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A state space model for predicting and controlling the temperature responses of indoor air zones

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Abstract

Indoor temperature distributions and air flows lead to the variation of local thermal comfort from place to place. To have more precise predictions and better control of the thermal conditions in the working zone where the room occupant sits and works, a both detailed and fast model of the dynamic indoor temperature distributions is needed. Unfortunately, very few papers studied such models due to the complexity of fluid (air) flows. This paper discusses a zonal model which is derived from computational fluid dynamics (CFD) and the output of a CFD code. The model is validated with experimental results. In order to design better control systems, the zonal model is transformed into a state space representation form. One example is given on how the state space model can be used for temperature predictions and more precise temperature controls.

Keywords: Air temperature distribution; Dynamic zonal model; State space model; Temperature prediction and control: Commercial buildings; Comfort; Europe; Simulation; Laboratory; Indoor temperature distribution; Air zonal model

1. Introduction

It is well-known that there exist indoor temperature distributions in building rooms (Fig. 1). The temperature of the 'working zone' where the room occupant sits and works is usually different from that of the 'discharge zone' of the conditioning air and that of the 'temperature sensor zone'. Our literature study shows that almost all air-conditioning control systems have not taken such temperature distributions into consideration since a dynamic and applicable model in such respect is simply not available. That is, the indoor air volume is still modeled as 'one point' or 'perfectly mixed' until recently. This negligence of indoor temperature distributions is one of the main reasons of the malfunction of the air-conditioning control system, which in turn causes the complaints of the room occupant about indoor thermal comfort. Therefore to have more precise predictions of indoor thermal comfort and better control of indoor thermal conditions, a both detailed and fast model of the dynamic indoor temperature distributions is needed.

So far, CFD (Computational Fluid Dynamics) is the best tool for predicting detailed indoor temperature distributions. But it is both too time consuming and not cost effective for studying the performances of control systems. Further, a detailed model of indoor temperature distributions like CFD is really not necessary since people are not sensitive to minor temperature changes (e.g., <0.5°C). Therefore, people have been trying to find a zonal model for the indoor temperature responses in the past years [1–4]. The basic idea of the zonal model is to divide the room air volume into several air zones each of which is assumed to be perfectly-mixed and is assigned one temperature. But the critical point of the zonal model is that the exact mass and heat exchanges between two adjacent air zones can not be determined realistically. In the models presented by Hemmi [1] and van der Kooi and Förch [2], such air mass exchange had to be determined by measurement, which was and is very difficult to carry out in reality. The zonal models given by Dalicieux [3] and Inard et al. [4] were based on the degradation of fluid mechanics equations. All these zonal models were not presented for control system designs, i.e., they did not have the form of either transfer function or state space representation which is required when a control system is designed.

Refs. [5,6] studied the prediction of indoor dynamic temperature distributions by using a fixed-flow-field obtained from a CFD calculation. Although good agreement between calculations and measurements was obtained and a great

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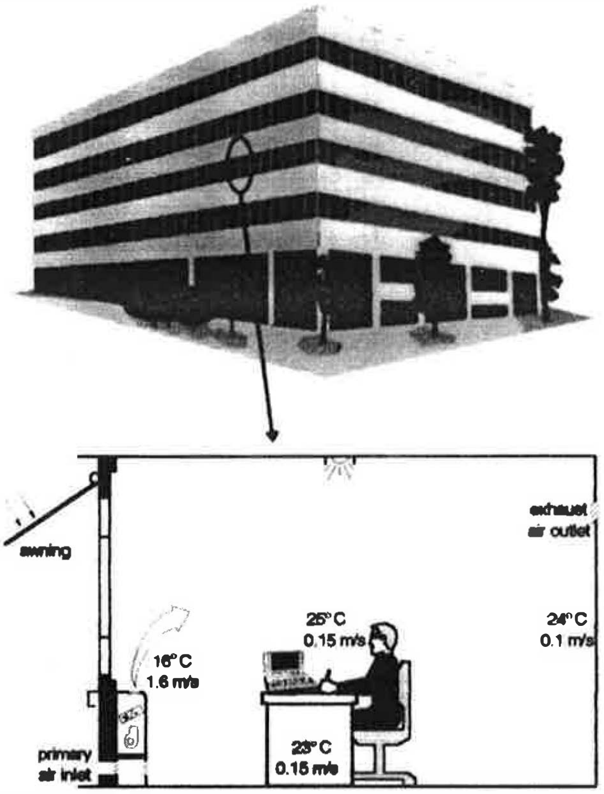

Fig. 1. Indoor temperature distribution of a room.

amount of computing time was saved, the model developed could still not be used by control system designers.

