Indoor Air 1999; 9: 57–62
Printed in Denmark. All rights reserved

Evaluating Age from Arbitrary Forms of Injection Functions of Tracer

F. BONTHOUX[1], J. M. DESSAGNE[1] AND G. AUBERTIN[1]

Abstract The age of the air in a room is normally determined either from a pulse response or from a step change response (up or down). There are a certain number of problems involved in applying these two theoretical models, especially those associated with the duration of the injection, which must either be infinitely short or infinitely long. A hybrid method that consists of injecting a known quantity of tracer for a given time offers the advantages of both methods. The equation for calculating age is exact, regardless of the type of flow considered, and is derived from the expressions already established for a pulse response to which a correction is included to account for the tracer generation function. If a rectangular pulse is used for the injection, the solution is particularly simple.

Key words General ventilation; Age of air; Air change efficiency; Measurement; Tracer gas; Finite injection.

Received 21 May 1997. Accepted for publication 2 February 1998.

Nomenclature

τ_i	Age of the air at any given point
τ_e	Age of the air measured in the exhaust
$<\tau>$	Room mean age of the air
ε	Air exchange efficiency
$c(t)$	Signal of measured concentration
$C(t)$	Concentration signal after conditioning
$\mu^{(i)}$	ith order moment of $C(t)$
$X(t)$	Concentration signal obtained for a pulse
$\mu_{pu}^{(i)}$	ith order moment of a pulse response $X(t)$
$E(t)$	Injection function
$\psi^{(i)}$	ith order moment of $E(t)$
$Y(t)$	Signal obtained for an injection $E(t)$
n	Duration of an injection

Concept of Age and Air Exchange Efficiency

Stemming from the research carried out in the area of chemical reactor engineering (Danckwerts, 1953; Villermaux, 1982), the concept of age has been in use since the 1980s (Sandberg, 1981) to evaluate the performance of general ventilation systems. The age of the air (or the time elapsed since it entered in the room) at a given point in the room (τ_i), or more particularly in the extraction system (τ_e), can be measured using a tracer. A comparison of the internal ages can be used to identify stagnant zones (Roulet, 1991). It is possible to estimate the mean age of the air in the room ($<\tau>$), as well as the air exchange efficiency $\left(\varepsilon=\frac{\tau_e}{2<\tau>}\right)$ (Sandberg and Sjöberg, 1983; Niemela, 1992). This efficiency characterizes the type of air exchange in the room; a value of 100% indicating a piston type flow; more than 50% indicating a tendency to displacement; and less than 50% indicating the existence of shortcircuiting between the air inlet and outlet.

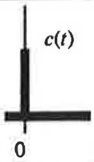
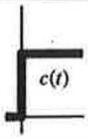
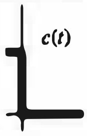
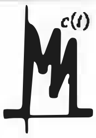
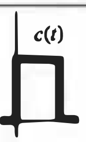
Age Measurement - Practical Considerations

Ages are calculated by recording the concentration response to an injection of tracer. Until now, equations for these calculations have only been established for pulse and step change injection modes (see upper part of Table 1). These different methods, each of which has its particular advantages and drawbacks, have been examined in comparative studies (Sutcliffe and Waters, 1990; Breum, 1992; Roulet and Cretton, 1992; Jung and Zeller, 1994), and the conclusions differ on certain aspects.

