually as intended (e.g., from the floor to the ceiling). Undesirable airflow patterns may lead to poor indoor air quality and increased energy usage.

Numerous parameters intended to characterize airflow patterns have been proposed, but most are variants of one of two basic concepts. The first concept is derived from temporal mixing theory, and uses ratios of age-of-air to characterize the airflow pattern. The age-of-air is the mean time that it takes a particle (e.g., a molecule) to travel from an inlet point, such as the outdoor air intake, to the measurement point. Sandberg and Sjoberg (1983) defined a ventilation performance measure called relative air diffusion efficiency as the ratio of the nominal time constant to the mean age of air. This performance measure was modified by Etheridge and Sandberg (1996) and renamed air-exchange efficiency.

The second concept used to characterize airflow patterns is the notion of an artificial bypass. Janssen (1984) developed a stratification model in which the airflow pattern is characterized by two parameters, the bypass fraction and the ventilation efficiency. The bypass fraction is the fraction of the air that is said to bypass or short-circuit the occupied portion of the zone. The ventilation efficiency is a measure of how effectively out-

door air is utilized. Janssen (1984) only analyzed single-zone buildings. Rask et al. (1988) modeled ineffective ventilation of multi-zone buildings using the bypass fraction method. A performance measure called the ventilation efficiency was then defined as one minus the bypass fraction.

A common feature of mechanically-ventilated buildings is that some of the return air is recirculated or recycled. This is usually done because the flow rates required for ventilation are too low for heating or cooling purposes. In order to meet the heating or cooling heat transfer rates and keep the supply temperatures within a reasonable range, higher supply flow rates are needed. Recirculation poses a problem for evaluating ventilation performance using measures derived from temporal mixing theory (such as the relative air diffusion efficiency) because they were developed under the assumption that "neither is it possible for a molecule either to return upstream once it has entered the room, or to re-enter the room once it has left it" (Sandberg and Sjoberg, 1983). Practical methods of characterizing airflow patterns must be able to accommodate recirculation.

A common feature of nearly all buildings, not just mechanically-ventilated buildings, is that they contain more than one room, chamber, or zone, and that the chambers exchange air directly through doorways and permeable ceilings in addition to any air exchange that may occur by recirculation. Practical methods of characterizing airflow patterns must also be able to accommodate multiple, interacting zones.

In this paper, calculation methods for characterizing the airflow pattern of indoor air are described. They are based on results from temporal mixing theory. The methods may be applied to buildings with or without mechanical ventilation, with or without recirculation, and with more than one zone. In the next section, ventilation performance measures for single-zone buildings are discussed because they are sufficiently simple for relations between important system parameters and variables to be displayed graphically, which provides important intuitive knowledge for analyzing multi-zone systems. Theoretical results for perfect-mixing and plug-flow systems are derived. Relations between ventilation performance measures described in this paper and those described in other papers are derived. In the section entitled Multi-Zone Systems, general calculation methods for multi-zone buildings are described. This is followed by a discussion of the implication of the theoretical results on previously published findings. Data from a previously published paper are used to illustrate the results.

Single-Zone Buildings

In this section, relations between the mean age of air and relevant system parameters will be described. Then the relation between air-change effectiveness and the system parameters will be described.

Age of Air

Consider the ventilation system depicted in Figure 1 in which a single zone is supplied by air that is a combination of recirculated return air and outdoor air. The fraction of return air that is recirculated will be denoted as R . The dotted line is used to show that the bypass is artificial. The bypass fraction is denoted as S . The age of air at a point in the room is the length of time that it takes a particle (e.g., a molecule) entering the system from outdoors to reach that point. The age of air will be denoted as α . The volumetric mean age of air will be denoted as $\bar{\alpha}$.

First, the age of air relations for two abstract systems will be described. These relations have been described by Federspiel (1996a). The nominal turnover time, denoted as τ_n , is defined as the ratio of the mass of air in the zone to the outdoor air mass flow rate:

$$\tau_n \equiv \frac{M}{f_o} \tag{1}$$

Note that all flow rates in this paper are mass flow rates unless specified otherwise. When the air in a single-zone system is perfectly mixed, then the volumetric mean age of the air is equal to the nominal turnover time (Sandberg and Sjoberg, 1983).

$$\bar{\alpha} = \tau_n \tag{2}$$

For a perfect-mixing system, the mean age of the air is independent of the recirculation fraction.

Another kind of abstract system that is commonly considered when analyzing ventilation performance is the system with plug flow and no diffusion (PFND).

