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A CONTINUOS TIME DYNAMIC MODEL OF TEMPERATURE IN AIR CONDITIONED ROOMS

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Abstract

The numerical evaluation of room air movement is made by systematic discretization of space and the dependent variables. This makes possible to replace the governing differential equations with simple algebraic equation. The dynamic model of the temperature is based on the energy balance equation, considering a *given flow field*. The temperature in a given control volume depends on the temperatures of its corresponding neighbours. This form of the model is not appropriate for control theory. However, it is a linear time-invariant differential equation system with respect to time which can be represented as a set of first order differential equation. The model becomes a continuos-time discrete-space system. An important advantage of this form is that the numerical integration method becomes a free choise. This system can be written in matrix or ***state-space*** form. The way in which this form can be achieved is presented in this paper.

1. Introduction

The numerical models of heat transfer, fluid flow, and other related processes can be achieved when the laws governing these processes have been expressed in mathematical form, generally in terms of differential equations [4]. The individual equations express a certain conservation principle.

The dynamic model of the temperature is based on the energy balance equation. In its most general form, the energy equation contains a large number of influences. For a steady low velocity flow, with negligible viscous dissipation, the energy equation can be written as:

$$\frac{\partial}{\partial t}(\rho h) + div(\rho \vec{V} h) = div(k\, grad\,(\theta)) + S_h \qquad (1)$$

where h is the specific enthalpy, k is the thermal conductivity, θ is the temperature, S_h is the volumetric rate of heat generation and $\vec{V}$ is the velocity. The term $div(k\ grad\ (\theta))$ represents the influence of conduction heat transfer within the fluid, according to Fourier law of conduction. For ideal gases and for solids and liquids:

$$c\ grad(\theta)=grad(h) \qquad (2)$$

where c is the constant-pressure specific heat. The energy equation becomes:

$$\frac{\partial}{\partial t}(\rho h) + div(\rho \bar{V} h) = div\left(\frac{k}{c}\ grad\,(h)\right) + S_h \qquad (3)$$

If c is constant, then:

$$h = c\theta \qquad (4)$$

which leads to:

$$\frac{\partial}{\partial t}(\rho\theta) + div(\rho \bar{V}\theta) = div\left(\frac{k}{c}\ grad\,(\theta)\right) + \frac{S_h}{c} \qquad (5)$$

The convection is created by fluid flow. In this paper the solution for temperature is considered in the presence of ***a given flow field*** (i.e. given velocity components and density). The flow field may be from experiment, be given as an analytical solution or calculated by computational fluid dynamics (CFD) computer programs. The origin of the flow field information is not relevant here.

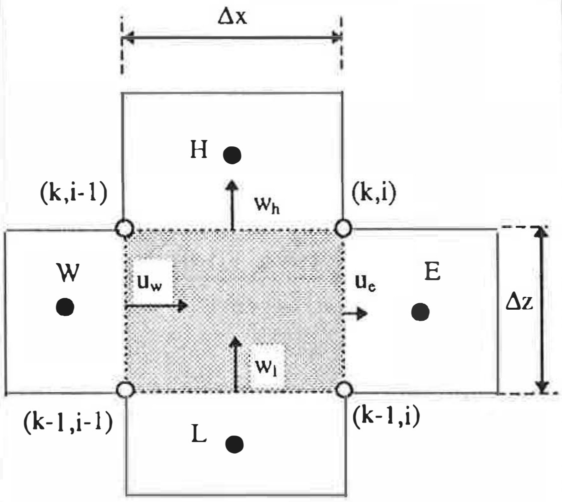

Figure 1 The control volume for a two dimensional cell

For an easier formulation, a solution in a two dimensional space is given. This space is the vertical plane (x, z), since there are many situations in which the three dimensional problem may be reduced to a two dimensional problem in the vertical plane [5]. In fact, the solution in three dimensional space is similar and can be easily obtained.

The model will be considered in continuous time and discrete space. This means that the derivate in time is considered continuous derivate and the derivate in space is discretized. Thus, equation (5) becomes an ordinary differential equation with respect to time.

