ELSEVIER

Energy and Buildings 29 (1998) 17–22

ENERGY AND BUILDINGS

# Criterial number by naturally ventilated tunnels

Jurij Modic *

*Faculty of Mechanical Engineering, University of Ljubljana, Aškerčeva 6, Ljubljana, Slovenia*

Received 1 December 1997; accepted 9 January 1998

**Abstract**

All internal combustion engines produce exhaust gases containing noxious compounds: carbon monoxide (CO), nitrogen oxides ($NO_x$), carbon oxides ($C_xHy$) and smoke. With the help of a mathematical model the concentrations of some dangerous substances at the end of the tunnel were calculated, and were replaced by a criterial number. A corresponding computer program was also developed thus enabling quick and simple calculations of some concentrations and the criterial number. Finally, a criteria for the border between natural and forced ventilation is presented as a correlation between the criterial number and other parameters.

*Keywords:* Tunnel; Ventilation; Emission; Mathematical model; Criterial number

## 1. Introduction

The number and aggregate length of road and rail tunnels in the world are growing all the time. In some cases, the tunnels are placed under the city or under the buildings in the city. On a less grandiose scale, many tunnels have been and are being built to ease traffic congestion and segregate hazardous goods traffic in urban areas. So we may say that the tunnel is also a part of the building. No matter what the method of construction or type of traffic carried, all tunnels designed to be used by human beings need effective ventilation. In the very shortest ones, or in a single bore tunnel with two lanes of unidirectional traffic, the piston effect of passing traffic may be adequate for the purpose, but in many other cases, fans are also needed to assist air exchange.

In normal operating conditions, a tunnel ventilation system has two main functions: to keep the air breathable and to keep it cool. The first of these is most important in the road tunnels. All internal combustion engines produce exhaust gases containing toxic compounds and smoke. Since the noxious substances in the vehicular tunnels affect the health of users, the amount of these substances should not exceed acceptable levels. Calculating methods were provided for adequate by diluting the concentration of noxious and dangerous contaminants. To dilute the concentrations of obnoxious or dangerous contaminants to acceptable levels, vehicular tunnels require ventilation, which may be provided by natural means, traffic induced 'piston effect', or mechanical equipment. Naturally ventilated and traffic induced systems are considered adequate for tunnels of relatively short length and low traffic volume (or density). Long and heavily travelled tunnels should have mechanical ventilation systems [1].

But even in tunnels used exclusively by electric trains, the heat emitted by traffic, stationary machinery and human bodies would soon cause temperatures to rise to intolerable levels without effective ventilation.

Naturally ventilated tunnels rely chiefly on meteorological conditions to maintain a satisfactory environment within the tunnel. The piston effect of traffic provides additional airflow when the traffic is moving. The chief meteorological condition affecting environment is the pressure differential between two portals of a tunnel created by differences in elevation, ambient temperatures, and/or wind. Unfortunately, none of these factors can be relied on for continued, consistent results. A sudden change in wind direction or velocity can negate all these natural effects, including the piston effect. The sum total of all pressures must be of sufficient magnitude to overcome the tunnel resistance, which is influenced by tunnel length, coefficient of friction, hydraulic radius, and air density [2].

In comparison with mechanical ventilation, in the case of natural ventilation the energy demand is also very low.

The main objective of this work is the presentation of circumstances in a road tunnel. For a general case, the governing differential equation was made and prepared for unidirectional traffic in the vehicular tunnel. Based on the effects between vehicles in hydrodynamical circumstances in the tunnel, the velocity of air and later the fields of concentration

---

* Tel.: +356 61 1771315; fax: +356 61 218567.

*PII* S0378-7788(98)00018-8

of three most important contaminants were calculated. Finally, the criterial number for determining the relation between the most important functions influencing the conditions at the end of the vehicular tunnel were made. A corresponding computer program was developed. The analysis made by the computer program and by the dimensionless number shoved, that the natural ventilation is sufficient for 2500 to 3500 m, depending on traffic and local circumstances, in some cases even up to 4500 m long.

