A STATE-SPACE REPRESENTATION OF THE DYNAMIC MODEL OF AIR TEMPERATURE DISTRIBUTIONS

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INTRODUCTION

Air flows and temperature distributions in rooms are significantly important in heating, refrigerating, ventilation and air conditioning. The Computational Fluid Dynamics (CFD) method gives us a very good tool to carry out studies on them. Unfortunately, the CFD simulation takes so much computing time that it is only suitable for some steady state calculations. However, information about the dynamic behaviour is necessary. To solve this contradiction, the air flow fields calculated from the CFD code can be thought as unchanging for some typical heating, cooling or ventilating situations (Ref. /5/) (see Figures 1 and 2). With fixed air flow fields, only the energy balance equation is left to be solved. The energy balance equation can be put in a state-space representation. This kind of representation is widely used in control theory. Thus a connection between CFD and control theory is made. The results from CFD calculations in steady state can be used in dynamical calculations of temperature needed by control theory. Practically, the flow fields calculated by CFD software (PHOENICS for example) can be used as input files for control related software (such as MATLAB).

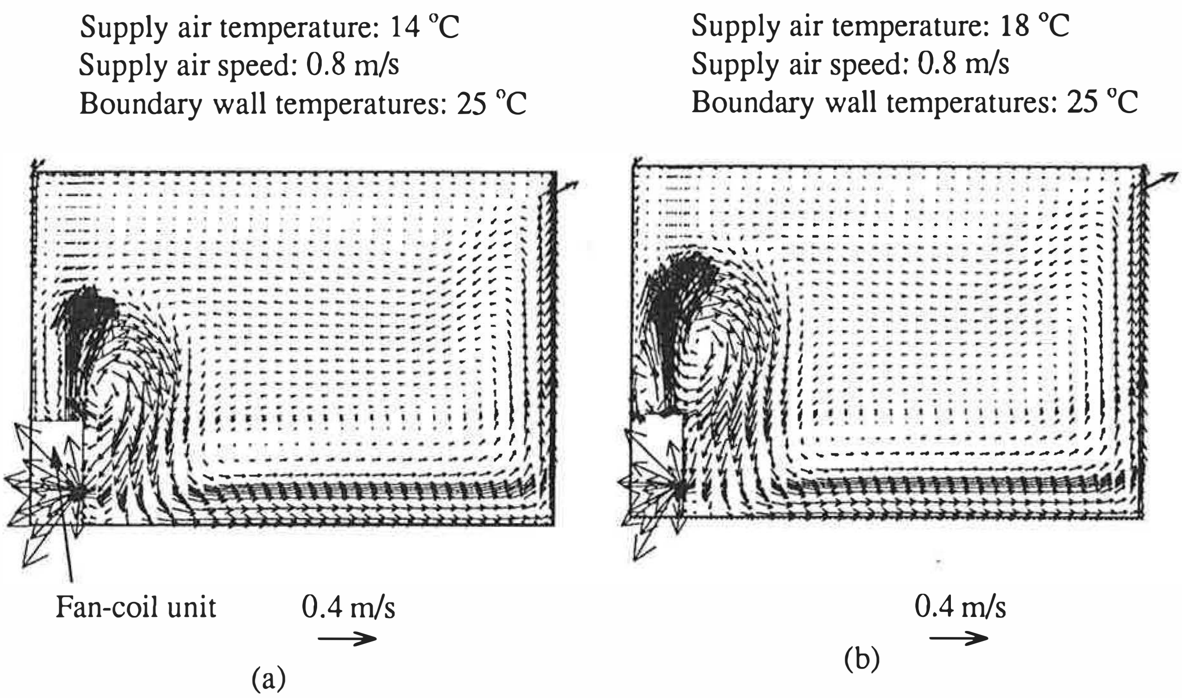

Figure 1 Flow fields in a room with fan-coil unit

ENERGY BALANCE EQUATION FOR FIXED FLOW FIELD

The dynamic temperature distribution of air is depicted by the following energy conservation equation:

$$\frac{\partial}{\partial t}(\rho\theta) + div(\rho\vec{V}\theta) = div(\Gamma grad\,(\theta)) + \frac{S_h}{c_p} \tag{1}$$

where θ is the air temperature, ρ is the density of air, t is time, $\vec{V}$ is the velocity vector, Γ is the effective thermal exchange coefficient, c_p is the specific heat at constant pressure, S_h is the heat source.

