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A STATE SPACE MODEL OF INDOOR TEMPERATURE DISTRIBUTIONS

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Abstract

To control the indoor thermal environment within the comfortable range, the dynamic temperature distributions and flows of room air must be correctly predicted. While the CFD (Computational Fluid Dynamics) technique can be used to carry out such a prediction task, its drawback is also obvious: too time-consuming. To solve this problem, the dynamic temperature distributions can be predicted with some fixed air flow fields calculated with CFD codes. That is, sacrifice the dynamics of indoor air flows and only preserve the dynamics of the temperature distributions. This paper discusses a state space model that can be used to predict the indoor dynamic temperature distributions. By dividing the room into several air zones and still using the fixed flow field, a much faster dynamic model of the indoor temperature distributions is constructed. This model can be easily used for designing and testing different indoor climate control systems.

1. Introduction

One of the most important factors that influences working efficiency is indoor thermal comfort which is a function of air temperature, air velocity, etc. To keep the indoor thermal environment within the comfortable range, the dynamics of air temperatures and velocities should be properly predicted and controlled. This is not easy since there exist temperature and velocity distributions in the room (Fig. 1) and in a real building room, there is usually only one temperature sensor that is almost always mounted on the wall near the door. Thus, without the correct dynamic modelling of indoor temperature distributions and air flows, the quality of the indoor thermal comfort control would be doubtful.

The fact of indoor temperature distributions easily makes us think of dividing the room into several air zones, each of which is supposed to be well mixed and represented by one temperature (Fig. 2). Since the temperature of one air zone is influenced by the adjacent air zones through the mass and heat exchanges, the quantities of these exchanges have to be determined before the temperatures of all air zones can be calculated. Until present, it seems that the most ideal method to determine these quantities is the CFD (Computational Fluid Dynamics) theory which takes the physical phenomena of both convection and diffusion into consideration. This is understandable since air is a kind of fluid. The different starting point lies in that instead of dividing the room air volume only into several air zones, the CFD theory demands it be divided into thousands of air zones or grid cells if we put them with CFD terms (Fig. 3). The CFD method increases not only the number of air zones but also the computing time drastically. For example, using the CFD method to predict the steady state air flow field in a room represented with 40x30x30 grid cells needs about 10 CPU hours of a SUN Sparc Station IPX or an equivalent 486-PC (number of iterations = 1500). If the dynamic air flow fields are to be simulated, hundreds of CPU hours will be needed. This is too time-consuming to be afforded by the air-conditioning field.