## 2. Basic equations

### 2.1. General

The scheme, shown in Fig. 1, represents the theoretical circumstances in a tunnel.

To obtain the common emission of noxious substances, we must know their specific emission, the number and the character of vehicles. On the basis of the vehicles emission, number of vehicles, their velocity, part of trucks and characteristics of the tunnel, we obtain the common emission, and on the base of allowable concentration, the air demand. So, there is an important connection between traffic density, tunnel length and velocity of vehicles.

To calculate the air demand, we also need traffic data and characteristics of the tunnel. So we first obtain the air velocity, and then on the basis of the tunnel cross-section also the air demand.

Finally, we compare the air demand, needed to dilute the obnoxious substances, with an air inlet, caused with induction. On this basis, we obtain the real concentrations of carbon monoxide, nitrogen oxides and soot (smoke).

So we need the tunnel characteristics (length $L$, cross-section $A$, road gradient $s$, altitude $H$, etc.), the data about traffic (number of personal cars $M_o$, number of trucks $M_t$, air resistance coefficients $c_{wo}$ and $c_{wt}$, front sections $A_{vo}$ and $A_{vt}$, etc.), emission data of noxious substances and the limit values for pollutants from internal standards $k_{\mathrm{COallow}}$, $k_{\mathrm{NOxallow}}$ and $k_{\mathrm{Sallow}}$.

We also need the data about tunnel and about traffic for obtain the air inlet.

### 2.2. Air velocity

The governing differential equation [3,4] is

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}t} =& -\left(\frac{\lambda}{D_h}+\frac{\Sigma\xi}{L}\right)\frac{u^2}{2}\mathrm{sign}\,u \pm \frac{\Delta p}{\rho L}\\ &-\frac{1}{LA}(c_{wo}{}^*A_{vo}N_{os}+c_{wt}{}^*A_{vt}N_{ts})\frac{u^2}{2}\mathrm{sign}\,u\\ &-\frac{1}{LA}(c_{wo}{}^*A_{vo}N_{o1}+c_{wt}{}^*A_{vt}N_{t1})\\ &\times\left(\frac{1-t}{L_x}|v_1|\right)\frac{(v_1+u)^2}{2}\mathrm{sign}(v_1+u)\\ &\times\frac{1}{LA}(c_{wo}{}^*A_{vo}N_{o2}+c_{wt}{}^*A_{vt}N_{t2})\\ &\times\left(1-\frac{t}{L-L_x}|v_2|\right)\frac{(v_2-u)^2}{2}\mathrm{sign}(v_2-u)\pm\frac{S_p}{\rho LA}\end{aligned} \tag{1}$$

If we take into account a single-bore tunnel with two lanes of unidirectional traffic, with the characteristics of tunnel (length $L$, cross section $A$, altitude $H$, road gradient $s$) and characteristics of traffic (part of trucks $p_t$, total numbers of vehicles $M$, coefficients of air resistance $c_{wo}$ and $c_{wt}$, length of vehicles $l_o$ and $l_t$, and frontal section of vehicles $A_{vo}$ and $A_{vt}$), from Eq. (1) [3] is

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}t} =& -\left(\frac{\lambda}{D_h}+\frac{\Sigma\xi}{L}\right)\frac{u^2}{2}+\frac{1}{LA}(c_{wo}{}^*A_{vo}N_o\\ &+c_{wt}{}^*A_{vt}N_t)\left(1-\frac{t}{L-L_x}|v|\right)\frac{(v-u)^2}{2}\end{aligned} \tag{2}$$