This paper presents a zonal model of dynamic indoor temperature responses. It is then transformed into a state space form for control system designs. One of the features of this model is that it does not ignore the important heat exchange between two air zones caused by turbulence, which was not considered by Refs. [1–3].

2. One basic assumption

We may have noticed such a phenomenon from our experience of daily life and experiments that for a typical heating (or cooling) situation, although the indoor air flow in some room locations may vary with time due to the existence of turbulence and disturbances, the prevailing air flow field that has dominant effects on air mass transportation and temperature distribution is relatively stable [5–7]. This phenomenon is especially apparent when the heating (or cooling) air is supplied through an air-conditioning unit with a fan, i.e., the indoor air movement is mainly driven by the supply air.

From the experience of using CFD codes, we also know that due to the nonlinearity of the momentum equations, under-relaxation has to be introduced in order to get converged results. This means that we intentionally slow down the solving process in order to get the correct air velocities. In fact, it is the solving process of the momentum equations (the velocity components) that consumes most of the computing time of a CFD simulation.

What we face now is such a situation: on the one hand, the solving process of velocities takes huge amount of time; on the other hand, the indoor air flow pattern does not change much for forced convection air flows. Then, why should we spend so much computing time on calculating the air velocities at every time step. If we could use a fixed air flow field for dynamic temperature calculations, then only the energy equation is left to be solved and a great of amount of computing time should be saved.

Three questions will have to be answered when the approximation of fixed-flow-field is made: (1) Can this method give satisfactory predictions of indoor dynamic temperature responses? (2) Can computing time be saved significantly, or can the computing time satisfy our requirement on dynamic simulations of temperature responses? (3) Can this model be used for control system designs? According to Refs. [5–7], the answers to the first two questions are *yes*. The answer to the third question is *no*.

From CFD theory, we know that the energy conservation equation is:

$$\frac{\partial \rho \theta}{\partial t}+\mathrm{div}\left(\rho \vec{V}\theta-\Gamma_{\mathrm{H,eff}}\,\mathrm{grad}\;\theta\right)=\frac{S_{\mathrm{H}}}{C_p} \tag{1}$$

where $\Gamma_{\mathrm{H,eff}}=\lambda/C_p+\rho\nu_t/\sigma_{\mathrm{H}}$ is the effective heat exchange coefficient.

When the air flow domain is divided into thousands of finite air volumes (a grid is constructed) and Eq. (1) is discretized, the following difference equation can be obtained:

$$\begin{aligned} a_{\mathrm{P}}\theta_{\mathrm{P}}(t)=&a_{\mathrm{E}}\theta_{\mathrm{E}}(t)+a_{\mathrm{W}}\theta_{\mathrm{W}}(t)+a_{\mathrm{N}}\theta_{\mathrm{N}}(t)+a_{\mathrm{S}}\theta_{\mathrm{S}}(t)\\ &+a_{\mathrm{T}}\theta_{\mathrm{T}}(t)+a_{\mathrm{B}}\theta_{\mathrm{B}}(t)+b \end{aligned} \tag{2}$$

Detailed descriptions of the coefficients a_{P}, a_{E}, a_{W}, a_{N}, a_{S}, a_{T}, a_{B} and b can be found in Patankar [8].

What we should remember is that the velocity field

$$\vec{V}$$

and turbulence viscosity ν_t of a steady-state air flow field are precalculated with a CFD code.

3. Air zonal model

According to the CFD theory, if the mass is balanced for every single finite air volume of the grid, then the mass is also balanced for any arbitrary group of finite air volumes. Starting from this point, we may reorganize the thousands of finite volumes used by CFD simulations and transform them into several air zones.