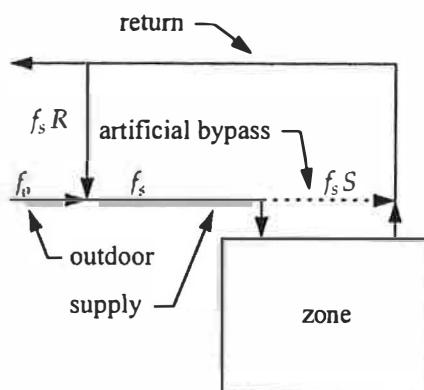


Fig. 1 Schematic diagram of a single-zone building

When f_o and R are constant, the PFND system is a linear, time-invariant transport delay. The zone delay is the time that it takes a particle to cross the zone from the supply to the return. It is equal to the local nominal turnover time of the zone, denoted as T .

$$T = \frac{M}{f_s} \quad (3)$$

It can be shown (see Appendix A) that, for a PFND system, the relation between R , T , and $\bar{\alpha}$ is as follows:

$$\bar{\alpha} = \frac{T(1+R)}{2(1-R)} \quad (4)$$

which illustrates that the age of air depends on the recirculation fraction for a PFND system.

Figure 2 demonstrates how recirculation affects the age of the air. The figure shows the normalized average concentration decay curves for a perfect mixing system and several PFND systems with different amounts of recirculation. The concentration is normalized by dividing by the initial concentration, and the time is normalized by dividing by the nominal turnover time. As the recirculation fraction increases, the decay response moves closer and closer to the response of the perfect-mixing system.

In practice, the air will not behave like it does in these abstract systems. Since the age of return air for systems with no recirculation is independent of the airflow pattern and is equal to T (Sandberg and Sjoberg, 1983; Mantegna, 1993), one can show (see Appendix A) that the mean age of air in any single-zone system is:

$$\bar{\alpha} = \bar{\alpha}_{R=0} + \frac{TR}{1+R} \quad (5)$$

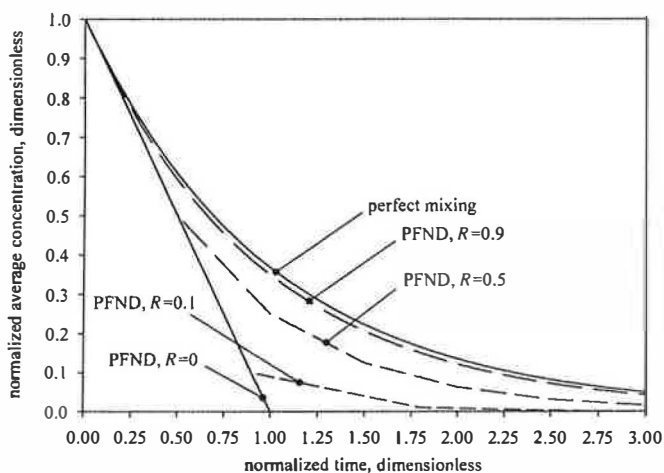


Fig. 2 Tracer gas decay for a perfect mixing system and several PFND systems

where $\bar{\alpha}_{R=0}$ is the mean age of air that would exist if $R=0$.

Air-Change Effectiveness

Sandberg and Sjoberg (1983) defined the air diffusion efficiency as the ratio of the nominal turnover time to the volumetric mean age of air.

$$\varepsilon_a \equiv \frac{\tau_n}{\bar{\alpha}} \quad (6)$$

This performance metric was modified by Etheridge and Sandberg (1996) and renamed air-exchange efficiency. The air-exchange efficiency is defined as the ratio of the nominal turnover time to the average time of exchange of air in the room.

$$\varepsilon_a \equiv \frac{\tau_n}{\tau_{exc}} \quad (7)$$

In the remainder of this paper, Equation (7) will be used rather than Equation (6) unless otherwise noted.

The average time of exchange of air, τ_{exc} , is equal to twice the mean age of air. Therefore, Equation (7) becomes

$$\varepsilon_a = \frac{\tau_n}{2\bar{\alpha}} \quad (8)$$

In other words, the air-exchange efficiency is simply half of the air diffusion efficiency.