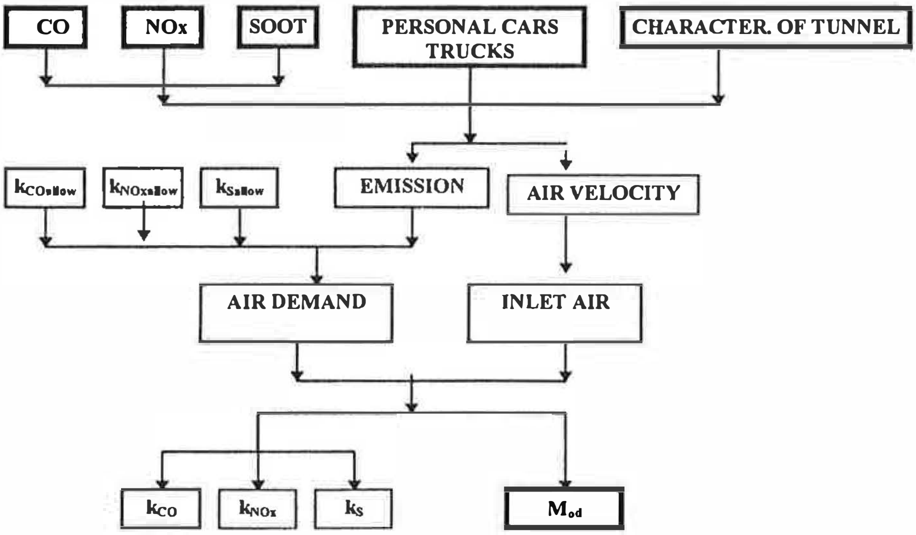

Fig. 1. Scheme—circumstances in tunnel.

If we consider momentous circumstances in Eq. (2) for unidirectional traffic, we may predict a quasi steady state conditions, that means

$$t=0 \text{ and } \frac{\mathrm{d}u}{\mathrm{d}t}=0$$

So, we obtain the following equation from Eq. (2)

$$\frac{1}{LA}(c_{\mathrm{wo}}{}^{*}A_{\mathrm{vo}}N_{\mathrm{o}}+c_{\mathrm{wt}}{}^{*}A_{\mathrm{vt}}N_{\mathrm{t}})\frac{(v-u)^2}{2} -\left(\frac{\lambda}{D_{\mathrm{h}}}+\frac{\Sigma\xi}{L}\right)\frac{u^2}{2}\left(\pm\frac{\Delta p}{L\rho}\right)=0 \qquad (3)$$

and air velocity $u$ as effect of induction [5], is

$$\begin{aligned}&u^2\left\{\frac{1}{2LA}\left[\left(1-\frac{p_t}{100}\right)\frac{M}{L}l_{\mathrm{o}}c_{\mathrm{wo}}{}^{*}A_{\mathrm{vo}}+\frac{p_{\mathrm{t}}}{100}\frac{M}{L}l_{t}c_{\mathrm{wt}}{}^{*}A_{\mathrm{vt}}\right]-0.50\left(\frac{\lambda}{D_{\mathrm{h}}}+\frac{1.35}{L}\right)\right\}\\&-u\left\{\frac{1}{LA}\left[\left(1-\frac{p_{\mathrm{t}}}{100}\right)\frac{M}{L}l_{\mathrm{o}}c_{\mathrm{wo}}{}^{*}A_{\mathrm{vo}}+\frac{p_{\mathrm{t}}}{100}\frac{M}{L}l_{\mathrm{t}}c_{\mathrm{wt}}{}^{*}A_{\mathrm{vt}}\right]v\right\}\\&+v^2\left\{\frac{1}{2LA}\left[\left(1-\frac{p_{\mathrm{t}}}{100}\right)\frac{M}{L}l_{\mathrm{o}}c_{\mathrm{wo}}{}^{*}A_{\mathrm{vo}}+\frac{p_{\mathrm{t}}}{100}\frac{M}{L}l_{t}c_{\mathrm{wt}}{}^{*}A_{\mathrm{vt}}\right]\right\}\pm\frac{\Delta p}{L\rho}=0\end{aligned} \qquad (4)$$