Fig. 2 shows a cooling situation in a typical air-conditioned room. The air flow pattern is precalculated with a CFD code. The division of the room air volume into air zones is obtained

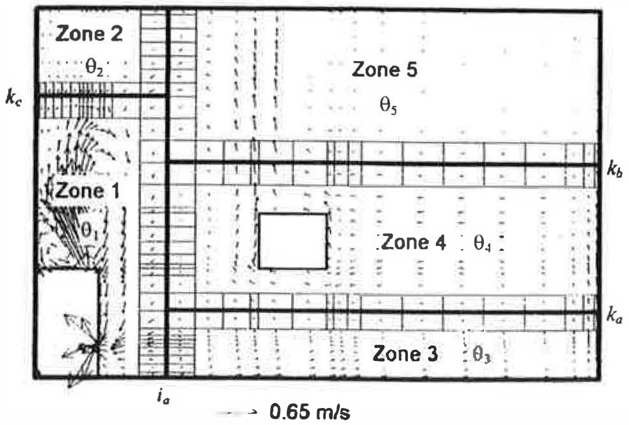

Fig. 2. Air zones in a room for a cooling case.

based on the air flow pattern and the steady state temperature field calculated with the CFD code.

By supposing that each air zone is perfectly mixed and is depicted with one temperature, the room thermal response model can be represented with several temperatures and thus enormously simplified.

It is obvious that the border between two adjacent air zones is also the border of the grid cells that are located on both sides of the border. The velocities $\vec{V}$ and exchange coefficients $\Gamma_{\mathrm{H,eff}}$ of these cells can be used to calculate the mass and heat exchanges between two air zones.

Suppose each air zone is well mixed and only has one temperature. For zones 1 to 5, which have air temperatures θ_1, θ_2, θ_3, θ_4 and θ_5, and air volumes V_1, V_2, V_3, V_4 and V_5, we have:

$$\frac{\partial V_1 \rho C_p \theta_1}{\partial t} + a_{\mathrm{P},1}\theta_1 = a_{\mathrm{T},12}\theta_2 + a_{\mathrm{E},13}\theta_3 + a_{\mathrm{E},14}\theta_4 + a_{\mathrm{E},15}\theta_5 + b_1 \quad (3)$$

$$\frac{\partial V_2 \rho C_p \theta_2}{\partial t} + a_{\mathrm{P},2}\theta_2 = a_{\mathrm{B},21}\theta_1 + a_{\mathrm{E},25}\theta_5 + b_2 \quad (4)$$

$$\frac{\partial V_3 \rho C_p \theta_3}{\partial t} + a_{\mathrm{P},3}\theta_3 = a_{\mathrm{W},31}\theta_1 + a_{\mathrm{T},34}\theta_4 + b_3 \quad (5)$$

$$\frac{\partial V_4 \rho C_p \theta_4}{\partial t} + a_{\mathrm{P},4}\theta_4 = a_{\mathrm{W},41}\theta_1 + a_{\mathrm{B},43}\theta_3 + a_{\mathrm{T},45}\theta_5 + b_4 \quad (6)$$

$$\frac{\partial V_5 \rho C_p \theta_5}{\partial t} + a_{\mathrm{P},5}\theta_5 = a_{\mathrm{W},51}\theta_1 + a_{\mathrm{W},52}\theta_2 + a_{\mathrm{B},54}\theta_4 + b_5 \quad (7)$$

Detailed descriptions of the coefficients in the above equations can be found in Ref. [7].

4. State space representation of the air-zonal model

Whenever a control system is designed, a model in the form of either transfer function or state space is often needed. The air-zonal model presented in the previous section can be easily converted into a state space model.

To be consistent with the control theory, we now define x as a state in a space and let it denote the temperature of an air zone:

$$x_1 = \theta_1,\ x_2 = \theta_2,\ x_3 = \theta_3,\ x_4 = \theta_4,\ x_5 = \theta_5, \quad (8)$$

$$\dot{x}_1 = \frac{\partial \theta_1}{\partial t},\ \dot{x}_2 = \frac{\partial \theta_2}{\partial t},\ \dot{x}_3 = \frac{\partial \theta_3}{\partial t},\ \dot{x}_4 = \frac{\partial \theta_4}{\partial t},\ \dot{x}_5 = \frac{\partial \theta_5}{\partial t}, \quad (9)$$

$$a_1 = V_1 \rho C_p,\ a_2 = V_2 \rho C_p,\ a_3 = V_3 \rho C_p,\ a_4 = V_4 \rho C_p,\ a_5 = V_5 \rho C_p, \quad (10)$$