Discretizing the above equation will yield (for an easier formulation, a solution in a two dimensional space (x-z) is given in this paper):

$$\rho\frac{\partial\theta}{\partial t} = -\rho\left[\frac{(u\theta)_e - (u\theta)_w}{\Delta x} + \frac{(w\theta)_h - (w\theta)_l}{\Delta z}\right] +$$

$$\frac{\left(\Gamma\frac{\partial\theta}{\partial t}\right)_e - \left(\Gamma\frac{\partial\theta}{\partial t}\right)_w}{\Delta x} + \frac{\left(\Gamma\frac{\partial\theta}{\partial t}\right)_h - \left(\Gamma\frac{\partial\theta}{\partial t}\right)_l}{\Delta z} + \frac{S_h}{c_p} \tag{2}$$

where e, w, h and l represent east, west, high and low (figure 1), u and w are the velocity components of $\vec{V}$ in the directions x and z.

The model is considered in continuous time and discrete space. This means that the derivate in time are considered as continuous and the derivate in space are discretized. Thus, Eq. (1) becomes an ordinary differential equation with respect to time.

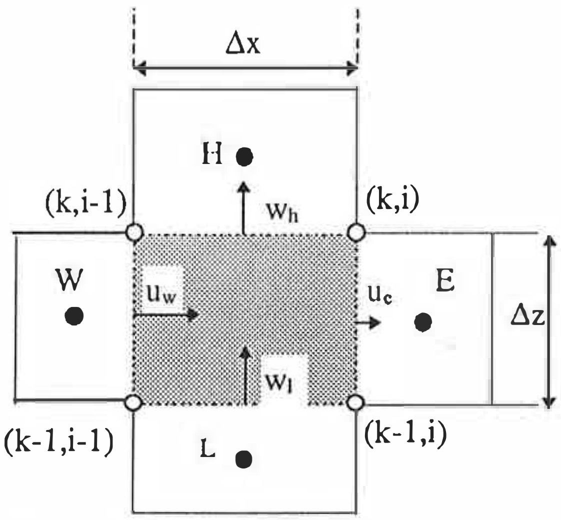

Figure 2 The control volume for a two dimensional cell

The formulation of the discretized equation from the differential equation follows the *finite volume method* presented in Ref. /1/ and Ref. /3/. Here the upwind difference scheme is used for temperature at all grid cells. The staggered grid method is used for the velocity components, i.e. the velocity components of a grid cell are supposed to lie on the faces of the control volume (Figure 2). Discretizing the right-hand side of equation (2) further, we have:

$$\dot{\theta} = a\theta + a_W\theta_W + a_E\theta_E + a_L\theta_L + a_H\theta_H + b \tag{3}$$

where θ_W , θ_E , θ_L , θ_H and θ are the temperatures of the five neighbouring cells in Figure 1, coefficients a_W , a_E , a_L , a_H and a are functions of ρ, c_p , u, w, Γ, Δx and Δz and b represents the source term.

Equation (3) can be used to calculate the dynamic temperature distributions. But the relation among the inputs, outputs and intermediate variables is not clearly expressed. For controller designs, the input-output relation is often required to be represented in a clear manner. This is because the controller design is usually based on the frequency-domain method or the state-space method. According the appearance of equation of (3), it is more suitable to be written in the state-space formulation.

Representing the system in the state-space is very convenient for the analysis of the system. In fact, the state-space models are widely used in control theory. Writing the system in a condensed matrix form makes it possible an easier manipulation of the model, with all the benefits of this. Furthermore, the state-space can be easily modified in other representations: transfer function, zero-pole-gain or partial fractions.