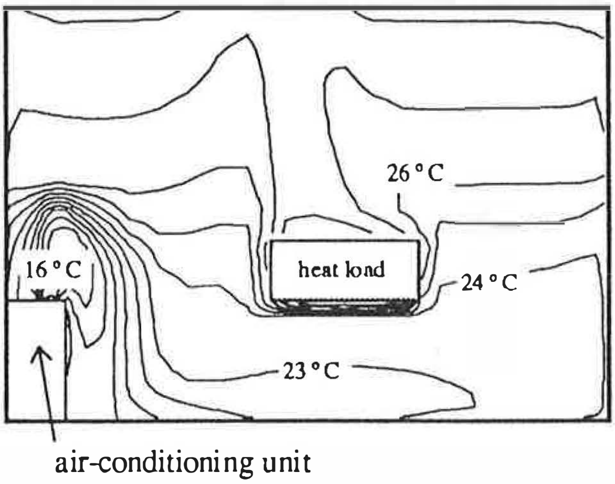

Fig. 1 The temperature distribution of an air-conditioned room

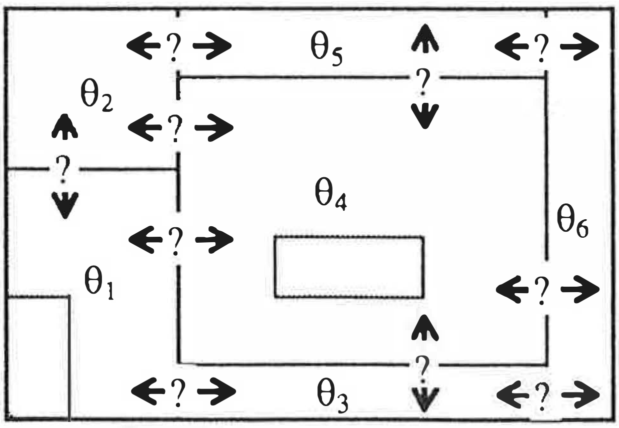

Fig. 2 Dividing the room into air zones

But whenever we talk about control, the dynamic control is almost always implied. To control the indoor thermal environment, the dynamic temperature distributions of air must be known. This requirement and the above time-consuming fact of CFD simulations challenge us with a contradictory problem. This problem can not be solved until the much faster yet cheap computers come into reality. At the moment, we have to find out some trade-off methods to solve this problem.

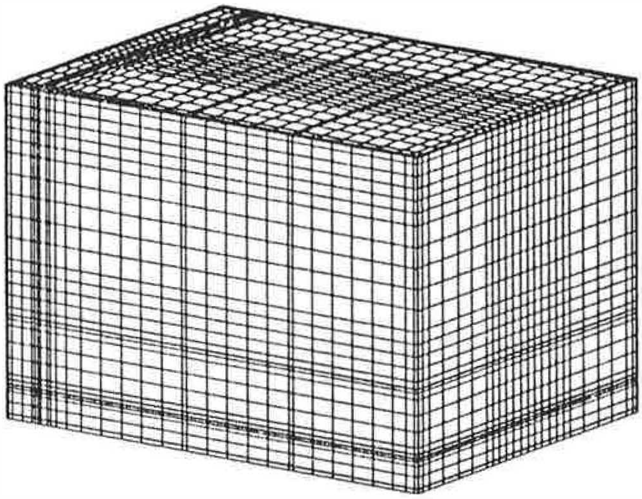

Fig. 3 Grid cells needed by a CFD simulation

Many of us might have noticed such a phenomenon: for some typical heating (or cooling) situations, although the indoor air flow field may vary with time due to the changes of heating (or cooling) power and the boundary conditions, the prevailing air flow field that has dominant effects on air temperature distributions is relatively stable. This reminds us that the flow field might be fixed and used for the dynamic temperature calculations. That is, in the following energy balance equation:

$$\frac{\partial \rho H}{\partial t} + \mathbf{div}\left(\rho \vec{V} H - \Gamma_{H,eff}\,\mathrm{grad} H\right) = S_H \tag{1}$$

where

t = time,

ρ = density of air,

H = enthalpy of air, and $H = C_p\theta$,

C_p = specific heat of air,

θ = temperature of air,

$\vec{V}$ = air velocity vector,

$\Gamma_{H,eff}$ = effective exchange coefficient of energy,

S_H = source rate of energy,

if $\vec{V}$ and $\Gamma_{H,eff}$ are precalculated with a CFD code and fixed, only H is left to be calculated and much computing time will be saved. The price paid with this method is the sacrifice of the dynamics of indoor air flows. The benefit is that a fast computation is obtained. This idea has been validated to be feasible by the authors of references 1 and 2.

From the standpoints of faster predictions of indoor dynamic temperature distributions and more accurate evaluations of building heating (or cooling) loads, the above method really gives more realistic results than a one point model, which assumes that the indoor air temperature distribution is homogeneous. But, if our objective is to control indoor dynamic temperature responses, this method will still be too time consuming since the time step has to be short (e.g. 10 minutes) and the temperatures of thousands of grid cells have to be calculated. Besides, this type of model is not suitable for control system designs.

As a matter of fact, we are not interested in too detailed information of indoor temperature distributions since most people are insensitive to minor temperature changes (e.g. < 0.5) inside the room. The division of the room into several air zones would give enough accuracy for the control system design if their temperatures could be calculated correctly. If this could be done, the computing time would be further reduced significantly and a much simpler dynamic model would be available.

2. Representing the indoor temperature distribution with air zones

According to the CFD theory, the calculation of temperature, velocity, etc. of a grid cell is based on the solution of the mass, momentum and energy balance equations. Obviously, if the mass is balanced for every single cell of all cells, then the mass is also balanced for any arbitrary patch or group of cells. Starting from this point, we may reorganise the thousands of grid cells used by CFD simulations and transform them into several air zones containing many cells (Fig. 4). By supposing each air zone is well mixed and is depicted with one temperature, the room thermal response model will be much simplified.