The formulation of the discretized equation from the differential equation follows the *finite volume method* presented in [1] and [4]. A computational grid is used with the temperature to be evaluated given discrete values at the grid points. A staggered grid is used for the velocity components. The velocity components are given for the points that lie on the faces of the control volume. In the grid convention shown in Figure 1 and Figure 2, x increases from W (west) to E (east) and z increases from L (low) to H (high). The dashed lines represent the faces of the control volume. They are denoted by **w, e, l, h** for west, east, low and high respectively.

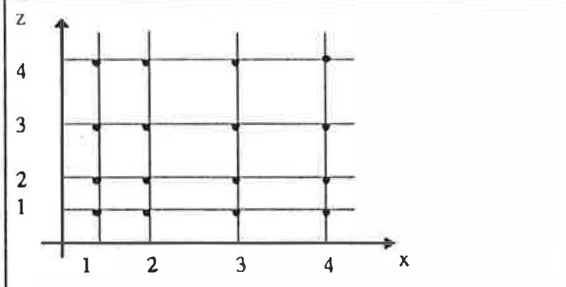

Figure 2 The grid for a two dimensional field

Integrating equation (5) over the control volume it results:

$$\rho c\frac{\partial\theta}{\partial t}=-\rho c\left[\frac{(u\theta)_e-(u\theta)_w}{\Delta x}+\frac{(w\theta)_h-(w\theta)_l}{\Delta z}\right]+\frac{\left(\Gamma\frac{\partial\theta}{\partial x}\right)_e-\left(\Gamma\frac{\partial\theta}{\partial x}\right)_w}{\Delta x}+\frac{\left(\Gamma\frac{\partial\theta}{\partial z}\right)_h-\left(\Gamma\frac{\partial\theta}{\partial z}\right)_l}{\Delta z} \qquad (6)$$

The discretization of equation (6) requires the application of a finite difference scheme. The *upwind difference scheme* or *the donor cell method* is considered here. In this scheme, the value of θ at the control surface (i.e. *w, e, l, h* surfaces) is taken as the value of upstream node point:

$$\theta_e=\theta \text{ for } u_e>0; \quad \theta_e=\theta_E \text{ for } u_e<0; \qquad (7)$$

and similarly for θ_w, θ_l, and θ_h. Using the notation $|[A]|$ to denote the greater of *A* and *0*, the first right term of equation (6) may be written:

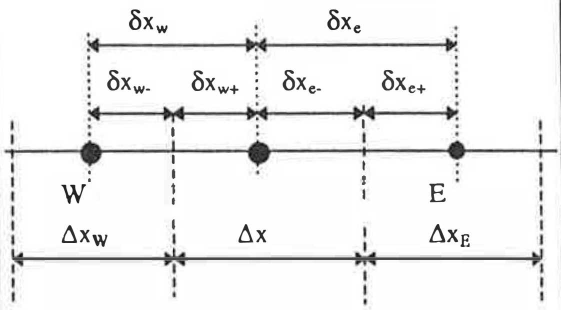

Figure 3 Grid points for *x* direction

$$-\rho c\left[\frac{(u\theta)_e-(u\theta)_w}{\Delta x}+\frac{(w\theta)_h-(w\theta)_l}{\Delta z}\right]=$$

$$\rho c\left[\frac{1}{\Delta x}\left(|[u_w]|\theta_W-|[-u_w]|\theta-|[u_e]|\theta+|[-u_e]|\theta_E\right)\right]+\rho c\left[\frac{1}{\Delta z}\left(|[w_l]|\theta_L-|[-w_l]|\theta-|[w_h]|\theta+|[-w_h]|\theta_H\right)\right] \qquad (8)$$

The second and the third right terms of equation (6) may be written as (see Figure 3):

$$\frac{\left(\Gamma\frac{\partial\theta}{\partial x}\right)_e-\left(\Gamma\frac{\partial\theta}{\partial x}\right)_w}{\Delta x}\cong\frac{1}{\Delta x}\left(\Gamma_e\frac{\theta_E-\theta}{\delta x_e}-\Gamma_w\frac{\theta-\theta_W}{\delta x_w}\right) \text{ and} \qquad (9)$$

$$\frac{\left(\Gamma\frac{\partial\theta}{\partial z}\right)_h-\left(\Gamma\frac{\partial\theta}{\partial z}\right)_l}{\Delta z}\cong\frac{1}{\Delta z}\left(\Gamma_h\frac{\theta_H-\theta}{\delta z_h}-\Gamma_l\frac{\theta-\theta_L}{\delta z_l}\right)$$

