

BRE Building Research Establishment Ltd

Modelling and measurement of soil gas flow

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Modelling and measurement of soil gas flow

Summary

There are two main soil gases of current concern to human health. These are radon, which is a carcinogen, and landfill gas, which is explosive and toxic. Both can be found at significant levels in the soil below buildings in certain locations in the country. It is a responsibility of the Building Research Establishment to find cost effective ways to protect new and existing buildings from the entry of these gases into buildings. These findings are applied in real housing through advice given to builders and householders and through changes to the Building Regulations.

As part of the process of designing effective remedial measures it is necessary to understand how these gases move within the soil, how they enter buildings, and how these processes can be controlled. Modelling and experiments play an important part in this process alongside direct field trials of remedial measures.

This report extends and develops various theoretical modelling studies to evaluate experiments carried out at BRE and on other sites. They cover a range of different aspects of soil gas flow, and can have potential applications in related fields, particularly heat transfer.

The work is divided into three main parts. These are:

- 1) Flow under natural driving forces in steady state
- 2) The effect of high pressure extraction on soil gas flow
- 3) Time dependent effects on soil gas flow.

In each case a mixture of analytical and numerical methods is used to study the behaviour of the flow, and the results from these are compared with experimental data from laboratory or field tests.

The conclusions give a summary of the understanding gained from the modelling and

experimental process, and indicate areas of uncertainty remaining.

Scope and background information

Scope

This work looks at the flow of gases in soil and how they move into buildings, and houses in particular. The aim is to understand the way in which the processes occur, in order to be able to prevent or minimise the risks to health in a cost effective way. As such it plays a supporting role to the work of the Building Research Establishment in its advice on how hazardous gases can be kept out of buildings.

In order to understand soil gas flow, three types of work have been used. These are analytical modelling, numerical modelling and experimental measurements. Generally the analytical modelling only applies to a simplified problem, but one which often gives insight into the real flow processes. A numerical model will generally solve most mathematical problems, provided (an important limitation) enough information is available. Experimental work consists of laboratory type tests to look at details of how flow occurs, and measurements on real buildings to give information on the whole system.

The modelling work has all been carried out by the author, some of it developing from work elsewhere. The experimental work has involved a wider group of people, including some outside of BRE. In some cases the tests were designed to support the modelling studies directly, in others the data were collected during some other work.

Aspects relating to the creation of soil gases, their health effects, how the gases are measured and the details of how they are excluded from dwellings are beyond the scope of this work. They are discussed briefly in the following background information, which is only meant to give an outline of the subjects.

In general the work has been carried out in support of the work of colleagues at BRE with frontline responsibility for carrying out remedial measures and giving advice. It is

therefore concerned more with the underlying processes involved in soil gas flows than the detail of remedial action. Our overall need is to increase our understanding of soil gas flows, so that we can provide cost effective solutions. It is therefore harder to see the direct link to applications from this work than in some other projects. Nevertheless there is a significant role for modelling in improving remedial measures, and this is recognised by those involved. The link between the modelling and real situations is discussed throughout the text.

Background information on radon and landfill gas

Radon

Radon is a heavy and chemically un-reactive gas, formed by the radioactive decay of uranium. Because it is only present in very small concentrations it would be of only academic interest if it were not radioactive. This section gives a brief introduction to how and where it is found, why it is a health issue and what can be done about removing it from homes.

Sources of radon

Radon is created in the radioactive decay chain of uranium, as discussed at length elsewhere [Loureiro 87]. Because of this whenever uranium is present in the rock then there will also be radon produced, with thorium and radium as intermediate stages. Because radon is a gas it is possible for it to move within the soil, and thereby reach the surface and come into contact with people.

The level of uranium in rock is of interest to those who wish to mine it for energy or weapons usage so maps of uranium concentrations in rock exist for many parts of the world. Generally the areas with the highest uranium concentrations have the highest radon levels too. However the radon concentrations near the surface are affected by a number of other features of the rock types and soil. These include the permeability, the level of the water table and the degree of fissuring of the rock. These effects mean that just measuring uranium levels give only an indication of where radon is likely to be a problem. Within the UK the areas most affected by radon have been investigated in detail by the National Radiation Protection Board, hereafter abbreviated to NRPB. They have published maps of where radon is most likely to be found in houses. The techniques used for this mapping are discussed by Miles and others, for example [Miles 94], while the results of the mapping are used in defining the affected areas [BRE 91].

The areas most affected are Cornwall and Devon, parts of Northamptonshire, Derbyshire and Somerset. In addition there are small pockets of high radon levels in various other parts of the country. In Devon and Cornwall the radon levels are due to high uranium levels in the dominant granite rock. However in Derbyshire the rocks tend to be more fractured, and although the uranium levels are not as high, the radon is able to travel further before decaying. The position is more complicated in Northamptonshire with a very wide variety of rock types present. Work is ongoing by NRPB and the British Geological Survey on whether it is possible and practical to use geology to help predict likely radon levels in houses. Other areas of the country are also being investigated.

Measuring and typical levels

Radon can be measured most easily by using the fact that it is radioactive. There are a number of different techniques, the details of which are beyond the scope of this work. They involve counting the alpha particles produced by the decay of either the radon or its decay products (radon daughters).

In the UK the most widely used technique is 'Etch-track', a special film left in the house for a long period (typically three months), which is developed and the marks on it counted. With a largely automated process NRPB are able to process some tens of thousands of these per year.

In addition to the above passive detector there are a number of active detection techniques which involve taking a sample of air and counting the decays as they occur, using scintillation, ion capture or other effects. These machines cost more money, but allow the dynamic effects to be considered because they allow a measurement to be taken in an hour or less.

In the UK we use the SI unit for radon concentrations, the becquerel (Bq), usually expressed as (Bqm⁻³). One becquerel produces 1 radioactive decay per second, so a level of 1 (Bqm⁻³) would produce 1 radioactive decay per second in each cubic metre of air.

A level of 200 Bqm⁻³ has been chosen in the UK as the 'action level' for homes [DoE 95]. At this level there is a best estimate of the lifetime risk of lung cancer of 3%. At levels above this householders are strongly encouraged to take action to reduce radon levels. Statistics show [Miles 94] that around 100,000 homes in the UK should have levels above the action level.

Effects of radon

It is not appropriate to discuss health effects in this work. There has been a vast amount of research into the health effects of radon. Although the debate about the level of the risk continues, there is a clear link between elevated radon levels and the risk of contracting lung cancer. No other effects have been proven. For more information at a basic level see [DoE 95], or search the considerable Health Physics literature.

Remedial measures for radon

The principal involvement of the Building Research Establishment (BRE) has been in the field of measures to protect new and existing buildings against the entry of radon. Our experience has been published in a series of guides describing how to carry out measures in existing dwellings [BRE 92, Scivyer 93-1, Scivyer 93-2, Welsh 94, Pye 93, BRE 94, Scivyer 95, Stephen 95], how to choose which measure is appropriate for any particular construction type [Scivyer 93-1], and how to protect new buildings against radon [BRE 91]. This last report is an approved document within the Building Regulations in England and Wales, and therefore represents the legal requirements for building houses in affected areas. Most of our work on radon has been supported by the Department of the Environment (DoE). Work on non-domestic buildings is on going, under the control of the Health and Safety Executive.