Not to mention the inevitable problems of mixing tracer and non-controlled air entries, step change

[1]*Heat Transfer and Ventilation Department, Institut National de Recherche et de Sécurité, Centre de Recherche et de Formaion, Avenue de Bougogne, B. P. N° 27, 54501 Vandoeuvre Cédex, France, Tel: 03-83-50-20-00, Fax: 03-83-50-21-84.*

Table 1 Expressions for calculating ages

Method of Injection		Values calculated from $\mu^{(i)}=\int_0^\infty C(t)\cdot t^i dt$		
		Function $C(t)$	Age τ_i (τ_e if extraction)	Spatial average of ages in the room $<\tau>$
Pulse		$c(t)$	$\frac{\mu^{(1)}}{\mu^{(0)}}$ in this case μ is μ_{pu}	$\frac{1}{2}\frac{\mu^{(2)}}{\mu^{(1)}}$
Step-up		$1-\frac{c(t)}{c(\infty)}$	$\mu^{(0)}$	$\frac{\mu^{(1)}}{\mu^{(0)}}$
Step-down		$\frac{c(t)}{c(0)}$		
Given injection function $E(t)$ of known form for interval $[0,n]$ and zero elsewhere		$c(t)$	$\frac{\mu^{(1)}-\frac{\psi^{(1)}}{\psi^{(0)}}\mu^{(0)}}{\mu^{(0)}}$ with $\psi^{(i)}=\int_0^n \mu^i E(u)du$	$\frac{1}{2}\frac{\mu^{(2)}-2\frac{\psi^{(1)}}{\psi^{(0)}}\mu^{(1)}+\left(2\left(\frac{\psi^{(1)}}{\psi^{(0)}}\right)-\frac{\psi^{(2)}}{\psi^{(0)}}\right)\mu^{(0)}}{\mu^{(1)}-\frac{\psi^{(1)}}{\psi^{(0)}}\mu^{(0)}}$
Rectangular pulse of duration N: $c(t)$=cst for interval $[0,n]$ and zero elsewhere		$c(t)$	$\frac{\mu^{(1)}-\frac{n}{2}\mu^{(0)}}{\mu^{(0)}}$	$\frac{1}{2}\frac{\mu^{(2)}-n\mu^{(1)}-\frac{n^2}{6}\mu^{(0)}}{\mu^{(1)}-\frac{n}{2}\mu^{(0)}}$

modes (up or down) seem to be the most reliable on account of the smoothed signal associated with the integrating effect of a continuous injection. The absence of any sudden variations limits the errors introduced during the numerical integration steps, an effect which is even more important when the sampling frequency is relatively low. On the other hand, these methods are costly in terms of time and tracer, and require a stable tracer generator.

Pulse injections have practical limitations. As it is not easy to predict the maximum concentration that will be observed, and in order to avoid saturating the analyser, the quantity of tracer used is usually reduced, meaning that the acquired signal is low in amplitude. In the case of tracers that can only be detected at high concentrations it is necessary to inject large quantities, which requires the use of either a high flow rate flowmeter or a volumetric buffer. Even if the exact quantity of tracer injected is not required to calculate the age distribution, it is needed to obtain other information such as transfer coefficients. However, these values will always be highly uncertain if the tracer pulse is generated for only a very short time with a flowmeter.

In this context, the advantages of a hybrid method combining both step and pulse injections are obvious. Such a method would allow a quantity of tracer to be generated for a given time without reaching a steady state. It would then be possible to use a reasonable amount of tracer to measure the concentrations with sufficient accuracy and, above all, to have a good idea beforehand of the maximum level of concentration that will be obtained (close to that obtained with a step change, the value of which can be estimated using the ventilation flow rate).

An Alternative to "Pulse" and "Step Change" Methods

The aim of this work is to present an alternative to the "pulse" and "step change" methods used for evalu-

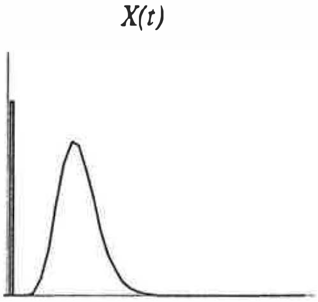


Fig. 1 Pulse response

ating the ages of air. The method proposed here is easy to implement, and can be used to perform this evaluation by means of an injection of finite duration (IFD) and of any known form. The calculations associated with this method are based on those already established for a pulse response, but do not require the use of the delicate deconvolution stage (Demoment, 1993; Max, 1972) needed to deduce the pulse response from the response to an IFD. A pulse response can be used to perform a detailed analysis of the air flow pattern when using, for example, the Residence Time Distribution model (Olander et al. 1995), but is not essential for age evaluation based on calculating the moments of the distribution (see upper part of Table 1).