Then Eqs. (3)–(7) become:

$$\dot{x}_1 = -\frac{a_{\mathrm{P},1}}{a_1}x_1 + \frac{a_{\mathrm{T},12}}{a_1}x_2 + \frac{a_{\mathrm{E},13}}{a_1}x_3 + \frac{a_{\mathrm{E},14}}{a_1} + \frac{a_{\mathrm{E},15}}{a_1}x_4 + \frac{b_1}{a_1} \quad (11)$$

$$\dot{x}_2 = \frac{a_{\mathrm{B},21}}{a_2}x_1 - \frac{a_{\mathrm{P},2}}{a_2}x_2 + \frac{a_{\mathrm{E},25}}{a_2}x_5 + \frac{b_2}{a_2} \quad (12)$$

$$\dot{x}_3 = \frac{a_{\mathrm{W},31}}{a_3}x_1 - \frac{a_{\mathrm{P},3}}{a_3}x_3 + \frac{a_{\mathrm{T},34}}{a_3}x_4 + \frac{b_3}{a_3} \quad (13)$$

$$\dot{x}_4 = \frac{a_{\mathrm{W},41}}{a_4}x_1 + \frac{a_{\mathrm{B},43}}{a_4}x_3 - \frac{a_{\mathrm{P},4}}{a_4}x_4 + \frac{a_{\mathrm{T},45}}{a_4}x_5 + \frac{b_4}{a_4} \quad (14)$$

$$\dot{x}_5 = \frac{a_{\mathrm{W},51}}{a_5}x_1 + \frac{a_{\mathrm{W},52}}{a_5}x_2 + \frac{a_{\mathrm{B},54}}{a_5}x_4 - \frac{a_{\mathrm{P},5}}{a_5}x_5 + \frac{b_5}{a_5} \quad (15)$$

Representing Eqs. (11)–(15) with a state space model we have:

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u} \quad (16)$$

where

$$\mathbf{A} = \begin{bmatrix} -\frac{a_{\mathrm{P},1}}{a_1} & \frac{a_{\mathrm{T},12}}{a_1} & \frac{a_{\mathrm{E},13}}{a_1} & \frac{a_{\mathrm{E},14}}{a_1} & \frac{a_{\mathrm{E},15}}{a_1} \\ \frac{a_{\mathrm{B},21}}{a_2} & -\frac{a_{\mathrm{P},2}}{a_2} & 0 & 0 & \frac{a_{\mathrm{E},25}}{a_2} \\ \frac{a_{\mathrm{W},31}}{a_3} & 0 & -\frac{a_{\mathrm{P},3}}{a_3} & \frac{a_{\mathrm{T},34}}{a_3} & 0 \\ \frac{a_{\mathrm{W},41}}{a_4} & 0 & \frac{a_{\mathrm{B},43}}{a_4} & -\frac{a_{\mathrm{P},4}}{a_4} & \frac{a_{\mathrm{T},45}}{a_4} \\ \frac{a_{\mathrm{W},51}}{a_5} & \frac{a_{\mathrm{W},52}}{a_5} & 0 & \frac{a_{\mathrm{B},54}}{a_5} & -\frac{a_{\mathrm{P},5}}{a_5} \end{bmatrix} \quad (17)$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{a_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{a_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{a_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{a_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{a_5} \end{bmatrix} \quad (18)$$

$$x=[x_1, x_2, x_3, x_4, x_5]^T \tag{19}$$

$$u=[b_1, b_2, b_3, b_4, b_5]^T \tag{20}$$

where T represents transpose. **A** is usually called system matrix. **B** is named as input matrix. Vector u contains the heat flux from the supply air.

The output vector is normally defined as:

$$y=\mathbf{C}x \tag{21}$$

where

$$y=[y_1, y_2, y_3, y_4, y_5]^T \tag{22}$$

$$\mathbf{C}=\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix} \tag{23}$$

The elements of the output (or measurement) matrix **C**, c_{ij} ($i,j=1,2,\ldots,5$), are determined according to the relation between the outputs and the states. For example, if the output vector y is equal to the state vector x, then $\mathbf{C}=\mathbf{I}$, where **I** is the unit matrix. If the system only has a single output, then **C** may be simply written as:

$$\mathbf{C}=[c_1, c_2, c_3, c_4, c_5] \tag{24}$$

If the system output is the air temperature of zone 4, then:

$$\mathbf{C}=[0, 0, 0, 1, 0] \tag{25}$$

If the output is the mean value of the air temperatures of zone 3 and zone 4, then:

$$\mathbf{C}=[0, 0, 0.5, 0.5, 0] \tag{26}$$

The elements of matrix **A** in the state space model contain the effect of the heat transfer due to air mass flows as well as the heat exchange effects due to turbulence and the free movement of air molecules.