If we remember that the border between two adjacent air zones is also the border of the grid cells that are located on both sides of the border, then we may still take advantage of the precalculated flow field. That is, the values of $\bar{V}$ and $\Gamma_{\mathrm{H,eff}}$ of the grid cells situated next to the air-zone borders can be used to calculate the mass and heat exchanges between one air zone and another (Fig. 5).

From CFD theory, we know that the final disrctized form of equation (1) has the following form:

$$a_{\mathrm{P},i,j,k}\theta_{i,j,k}(t) = a_{\mathrm{E},i,j,k}\theta_{i+1,j,k}(t) + a_{\mathrm{W},i,j,k}\theta_{i-1,j,k}(t) + a_{\mathrm{N},i,j,k}\theta_{i,j+1,k}(t)$$

$$+a_{\mathrm{S},i,j,k}\theta_{i,j-1,k}(t) + a_{\mathrm{T},i,j,k}\theta_{i,j,k+1}(t) + a_{\mathrm{B},i,j,k}\theta_{i,j,k-1}(t) + b \qquad (2)$$

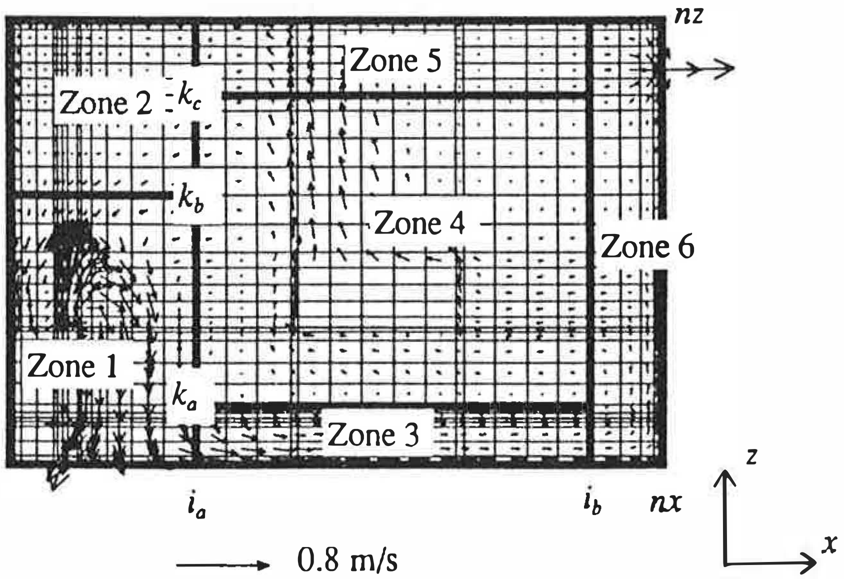

Fig.4 Reorganising grid cells into air zones

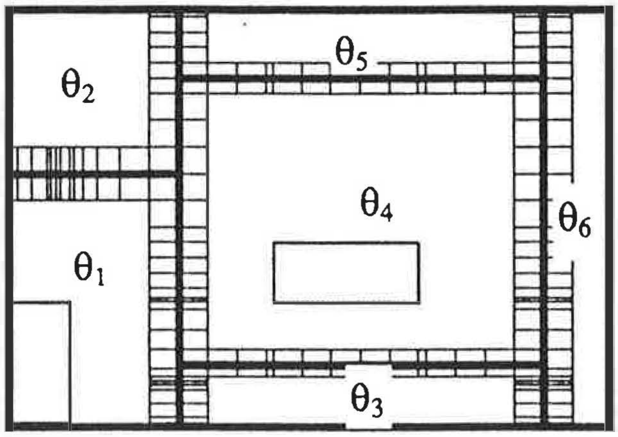

Fig. 5 Borders of air zones

where

$$a_{\mathrm{E},i,j,k} = \mathrm{C_p}(\Delta y\Delta z)_{i,j,k}\left(\mathrm{D}_{\mathrm{e},i,j,k} + \rho\mathrm{MAX}\left[-u_{i,j,k},0\right]\right) \quad (3)$$