Given that remedial techniques have been discussed at such length elsewhere, and the

relative effectiveness of them compared [Cliff 86, Cliff 91], there is no need to describe them in detail. However the key components of the main techniques are important in understanding the purpose behind modelling and experimental work, so they are discussed briefly here.

There are five classes of remedial measure, although sometimes more than one may be used on the same building.

i) Sealing

The dominant source of radon is usually the soil below the house. Hence if the paths for this entry can be sealed up, the radon levels should be reduced. This can be attempted by using a filler in any cracks found in the floor, by covering the floor with an airtight barrier, or, in extreme circumstances, by replacing the whole floor. This process is generally found to be fairly ineffective, and the explanation of this is one of the first successes of the modelling process. More information on sealing is given in [Pye 93].

ii) Sumps or sub-slab ventilation

In the UK we refer to sumps, but in the USA a sump is a water drainage item, and they use the term sub-slab ventilation or depressurisation system (SSV). Radon entry is generally caused by the pressure difference between inside the house and outside [Nazaroff 85, 87]. This pressure difference is generally about 1 or 2 Pascals, and is caused by temperature differences and the wind blowing on the house. Although this pressure is small it causes flow from below the floor to above it, carrying radon with it.

The sump is a void created below the floor slab, and air is removed from the void using a fan, or a passive stack ventilation system. The aim is to reverse the direction of flow of air across the floor, so that there is no bulk flow of air from below to above the floor. If this is achieved across all of the floor then little radon will enter the house, [BRE 92].

This measure is generally the most effective in houses with solid floors, and has achieved very large reductions in radon levels. The main problem is the cost. This comes from the

installation costs, from disturbing the floor, fitting a fan and pipe work, and the running costs of the fan. Nevertheless for houses with very high radon levels it is thought to be worthwhile.

In some cases the flow through the fan is reversed, so that air is pushed into the ground from the sump. This has the effect of pushing the radon away from the building, and can work in some conditions where the suction sump does not.



Figure 1: Schematic diagram of a radon 'sump'

iii) Underfloor ventilation

In houses with a suspended floor it can be effective to increase the ventilation rate of the void below the floor. This works for one of two reasons. Either the air in the subfloor space is diluted, so that less radon enters the house from underneath the floor, or the pressures in the underfloor space are changed so that there is less flow from the ground into the underfloor space, or less flow from the underfloor space into the house. The change can be achieved with passive measures, just increasing the number of airbricks, or with a fan. These methods are discussed elsewhere [Welsh 94], and can be very effective under some circumstances.

iv) Positive pressurisation

In some houses it is possible to use a small fan to blow air into the house and slightly

increase the pressure within it. This can have the same effect as a sump in reversing the normal pressure difference across the floor, and thereby prevent radon from entering the house. It also increases the ventilation rate, giving a dilution effect. However it is not as effective as a sump, and causes unwelcome drafts in many homes, so is not widely used. More information is given in [Stephen 95].

v) Ventilation

If the radon level in a house is not particularly high it may be possible to reduce it to an acceptable level by increasing or controlling the ventilation within the house. This may be appropriate where a house has inadequate ventilation, or a chimney which is no longer used. However it is rarely possible to achieve much by this method, although it has the benefit of being cheap.

Neither of the house ventilation options iv) or v) will be discussed further in this work.

Landfill gas

Sources of landfill gas

As the name implies landfill gas is gas produced in a landfill site, loosely defined as any hole which has been (re-)filled with waste of some type. If the waste contains significant organic matter of any type, then the breakdown of that material will produce the mixture of gases known as landfill gas.

The principal components of landfill gas are air, methane and carbon dioxide, in proportions which vary from 100% to 0% air and 0% to 50% or more of methane and carbon dioxide.

There are a many landfill sites in the UK, and many of the older ones were poorly set up. This means they may have little or no provision for protecting against the presence of gas; indeed the content of the waste and even the location of the site may not be known. Modern sites are better controlled, and generally protected against gas migration. Some

sites collect gas deliberately and burn the methane off, sometimes to generate electricity.

The bio-chemistry of the production of landfill gas is beyond the scope of this work, but has been discussed widely elsewhere.

Measuring and typical levels

There are several techniques for measuring landfill gases, including infra-red detectors, flame ionisation (combustible gases only) and gas chromatography. The techniques are not relevant to this work.

Concentrations are best expressed as concentrations by volume, for example 3 ppm means that of every million molecules, three will be of the chosen gas. In this way the weight of the gas is not important. Sometimes methane levels are given as %LEL, meaning percentage of the Lower Explosive Limit. Methane can explode above 5% by volume, so 50% LEL means 2.5% by volume. Whenever used in this work the percentage by volume will be used.

Levels of carbon dioxide are more complex because it is produced by people and by combustion. Levels up to 1000 ppm indoors are not unusual, and 'clean' outdoor air generally has around 300 ppm.

Effects of landfill gas

The two main constituents of landfill gas which affect people are methane and carbon dioxide. There may be other gases in the mixture due to contamination of the waste on the site but they are harder to quantify and less commonly observed.

Because of the explosive limit given above, a concentration anywhere in the building of 1% by volume of methane is often used as a trigger level to evacuate a building. There have been a few occasions when explosions have been linked to landfill sites, of which the best known was at Loscoe [Derbyshire CC].

Carbon dioxide is held to be an indication of poor air quality above 1000 ppm. In the UK the Health and Safety Executive (HSE) exposure limit for 8 hours is 5000 ppm, whilst the limit for a 10 minute exposure is 15,000 ppm. Levels above this cause nausea and can cause death [Edwards 89].

The presence of these gases has become more common as a result of ever increasing pressure on building land. Buildings have been constructed near to and even on top of old landfill sites, and this trend is likely to continue in the future.

Remedial action for landfill gas

The measures discussed for radon all apply in principle to landfill gas. However the main measure used has been to prevent the building of houses on landfill sites, thereby removing the need for protective measures. If a non domestic building is necessary on a landfill site there are guidelines for protecting it [Hartless 91], which as with the radon case form a part of the Building Regulations.

Because of the danger of explosion with landfill gas a feature of protective measures is an alarm system to alert the building operators to the presence of gas. The presence of a skilled building manager is one of the reasons why larger buildings are allowed on a site, when housing would not be.

The focus of modelling studies specific to landfill gas has been more fundamental than for radon. I have looked more at the processes driving the flow than the details of gas entry to buildings or how remedial measures work. This reflects the less advanced state of the subject area.

Outline of the work in this thesis

The requirement on the Building Research Establishment has been to produce practical, successful and cost effective solutions to the problems of soil gases. This thesis reports on much of the modelling work carried out to provide understanding of the processes going on in soil gas flows. Where possible this is related to experiments carried out either

in the laboratory or on site. Although all of the themes are linked, the work is divided up into three parts, covering natural driven flows, fan driven flows and time variation.