Sandberg and Sjoberg (1983) explicitly state that air diffusion efficiency, and therefore air-exchange efficiency, is intended for use with systems that do not recirculate air. Nevertheless, Equations (6) and (7) have been applied to systems that do recirculate air (Offermann, 1988; Persily et al., 1994; Sekhar et al., 1997). For systems that recirculate air, theoretical relations between the air-exchange efficiency and the recirculation fraction can be formulated. For a PFND system it can be shown (see Appendix B) that the air-exchange efficiency is:

$$\varepsilon_a = \frac{1}{1+R} \quad (9)$$

In general, the air-exchange efficiency is:

$$\varepsilon_a = \frac{\tau_n}{\bar{\alpha}_{R=0} + \tau_n R} \quad (10)$$

If there were no recirculation, then the air-exchange efficiency would provide a measure of the airflow pattern in the room. Therefore, another ventilation performance measure, which will be referred to as the relative air-change effectiveness, is defined as the value of the air-exchange efficiency that would have

been calculated had the recirculation fraction been zero.

$$\epsilon_r \equiv \frac{T}{2\bar{\alpha}_{R=0}} \quad (11)$$

For a single-zone system, it can be shown (see Appendix B) that:

$$\epsilon_r \equiv \frac{\tau_n - \alpha_s}{2(\bar{\alpha} - \alpha_s)} \quad (12)$$

where α_s is the age of the supply air. Equation (12) demonstrates the similarity between ϵ_r and the relative ventilation efficiency defined by Kim and Homma (1992) and the contaminant removal effectiveness defined by Etheridge and Sandberg (1996). The key similarity to these ventilation performance measures is that the supply air conditions are included in the calculations.

As with the air-exchange efficiency, the relative air-change effectiveness can be computed at a point by substituting the age of air at a point for the average age of air in Equation (12). When there is no recirculation, the age of the supply air is zero, so the relative air-change effectiveness becomes the same as the air-exchange efficiency. For single-zone PFND systems, the relative air-change effectiveness is equal to one regardless of the recirculation fraction. For perfectly mixed systems it is equal to 1/2.

Since the nominal turnover time is equal to the age of the supply air plus the local turnover time, the following alternative methods, which have been described by Federspiel (1997), can also be used to calculate ϵ_r :

$$\epsilon_r = \frac{T}{2(\bar{\alpha} - \alpha_s)} \quad (13)$$

$$\epsilon_r = \frac{T}{2(\bar{\alpha} - \tau_n + T)} \quad (14)$$

Equation (13) may be a useful calculation method when the age of air leaving the chamber cannot be measured, and Equation (14) may be useful when the age of air entering the chamber cannot be measured. In either case, one would calculate from measured values of M and f_s .

It can be shown (see Appendix B) that for a single-zone system, the relation between ϵ_a , ϵ_r and R is as follows:

$$\epsilon_a = \frac{\epsilon_r}{1 - R + 2R\epsilon_r} \quad (15)$$

This relation is shown in Figure 3. A similar relation between the air diffusion efficiency, ϵ_r and R has been

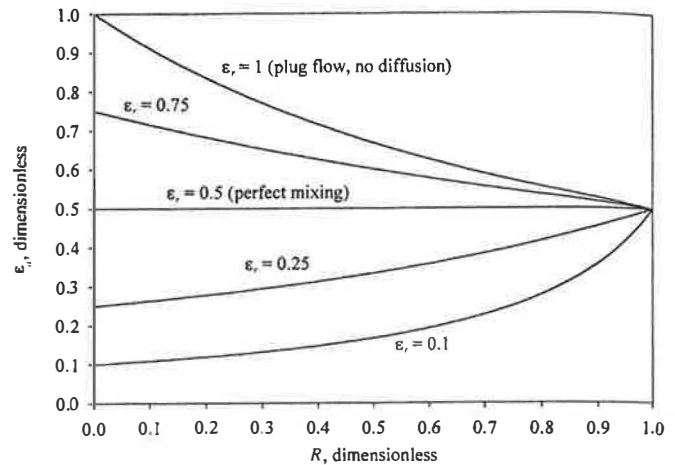


Fig. 3 Relation between ϵ_a , ϵ_r and R

described by Federspiel (1996b). Figure 3 shows that as the recirculation fraction approaches 1, ϵ_a approaches 1/2 regardless of the value of ϵ_r . It also shows that by itself, ϵ_a only provides a qualitative description of the air distribution pattern. If $\epsilon_a < 1/2$, then one can say that there is some short-circuiting, but one cannot say how much without also knowing R or by knowing ϵ_r .

Sometimes ineffective ventilation is modeled by assuming that some of the supply air bypasses the breathing zone and passes directly to the return duct (Janssen, 1984; Rask et al., 1988). This can be modeled by assuming that in $S < 0$ in Figure 1. It can be shown that for this system:

$$\epsilon_a = \frac{1 - S}{2(1 - RS)} \quad (16)$$

The air-exchange efficiency is a measure of how well outdoor air is utilized. In Rask et al. (1988), a ventilation performance measure called ventilation efficiency is defined as:

$$\eta \equiv 1 - S \quad (17)$$

It can be shown that:

$$\epsilon_r = \frac{\eta}{2} \quad (18)$$

In other words, the relative air-change effectiveness is measure of how effectively supply air is used to ventilate the zone. A problem with the bypass model is that some ventilation systems (e.g., displacement ventilation systems) make more effective use of ventilation air than perfect-mixing systems. When this happens, the bypass factor, S , must be negative. A problem with the bypass model is that a negative value of S does not have a physical interpretation.