If in Eq. (4) same coefficients we change with symbols

$$f_1=\left\{\frac{1}{2LA}\left[\left(1-\frac{p_t}{100}\right)\frac{M}{L}l_{\mathrm{o}}c_{\mathrm{wo}}{}^{*}A_{\mathrm{vo}}+\frac{p_{\mathrm{t}}}{100}\frac{M}{L}l_{\mathrm{t}}c_{wt}{}^{*}A_{vt}\right]-0.50\left(\frac{\lambda}{D_h}+\frac{1.35}{L}\right)\right\} \qquad (5)$$

$$f_2=\left\{\frac{1}{LA}\left[\left(1-\frac{p_{\mathrm{t}}}{100}\right)\frac{M}{L}l_{\mathrm{o}}c_{\mathrm{wo}}{}^{*}A_{\mathrm{vo}}+\frac{p_{\mathrm{t}}}{100}\frac{M}{L}l_{\mathrm{t}}c_{\mathrm{wt}}{}^{*}A_{\mathrm{vt}}\right]v\right\} \qquad (6)$$

$$f_3=\left\{\frac{1}{2LA}\left[\left(1-\frac{p_{\mathrm{t}}}{100}\right)\frac{M}{L}l_{\mathrm{o}}c_{\mathrm{wo}}{}^{*}A_{\mathrm{vo}}+\frac{p_{\mathrm{t}}}{100}\frac{M}{L}l_{\mathrm{t}}c_{\mathrm{wt}}{}^{*}A_{\mathrm{vt}}\right]v^2\right\} \qquad (7)$$

the simplified solution of equation for air velocity is

$$u=\frac{-\frac{f_2}{2}+\left[\left(\frac{f_2}{2}\right)^2-f_1f_3\right]^{0.50}}{f_1} \qquad (8)$$

From Eqs. (5)–(7) we see, that the velocity of air $u$ is a function of more parameters:

$$u=u(L, A, D_{\mathrm{h}}, \lambda, \Sigma\xi, \Delta p, p_{\mathrm{t}}, c_{\mathrm{wo}}{}^{*}, c_{\mathrm{wt}}{}^{*}, M, l_o, l_{\mathrm{t}}) \qquad (9)$$

### 2.3. Fresh air inlet

By multiplying velocity $u$ and tunnel cross section $A$ we obtain the ratio of inlet air $Z_u$

$$Z_u=A\,u=A\frac{-\frac{f_2}{2}+\left[\left(\frac{f_2}{2}\right)^2-f_1f_3\right]^{0.50}}{f_1} \qquad (10)$$

### 2.4. Allowable concentrations

Referring to international standards the limit values for pollutants in the vehicular tunnel atmosphere are as follows:
$k_{\mathrm{CO\ allow}}=100$ ppm
$k_{\mathrm{NO}_x\ \mathrm{allow}}=25$ ppm
$k_{\mathrm{S\ allow}}=7\cdot10^{-3}$ m

### 2.5. Air demand

Thus air demand for CO, $NO_x$ and soot dilution [3], is

$$Z=\sum_n\left[\left(Q_{\mathrm{CO}}\frac{278}{k_{\mathrm{CO\ allow}}}+Q_{\mathrm{NO}_x}\frac{278}{k_{\mathrm{NO}_x\ \mathrm{allow}}}\right)+Q_{\mathrm{d}}\frac{278}{10^{6}k_{\mathrm{S\ allow}}}\right] \qquad (11)$$

The function dependence of $Z$ is

$$Z=Z(q_{\mathrm{CO}}, f_L, f_{\mathrm{v}}, f_{\mathrm{s}}, f_{\mathrm{H}}, f_{\mathrm{a}}, D, k_{\mathrm{CO\ allow}}, \ldots) \qquad (12)$$

## 3. Criterial number

All the important functions are found in equations for $Z_u$ (Eq. (10)) and $Z$ (Eq. (11)), influencing the circumstances in the tunnel. In both cases they have the same dimension. Thus we may introduce ratio, or the criterial number

$$M_{\mathrm{od}}=\frac{Z_u}{Z} \tag{13}$$

or

$$M_{\mathrm{od}}=\frac{A\dfrac{\dfrac{f_2}{2}+\left[\left(\dfrac{f_2}{2}\right)^2-f_1f_3\right]^{0.50}}{f_1}}{\sum_n\left[\left(Q_{\mathrm{CO}}\dfrac{278}{k_{\mathrm{CO\,allow}}}+Q_{\mathrm{NO}_x}\dfrac{278}{k_{\mathrm{NO}_x\,\mathrm{allow}}}\right)+Q_{\mathrm{d}}\dfrac{278}{10^{*}k_{\mathrm{S\,allow}}}\right]} \tag{14}$$

Eqs. (13) and (14) are formally very simple, but we must taking into account, that instead of $f_1$, $f_2$ and $f_3$ the Eq. (5), (63), and Eq. (7) are needed. This problem we may simplify by a developed computer program. All the results were proved by experiments [3].