Part 1: Flow under natural driving forces in steady state

In the part covering natural driven flows the processes causing the entry of soil gases into houses are discussed. Then in order to understand these better the equation governing the pressure field in the soil - Laplace's Equation - has been studied. A series of solutions to problems which become closer to the 'real' situation of a house are considered. In order to see how well these relate to real flows they are compared to an experiment carried out on a BRE test hut, which shows that the theory gives a good estimate of the real flow.

The solutions were analytical solutions which provide a result for a useful but restricted geometry. To give a general solution a numerical method is needed, and in the later part of the natural flow part this is discussed. The numerical model was developed from one used at LBL in the USA. It has the benefit of allowing the radon concentration equation to be solved as well as the pressure equation, which is useful in some circumstances. The radon equation can only be solved exactly in very restricted circumstances.

Part 2: The effect of high pressure extraction on soil gas flow

As described earlier, the most effective remedial measure for radon involves the use of a fan to extract air from below a floor slab. This results in higher pressures than can occur naturally. My early work at BRE used the numerical model developed in Part 1 to study this process, and this starts Part 2. It gave insight into how these 'sumps' work, so was useful at the time.

Subsequent work elsewhere [Bonnefous 92-1] has shown that it is necessary to consider the effect of the higher speed flow on the pressure field. This complicates the solution process because the equation relating pressure change to velocity is then non-linear. This was considered by using data collected from a series of tests of 'pressure extension'. These consist of measuring the pressure field generated by sucking from below a floor slab. The suction can be provided by a normal sump fan, or a vacuum cleaner attached as a temporary measure.

Part 3: Time dependent effects on soil gas flow

Although in some circumstances it is possible to neglect the changes in a system which occur with time, it is necessary to check whether this is the case. There are a number of time dependent effects, and these need to be considered when investigating soil gas movement. Some examples are given in the table below, along with their approximate timescale:

Cause	Timescale
Atmospheric pressure changes	hours
Wind fluctuations	seconds
Atmospheric tides	12 hours
Moisture level in landfill site	days
Biological change in landfill site	months to years
Change in water table	days to months
Plant cover	months
Indoor temperature	24 hours and seasonal
Human behaviour	minutes to days to years

Table 1: Influences on soil gas which can vary with time

These can all effect soil gas to some extent, and it is not practical to address all of them. In this work the atmospheric pressure is the main subject considered, since it has a direct effect on the amount of gas which emerges from a landfill site. The levels of radon found in houses also vary with time, but the interactions are much more complicated than those needed to understand the main effects on a landfill site. The processes involved are investigated with a mixture of analytical and numerical models, and these are compared with data from BRE experiments.

Part 1: Flow under natural driving forces in steady state

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Appendices A-D: Outline derivations of approximate results for aspects of the conformal mapping solution.

Chapter 1: Introduction to natural forces

In order to understand how soil gases enter buildings, the driving forces for this flow have to be understood. There are two processes for soil gas movement, namely diffusion and pressure driven flow. It is necessary to consider both, although in many cases one will dominate and the other can be neglected.

The flow due to diffusion occurs because of the difference in concentration between the gas in the air inside the house and the gas in the soil. Fluids always diffuse down a concentration gradient, and so there is a gradual movement of gas from soil to air. In early radon modelling work it was thought that this process would explain indoor radon levels [Dimbylow 85]. However other work [Nazaroff 87] has shown that under most conditions diffusion alone is not enough to achieve the levels found in houses.

The second flow process is pressure driven or advective flow. If there is a pressure difference then soil gas will be transported by it, with the flow directed from high to low pressure. There are a number of possible causes of this pressure difference, all of which can be significant.

In a normal UK house the pressure inside is slightly less than that outside, because of the combined effect of the wind and temperature differences between outside and in. The wind causes positive pressures on the windward side of the house, and negative pressures on the leeward side. Usually the negative part covers a larger area than the positive, and this results in a negative pressure indoors.



Figure 1.1: The effect of the wind

The temperature effect is generally called the stack effect, and gives a pressure difference due to the different weights of two columns of air at different temperatures. When the indoor air is warmer than the outdoor air this results in the indoor floor level pressure being lower than the outdoor, floor level pressure. Air enters through the bottom half of the house (including the floor) and leaves through the top (including the roof).



Figure 1.2: The stack effect

The effect of these two mechanisms is to give an indoor pressure at floor level which is nearly always slightly less than that outside. Using a ventilation model, for example BREVENT [Cripps 92], or by direct measurement it is found that this pressure difference is usually less than 5 Pa, and typically only 1 Pa. However it is still able to generate a significant flow rate. This natural driven flow is the subject of this chapter.

There are three other mechanisms for driving the flow. One is the presence of a mechanical ventilation system in the house, which can enhance or reduce the natural pressure driven flow. The second is a permanent driving force due to the production of soil gas in the soil. This applies to landfill gas when a building is built on top of a landfill site, or near to an unprotected site. This is beyond the scope of the present work. The third process is caused by changes in atmospheric pressure. This is discussed in the chapter on time dependent effects.

Timber Floors

Timber is the dominant flooring material in the UK, with over 90% of homes having some part of the floor as suspended timber. This type of flooring consists of a timber floor suspended on joists above a small air gap, of 10 to 50 cm depth. This space should be ventilated using 'airbricks' (bricks with holes in them, or preformed plastic equivalents), but isn't in all cases. This space is known as the (subfloor) void. In some houses there is an oversite layer of concrete on top of the soil below the air of the void, but this is often of poor quality, and so does not greatly affect the flow of gas. In other cases, particularly in older houses there is no covering over the soil at all, and air can pass freely from the soil into the void. Figure 1.3 below shows the main elements of the construction.



Figure 1.3: Diagram of suspended timber floor

As a result of the use of timber there is an easy entry path for radon through the soil, into the void and through the many cracks in the timber floor. The methods used in the UK to remediate these houses are discussed in detail elsewhere [Welsh 94], [Welsh 93], but mainly involve improving the ventilation of the void.

Outline of this part

In this part the equations which describe the flow of gas into the void below a floor are discussed, and analytical solutions found for them under a number of different conditions. These conditions produce problems which are gradually more difficult to solve, but are also closer to reality.

The analytical result is then compared with the results from an experiment on flow into a void, and also with results derived by other workers.

Chapter 2: Theory and Literature review

Pressure field equation

The key equation used is Darcy's Law given by

$$Q = -\frac{k}{\mu} \cdot A \cdot \frac{dP}{dx}$$
 or $-\frac{k}{\mu} \cdot A \cdot \nabla P$ in 3 dimensions (2.1)

where

Q is the flow rate $(m^3 s^{-1})$, k is the permeability of the soil (m^2) , μ is the viscosity of the fluid flowing (Pa.s), A is the area of flow (m^2) , P is the excess pressure of the fluid compared to ambient (Pa), x is the length over which flow occurs (m).