where indexes *W, E, L, H* represent the values in the West, East, Low and High neighbour control volumes, respectively.
With:

$$\delta x_e=\delta x_{e-}+\delta x_{e+}=\frac{\Delta x}{2}+\frac{\Delta x_E}{2}; \qquad \delta x_w=\delta x_{w-}+\delta x_{w+}=\frac{\Delta x}{2}+\frac{\Delta x_W}{2}; \qquad (10)$$

$$\delta z_h=\delta z_{h-}+\delta z_{h+}=\frac{\Delta z}{2}+\frac{\Delta z_H}{2}; \qquad \delta z_l=\delta z_{l-}+\delta z_{l+}=\frac{\Delta z}{2}+\frac{\Delta z_L}{2};$$

The diffusion coefficients at the control surfaces are represented by:

$$\Gamma_w = f(\Gamma,\ \Gamma_W);\ \ \Gamma_e = f(\Gamma,\ \Gamma_E);\ \ \Gamma_h = f(\Gamma,\ \Gamma_H);\ \ \Gamma_l = f(\Gamma,\ \Gamma_L); \tag{11}$$

Different functions *f(.)* are proposed in literature. Let us assume the functions:

$$\Gamma_w = \frac{\Gamma + \Gamma_W}{2};\ \ \Gamma_e = \frac{\Gamma + \Gamma_E}{2};\ \ \Gamma_l = \frac{\Gamma + \Gamma_L}{2};\ \ \Gamma_h = \frac{\Gamma + \Gamma_H}{2}; \tag{12}$$

Then,

$$\frac{\left(\Gamma \frac{\partial \theta}{\partial x}\right)_e - \left(\Gamma \frac{\partial \theta}{\partial x}\right)_w}{\Delta x} \cong \frac{1}{\Delta x}\left(\frac{\Gamma_E + \Gamma}{\Delta x_E + \Delta x}(\theta_E - \theta) - \frac{\Gamma_W + \Gamma}{\Delta x_W + \Delta x}(\theta - \theta_W)\right)$$

$$\frac{\left(\Gamma \frac{\partial \theta}{\partial z}\right)_h - \left(\Gamma \frac{\partial \theta}{\partial z}\right)_l}{\Delta z} \cong \frac{1}{\Delta z}\left(\frac{\Gamma_H + \Gamma}{\Delta z_H + \Delta z}(\theta_H - \theta) - \frac{\Gamma_L + \Gamma}{\Delta z_L + \Delta z}(\theta - \theta_L)\right) \tag{13}$$

With the notation:

$$\frac{d}{dt}\theta = \dot{\theta} \tag{14}$$

the discrete form of equation (6) is then:

$$\dot{\theta} = \frac{1}{\Delta x}\left(|[u_w]|\theta_W - |[-u_w]|\theta - |[u_e]|\theta + |[-u_e]|\theta_E\right) + \frac{1}{\Delta z}\left(|[w_l]|\theta_L - |[-w_l]|\theta - |[w_h]|\theta + |[-w_h]|\theta_H\right) +$$

$$+\frac{1}{\rho c \Delta x}\left(\frac{\Gamma_E + \Gamma}{\Delta x_E + \Delta x}(\theta_E - \theta) - \frac{\Gamma_W + \Gamma}{\Delta x_W + \Delta x}(\theta - \theta_W)\right) + \frac{1}{\rho c \Delta z}\left(\frac{\Gamma_H + \Gamma}{\Delta z_H + \Delta x}(\theta_H - \theta) - \frac{\Gamma_L + \Gamma}{\Delta x_L + \Delta x}(\theta - \theta_L)\right) \tag{15}$$

2. Correction term in energy balance equation

If the mass balance on the *control volume* is not respected, that is:

$$\Delta x(w_l - w_h) + \Delta z(u_w - u_e) \neq 0 \qquad \text{when } \rho = \text{constant}, \tag{16}$$

then a correction term should be introduced. This term may be seen as a flow in an additional direction in space, orthogonal on the axes of the given space In a two dimensional space *(x,z)* this direction may be *y*, so that:

$$\Delta x \Delta y(w_l - w_h) + \Delta x \Delta z(v_s - v_n) + \Delta z \Delta y(u_w - u_e) = 0 \tag{17}$$

or:

$$\frac{u_e - u_w}{\Delta x} + \frac{v_s - v_n}{\Delta y} + \frac{w_l - w_h}{\Delta z} = 0 \tag{18}$$

where *s* and *n* are indexes for the *south* and *north* surfaces respectively.