Outline of the Method

If $X(t)$ is the pulse response (see Figure 1), the response, $Y(t)$, to an IFD, $E(t)$, can be evaluated by exploiting the properties of linear systems (addition, multiplication by a constant, and invariance to temporal translation). To do this, $E(t)$ is considered as resulting from a series of pulses, E_k, of different amplitudes, and $Y(t)$ is thus the weighted summation of the time-shifted responses $X(t)$ (see Figure 2). This operation is nothing more than the convolution of $X(t)$ by $E(t)$.

If, for reasons of mathematical stability, it is decided not to perform the deconvolution of $Y(t)$ by $E(t)$ to obtain the $X(t)$ required for the calculation of the moments, the problem is thus to find an expression to calculate the moments of $X(t)$.

It is possible to do so from the moments of $Y(t)$ by using the expression:

$$Y(t) = \sum_k E_k\, X(t-k)$$

The calculation of the moments

$$\int_0^\infty Y(t) \cdot t^i\, dt$$

then takes the form:

$$\sum_k E_k \int_0^\infty X(t-k) \cdot t^i\, dt$$

By choosing this last form it is possible to introduce the moments of $X(t)$. Indeed, transforming the integration variable $t' = t-k$ allows the integral:

$$\int_0^\infty X(t-k) \cdot t^i\, dt$$

to be written as:

$$\int_{-k}^\infty X(t') \cdot (t'+k)^i\, dt'$$

Furthermore, since $X(t)$ is zero for every time $t<0$, this latter integral is equivalent to:

$$\int_0^\infty X(t') \cdot (t'+k)^i\, dt'$$

Finally, if we develop $(t'+k)^i$, we obtain a combination of the moments of $X(t)$.

This short demonstration proves that it is possible to establish a relationship between the moments of $Y(t)$ and those of $X(t)$. The development that follows uses this same approach in a more complete way. The equations used to take an IFD into account were developed from the mathematical equation that defines the operation of the convolution of two signals. Amongst other things, this operation can be used to obtain the response of a linear system to a given in-

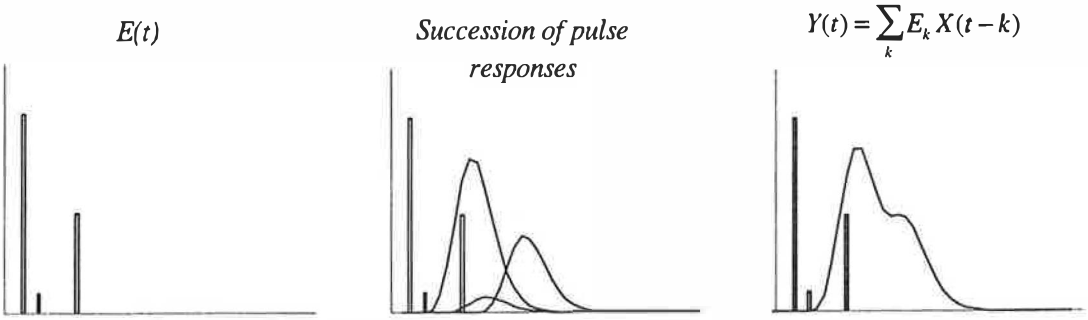


Fig. 2 Evaluation of the response to an IFD

put, based on the knowledge of its pulse response. This mathematical tool can be used to define the moments of the pulse response as a function of the moments of the IFD and the moments of the response of a system to this IFD. The knowledge of the moments of the pulse response can then be used to determine the age of the air.