Darcy developed this result from experiments in the 1850's. It has been widely used and investigated, and there are theoretical derivations available, eg [Bear 72]. It can be combined with the continuity equation, where v is the velocity of flow, defined as

$$\nabla v = 0 \tag{2.2}$$

to give Laplace's Equation [Mowris 86]

$$\nabla^2 P = 0 = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \quad in \text{ two dimensions.}$$
(2.3)

This equation describes the pressure field within a region of soil. If it can be solved, then the flow rate can be found from Darcy's Law. It is worth noting that there is some variation in the way that the terms permeability and Darcy's Law are used in different sciences. In particular in studies of liquid flows in porous media a different definition of permeability is often used, with units of m/s. Hence it is important to check which form of the equation is being used; most work on soil gas has used the form given here.

Hence Laplace's Equation is used to model the pressure in soil wherever the Darcy Law for gas flow is valid. This is a fortunate result, since there has been much work done on solving Laplace's equation in a wide range of geometries, as it appears in many different fields of science.

Work at Lawrence Berkeley Laboratory has shown that soil gas flow is not always described by a linear equation, [Bonnefous 92-1]. However this generally happens when there is a sub-slab ventilation system being used (or radon sump in the UK), and gas flow velocities become high, of order 0.1 ms⁻¹ or more. The problems considered here have velocities of order 10⁻⁴ ms⁻¹, so the Darcy Law remains valid. The cases when non-Darcy flow occurs are considered in Part 2 of this thesis on high pressure flows.

Radon concentration equation

It is sometimes sufficient just to calculate the flow rate of soil gas into a building. This is because from the flow rate a prediction of a likely radon entry rate can be made, by assuming the soil radon concentration. The worst case assumption is that the deep soil radon concentration applies at all levels.

However for more detailed work, or if the effect of diffusion is important, then a full equation for the radon concentration is needed. This needs to be solved after the pressure field equation, since it depends on the flow of soil gas.

There are 4 components to the radon concentration equation: a diffusion term, an advection (or pressure driven flow) term, a creation term and a radioactive decay term.

The Diffusion Equation is written in one dimension as

$$\frac{\partial C}{\partial t} = \frac{D}{\epsilon} \cdot \frac{\partial^2 C}{\partial z^2}$$
(2.4)

where

C is the concentration of gas (Bqm⁻³ for radon), t is the time (s), D is the diffusion coefficient of radon in air (m²/s), ϵ is the porosity of the soil () and z is distance in the soil (m).

Combining this with the advection, creation (ϕ , mol m⁻³ s⁻¹) and decay (λ , s⁻¹) terms gives an equation for radon transport, stated by [Clements 74], [Loureiro 87] and many others as

$$\frac{\partial C}{\partial t} = \frac{D}{\epsilon} \frac{\partial^2 C}{\partial z^2} - \frac{1}{\epsilon} \frac{\partial (vC)}{\partial z} - \lambda C + \phi . \qquad (2.5)$$

Solving this, and then using the result to calculate the flux of radon allows the calculation of radon entry rate into a building, and hence an estimate can be made of the likely indoor radon levels.

Brief review of modelling work into radon flows

Gadgil [Gadgil 91-1] has written a review of radon entry models, covering modelling work of all types up to 1991.

Pre-dating the work on modelling how buildings are affected by radon there was work on radon movement in soil. This dates back to the discovery of radon. Schroeder et al [Schroeder 65] measured the radon diffusion coefficients of different soils.

In looking at the effect of atmospheric pressure on the flow of radon out of the ground Clements [Clements 74] had already used the full radon concentration equation (2.5). His work was mostly in one dimension, and considered the time dependent effects of changes in atmospheric pressure. This is considered again in Part 3.

Schery et al at New Mexico Inst of Mining [Schery 88] studied the way in which radon moved through soils, and how cracks affect this in particular. This is a major subject in its own right and not covered in this thesis.

In early work it was thought that diffusion would dominate the entry of radon into buildings, so the pressure driven component was ignored. Work in the USA [Landman 82], and at the NRPB [Dimbylow 85] produced the likely entry rates due to diffusion. However the conclusion coming from this work was that under most condition diffusion alone could not provide enough radon entry to produce the radon levels observed in buildings.

This result was supported by important experimental work at LBL by Nazaroff et al [Nazaroff 87], who investigated the rate of entry into the basements of houses. They

measured the pressure differences across the basement floor, the pressure field in the surrounding soil, and the ventilation rate (using tracer gases). They concluded that the pressure driven flows are the main component of radon entry, and that increasing depressurisation of a basement will increase radon levels, in spite of increasing the ventilation rate.

Landman and Cohen [Landman 83] looked at the flow through a crack with diffusion enhanced by pressure driven flow. They used an analytical method using line sources. Landman and Delsante [Landman 86] also worked on heat flow, but as the equations are the same as those used for Darcy driven flow, the results can be transferred to gas flow. This is discussed further in Chapter 8, as they worked on a similar problem to those tackled in this work.

Dimbylow [Dimbylow 87] extended his previous work by looking at pressure driven flow, solving this first with a finite difference scheme, and then solving the radon equation afterwards. He predicted a considerable increase in radon entry compared to that due to diffusion alone, with an increase of up to a factor of 15.

Scott [Scott 93] has done work on the entry of radon, and the effectiveness of different remedial measures. He used a range of different techniques to compare these, but favoured simpler approximate models to the more involved computational techniques.

The largest amount of modelling work in this area has been done at Lawrence Berkeley Laboratory (LBL) in the USA. The first step was by Mowris [Mowris 86] or [Mowris 87], who used both simplified analytical solution and a finite difference model of the pressure flow of soil gas into a building. He investigated the importance of pressure difference, soil permeability and crack width on the entry of soil gas, but did not model the radon concentrations. It is his model of soil gas that the BRE work developed from.

The next work at LBL was by Loureiro [Loureiro 87], who took the next step of modelling both pressure driven flow and the full radon concentration equation. He did this in 3D, an improvement on the 2D models used previously, but with greater computing needs. His is a very thorough study of the impact of soil and crack

characteristics on radon entry rate. One particular prediction was that the entry rate of soil-gas increases linearly with permeability, until the width of the crack in the floor slab is restricting the entry rate. However the radon entry rate was predicted to be less dependent on soil permeability for permeabilities less than about 10⁻¹² (m²), because diffusion then dominates.

Developing from the work by Loureiro the next step was to consider the effect of nonlinear effects on the pressure field. When radon sumps, or sub-slab depressurisation systems, are used then the pressures present are much larger than those caused by natural effects. Bonnefous studied this extensively, and developed a model to include this non-Darcy flow. Papers covering this include [Bonnefous 92-1] and [Bonnefous 92-2] which consider the effectiveness of radon sumps, and [Bonnefous 93] and [Gadgil 91-2] which consider the impact of a subslab layer of permeable aggregate on radon entry and the effectiveness of radon sumps. It is a significant finding of this work that in the absence of a radon sump, adding a high permeability layer will probably increase radon entry. This material is considered further in the High Pressure Flow part.

Gee, Holford and Owczarski and others at Pacific Northwest Laboratory developed a three dimensional finite element computer code, Rn3D, to look at transport of radon in soil and into structures. There are a number of reports on this work, including [Holford 88] and [Owczarski 89], but I have not studied them in detail.