Considering that there is no diffusion in this new fictitious direction of space, equation (15) becomes:

$$\dot{\theta} = \frac{1}{\Delta x}\left(|[u_w]|\theta_W - |[-u_w]|\theta - |[u_e]|\theta + |[-u_e]|\theta_E\right) + \frac{1}{\Delta y}\left(|[v_s]|\theta_S - |[-u_s]|\theta - |[u_n]|\theta + |[-u_n]|\theta_N\right) +$$

$$+\frac{1}{\Delta z}\left(|[w_l]|\theta_L - |[-w_l]|\theta - |[w_h]|\theta + |[-w_h]|\theta_H\right) + \tag{19}$$

$$+\frac{1}{\rho c \Delta x}\left(\frac{\Gamma_E + \Gamma}{\Delta x_E + \Delta x}(\theta_E - \theta) - \frac{\Gamma_W + \Gamma}{\Delta x_W + \Delta x}(\theta - \theta_W)\right) + \frac{1}{\rho c \Delta z}\left(\frac{\Gamma_H + \Gamma}{\Delta z_H + \Delta x}(\theta_H - \theta) - \frac{\Gamma_L + \Gamma}{\Delta x_L + \Delta x}(\theta - \theta_L)\right)$$

Considering the temperatures being equal along the new direction and putting:

$$v \equiv \frac{u_e - u_w}{\Delta x} + \frac{w_h - w_l}{\Delta z} \tag{20}$$

equation (19) becomes:

$$\dot{\theta} = \frac{1}{\Delta x}\left(|[u_w]|\theta_W - |[-u_w]|\theta - |[u_e]|\theta + |[-u_e]|\theta_E\right) + \frac{1}{\Delta z}\left(|[w_l]|\theta_L - |[-w_l]|\theta - |[w_h]|\theta + |[-w_h]|\theta_H\right) +$$

$$+\frac{1}{\rho c \Delta x}\left(\frac{\Gamma_E + \Gamma}{\Delta x_E + \Delta x}(\theta_E - \theta) - \frac{\Gamma_W + \Gamma}{\Delta x_W + \Delta x}(\theta - \theta_W)\right) + \tag{21}$$

$$+\frac{1}{\rho c \Delta z}\left(\frac{\Gamma_H + \Gamma}{\Delta z_H + \Delta x}(\theta_H - \theta) - \frac{\Gamma_L + \Gamma}{\Delta x_L + \Delta x}(\theta - \theta_L)\right) + v\theta$$

In equation (21) the term $v\theta$ is the correction term for the unbalanced mass shown in equations (16) and (17).
With the notations:

$$a_W = \frac{|[u_w]|}{\Delta x} + \frac{1}{\rho c \Delta x}\frac{\Gamma_W + \Gamma}{\Delta x_W + \Delta x} \tag{22}$$

$$a_E = \frac{|[-u_e]|}{\Delta x} + \frac{1}{\rho c \Delta x}\frac{\Gamma_E + \Gamma}{\Delta x_E + \Delta x} \tag{23}$$

$$a_L = \frac{|[w_l]|}{\Delta z} + \frac{1}{\rho c \Delta z}\frac{\Gamma_L + \Gamma}{\Delta z_L + \Delta z} \tag{24}$$

$$a_H = \frac{|[-w_h]|}{\Delta z} + \frac{1}{\rho c \Delta z}\frac{\Gamma_H + \Gamma}{\Delta z_H + \Delta z} \tag{25}$$

$$a = -\frac{|[-u_w]|}{\Delta x} - \frac{|[u_e]|}{\Delta x} - \frac{|[-w_l]|}{\Delta z} - \frac{|[w_h]|}{\Delta z} - \frac{1}{\rho c \Delta x}\frac{\Gamma_E + \Gamma}{\Delta x_E + \Delta x} - \frac{1}{\rho c \Delta x}\frac{\Gamma_W + \Gamma}{\Delta x_W + \Delta x} -$$