Multi-Zone Systems

Most buildings contain a number of zones that interact both through recirculation in an air-handling unit and directly through passages such as doorways. Figure 4 shows some of the common flow paths in buildings with multiple zones and mechanical ventilation. These flow paths include both primary and secondary recirculation, direct air exchange between zones, and zones which are arranged in series as well as in parallel. The subscripts on the flow parameters in the figure should be read as "from-to, zone number". For example, $F_{pt,1}$ refers to the flow from the plenum to the terminal unit of zone 1.

The theory described above can be adapted to multi-zone buildings by modeling a multi-zone building as an interconnected set of mixing chambers, and using two rules from temporal mixing theory. The first rule is that at a junction point in the process (e.g., where two ducts combine into one) the age of air downstream of the junction is the flow-weighted average of the age of air of the ducts upstream of the junction. The second rule is that the mean age of air is additive for independent chambers in series. In other words, the age of air leaving a set chambers in series is equal to the sum of the age of air had the chambers not been arranged in series. These mathematical results and others are described in detail by Nauman and Buffham (1983).

The relation between the incoming age and the outgoing age for a chamber with m inputs and n outputs is as follows:

$$\frac{\sum_{k=1}^n F_{ek} \alpha_{ek}}{F} = \frac{\sum_{k=1}^m F_{ik} \alpha_{ik}}{F} + \frac{M}{F} \quad (19)$$

$$F = \sum_{k=1}^n F_{ek} = \sum_{k=1}^m F_{ik} \quad (20)$$

The subscripts e and i refer to exit and inlet, respectively. M refers to the mass of air. Equation (19) states that the flow-weighted average of the outgoing age is

equal to the flow-weighted average of the incoming age plus the age accumulation. For a chamber with just one input and one output, Equation (2) becomes the following:

$$\alpha_e = \alpha_i + \frac{M}{F} \quad (21)$$

Age distribution at junctions in ducts is modeled with Equations (19) and (20) by assuming that the junction is massless. Where a duct diverges into more than one duct, the ages in the branches downstream of the junction equal the age upstream of the junction. Where ducts converge into one duct, the age downstream of the junction is the flow-weighted average of the ages upstream.

There are three methods of calculating ϵ_r for multi-zone systems which are analogous to the three methods for single-zone systems that were described in the section on air-change effectiveness. For a chamber with m inputs and n outputs, ϵ_r may be computed as follows:

$$\epsilon_r = \frac{\sum_{k=1}^n F_{ek} \alpha_{ek} - \sum_{k=1}^m F_{ik} \alpha_{ik}}{2 \left(F \bar{\alpha} - \sum_{k=1}^m F_{ik} \alpha_{ik} \right)} \quad (22)$$

For a chamber with just one input and one output, Equation (22) becomes the following:

$$\epsilon_r = \frac{\alpha_e - \alpha_i}{2 (\bar{\alpha} - \alpha_i)} \quad (23)$$

The calculation methods for multi-zone systems which are analogous to Equations (13) and (14) are the following:

$$\epsilon_r = \frac{T}{2 \left(F \bar{\alpha} - \sum_{k=1}^m F_{ik} \alpha_{ik} \right)} \quad (24)$$

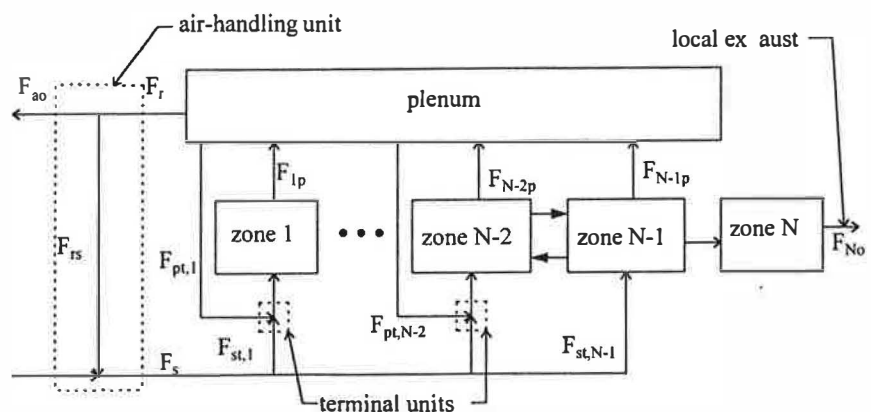


Fig. 4 Schematic diagram of a multi-zone ventilation system

$$\epsilon_r = \frac{T}{2 \left(F\bar{\alpha} - \sum_{k=1}^m F_{ek} \alpha_{ek} + T \right)} \quad (25)$$