Derivation of the Equations

The response $Y(t)$ of a linear system to an input signal $E(t)$ can be obtained through the convolution of the pulse response $X(t)$ and the input function.

$$Y(t)=E(t)\otimes X(t)=\int_0^t E(u).\, X(t-u)du \qquad (1)$$

The pulse response of a real system satisfies the following conditions:

$$X(t)=0 \text{ if } t\leq 0 \qquad (2)$$

and

$$X(t)\rightarrow 0 \text{ if } t\rightarrow\infty \qquad (3)$$

which will be replaced by $X(t)=0$ *if* $t> T$ for a sufficiently large T.

For the case studied here, where the injection is performed for a given period, the function $E(t)$ is defined as:

$$E(t)=0 \text{ if } t\leq 0 \qquad (4)$$

$E(t)=0$ *if* $t>n$ where n represents the duration of the injection. (5)

This last equation is used to reduce equation (1) in

$$Y(t)=\int_0^n E(u)\cdot X(t-u)du \qquad (6)$$

since, in this case:

$$\int_n^t E(u)\cdot X(t-u)\cdot du=0$$

From equations (3) and (6), it can be stated that $Y(t)=0$ if $t>T+n$ (7)

As is the case for a pulse response, it is possible to evaluate the *ith order moments* (from 0 to 2) *of the response* $Y(t)$.

$$\mu^{(i)}=\int_0^\infty Y(t)\cdot t^i\, dt \qquad (8)$$

Equation (7) can be used to show that:

$$\int_{T+n}^\infty Y(t)\cdot t^i\, dt=0$$

and to simplify equation (8) to:

$$\mu^{(i)}=\int_0^{T+n} Y(t)\cdot t^i\, dt$$

It can be observed that in order to evaluate $\mu^{(i)}$, if $X(t)\approx 0$ from time T, it is necessary to record $Y(t)$ for a period $T+n$.

Then, using equation (6), we obtain:

$$\mu^{(i)}=\int_0^{T+n} t^i \left(\int_0^n E(u)\cdot X(t-u)du\right)dt$$

and after rearrangement and permutation of the integrals:

$$\mu^{(i)}=\int_0^n E(u)\left(\int_0^{T+n} t^i\cdot X(t-u)dt\right)du$$

If we then replace the integration variable t by $t'+u$, the equation can be written:

$$\mu^{(i)}=\int_0^n E(u)\left(\int_{-u}^{T+n-u} (t'+u)^i\cdot X(t')dt'\right)du$$

For the sake of simplicity, the dummy variable t' can be called t once again. The integral $\int_{-u}^{T+n-u}\ldots$ can be separated into three components $\int_{-u}^{0}\ldots, \int_0^T\ldots \int_T^{T+n-u}\ldots$. Firstly, equation (2) shows that the quantity $\int_{-u}^{0}\ldots X(t)dt$ is always zero and, secondly, from equation (3) that $\int_0^n\left(\int_T^{T+n-u}\ldots X(t)dt\right)du$ is also zero since the t involved in the calculation of this integral are always greater than T. The equation can thus be written in the form:

$$\mu^{(i)}=\int_0^n E(u)\left(\int_0^T (t+u)^i\cdot X(t)dt\right)du$$

Using the equation to calculate the order moments 0, 1 and 2 of $Y(t)$ leads to the appearance of the terms:

$$\int_0^T X(t)dt,\ \int_0^T t\, X(t)dt,\ \int_0^T t^2\, X(t)dt$$

which are the moments of the pulse response $X(t)$ and which we will refer to as $\mu_{pu}^{(0)}, \mu_{pu}^{(1)}, \mu_{pu}^{(2)}$.