Blue et al [Blue 90] used a simplified analytical model to try to explain the weakness in the correlation between soil gas radon and indoor radon levels. They chose to consider diffusion in the soil, but pressure driven flow into the building, giving some insight into the levels observed in homes.

Mosley at the US Environmental Protection Agency (EPA) has carried out a number of modelling studies, and experiments in support of them, [Mosley 95].

Another large amount of work on radon modelling has been carried by Nielson and Rogers, and others from Rogers and Associates Engineering Corporation. A summary paper [Nielson 94] gives a good overview of their work, and the outputs it has produced

including the RAETRAD computer model. They used a two dimensional finite difference technique to model the movement of gas in soil and into dwellings. In order to account for the three dimensional aspects of flow they used an elliptical-cylindrical geometry to represent rectangular buildings. This is an improvement on two dimensional modelling, but doesn't model the corners correctly. They compared the predictions of their model with a large number of measurements of radon levels in dwellings where the soil characteristics were also measured. The correlation is generally good.

As a development from the RAETRAD model they used sensitivity analyses to generate a lumped parameter model [Nielson 93] which included the results from the full model into a simplified form. It allows the likely indoor radon level to be estimated quickly from soil and building data. This work was used to support the development of the Building Code for Florida where, because of high water tables, they tend not to have basements as in other parts of the USA.

In Canada Yuill and Wray, [Yuill 91], [Wray 91] used a computer model, CONAIR, to look at the effectiveness of sump systems. The model accounted for time changes in the weather, particularly through the wind speed. They found that whilst sub-slab depressurisation of the soil below a building produced considerable reductions in radon, in the Canadian climate it could well result in problems of freezing soil or backdraughting of boilers. They used their model together with experiments on the airtightness of the floor slabs to advise builders how to remediate more effectively.

Another type of modelling involves looking at statistical data for radon levels indoors and investigating the soil and building factors which cause them. This is beyond the scope of this thesis, but is covered by Gunby et al [Gunby 93] from the UK, and others.

Moving to Europe, the earliest work on radon was carried out in Finland and Sweden. Arvela and Winqvist at the Finnish Centre for Radiation and Nuclear Safety considered the effect of changing ventilation rates on radon levels [Arvela 86], but without modelling the soil. Much progress was made in Sweden on remedial measures and measurements, [Clavensjo 84] and others, but I am not aware of much modelling of the entry of gas into buildings until recent work by Hubbard at the Swedish Radiation Protection Institute (SSI), discussed in the Time dependent part.

The main areas of modelling work carried out in Europe were supported by an EC contract, and involved work in Denmark, Sweden, the Netherlands and Belgium. The contract was co-ordinated by Miles at the UK National Radiological Protection Board (NRPB), whose work on radon mapping is beyond the scope of this thesis.

Anderson at Riso National Laboratory, Denmark spent a year at LBL, and so was able to develop from the work which they did there. His work, [Anderson 92], [Anderson 93] and [Anderson 94] reports on comparisons of a two dimensional model with the experimental results from a test basement structure at Riso. The model displays the correct qualitative results, but as with most other modelling work there remain problems in getting quantitative agreement between a model and 'real' results.

Some of this work was carried out with Hubbard and others at SSI at Sweden. She has been looking at time variations in indoor and soil gas radon concentrations, with a view to improving the use of short term measurements. [Hubbard 95].

In Belgium, Cohilis and others at the Belgian Building Research Institute (CSTC) used an existing finite element thermal model to look at the movement of gas into a dwelling, [Cohilis 91]. It is a promising approach, provided the Darcy Law is valid. They put more effort into modelling the movement of radon once it has entered the building [Cohilis 92], [Ducarme 94]. This is an interesting area, but not within the scope of this thesis.

Last in this review a significant amount of modelling work has been carried out at a Dutch Research laboratory called KVI. De Meijer, van der Graaf and others have made comparisons between modelling results and data from a test cell [van der Graaf 91, 93-1, 93-2]. The idea is similar to that used by Mosley at EPA, in trying to understand fully a simplified situation before moving on to a more complex solution. They used a mixture of approximate analytical solutions and computational solutions [van der Spoel 93, 94], [van der Graaf 93- 3, 93-4], and generally found good agreements with their experimental results. The modelling techniques used included Laplace Transforms, as used in this work.

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Chapter 3: Sloping step problem

The problem solved in this chapter is a fairly straightforward solution to Laplace's Equation which is relevant to the more advanced problems considered later. Because the mathematics is fairly standard the solution is given with little derivation.

The problem represents a simple two dimensional building, with a fixed pressure inside and out, and a linear change in pressure across the walls. The parameters n and m are used to allow a general solution to be found. It is defined by the following pressure distribution on y=0, also shown as figure 3.1.

x > n,	P(x,0) = 0
m < x < n	P(x,0) = n - x
x < m	P(x,0) = n - m



Figure 3.1: Pressure distribution on y=0 for solution 1

The value of m represents the distance from the centre of the house to the inside edge of the wall, while n is the length to the outer edge of the wall. Using a Laplace transform method the solution is found to be

$$\pi P(x,y) = \left((n\pm x) \cdot \tan^{-1} \left(\frac{n\pm x}{y} \right) \right) - \left(\frac{y}{2} \cdot \ln(y^2 + (n\pm x)^2) \right)$$

$$- \left((m\pm x) \cdot \tan^{-1} \left(\frac{m\pm x}{y} \right) \right) - \left(\frac{y}{2} \cdot \ln(y^2 + (m\pm x)^2) \right)$$
(3.1)

Each of the four braced terms are to be repeated with plus and then minus, giving eight terms in all. This result for the pressure field is plotted in figure 3.2, with the factors m and n equal to 2 and 3 respectively, and the pressures normalised. This shows that the boundary conditions have been satisfied correctly, with a linear pressure change between m and n on the x axis.



Figure 3.2: Pressure contours for solution 1

Flow rate for solution 1

By differentiating (3.1) to give dP/dx, and then integrating this in Darcy's Law (2.1) with respect to x from -m to +m, gives the flow rate, Q, as

$$Q = \frac{2kP_0}{(n-m)\pi\mu} \cdot \left[(n+m).\ln(n+m) - (n-m).\ln(n-m) - 2m.\ln(2m) \right] .$$
(3.2)

Here the indoor pressure is set to P_0 , and the factor (n-m) is needed to account for this. Note that the flow is predicted to be linearly related to the indoor pressure, the permeability of the soil and a factor relating to the shape of the building.

This flow result is the same as that produced by Landman and Delsante [Landman 86]

but via an alternative method involving Fourier series. They gave their result in terms of different parameters, but they are exactly the same. Note that in the limit m tends to n, the first and third terms in the square bracket cancel and the second term simplifies to ln(n-m). This predicts an infinite flow rate, as can be found for the simpler 'Top Hat' problem.

The problem presented here corresponds to an assumed pressure distribution. The pressure is unlikely to fall off exactly linearly across a wall, but avoiding any error involved in this requires considerably more effort, as discussed in the next section. The solution found in this section can be compared with that in the next section, or used to estimate the flows in a similar physical situation, if the permeability of the soil is known.