Values for the criterial number can be different:

$M_{\mathrm{od}}=1$

In this case $Z_u$ equals $Z$, which means, that the concentrations of pollutants are just within allowable limits.

$M_{\mathrm{od}}>1$

$Z_u>Z$. The concentrations of pollutants are under allowable limits, but the energy consumption for movement of vehicles is greater (because of $u^2$).

$M_{\mathrm{od}}<1$

$Z_u<Z$. We need a mechanical ventilation, which means a great energy consumption for forced ventilation.

## 4. Computer program

The program calculate al the important parameters in the tunnel, except the criterial number, has possibility to, The algorithm is shown in Fig. 2.

We have the possibility of changing the basic data (($\lambda$, $D_h$, $c_{wo}$, $A_{vo}$, $l_o$, $c_{wt}$, $A_{vt}$ and $l_t$). In the beginning the basic values are introduced in the main program, but they can be changed. Following is the data input for the concrete tunnel: name, cross-section, altitude, year of opening, numbers of vehicles, road gradient, part of trucks, velocity of vehicles and length of tunnel. Finally, we get the results:

- air velocity [m/s]
- fresh air inlet [$m^3/s$]
- carbon monoxide emission [$m^3/h$]
- air demand for CO ($Z_{\mathrm{CO}}$) [$m^3/s$]
- carbon monoxide concentration [ppm]
- nitrogen oxides emission [$m^3/h$]
- air demand for $NO_x$ ($Z_{\mathrm{NO}x}$) [$m^3/s$]
- nitrogen oxides concentration [ppm]
- soot emission [$m^3/h$]
- air demand for soot ($Z_S$) [$m^3/s$]
- coefficient of extinction [$m^{-1}\cdot 10^{-3}$]
- total air demand [$m^3/s$]
- criterial number [ – ]
- maximal length of tunnel ($L_{\mathrm{Mod}}$) [m]

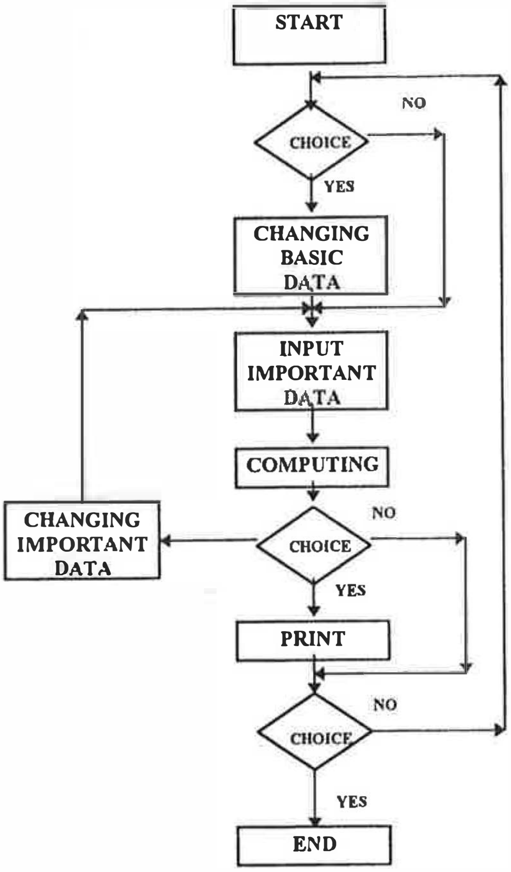

Fig. 2. Scheme of computer program.