$$-\frac{1}{\rho c \Delta z}\frac{\Gamma_H + \Gamma}{\Delta z_H + \Delta z} - \frac{1}{\rho c \Delta z}\frac{\Gamma_L + \Gamma}{\Delta z_L + \Delta z} + \frac{u_e - u_w}{\Delta x} + \frac{w_h - w_l}{\Delta z} = \tag{26}$$

$$= -(a_W + a_E + a_L + a_H)$$

equation (21) may be written as:

$$\dot{\theta} = a\theta + a_W\theta_W + a_E\theta_E + a_L\theta_L + a_H\theta_H \tag{27}$$

Equation (27) represents a continuos-time discrete-space model of the room temperature in the air conditioned rooms. If the system (27) is discretized using Euler method, the relation between the coefficients practically obeys the rule of coefficients stated in [4].

3. State space representation

The model expressed by equation (27) may be put in the state-space representation:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{28}$$

where the vectors u, x, and y are:

- the states x, which are the temperatures in each control volume;
- the inputs u, which are the limit conditions and other independent variables which change the states;
- the outputs y, which are, usually, the temperature of interest (for example, where the measuring device is located).

and matrices A and B express the coefficients from equation (27). On the other hand, matrices C and D have no direct counterparts in equation (27), since equation (27) represents only the first equation of system (28).

To obtain the state-space representation, the matrix of temperature should be written as a vector and the matrices of coefficients A and B should be modified accordingly. The matrices C and D are separate cases.

3.1. *Temperatures matrix and the state vector*

It is easy to obtain the state vector from the temperatures matrix. The matrix x is all the elements of θ regarded as a single column (Figure 4). However, a border of zeros is added to the matrix θ. This border is very useful as it allows a simpler formulation of the changes in order to obtain A and B matrices [2]. In Figure 4, a very simple example is considered. The order of elements in Figure 4 is rather oddly because a resembles with the discretized space is considered [3]. Having a border of zero's, it means that in fact only the cells denoted by numbers 5 and 8 are part of the initial system. The zero border makes the temperatures matrix

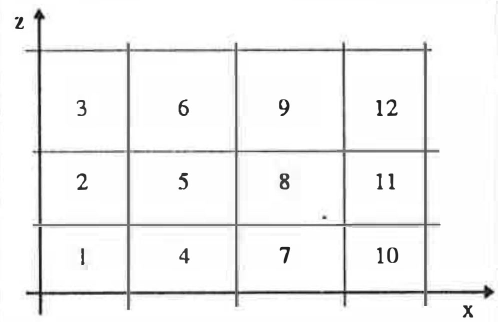

Figure 4 Two dimensional grid and the order of elements in the state vector

circular, so that the western neighbour of the first column is the last column, the eastern neighbour of the last column is the first column, the lower neighbour of the first row is the last row and the higher neighbour of the last row is the first row. For example, the control volumes *10*, *11*, *12* have the control volumes *1*, *2*, and *3*, as eastern neighbours, respectively; the lower neighbours of control volumes *1*, *4*, *7*, and *10* are the control volumes *3*, *6*, *9*, and *12*, respectively. The state vector, for this example is:

$$\begin{bmatrix} \theta_{31} & \theta_{32} & \theta_{33} & \theta_{34} \\ \theta_{21} & \theta_{22} & \theta_{23} & \theta_{24} \\ \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} \end{bmatrix} \leftrightarrow \begin{bmatrix} \theta_{11} & \theta_{21} & \theta_{31} & \theta_{12} & \theta_{22} & \theta_{32} & \theta_{13} & \theta_{23} & \theta_{33} & \theta_{14} & \theta_{24} & \theta_{34} \end{bmatrix}^T$$

$$\leftrightarrow \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} \end{bmatrix}^T$$