THE STATE SPACE REPRESENTATION

The model expressed by equation (3) may be put in the state-space representation:

$$\begin{cases} \dot{x} = \boldsymbol{A}x + \boldsymbol{B}u \\ y = \boldsymbol{C}x + \boldsymbol{D}u \end{cases} \tag{4}$$

where

- vector $\boldsymbol{x}$ represents states x_i , which are temperatures in each control volume;
- vector $\boldsymbol{u}$ represents inputs u_i , which are heat sources and boundary conditions;
- vector $\boldsymbol{y}$ represents outputs y_i , which are, usually, the temperature of interest (for example, where the temperature sensor is located).

Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ express all the coefficients from Eq. (3). But matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ have no direct counterparts in Eq. (3).

Next, there will be explained how to write the temperatures of all grid cells into a vector and how to write the coefficients into matrices.

The temperatures matrix and the state vector

The temperatures of all grid cells can be written as a matrix θ. The vector $\boldsymbol{x}$ is all the elements of θ regarded as a single column (Figure 3). However, a border of zeros is added to the matrix θ. This border is very useful as it allows a simpler formulation of the changes in order to obtain $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices. In Figure 3, a very simple example is considered. The order of elements in matrix θ (Figure 3) resembles the discretized space considered. Having a border of zero's, it means that in fact only the cells denoted by numbers *5* and *8* are part of the initial system.

The zero border makes the temperatures matrix *circular*, so that the western neighbour of the first column is the last column, the eastern neighbour of the last column is the first column, the lower neighbour of the first row is the last row and the higher neighbour of the last row is the first row.

The state vector, for this example is:

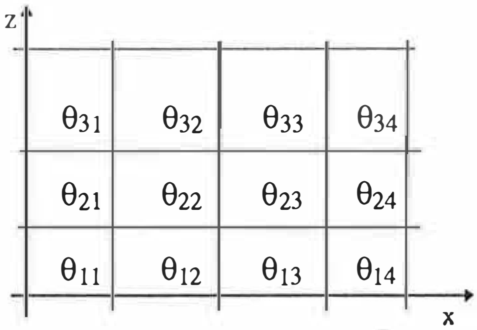

Figure 3 The a two dimensional grid and the order of elements in the state vector

$$\begin{bmatrix} \theta_{31} & \theta_{32} & \theta_{33} & \theta_{34} \\ \theta_{21} & \theta_{22} & \theta_{23} & \theta_{24} \\ \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} \end{bmatrix} \leftrightarrow [\theta_{11}\ \theta_{21}\ \theta_{31}\ \theta_{12}\ \theta_{22}\ \theta_{32}\ \theta_{13}\ \theta_{23}\ \theta_{33}\ \theta_{14}\ \theta_{24}\ \theta_{34}]^T$$

$$\leftrightarrow [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}\ x_{11}\ x_{12}]^T$$

The matrix A

The matrix $\boldsymbol{A}$ represents the coefficients which connect the temperature in a control volume with the temperature in the neighbouring control volumes (W, E, L, H).

The a coefficients will be on the first diagonal of matrix A. The a_W coefficients will be on the $-m$ diagonal, m being the number of rows of temperature matrix. Since the first m elements of vector x are zero and the corresponding a_W coefficients are zero, only the $-m$ diagonal is necessary. The a_E coefficients are treated analogous. They are on the m diagonal. The a_L and a_H take advantage of the zero border of matrix of temperatures. Since $a_{L11} = a_{L12} = a_{L13} = a_{L14} = 0$, the coefficients a_L are put on the -1 diagonal of matrix $\boldsymbol{A}$, starting with the second coefficient, i.e.: a_{L21}. The coefficients a_H are treated in the same way. They are on the $+1$ diagonal of the matrix.