where $T=M/F$. As an example, the relative air-change effectiveness of zone 1 in Figure 4 could be calculated as follows:

$$\epsilon_{r1} = \frac{\alpha_{e1} - \alpha_{i1}}{2 \left(\bar{\alpha}_1 - \alpha_{i1} \right)} \quad (26)$$

where α_{e1} is the age of air leaving zone 1 to the plenum, α_{e1} is the age of air entering zone 1 from the terminal unit, and $\bar{\alpha}_1$ is the volumetric mean age of air in zone 1. The relative air-change effectiveness of zone N could be calculated as follows:

$$\epsilon_{rN} = \frac{T_N}{2 \left(\bar{\alpha}_N - \alpha_{eN} + T_N \right)} \quad (27)$$

where:

$$T_N = \frac{M_N}{F_{No}} \quad (28)$$

and where $\bar{\alpha}_N$ is the volumetric mean age of air in zone N , and α_{eN} is the age of air leaving zone N . This method may be more appropriate for zone N because it would probably be difficult to measure α_{iN} , but relatively easy to measure F_{No} .

Discussion

Many published experiments on air-change effectiveness involve tests on systems that recirculate air. Typically the air diffusion efficiency or the air-exchange efficiency is reported, and the tests are often carried out with large recirculation fractions (e.g., minimum outdoor air conditions). The theory presented above predicts that under these conditions the air-exchange efficiency will be close to 1/2 (i.e., air diffusion efficiency will be close to 1). This prediction is consistent with many published findings such as Fisk et al. (1988), Persily and Dols (1991), Persily et al. (1994), and Sekhar et al. (1997), although some experiments such as those described by Offermann (1988) have shown that the air-exchange efficiency may be significantly less than 1/2 even with recirculation.

The results of the section on air-change effectiveness illustrate that it is not possible draw quantitative conclusions about the airflow pattern in single-zone systems from ϵ_a if air is recirculated. Only qualitative conclusions can be drawn. For example, the amount of

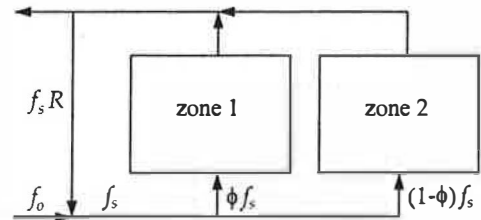


Fig. 5 Schematic diagram of a two-zone PFND system

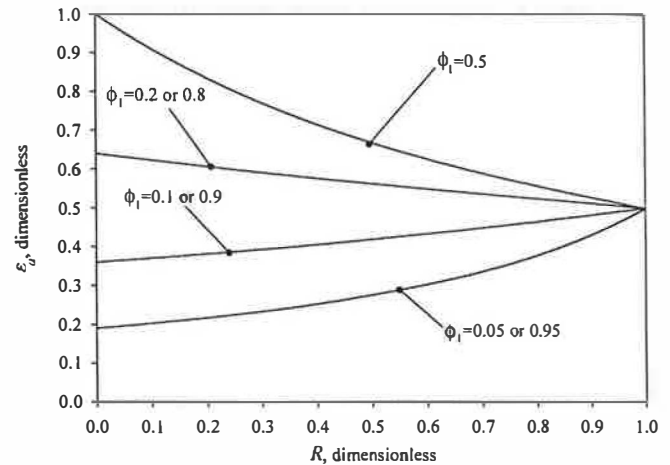


Fig. 6 Relation between ϵ_a , R , and ϕ_1 for the system shown in Figure 5

short-circuiting may be arbitrarily high (i.e., ϵ_r arbitrarily close to zero), and yet the value of ϵ_a may be arbitrarily close to 1/2 simply by choosing a sufficiently high recirculation fraction.

For multi-zone systems, nothing can be said about the "within-zone" airflow patterns from ϵ_a . Therefore, conclusions from tracer-gas tests such as "it is observed that the AEE values are usually close to one implying no serious problems of short circuiting of ventilation air" (Sekhar, 1997) may be incorrect. A value of ϵ_a close to 1/2 may be achieved even if there is arbitrarily large amounts of short circuiting or perfect plug flow in one or more zones simply by recirculating sufficiently large quantities of air. Furthermore, a value of ϵ_a lower than 1/2 may be due solely to the way that the supply air is distributed to the zones. In other words, the effects of airflow patterns on ϵ_a are confounded by differing supply air fractions and recirculation.