3.2. Matrix A

The matrix A represents the coefficients which connect the temperature in a control volume with the temperature in the neighbouring control volumes (W, E, L, H).
The a coefficients will be on the first diagonal of matrix A. The first term $\alpha\theta$ of equation (27) may be written as:

$$\begin{matrix} a_{11}.\theta_{11} \\ a_{21}.\theta_{21} \\ \vdots \\ a_{34}.\theta_{34} \end{matrix} \leftrightarrow \begin{bmatrix} a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{11} & a_{12} & a_{13} & a_{14} \end{bmatrix} \bullet * \begin{bmatrix} \theta_{31} & \theta_{32} & \theta_{33} & \theta_{34} \\ \theta_{21} & \theta_{22} & \theta_{23} & \theta_{24} \\ \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} \end{bmatrix} \leftrightarrow$$

$$\begin{bmatrix} a_{11} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & a_{21} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & a_{31} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & a_{12} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_{22} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & a_{32} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{13} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{23} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{33} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{14} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{24} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{34} \end{bmatrix} \bullet \begin{bmatrix} \theta_{11} \\ \theta_{21} \\ \theta_{31} \\ \theta_{12} \\ \theta_{22} \\ \theta_{32} \\ \theta_{13} \\ \theta_{23} \\ \theta_{33} \\ \theta_{14} \\ \theta_{24} \\ \theta_{34} \end{bmatrix}$$

where $\bullet *$ is the element-by-element product of the arrays [3]. The unspecified elements of matrices are zero.
The a_W coefficients will be on the $-m$ diagonal, m being the number of rows of temperature matrix. Since the first m elements of vector x are zero and the corresponding a_W coefficients are zero, only the $-m$ diagonal is necessary.

$$\begin{matrix} a_{W11}.\theta_{W11} \\ a_{W21}.\theta_{W21} \\ \vdots \\ a_{W34}.\theta_{W34} \end{matrix} \leftrightarrow \begin{bmatrix} a_{W31} & a_{W32} & a_{W33} & a_{W34} \\ a_{W21} & a_{W22} & a_{W23} & a_{W24} \\ a_{W11} & a_{W12} & a_{W13} & a_{W14} \end{bmatrix} \bullet * \begin{bmatrix} \theta_{34} & \theta_{31} & \theta_{32} & \theta_{33} \\ \theta_{24} & \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{14} & \theta_{11} & \theta_{12} & \theta_{13} \end{bmatrix} \leftrightarrow$$

$$\begin{bmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W11} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W21} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W31} \\
a_{W12} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & a_{W22} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & a_{W32} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & a_{W13} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & a_{W23} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & a_{W33} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W14} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W24} & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W34} & \cdot & \cdot & \cdot
\end{bmatrix} \bullet \begin{bmatrix} \theta_{11} \\ \theta_{21} \\ \theta_{31} \\ \theta_{12} \\ \theta_{22} \\ \theta_{32} \\ \theta_{13} \\ \theta_{23} \\ \theta_{33} \\ \theta_{14} \\ \theta_{24} \\ \theta_{34} \end{bmatrix}$$

The a_E coefficients are treated analogous. They are on the m diagonal.
The a_L and a_H take advantage of the zero border of matrix of temperatures:

$$\begin{matrix} a_{L11}.\theta_{L11} \\ a_{L21}.\theta_{L21} \\ \vdots \\ a_{L34}.\theta_{L34} \end{matrix} \leftrightarrow \begin{matrix} a_{L11}.\theta_{31} \\ a_{L21}.\theta_{11} \\ \vdots \\ a_{L34}.\theta_{24} \end{matrix} \leftrightarrow \begin{bmatrix} a_{L31} & a_{L32} & a_{L33} & a_{L34} \\ a_{L21} & a_{L22} & a_{L23} & a_{L24} \\ a_{L11} & a_{L12} & a_{L13} & a_{L14} \end{bmatrix} \bullet * \begin{bmatrix} \theta_{21} & \theta_{22} & \theta_{23} & \theta_{24} \\ \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} \\ \theta_{31} & \theta_{32} & \theta_{33} & \theta_{34} \end{bmatrix} \leftrightarrow$$

$$\begin{bmatrix}
\cdot & \cdot & a_{L11} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{L21} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & a_{L31} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 0 & \cdot & \cdot & a_{L12} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & a_{L22} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & a_{L32} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & a_{L13} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{L23} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{L33} & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & a_{L14} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{L24} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{L34} & \cdot
\end{bmatrix} \bullet \begin{bmatrix} \theta_{11} \\ \theta_{21} \\ \theta_{31} \\ \theta_{12} \\ \theta_{22} \\ \theta_{32} \\ \theta_{13} \\ \theta_{23} \\ \theta_{33} \\ \theta_{14} \\ \theta_{24} \\ \theta_{34} \end{bmatrix}$$

Since $a_{L11} = a_{L12} = a_{L13} = a_{L14} = 0$, the coefficients a_L are put on the -1 diagonal of matrix A, starting with the second coefficient, i.e.: a_{L21}.
The coefficients a_H are treated in the same way. They are on the $+1$ diagonal of the matrix.