Roughly, matrix $\boldsymbol{A}$ has the coefficients a, a_H, a_L, a_E, a_W on the diagonal 0, $+1$, -1, $+m$ and $-m$, respectively. The precise form of matrix $\boldsymbol{A}$ is:

$$
A = \begin{bmatrix}
a_{11} & a_{H11} & \cdot & a_{E11} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{L21} & a_{21} & a_{H21} & \cdot & a_{E21} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & a_{L31} & a_{31} & a_{H31} & \cdot & a_{E31} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{W12} & \cdot & a_{L12} & a_{12} & a_{H12} & \cdot & a_{E12} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & a_{W22} & \cdot & a_{L22} & a_{22} & a_{H22} & \cdot & a_{E22} & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & a_{W32} & \cdot & a_{L32} & a_{32} & a_{H32} & \cdot & a_{E32} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & a_{W13} & \cdot & a_{L13} & a_{13} & a_{H13} & \cdot & a_{E13} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & a_{W23} & \cdot & a_{L23} & a_{23} & a_{H23} & \cdot & a_{E23} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & a_{W33} & \cdot & a_{L33} & a_{33} & a_{H33} & \cdot & a_{E33} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W14} & \cdot & a_{L14} & a_{14} & a_{H14} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W24} & \cdot & a_{L24} & a_{24} & a_{H24} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{W34} & \cdot & a_{L34} & a_{34}
\end{bmatrix}
$$

So, adding a border of zeros to the initial system, discretized in space, matrix ***A*** in the space-state representation becomes a *quint diagonal* matrix, as it has only five non-zero diagonals. Matrix ***A*** has the elements on the diagonals *0, +1, -1, +m,* and *-m.* It is obvious that the dimension of the system increases considerably as the number of grids increases. If the discretized space has $l \times n$ control volumes, matrix ***A*** will have $l \times n$ rows and $l \times n$ columns. However, this matrix is a sparse matrix (i.e. it contains a lot of null elements).

The matrix B

This matrix contains the coefficients of the inputs. As the inputs are the temperatures of the walls, usually these coefficients are the heat convection coefficients. However, in air conditioned rooms, inputs are also the temperatures in the control volumes where the air is introduced in the room. For these control volumes, the corresponding coefficients in matrix ***A*** should be put to zero and their values should be given to corresponding elements in ***B*** matrix.

The matrix C

This matrix expresses the connection between the temperature in the control volumes (which are the states of the system) and the outputs. It simple contains *1's* as the coefficients of the temperatures of interest.

The matrix D

This matrix represents a direct feed through the system (i.e. the outputs are not delayed relatively to the inputs as they depend on the inputs at the same time). In this paper, the ***D*** matrix is identically null.

APPLICATION

The state-space model can be easily implemented in MATLAB. Then the simulations are carried out using the powerful graphic features and toolboxes (*SIMULINK*, Signal Processing, Control Systems and so on). An example on how to use a fixed flow field (achieved with PHOENICS) for the dynamic simulation is shown in Figure 4 and 5.