$$a_{\mathrm{W},i,j,k} = \mathrm{C_p}(\Delta y\Delta z)_{i,j,k}\left(\mathrm{D}_{\mathrm{w},i,j,k} + \rho\mathrm{MAX}\left[u_{i-1,j,k},0\right]\right) \quad (4)$$

$$a_{\mathrm{N},i,j,k} = \mathrm{C_p}(\Delta x\Delta z)_{i,j,k}\left(\mathrm{D}_{\mathrm{n},i,j,k} + \rho\mathrm{MAX}\left[-v_{i,j,k},0\right]\right) \quad (5)$$

$$a_{\mathrm{S},i,j,k} = \mathrm{C_p}(\Delta x\Delta z)_{i,j,k}\left(\mathrm{D}_{\mathrm{s},i,j,k} + \rho\mathrm{MAX}\left[v_{i,j-1,k},0\right]\right) \quad (6)$$

$$a_{\mathrm{T},i,j,k} = \mathrm{C_p}(\Delta x\Delta y)_{i,j,k}\left(\mathrm{D}_{\mathrm{t},i,j,k} + \rho\mathrm{MAX}\left[-w_{i,j,k},0\right]\right) \quad (7)$$

$$a_{\mathrm{B},i,j,k} = \mathrm{C_p}(\Delta x\Delta y)_{i,j,k}\left(\mathrm{D}_{\mathrm{b},i,j,k} + \rho\mathrm{MAX}\left[w_{i,j,k-1},0\right]\right) \quad (8)$$

$$a^0_{\mathrm{P},i,j,k} = \frac{\mathrm{C_p}\rho(\Delta x\Delta y\Delta z)_{i,j,k}}{\Delta t} \quad (9)$$

$$b = \mathrm{S_H}(\Delta x\Delta y\Delta z)_{i,j,k} + a^0_{\mathrm{P},i,j,k}\,\theta_{i,j,k}(t-1) \quad (10)$$

$$a_{\mathrm{P},i,j,k} = a_{\mathrm{E},i,j,k} + a_{\mathrm{W},i,j,k} + a_{\mathrm{N},i,j,k} + a_{\mathrm{S},i,j,k} + a_{\mathrm{T},i,j,k} + a_{\mathrm{B},i,j,k} + a^0_{i,j,k} \quad (11)$$

In the above formulae, Δx, Δy and Δz are the length, width and height of a grid cell; MAX[a, b] is equal to the bigger one between variables a and b; terms $\mathrm{D}_{\mathrm{e},i,j,k}$, $\mathrm{D}_{\mathrm{w},i,j,k}$, $\mathrm{D}_{\mathrm{n},i,j,k}$, $\mathrm{D}_{\mathrm{s},i,j,k}$, $\mathrm{D}_{\mathrm{t},i,j,k}$, $\mathrm{D}_{\mathrm{b},i,j,k}$ represent the effective diffusion effects of heat and are functions of the turbulent and laminar diffusivities of heat. Δt is the discrete time step. i, j and k are the discrete coordinates of the cell. E (e), W (w), N (n), S (s), T (t) and B (b) mean east, west, north, south, top and bottom respectively.

For air zone 1 with temperature θ_1 and air volume V_1 in Fig. 5, we have

$$\frac{\partial V_1\rho C_p\theta_1}{\partial t} = q_1 +$$

$$\sum_{k=k_b}^{k=k_b}\sum_{j=1}^{j=ny}\sum_{i=1}^{i=i_a}\mathrm{C_p}(\Delta x\Delta y)_{i,j,k}\left[\left(\mathrm{D}_{\mathrm{t},i,j,k} + \rho\mathrm{MAX}\left[-w_{i,j,k},0\right]\right)\theta_2 - \left(\mathrm{D}_{\mathrm{t},i,j,k} + \rho\mathrm{MAX}\left[w_{i,j,k},0\right]\right)\theta_1\right]$$