For example, consider the two-zone system in Figure 5. Assume that the zones are of equal size and that the airflow pattern in each zone is plug flow with no diffusion. Under these conditions, it can be shown that the relation between ϵ_a , R , and the fraction of the supply air delivered to zone 1 (denoted as ϕ_1) is as follows:

$$\varepsilon_a = \frac{4\phi_1(1-\phi_1)}{8R\phi_1(1-\phi_1) + (1-R)} \quad (29)$$

This relation is depicted in Figure 6. The figure illustrates that for this particular two-zone system, it is possible for the value of ε_a to be less than 1/2 even though the airflow pattern in each zone is PFND. In fact, as ϕ_1 approaches zero or one, the value of ε_a approaches 0. This example demonstrates the fact that short-circuiting may have nothing to do with ε_a being less than 1/2.

Fisk and Faulkner (1992) have proposed a ventilation performance measure called the air diffusion effectiveness (ADE) which eliminates some of the confounding in multi-zone systems. This is done by replacing the nominal turnover time in Equation 6 with the age of air leaving the zone (at the return grille) being tested. Doing so makes ADE a qualitative measure of the airflow pattern. It is only confounded by recirculation of air. In other words, there is a similar relation between ε_r , R , and ADE in multi-zone systems as there is between ε_r , R , and ε_a in single-zone systems.

A clear definition of the control volume in which the air-change effectiveness is to be determined is fundamentally important. When the age of the supply air is not included in the effectiveness calculation and when the volumetric mean age of air for the whole building is used in the calculation of air-change effectiveness, then the control volume is the building envelope. In this case, if the air-change effectiveness indicates short-circuiting it means that the short-circuiting may be occurring in any number of places including the ceiling plenum by virtue of duct leakage as well as across the ceiling of individual rooms. Note that uncompensated short-circuiting outside the building (an additional recirculation effect) will confound the detection of "whole-building" short-circuiting. If the age of air in just one location or space in the building is used in the calculation of air-change effectiveness, then the computed value is a complex function of how supply air is being distributed and of how well air is being mixed in each space. It is not a measure of short-circuiting.

To illustrate how the theory reported in the sections on Single- and Multi-Zone Systems may affect conclusions from a real experiment, the data reported in Offermann (1988) are re-evaluated, and values of ε_r are calculated. The article describes a tracer gas experiment on a zone in a multi-zone system which was divorced from the other zones. Although the zone was separated from the others, air was still passed through a ceiling plenum, which accumulated age. The HVAC system was operated in the heating mode, so the supply air was warmer than the zone air. Three air deliv-

ery configurations were tested. The first configuration was typical of North American construction where the supply and return were both in the ceiling. The second configuration was intended to function as a displacement ventilation system. The warm supply air was delivered at the ceiling, and the return air drawn from a low point on one wall. The third configuration was designed to intentionally induce short-circuiting. A jet of supply air was aimed at the return grille from a distance of approximately eight feet. Both the supply and return points of the third configuration were located at the ceiling level.

Table 1 shows the values of the breathing-zone air-exchange efficiency derived from Offermann (1988). These values were computed by dividing the air diffusion efficiency reported in the paper by two. The table also shows the relative air-change effectiveness calculated from information supplied in the article. The values of ε_r shown in the table were not calculated with Equation (23) because one of the configurations delivered air to a ceiling plenum rather than directly to the zone, and because there was some uncertainty about leakage of air into the return duct. Instead, ε_r was calculated as follows:

$$\varepsilon_r = \frac{T}{2(\alpha_b - \alpha_p - \alpha_s)} \quad (30)$$

where T was calculated from the reported nominal ventilation rate, volume of the space (total volume minus the volume of the ceiling plenum), and recirculation fraction (67%), α_b was the age of air in the breathing zone, α_p was the age accumulated in the ceiling plenum, and α_s was the age of the supply air, which was measured as it left the rooftop air-handling unit. For the typical configuration corresponding to H1 and H2, and for the short-circuiting configuration corresponding to H5, $\alpha_p = 0$ because the supply air was ducted directly to the room. In the displacement flow configuration corresponding to H3 and H4, the supply air first passed through the plenum and accumulated age before entering the room.