Hence, we can write:

$$\mu^{(0)}=\mu_{pu}^{(0)}\int_0^n E(u)du$$

$$\mu^{(1)}=\mu_{pu}^{(1)}\int_0^n E(u)du+\mu_{pu}^{(0)}\int_0^n uE(u)du$$

$$\mu^{(2)}=\mu_{pu}^{(2)}\int_0^n E(u)du+2\mu_{pu}^{(1)}\int_0^n uE(u)du+\mu_{pu}^{(0)}\int_0^n u^2\, E(u)du$$

Similarly, the moments 0 at 2 of the injection function $E(t)$ will be referred to as $\psi^{(0)}, \psi^{(1)}, \psi^{(2)}$

Hence, we can write

$$\mu^{(0)}=\mu_{pu}^{(0)}\psi^{(0)}$$

$$\mu^{(1} =\mu_{pu}^{(1)}\psi^{(0)}+\mu_{pu}^{(0)}\psi^{(1)}$$

$$\mu^{(2)}=\mu_{pu}^{(2)}\psi^{(0)}+2\mu_{pu}^{(1)}\psi^{(1)}+\mu_{pu}^{(0)}\psi^{(2)}$$

And it can be deduced that:

$$\mu_{pu}^{(0)}=\frac{1}{\psi^{(0)}}\mu^{(0)}$$

$$\mu_{pu}^{(1)}=\frac{1}{\psi^{(0)}}\left[\mu^{(1)}-\frac{\psi^{(1)}}{\psi^{(0)}}\mu^{(0)}\right]$$

$$\mu_{pu}^{(2)}=\frac{1}{\psi^{(0)}}\left[\mu^{(2)}-2\frac{\psi^{(1)}}{\psi^{(0)}}\mu^{(1)}+\left(2\left(\frac{\psi^{(1)}}{\psi^{(0)}}\right)^2-\frac{\psi^{(2)}}{\psi^{(0)}}\right)\mu^{(0)}\right]$$

Knowledge of the moments of the pulse response can then be used to determine the age of the air using the equations defined for a pulse response (see Table 1).

Results

The calculation of the ages from an injection $E(t)$ of any known form and duration n can be performed using the following expression:

$$\tau_i=\frac{\mu_{pu}^{(1)}}{\mu_{pu}^{(0)}}=\frac{\mu^{(1)}-\frac{\psi^{(1)}}{\psi^{(0)}}\mu^{(0)}}{\mu^{(0)}}$$

$$<\tau>=\frac{1}{2}\frac{\mu_{pu}^{(2)}}{\mu_{pu}^{(1)}}=\frac{1}{2}\frac{\mu^{(2)}-2\frac{\psi^{(1)}}{\psi^{(0)}}\mu^{(1)}+\left(2\left(\frac{\psi^{(1)}}{\psi^{(0)}}\right)^2-\frac{\psi^{(2)}}{\psi^{(0)}}\right)\mu^{(0)}}{\mu^{(1)}-\frac{\psi^{(1)}}{\psi^{(0)}}\mu^{(0)}}$$

Even if the form of the function can be chosen, it should be borne in mind that one must be able to do so with the tracer generator available. If the shape of the function is imposed, it must be known for the entire duration of the injection.

This latter case is one where the age measurement is performed using a tracer generator that delivers a variable flowrate. An example of such an application is a generator based on the evaporation of a liquid where the measurement from a pulse injection is not easy (tracer cannot be stored for a massive injection) and where the time needed to achieve stable operation and the autonomy of the generator could prevent the use of a step change method. In this case, the injection function could be obtained by continuous weighing.

In a more common situation, where the tracer generation can be programmed, the most realistic injection function is the constant rate injection. In addition to this case, which will be discussed below, ramp, triangular, and other functions are possible, but their usefulness remains to demonstrate given the higher costs that their implementation might entail.