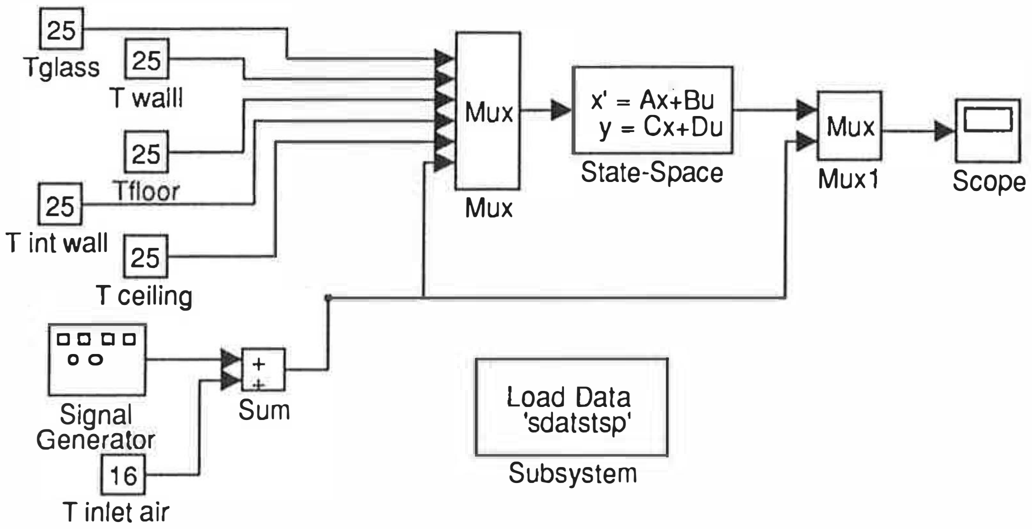

Figure 4 *SIMULINK* system using state-space model

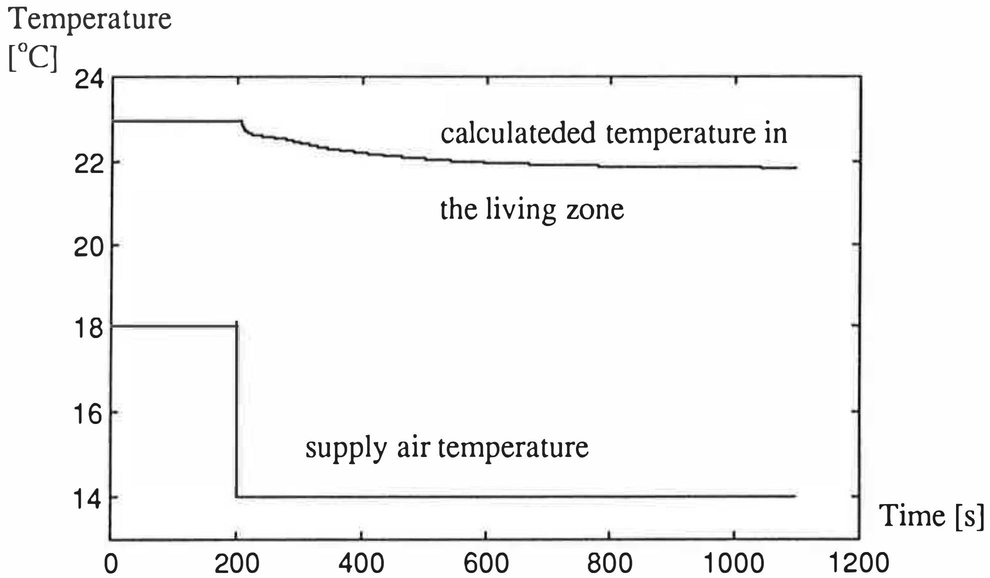

Figure 5 The results of the dynamic simulation

CONCLUSIONS

The dynamic temperature model of the flowing air can be derived from the energy balance equation when the flow field is fixed. It may be further easily put into a state-space representation..

This representation has the advantage of the easiness to connect with control theory. Thus various control strategies can be tested on it. The dynamic temperature distribution of the air can be easily obtained.

The state-space model contains many 'weak' dynamical parts that can be eliminated by model reduction methods. Through model reduction techniques (this can be easily done by using the MATLAB software), the scale of the model can be reduced while its 'strong' dynamics is preserved.

ACKNOWLEDGEMENT

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SUMMARY

Knowing the dynamic behaviour of the air temperature in air-conditioned rooms is important. The steady-state flow field and temperature distribution can be calculated by computer fluid dynamics (CFD) programs. To study the dynamics (in order to control the temperature or to know the response with time in different places) is prohibitively time consuming if CFD programs are to be used. For certain ranges of inlet temperature, the flow field could be considered as fixed. The output of CFD (the static flow field) can be used to achieve a state space model of the dynamic response of the air temperatures at various places in the room. The dynamic calculations can be then carried out much faster. In fact, this idea is accomplished through combining two softwares, PHOENICS for the CFD and MATLAB for control. Moreover, a lot of programs (toolboxes) specially designed for system analysis and controller design are available in MATLAB.