$$+\sum_{k=1}^{k=k_a}\sum_{j=1}^{j=ny}\sum_{i=i_a}^{i=i_a}\mathrm{C_p}(\Delta y\Delta z)_{i,j,k}\left[\left(\mathrm{D}_{\mathrm{e},i,j,k} + \rho\mathrm{MAX}\left[-u_{i,j,k},0\right]\right)\theta_3 - \left(\mathrm{D}_{\mathrm{e},i,j,k} + \rho\mathrm{MAX}\left[u_{i,j,k},0\right]\right)\theta_1\right]$$

$$+\sum_{k=k_a}^{k=k_b}\sum_{j=1}^{j=ny}\sum_{i=i_a}^{i=i_a}\mathrm{C_p}(\Delta y\Delta z)_{i,j,k}\left[\left(\mathrm{D}_{\mathrm{e},i,j,k} + \rho\mathrm{MAX}\left[-u_{i,j,k},0\right]\right)\theta_3 - \left(\mathrm{D}_{\mathrm{e},i,j,k} + \rho\mathrm{MAX}\left[u_{i,j,k},0\right]\right)\theta_1\right]$$

where q_1 is the total heat sources for zone 1. It is the summation of the heat sources that are located inside zone 1 such as computers, printers, room occupants, etc. and the heat coming from boundary walls. The above equation can be further written as:

$$\frac{\partial V_1 \rho C_p \theta_1}{\partial t} + a_{P,1}\theta_1 = a_{T,12}\theta_2 + a_{E,13}\theta_3 + a_{E,14}\theta_4 + q_1 \tag{12}$$

where

$$a_{T,12} = \sum_{k=k_b}^{k=k_b} \sum_{j=1}^{j=ny} \sum_{i=1}^{i=i_e} C_p(\Delta x \Delta y)_{i,j,k}\left(D_{t,i,j,k} + \rho MAX\left[-w_{i,j,k}, 0\right]\right) \tag{13}$$

$$a_{E,13} = \sum_{k=1}^{k=k_a} \sum_{j=1}^{j=ny} \sum_{i=i_e}^{i=i_e} C_p(\Delta y \Delta z)_{i,j,k}\left(D_{e,i,j,k} + \rho MAX\left[-u_{i,j,k}, 0\right]\right) \tag{14}$$

$$a_{E,14} = \sum_{k=k_a}^{k=k_b} \sum_{j=1}^{j=ny} \sum_{i=i_e}^{i=i_e} C_p(\Delta y \Delta z)_{i,j,k}\left(D_{e,i,j,k} + \rho MAX\left[-u_{i,j,k}, 0\right]\right) \tag{15}$$

$$a_{P,1} = a_{T,12} + a_{E,13} + a_{E,14} \tag{16}$$

Here the obtain of the formula of $a_{P,1}$ supposes that the air mass of zone 1 is balanced.

Similarly, for the air zones 2, 3, 4, 5 and 6, we have:

$$\frac{\partial V_2 \rho C_p \theta_2}{\partial t} + a_{P,2}\theta_2 = a_{B,21}\theta_1 + a_{E,24}\theta_4 + a_{E,25}\theta_5 + q_2 \tag{17}$$

$$\frac{\partial V_3 \rho C_p \theta_3}{\partial t} + a_{P,3}\theta_3 = a_{W,31}\theta_1 + a_{T,34}\theta_4 + a_{E,36}\theta_6 + q_3 \tag{18}$$

$$\frac{\partial V_4 \rho C_p \theta_4}{\partial t} + a_{P,4}\theta_4 = a_{W,41}\theta_1 + a_{W,42}\theta_2 + a_{B,43}\theta_3 + a_{T,45}\theta_5 + a_{E,46}\theta_6 + q_4 \tag{19}$$

$$\frac{\partial V_5 \rho C_p \theta_5}{\partial t} + a_{P,5}\theta_5 = a_{W,52}\theta_2 + a_{B,54}\theta_4 + a_{E,56}\theta_6 + q_5 \tag{20}$$

$$\frac{\partial V_6 \rho C_p \theta_6}{\partial t} + a_{P,6}\theta_6 = a_{W,63}\theta_3 + a_{W,64}\theta_4 + a_{W,65}\theta_5 + q_6 \tag{21}$$

The coefficients in equations (17) through (21) can be obtained in the same way as in equation (12).

Now we see that, by taking advantage of the CFD calculations, a simple dynamic model for predicting indoor temperature distributions is derived.