5. Model validation

To validate the above air zonal model, some dynamic measurements are made in a test room. Measures of the room are as follows (Fig. 3):

Internal ($L\times W\times H$) = 4.1 m × 3.1 m × 2.7 m

External ($L\times W\times H$) = 4.4 m × 3.4 m × 3.0 m

For walls, roof and floor: construction materials of walls and roof: polystyrene with steel plate layers on both sides. Construction materials of floor: soil, polystyrene, plywood with aluminum plate on the top.

Measures of the heat source box and fan-coil unit:

Hear source box ($L\times W\times H$) = 0.50 m × 0.60 m × 0.40 m

Fan-coil unit ($L\times W\times H$) = 0.45 m × 1.00 m × 0.80 m

The heat source is to emulate the heat generated from a computer and a seated person. An incandescent lamp of 150 W is mounted inside the heat source box.

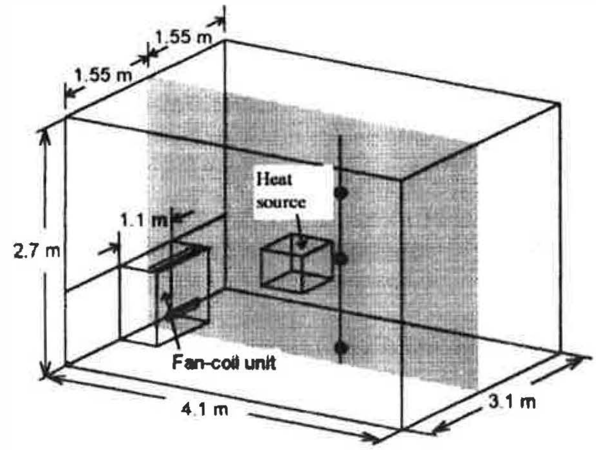

Fig. 3. Test room.

The fan-coil unit is installed below the window. Hot water and chilled water are supplied to the coil (heat exchanger). The flow rates of the hot and chilled water can be changed by controlling two proportional solenoid valves. The size of the supply air grill at the top of the fan-coil unit is 0.80 m × 0.13 m. The direction of the grill can be changed so that the supply air from the fan-coil unit can be guided either toward the room or toward the window.

Fig. 3 also shows the locations of three thermocouples in the central plane of the room. The heights of the thermocouples are arranged in such a way that the temperatures at the ankle and neck positions of the room occupant (when seated) and the temperature at the head position (when standing) should be measured. For clarity, the names 'high', 'mid' and 'low' are assigned to the thermocouples to reflect their positions.

To calculate the temperatures of indoor air, internal surface temperatures of the walls as well as the ceiling, floor and window need to be known. There are two ways to obtain the surface temperatures: through calculation and through measurement. To validate the model, any additional and unnecessary computation error should be reduced as much as possible. Thus the surface temperatures are measured here and are shown in Fig. 4.

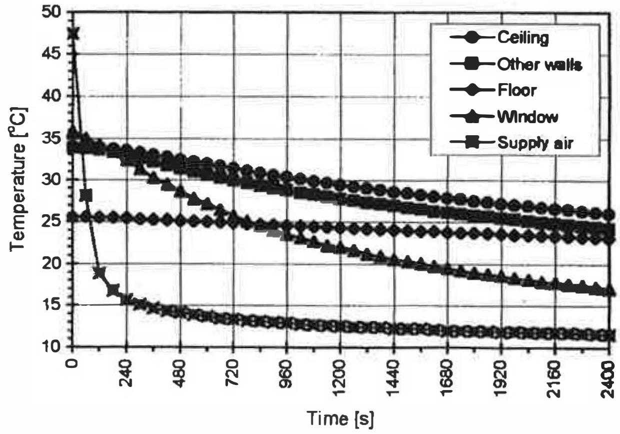

Fig. 4. Temperatures of wall surfaces and supply air.

The situation shown in Fig. 4 is a cooling case. Cooling air is supplied at low fan speed (the averaged supply air speed = 1.64 m/s) and is guided toward the window. The calculated steady-state air flow pattern at the center plane is shown in Fig. 2.