The re-analysis of the data demonstrates two points. First, the short-circuiting was much more severe than was indicated by the original analysis. Second, the

Table 1 Re-evaluation of the data in Offermann (1988)

configuration	ε_d	ε_r
H1: ceiling supply and return	0.37	0.15
H2: ceiling supply and return	0.33	0.17
H3: ceiling supply, floor return	0.38	0.28
H4: ceiling supply, floor return	0.36	0.23
H5: "short-circuiting" design	0.29	0.22

original analysis indicates that the configuration designed to induce short-circuiting had the lowest air-exchange efficiency. However, by calculating ϵ_r , one can see that the data actually suggests that the system with ceiling supply and return induced more short-circuiting than the system designed to induce it. This is probably because the supply air velocities were higher with the system designed for short-circuiting, and the higher velocities probably entrained surrounding air and induced more mixing.

Another implication of the theory described in previous sections pertains to the use of experimentally-determined effectiveness parameters in controlling ventilation systems. The results of the Section on Single-Zone buildings and the example described above illustrate that an experimentally-determined value of ϵ_a should not be used to modulate the nominal amount of outdoor air required to satisfy a ventilation requirement. In particular, the nominally required amount of outdoor air should not be multiplied by an experimentally-determined value of ϵ_a to determine the actual amount of outdoor air to be supplied because it varies as the supply and recirculation flow rates vary. If the goal of the ventilation system were to ensure that the mean age of air in a zone was maintained at or below a specified level, then experimentally-determined values of ϵ_r could be used as follows. First, the age of the supply air would be determined. If there is no secondary recirculation, then it is the product of the nominal turnover time for the entire building and the recirculation fraction. Then the mean age of air in each zone would be calculated by adding the age accumulated in the zone to the age of the supply air as follows:

$$\bar{\alpha}_i = \alpha_s + \frac{T_i}{\epsilon_{r_i}} \quad (31)$$

where the subscript i refers to the zone number. The value of T_i would be computed from a measured value of the supply flow rate to zone i and the volume of zone i . If the calculated value were greater than the requirement, then the amount of outdoor air would be increased. If the value in every zone were less than the requirement and if there were no conflict with other control requirements, then the amount of outdoor air would be decreased.

Although the assumption on which the theory in this paper is based on not very restrictive, it would still be useful to conduct a controlled experiment designed to validate or invalidate the theory. One useful experiment would be to test the independence of the residence time distributions by conducting a tracer gas test on a set of rooms arranged in series. Different types of passages be-

tween the rooms such as open doorways, closed doorways, and vents could be tested. Another good experiment would be to conduct a tracer gas test on a single-zone system with either displacement ventilation or extreme short-circuiting and determine if the predicted effect of recirculating air is close to the actual effect.

Conclusions

1. This paper extends the theory of ventilation performance measurement to any building that can be treated as a set of interconnected chambers. A ventilation effectiveness parameter called relative air-change effectiveness is defined.

2. Three methods of calculating relative air-change effectiveness are described. One method is based on age of air measurements at three locations, while the other two are based on two age of air measurements and a flow rate measurement. Relations between relative air-change effectiveness and ventilation parameters proposed previously are derived.

3. The results of the theory illustrate that published conclusions about airflow patterns in buildings that are based on tracer gas tests may be incorrect because of the effects of recirculated air or air moving between zones.

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Appendix A: Age of Air Relations

Equation (4)

Consider a single zone with recirculation such as the one depicted in Figure 7. If this is a PFND system, then the transport delay across the zone is equal to the height of the zone divided by the velocity:

$$T = \frac{H}{v} \quad (\text{A1})$$

where H is the distance from the supply to the return and is the velocity of air in the zone. If there were no recirculation, then the age of air at the point h would

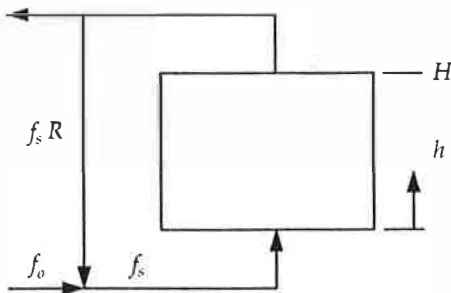


Fig. 7 Schematic diagram of a PFND system

simply be h/v . However, since only $1-R$ of the supply air is fresh while R of it is recirculated return air, the age of air at the point h is:

$$\alpha(h) = (1-R) \frac{h}{v} + R(1-R) \left(\frac{h}{v} + T \right) + R^2(1-R) \left(\frac{h}{v} + 2T \right) + R^3(1-R) \left(\frac{h}{v} + 3T \right) + \dots \quad (\text{A2})$$

which can be expressed as:

$$\alpha(h) = (1-R) \sum_{n=0}^{\infty} R^n \left(\frac{h}{v} + nT \right) \quad (\text{A3})$$