Chapter 4: Analytic solution to pressure field for a mixed boundary value problem

Introduction

A mixed boundary value problem is one where the type of boundary changes from fixed pressure to fixed flow rate (or pressure gradient). In particular in moving from open air to a solid wall we move from a pressure boundary to a no-flow boundary. This can be solved numerically, although the detail of the result near the change-over point can be difficult.

Here the problem is tackled analytically. It cannot be evaluated easily because the solution consists of integrals which have to be found numerically. However it has been possible to calculate the flow rate into the 'house' in a fairly simple way, and this result will prove useful.

The problem is a development from that in the previous chapter. Here no assumption is made about the pressure change across the wall of the house. The only assumption is of no flow vertically into the base of a wall. Although walls built with no footings are no longer allowed within building regulations, there are many houses built in the past which were built without foundations. Hence the problem has some validity in its own right. In addition it can be extended to a more general case with the technique discussed in the following chapter. Some of this material was published in [Cripps 93-2], but this chapter gives a more complete description of the study.

The bare soil house

From chapter 2 we assume we are trying to solve Laplace's Equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0 \tag{4.1}$$

in the half plane y < 0. The boundary conditions come from considering the problem:


Figure 4.1: Schematic diagram of 'house'

This house is then modelled by assuming some behaviour on the surface, defined as y=0. The conditions on y=0 are shown on figure 4.2 below. This represents the soil surface, P=0, no flow within solid walls, $\partial P/\partial y = 0$, and a fixed pressure inside a house, P=Po. This gives a good representation of the position in a house built with a timber floor with no concrete oversite, and the huts on the BRE radon pit.



Figure 4.2: The pressure field problem

In the method used the pressure problem is then represented using a complex variable form. Let w(z) be a function of the complex variable z = x + iy. Then w(z) can be written

$$w(z) = U(x,y) + i.V(x,y).$$
(4.2)

The use of a function of a complex variable is useful because of the fact that any function of a complex variable automatically satisfies Laplace's Equation. This means that the problem here is to find a solution which matches the boundary conditions. The Cauchy Riemann Equations state that

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y} \tag{4.3}$$

and

$$\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}.$$
(4.4)

Using these and the boundary conditions on the P problem in figure 4.2 allows us to define the boundary conditions on the related problem in w(z) as in figure 4.3 below.



Figure 4.3: Boundary conditions on the w problem

Here B is a constant to be determined. The boundary of the w region is called L. A part of the boundary where the real part is defined is labelled L', while those where the imaginary part are defined are labelled L". Note that since V is an even function of x, from the Cauchy-Riemann equations (4.3 and 4.4) U must be odd, hence the choice of $\pm B$ for the L" regions. Finding the constant B is a main part of the solution to the problem. The constant B is also important in finding the flow rate into the house.

So if we can solve for w we then take the imaginary part, which will give the pressure field V(x,y). The U part of the solution tells us about the pressure gradients.

Now [Muskhelishvili 46] gives the solution to this type of problem as, p. 281

$$w(z) = \frac{1}{\pi i} \sqrt{\left(\frac{(z-a_1)(z-a_2)}{(z-b_1)(z-b_2)}\right)} \int_L \sqrt{\left(\frac{(t-b_1)(t-b_2)}{(t-a_1)(t-a_2)}\right)} \cdot \frac{h(t)dt}{t-z} + C \cdot \sqrt{\left(\frac{(z-a_1)(z-a_2)}{(z-b_1)(z-b_2)}\right)}$$
(4.5)

ŝ

where

C is an arbitrary real constant to be determined, h(t) is given by the condition on w on the boundary L (i.e. y=0), the points a_j are where a boundary of type L' changes to one of type L', the points b_j have the opposite change, L' to L''.

For this problem the a_1 , a_2 have values -m and +1, and b_1 , b_2 terms -1 and +m. Inserting these in (4.5) defines the solution to the problem. However the value of h(t) has to be inserted into the expression, and this results in 5 different terms to the result, because there are 5 different regions to be considered. However for two of these it is zero so they

make no contribution.

h(t) is defined as follows:

$$\begin{split} t < -m & h(t) = 0, \\ -m < t < -1 & h(t) = -B, \\ -1 < t < +1 & h(t) = i.P_{0,} \\ +1 < t < +m & h(t) = +B, \\ t > +m & h(t) = 0. \end{split}$$

The other problem to consider at this stage is the argument of expressions containing t within the root sign in the integral. As the value of t changes the expression in the root changes sign, and hence the argument of the root of it needs to be considered separately. The arguments are defined as follows:

x=	-m x=	-1 x	=1	x=m
$arg t+m = \pi$ $arg t+1 = \pi$ $arg t-1 = \pi$ $arg t-m = \pi$	arg t+m = 0 $arg t+1 = \pi$ $arg t-1 = \pi$ $arg t-m = \pi$	arg t+m = 0 arg t+1 = 0 $arg t-1 = \pi$ $arg t-m = \pi$	arg t+m = 0 arg t+1 = 0 arg t-1 = 0 arg t-m = 0	0 $\arg t+m = 0$ 0 $\arg t+1 = 0$ $\arg t-1 = 0$ $\arg t-m = 0$



Using these allows the correct calculation of the root terms in equation (4.5). That the solution defined above meets the boundary condition can be checked by calculating the values on the boundary by examining the terms in (4.5). Some care needs to be taken in treating the principal values of integrals where the integrands diverge at some point on the boundary. The solution does match the condition, but the confirmation of this is too long to reproduce here.

The next step is to find the constants B and C, using the fact that the expressions for w must be finite. Concentrating on the x axis, where the boundary is defined, we re-write equation (4.5) in terms of x instead of z, and substitute for the a and b terms, with im $z \rightarrow 0+$ in the integrand. Under this limiting process, (4.6) yields a principal value integral and a term from the pole at t=z which does not contribute to (4.7). Hence

$$w(x) = \frac{1}{\pi i} \sqrt{\left(\frac{(x+m)(x-1)}{(x+1)(x-m)}\right)} \begin{bmatrix} \int_{-m}^{+m} \sqrt{\left(\frac{(t+1)(t-m)}{(t+m)(t-1)}\right)} & \frac{h(t)dt}{t-x} + iC\pi \end{bmatrix}.$$
 (4.6)

From equation (4.6) at x = m, or at x = -1 the outer factor goes to infinity, so the inner terms must also total zero or w will diverge. Hence at x=m, expanding (4.6) into the 3 parts, including the effect of the arguments of t, which produces an i term for each part which all cancel

$$0 = \int_{-m}^{-1} \sqrt{\left(\frac{(-1-t)(m-t)}{(t+m)(1-t)}\right)} \frac{-Bdt}{t-m} + \int_{-1}^{+1} \sqrt{\left(\frac{(t+1)(m-t)}{(t+m)(1-t)}\right)} \frac{P_0 dt}{t-m} +$$

$$\int_{-m}^{+m} \sqrt{\left(\frac{(t+1)(m-t)}{(t+m)(t-1)}\right)} \frac{Bdt}{t-m} + C\pi .$$
(4.7)