With the calculated velocity field and effective exchange coefficients, the coefficients of the energy Eq. (1) can be calculated. The indoor dynamic air temperature distributions are then calculated with Eq. (2). Here, the value of 3.0 W/(m^2 K) is adopted for the convective heat transfer coefficients between the wall surfaces and the adjacent air [9].

For the measured temperature responses, the effect of the mean radiant temperatures (MRT) of the surrounding surfaces on the three thermocouples have also been taken into account at every time step (Fig. 5). Details on such consideration can be found in Ref. [7].

The room is divided into air zones as follows (length by width by height) (Fig. 2):

Zone 1	1.14 m×3.1 m×2.15 m
Zone 2	1.14 m×3.1 m×0.55 m
Zone 3	2.96 m×3.1 m×0.62 m
Zone 4	2.96 m×3.1 m×1.11 m
Zone 5	2.96 m×3.1 m×0.97 m

The comparisons between the calculated results and the measured results are shown in Figs. (6)–(8).

From the compared results, we may see that the indoor dynamic temperature distributions are well predicted with the zonal model. But the computing time is reduced to about 1 CPU second on a PC-486 (66 MHz) computer, while a similar dynamic CFD simulation will take dozens of CPU hours.

6. Controlling the temperature of an air zone based on its predicted values—an example of better control system design

Since the air zonal model has been transformed into a state space model, we now give an example of how to design a better control system of the temperature of an air zone.

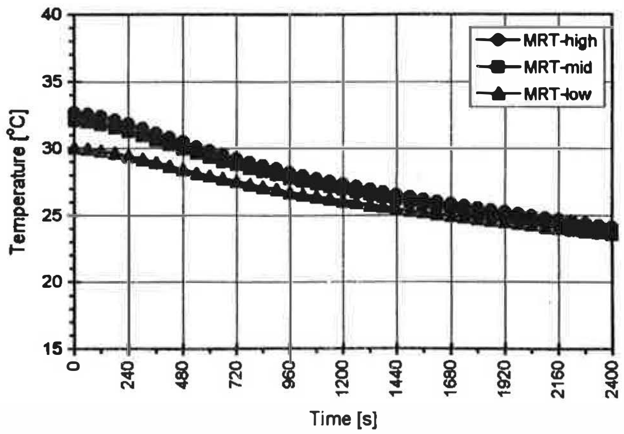

Fig. 5. Mean-Radiant-Temperatures of wall surfaces to the thermocouples at 'high', 'middle' and 'low'.

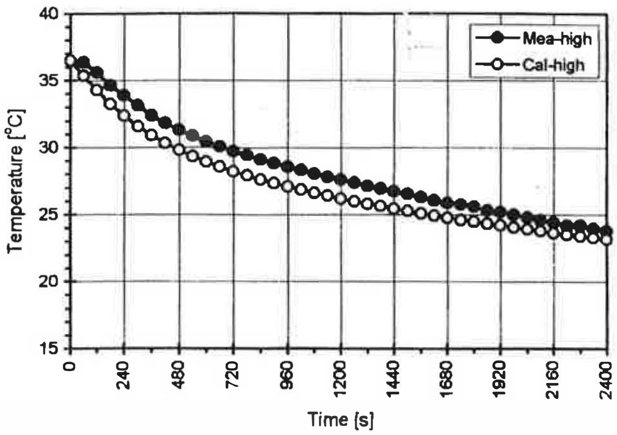

Fig. 6. Calculation vs. measurement at zone 5.

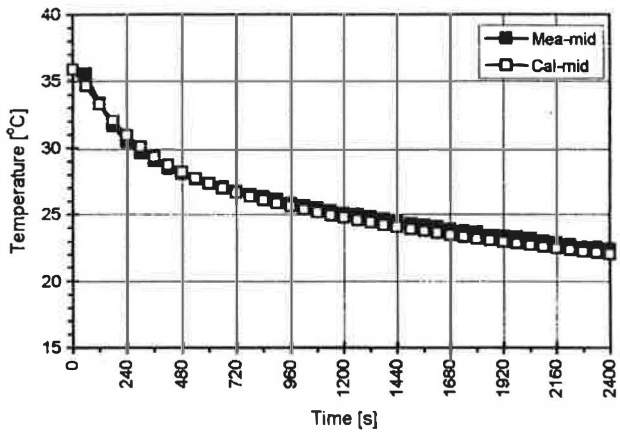

Fig. 7. Calculation vs. measurement at zone 4.