## 5. Computation results

### 5.1. Carbon monoxide concentration

We can compute the concentration of carbon monoxide, concentration of nitrogen oxides and concentration of soot at

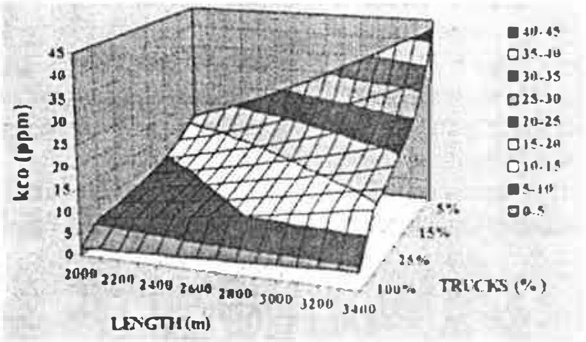

Fig. 3. CO concentration as function of $p_t$, $v = 60$ km/h, $s = 2\%$, $y = 2000$, $M = 600$ h$^{-1}$.

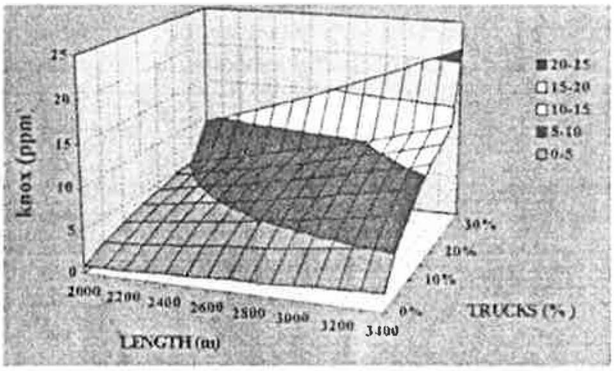

Fig. 4. NO$_x$ concentration as function of $p_t$, $v = 60$ km/h, $s = 2\%$, $y = 2000$, $M = 600$ h$^{-1}$.

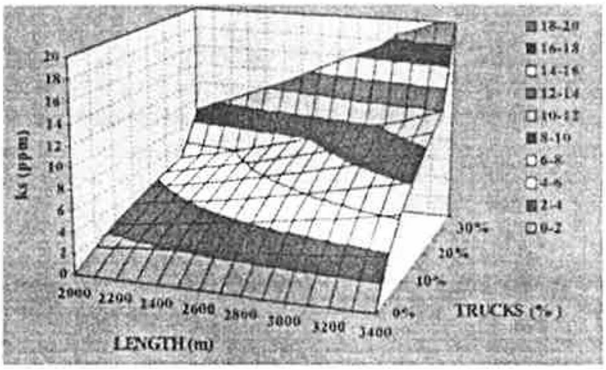

Fig. 5. Soot concentration as function of $p_t$, $v = 60$ km/h, $s = 2\%$, $y = 2000$, $M = 600$ h$^{-1}$.

the end of the tunnels with different length, with the application (referring to [3]). The basic values are: density of traffic $M = 600$ h$^{-1}$, velocity of vehicles $v = 60$ km/h, road gradient $s$2%, year of tunnel opening $y = 2000$, part of trucks $p_t = 0\% \div 100\%$, cross section of the tunnel $A = 55$ m$^2$, and altitude $H = 500$ m. If $p_t$ is increasing, the concentration of carbon monoxide decreases. The results are shown in Fig. 3.

### 5.2. Nitrogen oxides concentration

If $p_t$ is increasing, the concentration of nitrogen oxides increases. The results are shown in Fig. 4.

### 5.3. Soot concentration

If $p_t$ is increasing, the concentration of soot increases. The results are shown in Fig. 5.

### 5.4. Using the criterial number

A great advantage in using the criterial number is that we need only one figure to analyse the circumstances in the tunnel. But in this case, we do not differentiate between particular concentrations. We notice in our case, that the concentration of smoke represents the greatest danger. This concentration exceeds the critical value by $p_t = 20\%$ at a 2700-m long tunnel, but other concentrations are under allowable values. The value of the critical number is 0.93, which means, that at least one concentration is over allowable limit. All concentrations are computed by vehicles travelling with velocity of 60 km/h.