Roughly, matrix A has the coefficients a, a_H, a_L, a_E, a_W on the diagonal 0, $+1$, -1, $+m$ and $-m$, respectively. The precise form of matrix A is:

$$A = \begin{bmatrix}
a_{11} & a_{H11} & \cdot & a_{E11} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{L21} & a_{21} & a_{H21} & \cdot & a_{E21} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & a_{L31} & a_{31} & a_{H31} & \cdot & a_{E31} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{W12} & \cdot & a_{L12} & a_{12} & a_{H12} & \cdot & a_{E12} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & a_{W22} & \cdot & a_{L22} & a_{22} & a_{H22} & \cdot & a_{E22} & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & a_{W32} & \cdot & a_{L32} & a_{32} & a_{H32} & \cdot & a_{E32} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & a_{W13} & \cdot & a_{L13} & a_{13} & a_{H13} & \cdot & a_{E13} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & a_{W23} & \cdot & a_{L23} & a_{23} & a_{H23} & \cdot & a_{E23} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & a_{W33} & \cdot & a_{L33} & a_{33} & a_{H33} & \cdot & a_{E33} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W14} & \cdot & a_{L14} & a_{14} & a_{H14} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W24} & \cdot & a_{L24} & a_{24} & a_{H24} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W34} & \cdot & a_{L34} & a_{34}
\end{bmatrix} \qquad (29)$$

3.3. Matrix B

This matrix contains the coefficients of the inputs. As the inputs are the temperatures of the walls, usually these coefficients are the heat convection coefficients. However, in air conditioned rooms, inputs are also the temperatures in the control volumes where the air is introduced in the room and the internal heat sources. For these control volumes, the corresponding coefficients in matrix A should be put to zero and their values should be given to corresponding elements in B matrix [2].

3.4. Matrix C

This matrix expresses the connection between the temperature in the control volumes (which are the states of the system) and the outputs. It simple contains *1's* as the coefficients of the temperatures of interest.

3.5. Matrix D

This matrix represents a direct feed through the system (i.e. the outputs are not delayed relatively to the inputs as they depend on the inputs at the same time). In this paper, the D matrix is identically null.

4. Conclusions

The dynamic of the temperatures in the air conditioned rooms, as derived from the energy balance equation, may be written in continuos-time discrete-space form. Then this form can be easily put in state-space representation. Adding a border of zeros to the initial system, discretized in space, matrix A in the space-state representation becomes a *quint diagonal* matrix, as it bas only five non-zero diagonals. Matrix A has the elements on the diagonals 0, $+1$, -1, $+m$, and $-m$. Some of these elements are put to zero in matrix A and their values are given to elements in matrix B. The matrix C simply connects the temperature in the points of interest to the output.

It is obvious that the dimension of the system increases considerable. If the discretized space has $l*n$ control volumes, matrix A becomes an $l*n$ rows by $l*n$ columns one. For usual systems, matrix A can easily reach size of thousands by thousands. However, this matrix is a sparse matrix (i.e. it contains a lot of null elements). In fact matrices B and C are also sparse. Taking advantages of this fact, the memory needed for the space-state representation, although larger then the initial one, is no longer a problem [2].

Although it seems cumbersome, representing the system in the state-space is very convenient for the

analysis of the system. In fact, the state-space models are widely used in control theory. Writing the system in a condensed matrix form makes it possible an easier manipulation of the model, with all the benefits of this. Furthermore, the state-space representation can be easily modified in other representations: transfer function, zero-pole-gain or partial fractions. Practically, this representation allows the analysis and simulation of the system with all the tools available in *MATLAB* software.
The continuos-time form of the dynamic model allows any numerical integration method to be choosen. This freedom of choice is useful as numerical integration methods like predictor-corrector or Euler may be desired in different applications.

5. Aknowledgements

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