In the case of a constant injection, $E(t)=b$,

we obtain $\psi^{(0)}=b\cdot n$, $\psi^{(1)}=b\cdot\frac{n2}{2}$, $\psi^{(2)}=b\cdot\frac{n3}{3}$

and hence:

$$\tau_i=\frac{\mu^{(1)}-\frac{n}{2}\mu^{(0)}}{\mu^{(0)}}$$

and

$$<\tau>=\frac{1}{2}\frac{\mu^{(2)}-n\mu^{(1)}+\frac{n^2}{6}\mu^{(0)}}{\mu^{(1)}-\frac{n}{2}\mu^{(0)}}$$

It can be seen that if $n\to 0$, the ages tend to the values described for a pulse injection.

Table 1 presents the expressions used to calculate the age of air: the upper part deals with the conventional methods whereas the lower part covers the methods presented in this paper.

Conclusion

The new method presented in this paper offers a number of undeniable practical advantages both for the generation of a tracer and the measurement of its concentration. It is possible to generate the tracer for a fixed duration, and to account for any possible generation instabilities. The quantity of tracer used and the injection duration can be adapted to make the most of the measurement devices. The additional numerical operations that need to be carried out do not have any financial impact on the measurements since the use of computers is already required for applying conventional methods.

In a number of cases, employing this new method could improve measurement accuracy. Therefore, the evaluation of measurement uncertainties should be examined in more detail in a future study. In the case of tracer generation at a constant rate, the optimal value of the *"room time constant/injection duration"* ratio as a function of the performance capabilities of the equipment used is an essential feature.

In addition, this method is of great interest when the tracers used do not have the flexibility of tracer gas. Finally, the concept of age is not limited to the study of ventilation, and these results could be applied to other fields.

References

Breum, N.O. (1992) "A comparison in measurement of local and room mean age of air", *Air Infiltration Review,* **13**(2), 8–9.

Danckwerts, P.V. (1953) "Continuous flow systems - Distribution of Residences Times", *Chemical Engineering Science,* **2**(1), 1–13.

Demoment, G. (1993) "Problèmes inverses et problèmes mal-posés", *La revue des Laboratoires d'essais,* Vol. **34**, Février, 7–10.

Jung, A. and Zeller, M. (1994) "An analysis of different tracer gas techniques to determine the air exchange efficiency in a mechanically ventilated room". In: *Proceedings of Roomvent' 94,* Krakow, Poland, Vol. **4**, pp. 316–322.

Niemela, R. (1992) "Fled measurements of air change efficiency in large reinforced plastics plants". In: *Proceedings of Roomvent' 92,* Aalborg, Denmark, Vol. **4**, pp. 283–292.

Max, J. (1972) *Méthodes et techniques de traitement du signal et applications aux mesures physiques,* Paris, Masson et Cie.

Olander, L., Dessagne, J.M., Bonthoux F. and Leclerc J. P. (1995) "Residence time distribution characterises ventilation in industrial premises", *Chemical Technology Europe,* Vol. 2, No. 3.

Roulet, C.A. and Cretton, P. "Field comparison of age of air measurement techniques". In: *Proceedings of Roomvent' 92,* Aalborg, Denmark, Vol. **3**, pp. 213–229.

Roulet, C.-A., Compagnon, R. and Jakob, M. (1991) "A simple method using tracer gas to identify the main airflow and contaminant path within a room", *Indoor Air,* **3**, 311–322.

Sandberg, M. (1981) "What is ventilation efficiency", *Build and Environment,* **16**(2), 123–135.

Sandberg, M. and Sjöberg, M. (1983) "The use of moments for assessing air quality in ventilated rooms", *Build and Environment,* **18**, 4, 181–197.

Sutcliffe, H.C. and Waters, J.R. (1990) "Error in the measurement of local and room mean age using tracer gas methods". In: *Proceedings of 11th AIVC Conference,* Belgirate, Italy, Vol. **2**, pp. 279–292.

Villermaux, J. (1982) *Génie de la réaction chimique conception et fonctionnement des réacteurs,* Paris, Technique et documentation Lavoisier.