We compute the criterial number for the same conditions. The important results are shown in Fig. 6, which substitutes Figs. 3–5. The criterial number by $p_t = 20\%$ is smaller as 1 at a length of 2700 m.

If we consider $p_t = \text{const}$, we may show the criterial number as a function of velocity. If the $p_t = 20\%$, we obtain the results shown in Fig. 7. With velocity 40 km/h, the maximal length is 2200 m.

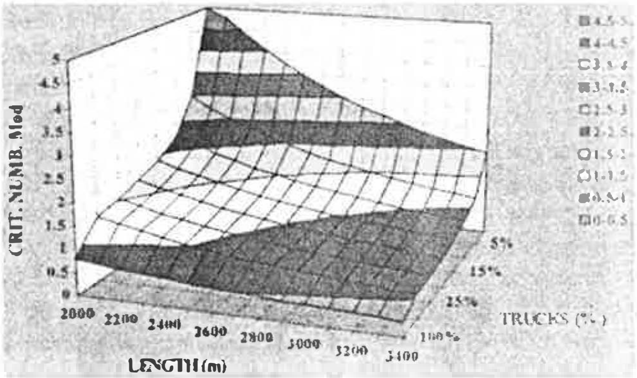

Fig. 6. Criterial number $M_{od}$ as a function of $p_t$, $v = 60$ km/h, $s = 2\%$, $y = 2000$, $M = 600$ h$^{-1}$.

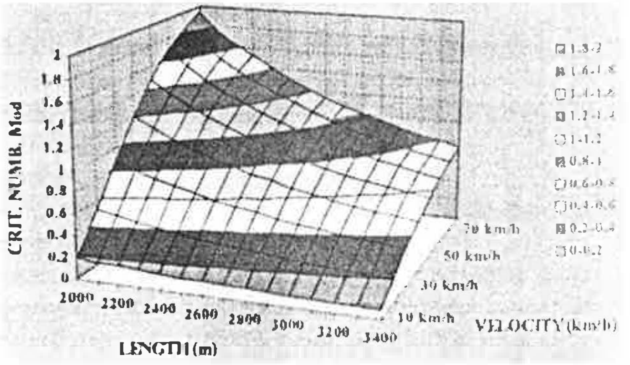

Fig. 7. Criterial number $M_{od}$ as a function of velocity, $p_t = 20\%$, $s = 2\%$, $y = 2000$, $M = 600$ h$^{-1}$.

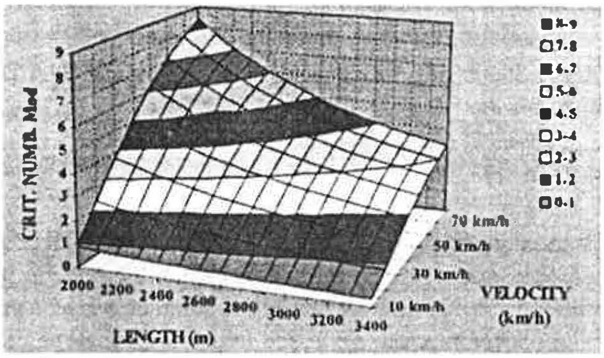

Fig. 8. Criterial number $M_{od}$ as a function of velocity, $p_t = 15\%$, $s = 2\%$, $y = 2000$, $M = 600\ h^{-1}$.

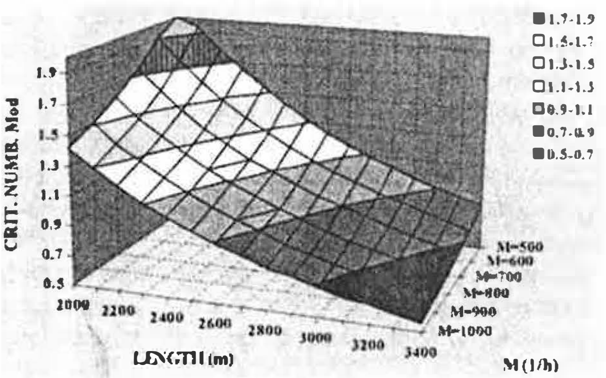

Fig. 9. Criterial number $M_{od}$ as a function of $M$, $v = 60$ km/h, $p_t = 15\%$, $s = 2\%$, $y = 2000$.