Similarly at x = -1

$$0 = \int_{-m}^{-1} \sqrt{\left(\frac{(-1-t)(m-t)}{(t+m)(1-t)}\right)} \frac{-Bdt}{t+1} + \int_{-1}^{+1} \sqrt{\left(\frac{(t+1)(m-t)}{(t+m)(1-t)}\right)} \frac{P_0 dt}{t+1} +$$

$$\int_{-m}^{+m} \sqrt{\left(\frac{(t+1)(m-t)}{(t+m)(t-1)}\right)} \frac{Bdt}{t+1} + C\pi .$$
(4.8)

By subtracting (4.8) from (4.7) to remove the C term, an expression for B can be found. After some rearrangement it is given by

$$B = -P_0 \cdot \frac{\int_0^1 \frac{dt}{\sqrt{(1-t^2)(m^2-t^2)}}}{\int_0^m \frac{dt}{\sqrt{(t^2-1)(m^2-t^2)}}}.$$
(4.9)

Using this expression within equation (4.7) or (4.8) leads to the simple result for C C = 0. (4.10)

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The result for B is defined by two elliptical integrals. These are given in tables, or can be found numerically. Some values for B for different values of the parameter m are given below. For values of m near to 1, or large values of m an approximate result can be found for B. These are given in Appendix A to this part.

m	B/P ₀
1.1	1.41
1.2	1.20
1.5	0.95
2.0	0.78

Table 4.1: Values of B/P_0 for different values of m

Complete expression for the solution

Hence combining the previous results the full result is

$$w(z) = \frac{1}{\pi i} \sqrt{\left(\frac{(z+m)(z-1)}{(z+1)(z-m)}\right)} .$$

$$\left[\int_{-m}^{-1} \sqrt{\left(\frac{(-1-t)(m-t)}{(t+m)(1-t)}\right)} \frac{-Bdt}{t-z} + \int_{-1}^{+1} \sqrt{\left(\frac{(t+1)(m-t)}{(t+m)(1-t)}\right)} \frac{P_0 dt}{t-z} + \int_{+1}^{+m} \sqrt{\left(\frac{(t+1)(m-t)}{(t+m)(t-1)}\right)} \frac{Bdt}{t-z}\right].$$
(4.11)

Using the result

In order to evaluate the result from equation (4.11) and the definition of B a program was needed to calculate the integrals numerically. This required careful handling of the regions near to the changes between types of boundary condition, since at these points the terms involved become large.

This method was used to produce the plots shown in figures 4.5, 4.6 and 4.7. These give a result which meets the boundary conditions, and appear reasonable.

In figures 4.5 and 4.6 the parameter m had value 2. This is an unrealistic situation, since

it implies walls half the width of the space they enclose. However it shows the boundary conditions better than an example with smaller m.

In figure 4.5 the whole region is shown as a pressure contour plot. The flow of gas would be perpendicular to the pressure contours at all points, with the rate proportional to the pressure gradient or inversely proportional to the separation of the lines. This can be thought of as a marble rolling down a hill. In this plot the main pressure boundary conditions are seen to be met correctly.



Figure 4.5: Pressure contours when m=2



Figure 4.6: Pressure contours when m=2: enlargement of 'wall' region

Figure 4.6 is a close up of the right hand wall of figure 4.5. This shows the pressure contours meeting the y=0 line perpendicular to it. Hence because no flow occurs along pressure contours, no flow occurs across y=0 between x=1 and x=2 as required by the boundary conditions. Note that the kinks in the lines are due to not calculating enough data points for all regions, rather than representing any strange behaviour.

The non 'straight line' pressure drop between x=1 and x=2 is the main difference between this result and the one found earlier in chapter 3. There the pressure drop across the wall was forced to be linear, but this implied flow within the wall, which may or may not be valid.

Figure 4.7 shows the situation when m = 1.1. This means the walls are one tenth of the width of the room, which is more realistic than the previous result. The pressure contours are squeezed closer together at the walls, which means faster flow in those regions.



Figure 4.7: Pressure contours when m=1.1

Flow produced by the pressure distribution

A key result is the flow rate into a house produced by a given pressure distribution. This is given, assuming linear i.e. Darcy flow, by the integral of the pressure gradient between the two walls. This gives the result for the flow as

$$flow = \int_{-1}^{+1} -\frac{k}{\mu} \cdot \frac{\partial P}{\partial y}\Big|_{y=0} \cdot dx = \int_{-1}^{+1} -\frac{k}{\mu} \cdot \frac{\partial V}{\partial y}\Big|_{y=0} \cdot dx .$$
(4.12)

From the Cauchy Riemann equations (4.3 and 4.4) this is then equal to

$$flow = \int_{-1}^{+1} -\frac{k}{\mu} \cdot \frac{\partial U}{\partial x}\Big|_{y=0} \cdot dx .$$
(4.13)

But this is easily evaluated, since U is known at -1 and 1 as B and -B respectively. Hence the flow rate is simply given by

$$flow = -\frac{2Bk}{\mu} . \tag{4.14}$$

Here the flow is given in m³/s per metre of wall. Using this and the known values of B found earlier we can predict the flow rate expected for different geometries. This is returned to later. Note that as for the simpler solution of chapter 3 the solution for the flow rate is proportional to the pressure and a function of the geometry, although this function is different here.

Chapter 5: Conformal mapping of the mixed boundary value solution

Introduction

Conformal Mapping is a process whereby a solution to a problem found in one co-ordinate set is transferred to another co-ordinate set. This gives a solution to a problem which we may not have been able to solve in another way, or may be simpler than a direct method. It is discussed in most standard maths for degree level scientists text books, for example [Boas 83], or [Arfken 85].

Therefore, because we have solved a mixed boundary value problem in the previous chapter, we can use that solution to find the solution to another mixed boundary value problem by mapping the first onto a different co-ordinate set. The first step was to tackle a problem with thin foundation walls extending into the ground. The walls were assumed to allow no flow through them, which is a reasonable approximation in permeable soils. Then the method was extended to problems where the walls have thickness as well as depth. This problem is more general than the first and only slightly more difficult to evaluate. However it is easier to make approximations to the flow result in the thin wall case.

The methods used predict both the flow rate caused by the defined pressure distribution and the pressure field in the soil. The result for the flow rate is the simpler, and will be the most useful. The pressure field description involves numerical integrations, and is therefore harder to use. However it will be useful for model inter-comparisons and understanding experimental data.

Because the equation solved, Laplace's Equation, is applicable in other fields, these results may be of interest to those working outside of soil gas modelling. Flow of heat is the most obvious example, since heat loss through ground floors is very closely analogous to these problems.

Pressure field for house with bare soil floor and thin footings

Problem definition

The mixed boundary configuration of chapter 4 gives the solution to a problem where the boundary condition was defined all along the soil surface. It consisted of areas where the pressure was set to 0 (outside), areas where no flow could occur (walls) and areas where the pressure was set to P_o (inside). The majority of buildings have walls which extend down into the ground, so we would like to be able to solve for this type of problem as well. As a first step we will look at a thin wall extending down into the ground as shown in figure 5.1 below.