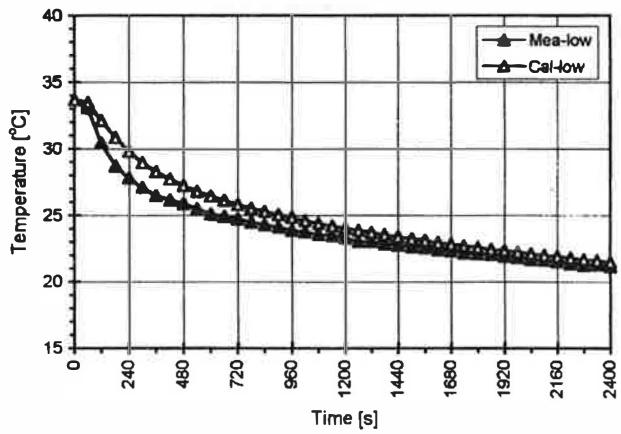

Fig. 8. Calculation vs. measurement at zone 3

It is self-evident that, in the real-time control of an industrial process, the more information is fedback to the controller, the better the process will be controlled. In classical control theory, only the system output (the measured process variable) is used as feedback. In modern control theory, not only the measured variable, but also other state variables can be used as feedbacks as long as they are known.

But a commonly seen problem in designing a controller is that some variables to be controlled or monitored are either impossible to measure or too expensive to measure. For instance, the idea of mounting five temperature sensors in a room is obvious not feasible. The normal practice is that only one temperature sensor is used for one room. Thus it only measures the temperature of the air zone where it is located. If the room is depicted with several air zones and the temperatures of the zones other than the sensor-located zone need to be known, then they have to be estimated in some way in real time. The estimated temperatures can be used as feedback information to the controller and then compared with the set point so that better control actions can be taken.

The real-time prediction task of temperatures can be fulfilled by a state estimator whose design can be found in many books on modern control theory. Here we only give the scheme. Fig. 9 shows block diagram of the reduced-order estimator for indoor temperature predictions. x_n denotes the temperature that can be measured by a sensor. x_e is a vector containing the air zone temperatures which can not be measured but have to be estimated. $\hat{x}_e$ is the output of the estimator and contains the estimated temperatures.

Fig. 10 shows the temperature control system which is designed based on the estimation–feedback-control principle. In Fig. 10, x_5 is the temperature of the recirculated air and is the measured temperature. x_2 is the air temperature of the working zone that needs to be controlled but is not measured. The estimation of x_2, $\hat{x}_2$, is fedback to the controller, compared with the set point r and controlled.

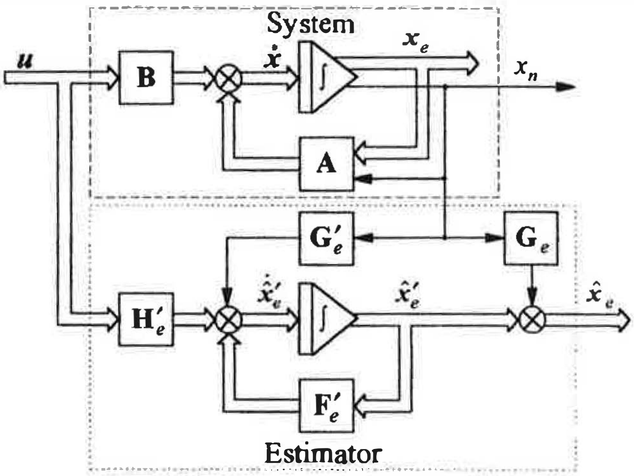

Fig. 9. Scheme of reduced-order estimator.

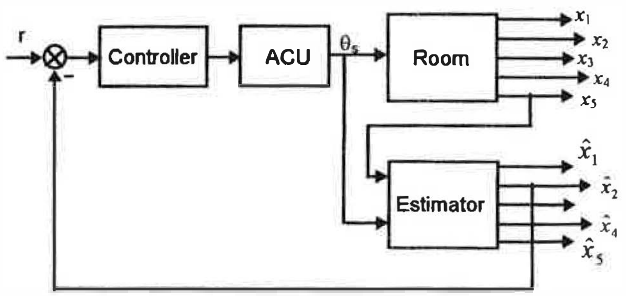

Fig. 10. Control system scheme based on the estimation–feedback-control principle.