This as an algebraic-geometric series which converges to:

$$\alpha(h) = \frac{h}{v} + \frac{TR}{1-R} \quad (\text{A4})$$

if $|R| < 1$. Since $0 < R < 1$, the age of air at a point in the zone is described by Equation (A4). The mean age of air for a PFND system is determined by integrating the age of air at each point from the supply to the return and dividing by the distance as follows:

$$\bar{\alpha} = \frac{1}{H} \int_0^H \alpha(h) dh \quad (\text{A5})$$

Substituting Equation (A4) into Equation (A5) and integrating yields Equation (4).

Equation (5)

The age of air in the return duct is equal to τ_r . Therefore, the age of air in the supply duct is:

$$\alpha_s = \tau_r R \quad (\text{A6})$$

Substituting Definition 1 into Equation (A6) gives:

$$\alpha_s = \frac{M}{f_o} R \quad (\text{A7})$$

The supply flow rate and the outdoor air flow rate are related as follows:

$$f_o = f_s(1-R) \quad (\text{A8})$$

Combining Equations (3), (A7), and (A8) gives the following:

$$\alpha_s = \frac{TR}{1-R} \quad (\text{A9})$$

The age at a point in a system with an arbitrary airflow pattern is:

$$\alpha = \alpha_{R=0} + \frac{TR}{1-R} \quad (\text{A10})$$

due to the additivity property of age of air. Using the same argument, the volumetric mean age of air in a system with an arbitrary airflow pattern is given by Equation (5).

Appendix B: Air-Change Effectiveness Relations

Equation (9)

Substituting Equation (4) into Equation (8) yields the following:

$$\varepsilon_a = \frac{\tau_n (1-R)}{T (1+R)} \quad (\text{B1})$$

The relation between τ_n and T is as follows:

$$\tau_n = \frac{T}{1-R} \quad (\text{B2})$$

Substituting Equation (B2) into Equation (B1) yields Equation (9).

Equation (12)

From the additivity of age of air, the nominal turnover time is the sum of the age of the supply air and the local turnover time:

$$\tau_n = \alpha_s + T \quad (\text{B3})$$

Substituting T from Equation (B3) into the numerator of Equation (11) gives the following:

$$\varepsilon_r = \frac{\tau_n - \alpha_s}{2\bar{\alpha}_{R=0}} \quad (\text{B4})$$

Substitution $\bar{\alpha}_{R=0}$ from Equation (5) into Equation (B4) gives the following:

$$\varepsilon_r = \frac{\tau_n - \alpha_s}{2 \left(\bar{\alpha} - \frac{TR}{1+R} \right)} \quad (\text{B5})$$

Substituting Equation (A9) into Equation (B5) leads to Equation (12).

Equation (15)

Substituting the quantity $2\bar{\alpha}$ from Equation (12) into Equation (8) gives the following:

$$\varepsilon_a = \frac{\tau_n \varepsilon_r}{\tau_n - \alpha_s + 2\alpha_s \varepsilon_r} \quad (\text{B6})$$

Substituting α_s from Equation (A6) into Equation (B6) gives the following:

$$\varepsilon_a = \frac{\tau_n \varepsilon_r}{\tau_n - R\tau_n + 2R\tau_n \varepsilon_r} \quad (\text{B7})$$

Canceling τ_n in Equation (B7) yields Equation (15).

Equation (16)

For the system shown in Figure 1 with an artificial bypass, the nominal turnover time is related to the bypass factor as follows:

$$\tau_n = S\alpha_s + (1-S)\bar{\alpha} \quad (\text{B8})$$

Substituting $\bar{\alpha}$ from Equation (B8) into Equation (8) leads to the following:

$$\varepsilon_a = \frac{\tau_n (1-S)}{2(\tau_n - S\alpha_s)} \quad (\text{B9})$$

Substituting α_s from Equation (A6) into Equation (B9) and canceling τ_n leads to Equation (16).

Equation (18)

For the bypass system, the air in the zone is perfectly mixed. Therefore, the mean age of air in the zone is related to the bypass factor, the age of the supply air, and the local turnover time as follows:

$$\bar{\alpha} = \alpha_s + \frac{T}{1-S} \quad (\text{B10})$$

Solving for the quantity $1-S$ in Equation (B10) leads to the following:

$$\eta = \frac{T}{\bar{\alpha} - \alpha_s} \quad (\text{B11})$$

Comparing the right hand side of Equation (B11) with (Equation 13) shows that η is twice ε_r .