3. State space model for control system designs

Whenever a control system is designed, a model of the form of either transfer function or state space is often needed. While transfer functions are suitable for SISO (Single Input Single Output) system designs with classical control theory, state space models are usually used for MIMO (Multi Input Multi Output) systems in modern control theory. With more than indoor temperature to be predicted, the indoor thermal environment is obviously an MIMO plant that has to be represented a state space model.

To be consistent with the modern control theory, we now change the definition of x from coordinate to state in a space, and denote:

$$x_1 = \theta_1,\ x_2 = \theta_2,\ x_3 = \theta_3,\ x_4 = \theta_4,\ x_5 = \theta_5,\ x_6 = \theta_6, \tag{22}$$

$$\dot{x}_1 = \frac{\partial\theta_1}{\partial t},\ \dot{x}_2 = \frac{\partial\theta_2}{\partial t},\ \dot{x}_3 = \frac{\partial\theta_3}{\partial t},\ \dot{x}_4 = \frac{\partial\theta_4}{\partial t},\ \dot{x}_5 = \frac{\partial\theta_5}{\partial t},\ \dot{x}_6 = \frac{\partial\theta_6}{\partial t}, \tag{23}$$

$$x = [x_1, x_2, x_3, x_4, x_5, x_6]^T \tag{24}$$

where T represents transpose. Equations (12) and (17) through (21) can be written as a state space model:

$$\dot{x} = Ax + Bu \tag{25}$$

where

$$A = \begin{bmatrix} -\frac{a_{P,1}}{a_1} & \frac{a_{T,12}}{a_1} & \frac{a_{E,13}}{a_1} & \frac{a_{E,14}}{a_1} & 0 & 0 \\ \frac{a_{B,21}}{a_2} & -\frac{a_{P,2}}{a_2} & 0 & \frac{a_{E,24}}{a_2} & \frac{a_{E,25}}{a_2} & 0 \\ \frac{a_{W,31}}{a_3} & 0 & -\frac{a_{P,3}}{a_3} & \frac{a_{T,34}}{a_3} & 0 & \frac{a_{E,36}}{a_3} \\ \frac{a_{W,41}}{a_4} & \frac{a_{W,42}}{a_4} & \frac{a_{B,43}}{a_4} & -\frac{a_{P,4}}{a_4} & \frac{a_{T,45}}{a_4} & \frac{a_{E,46}}{a_4} \\ 0 & \frac{a_{W,52}}{a_5} & 0 & \frac{a_{B,54}}{a_5} & -\frac{a_{P,5}}{a_5} & \frac{a_{E,56}}{a_5} \\ 0 & 0 & \frac{a_{W,63}}{a_6} & \frac{a_{W,64}}{a_6} & \frac{a_{W,65}}{a_6} & -\frac{a_{P,6}}{a_6} \end{bmatrix}$$

$$B = \left[\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4}, \frac{1}{a_5}, \frac{1}{a_6}\right], \qquad u = [q_1, q_2, q_3, q_4, q_5, q_6]^T$$

$$a_1 = V_1\rho C_p,\ a_2 = V_2\rho C_p,\ a_3 = V_3\rho C_p,\ a_4 = V_4\rho C_p,\ a_5 = V_5\rho C_p,\ a_6 = V_6\rho C_p.$$

4. Conclusions and discussions

To control the indoor thermal environment within the comfortable range, a dynamic model that should both reflect the air temperature distributions and be simple enough is needed for control system designs. Combining the CFD theory, this paper discusses a state space model with the air-zone method.

Given the boundary conditions and heat sources, the steady state indoor air flow field is first calculated. Then the room is divided into several air zones according to the air flow pattern. By supposing each air zone is well mixed and is represented by one temperature, the energy balance equation is easily obtained for each air zone. The mass and energy exchanges between one air zone and another is given by the precalculated flow field.

It is obvious that this method relies upon the CFD calculation. A number of flow fields should be prepared to deal with the significant changes of the flow field. Fortunately, the number of flow patterns is limited for most real building rooms. These flow fields can be precalculated with CFD codes and saved as a database.

The state space model derived in this paper can be easily used for control system designs.

5. References

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