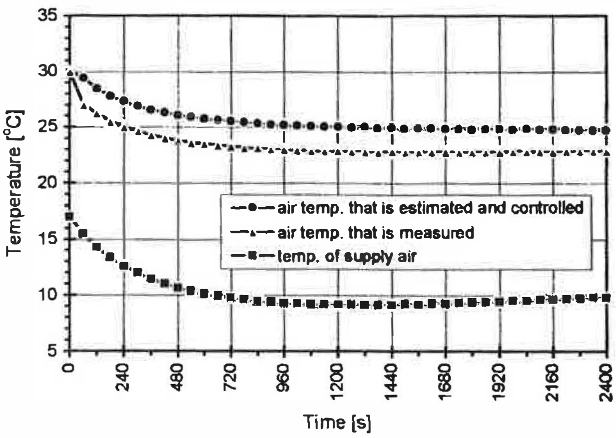

Fig. 11. Simulation results of the control system designed

Fig. 11 shows the simulated control results by using the above control scheme. The set point for the air temperature of the working zone is 25°C in a summer season. We see from the figure, that with this control scheme, the air temperature of the working zone can be precisely controlled albeit it is not measured.

7. Conclusions

The traditional modeling assumption of perfectly-mixed indoor air volume can no longer satisfy the requirement of better control system designs of indoor thermal conditions.

For the depiction of dynamic responses of indoor temperature distributions, CFD method should give the best prediction results. But it takes too much computing time and is not cost effective for the dynamic control investigation of indoor thermal conditions.

For rooms installed with air-conditioning units, the indoor air flow patterns are mainly determined by the speed and direction of the supply air. If these two factors do not change, then the indoor air flow field can be thought of as time invariant (fixed). Based on this assumption, the air flow field needs to be solved only once. What is left to be solved is the energy equation.

In this paper, a simplified air-zonal model is presented for dynamic indoor temperature predictions. The model is then transformed into a state space model for control system designs. The model is validated with experimental results. The validations indicate that the model can give satisfactory and realistic predictions of the dynamic indoor temperature distributions. The total computing time used by the air-zonal model (also state space model) is much less than that used by the dynamic CFD simulation.

Another advantage of the state space model is that it can be easily transformed into transfer functions so that the frequency response analysis can be carried out on temperature responses.

Unlike the simplified models of indoor temperature distributions already published in literature, the zonal model developed in this paper does not ignore the important turbulence effect on heat exchanges among air zones. Each element of matrix **A** in the state space model contains the effect of the heat transfer due to air mass flows as well as the heat exchange effects due to turbulence and the free movement of air molecules. Therefore, the elements of matrix **A** can be artificially changed within a certain extent without influencing the mass balance condition of each air zone. The noteworthy point is that not one but two symmetric elements in matrix **A** should be changed simultaneously. The equivalent result is that the turbulent effect is enhanced or attenuated. Such changes might be necessary for model fitting (curve fitting).

Possible applications of the state space model developed in this thesis include:

1. Fast predictions of dynamic indoor temperature distributions.
2. Detailed and fast predictions indoor thermal comfort levels at different room locations.
3. Control system designs and simulations
4. Finding the transfer functions relating the supply air temperature and the temperatures at various room locations.
5. Finding the optimal control of the temperature sensor.
6. Finding control rules of a fuzzy logic controller

One example is given in this paper to show how the temperatures of air zones can be estimated, fedback and then controlled. The results indicate that the temperature of the working zone can be precisely controlled by taking advantage of the state space model and the state estimation theory.

8. Nomenclature

θ	temperature of air (K)
ρ	density of air (kg/m^3)
t	time (s)
S_H	heat generation rate per unit volume (W/m^3)
C_p	specific heat of air [J/(kg K)]
λ	thermal conductivity of air [(W/(m K)]
ν_t	turbulent viscosity (m^2/s)
σ_H	turbulent Prandtl number (dimensionless)
E, W, S, N, T and B	subscriptions of the coefficients a's, and means East, West, South, North, Top and Bottom
r	temperature set point of the control system
ACU	air-conditioning unit
θ_s	temperature of the supply air
$x_1,\ldots,x_5$	temperatures of air zones 1,...,5
$\tilde{x}_1,\ldots,\tilde{x}_5$	estimations of $x_1,\ldots,x_5$

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