If the part of truck is $p_t = 15\%$, road gradient $s = \pm 0\%$, and the year of opening $y = 2020$, we obtain the results as shown in Fig. 8. We see that the maximal length of the tunnel with a velocity of 30 km/h, they would be more than 3400 m.

If we consider $v = \text{const}$ (60 km/h), and: $p_t = 20\%$, $s = 2\%$, $y = 2000$, $M = 500\ h^{-1} \div 1000\ h^{-1}$, the air velocity $u$ is a function of $M$ (Fig. 9).

By number of the vehicles $M = 500\ h^{-1}$ and with the velocity 60 km/h the maximal length of the tunnel is about 3000 m, and by $M = 1000\ h^{-1}$ and the same velocity the maximal length is 2400 m.

It is also possible to take other function relations into account.

## 6. Conclusion

We find out, that by analysing air velocities, concentrations of pollutants and other significant parameters we need many particulars and calculation proceedings to calculate circumstances in the tunnel. However, by using the criterial number, the proceedings are minimised. This is a simple way to get the most important data and the length where the natural ventilation of vehicular tunnel is sufficient. The analysis of circumstances in the tunnel with the help of the criterial number is quick and simple. It is not necessary to compute three different concentrations, it is enough to compute the criterial number $M_{od}$. If $M_{od} > 1$, all the concentrations of pollutants are under allowable limits and forced ventilation is not necessary. If $M_{od} < 1$, one or more concentrations are at the upper allowable limit and forced ventilation is necessary.

From expressions (9) and (12) it is obvious, that a tunnel with unidirectional traffic has some advantages. All calculations, their graphic presentations and comparison by known models show, that this is a convenient base for controlling air velocity and estimating a possibility of natural ventilating. If the density of traffic is under 600 $h^{-1}$, and the velocity is exceeding 40 km/h, the concentrations of pollutants are under an allowable limit by a length of the tunnel up to 2400 m. In this case the road gradient is not important. If the density of traffic is higher than 70 $h^{-1}$ and the road gradient higher than +4%, the limit of natural ventilation is under a length of 3000 m, but higher than 2500 m.

## 7. Nomenclature

| | | |
|---|---|---|
| $A$ | Cross-section | $m^2$ |
| $A_{vo}$ | Front section (p. c.) | $m^2$ |
| $A_{vt}$ | Front section (truck) | $m^2$ |
| $c_{wo}$ | Air resistance coef. (p. c.) | – |
| $c_{wt}$ | Air resistance coeff. (truck) | – |
| $H$ | Tunnel altitude | m |
| $L$ | Tunnel length | m |
| $l_o$ | Personal car length | m |
| $l_t$ | Truck length | m |
| $M$ | Total number of vehicles | $h^{-1}$ |
| $p_t$ | Truck part | % |
| $u$ | Air velocity | m/s |
| $v$ | Vehicle velocity | km/h |
| $y$ | Year of tunnel opening | – |
| $Z$ | Fresh air demand | $m^3/s$ |
| $Z_u$ | Inlet air ratio | $m^3/s$ |

## References

[1] I. Johansson, Flexibility in Motion Tunnels and Tunnelling, April 1990, pp. 63–65.

[2] Enclosed Vehicular Facilities ASHRAE Applications Handbook (SI), 1995.

[3] J. Modic, Velocity Field and Concentration of Noxious Substances in Tunnels Dissertation, Faculty of Mechanical Engineering, Ljubljana, July 1997, Mentor prof. dr. P. Novak.

[4] K. Pucher, Abbremsung der Luftbewegung bei einem Brand in einem mit Strahlventilatoren belüftenten Tunnel, ÖIAZ 3/1979, Graz.

[5] J. Modic, P. Novak, Generalised Model of Air Movement in Tunnel, Journal of Mechanical Engineering, Ljubljana, Slovenia, 1977/79, pp. 375–384.