Figure 5.1: Diagram for the mapped pressure field problem, the Z(X,Y) plane

Note that d and c are positive, real numbers.

At first sight the problems described by figures 5.1 and 4.2 may not appear very similar. However following the path from points 1 to 7 of figure 5.1 and comparing it to moving from left to right across figure 4.2 shows the similarity. First there is region (from 1 to 2 for figure 5.1) with fixed pressure 0, then a region (from 2 to 3 to 4) with no flow perpendicular to the surface, then a region of fixed pressure P_o (4 to 5), another region of no flow (5 to 7) and finally another region of fixed pressure 0 (7 to 8). Points 1 and 8 are both taken to be at infinity, and so are essentially the same point. The problem then is to find the transformation that maps points from figure 4.2 onto figure 5.1, so that we can find the pressure field for figure 5.1 without further solving of the basic equations.

Unfortunately the transformation is somewhat complicated. The layout we are trying to map from is shown as figure 5.2. The points 1' to 7' on figure 5.2 are the points in the z(x,y) plane which will transform to the Z(X,Y) plane as points 1 to 7.



Figure 5.2: Diagram of the plane mapped from, the z(x,y) plane.

Definition of the transformation

The mapping used is called the Schwarz-Christoffel transformation [Carrier 66]. It gives the transformation which maps a plane onto a closed polygon, if the positions of the corners and angles at each corner are known. It is

$$Z(X,Y) = f(z) = a \int_{z_0}^{z} \sum_{j=1}^{n} (z'-e_j)^{(\alpha_j/\pi)-1} dz' + b$$
(5.1)

where

a, b and z_0 are constants to be determined, but we can choose $b = z_0 = 0$,

 α_i are the angles at the points in the Z plane,

e_i are the positions of the corresponding points in the z plane,

n is the number of points on the x axis, here 7 since points 1 and 8 are the same.

In this case the angles α_j are $\pi/2$, 2π , $\pi/2$, $\pi/2$, 2π , $\pi/2$ respectively for the points 2 to 7. The e_j are the points on the Y axis -m, - λ , -1, 1, λ , m.

Hence the transformation needed is

$$Z = f(z) = am \cdot \left(\int_{0}^{z} \frac{(z'^{2} - \lambda^{2})dz'}{\sqrt{[(z'^{2} - 1) \cdot (m^{2}z'^{2} - 1)]}} \right)$$

This can usefully be rearranged to

$$f(z) = am^{2} \cdot \left(\int_{0}^{z} \frac{\sqrt{(z'^{2}-1)}}{\sqrt{(z'^{2}-1)}} \cdot dz' + (1-\frac{\lambda^{2}}{m^{2}}) \cdot \int_{0}^{z} \frac{dz'}{\sqrt{[(\frac{z'^{2}}{m^{2}}-1).(z'^{2}-1)]}} \right) \cdot dz' + (1-\frac{\lambda^{2}}{m^{2}}) \cdot \int_{0}^{z} \frac{dz'}{\sqrt{[(\frac{z'^{2}}{m^{2}}-1).(z'^{2}-1)]}} dz'$$

(5.2)

This expression contains commonly occurring elliptic integrals which have to calculated numerically for nearly all values of m and z. The values are found in tables, for example [Abramowitz 65], or by numerical integration. Hence (5.2) can be written in a shorter form using notation for the elliptic integrals,

$$Z = am^2 \cdot \left[E(\frac{1}{m}, z) + (1 - \frac{\lambda^2}{m^2}) \cdot F(\frac{1}{m}, z) \right]$$
(5.3)

where

F(1/m, z) is the elliptic integral of the first kind,

E(1/m, z) is the elliptic integral of the second kind.

Finding the parameters in the transformation

There are then three unknowns remaining, a, λ and m. However there are three points where the result of the transformation is known:

f(1) = d	corresponds to points 5 and 5'	(5.4)

$$f(\lambda) = d - ic$$
 corresponds to points 6 and 6' (5.5)

$$f(m) = d$$
 corresponds to points 7 and 7'. (5.6)

We could equally well have used the points 2, 3 and 4, but the symmetry used earlier means no further information can be gained from that.

Hence we have a fully defined result, but its use is not straightforward. In order to use equation (5.2) the conditions (5.4) to (5.6) need to be used with the full expressions for

the integrals from (5.2) or the shorthand form (5.3).

Using (5.4) in (5.3) we obtain

$$Z = f(1) = am^{2} \left[E(\frac{1}{m}, \ 0 \rightarrow 1) + (1 - \frac{\lambda^{2}}{m^{2}}) \cdot F(\frac{1}{m}, \ 0 \rightarrow 1) \right] = d$$
(5.7)

where the E and F terms are integrated from 0 to 1. Similarly from (5.5) we have

$$Z = f(\lambda) = am^{2} [E(\frac{1}{m}, 0 - \lambda) + (1 - \frac{\lambda^{2}}{m^{2}}) \cdot F(\frac{1}{m}, 0 - \lambda)] = d - ic$$
(5.8)

and from (5.6)

$$Z = f(m) = am^{2} \left[E(\frac{1}{m}, \ 0 \neg m) + (1 - \frac{\lambda^{2}}{m^{2}}) \cdot F(\frac{1}{m}, \ 0 \neg m) \right] = d .$$
 (5.9)

The path of the second (5.8) and third (5.9) integrals includes the earlier ones; for (5.8) we can separate out the first part of the path (0 to 1), and so look at the second part (1 to λ) as

$$f(1) + am^{2}[E(\frac{1}{m}, 1 - \lambda) + (1 - \frac{\lambda^{2}}{m^{2}}) \cdot F(\frac{1}{m}, 1 - \lambda)] = d - ic .$$
(5.10)

But f(1) = d so (5.10) simplifies further to define c in terms of a by

$$am^{2}[E(\frac{1}{m}, 1 \rightarrow \lambda) + (1 - \frac{\lambda^{2}}{m^{2}}) \cdot F(\frac{1}{m}, 1 \rightarrow \lambda)] = -ic$$
 (5.11)

Similarly from (5.9) we obtain

$$f(\lambda) + am^2 [E(\frac{1}{m}, \lambda - m) + (1 - \frac{\lambda^2}{m^2}) \cdot F(\frac{1}{m}, \lambda - m)] = d$$
 (5.12)

But from (5.8), $f(\lambda) = d$ - ic, so that

$$am^2[E(\frac{1}{m}, \lambda \neg m) + (1 - \frac{\lambda^2}{m^2}) \cdot F(\frac{1}{m}, \lambda \neg m)] = ic$$
 (5.13)

Comparing the equations (5.11) and (5.13) indicates that there is a relation between the sizes of m and λ . They are dependent on each other, and so cannot be chosen separately. Since for this problem m is of more use I will assume a value of m and find λ from it. Either (5.11) or (5.13) gives a relationship between a and c, while (5.7) or (5.9) relates a and d. Hence c can be given in terms of d by dividing (5.11) by (5.7), or whichever form is most convenient. In order to choose a pair of values of c and d and find the values of

m and λ which correspond to them it is necessary to use an iteration method to find the appropriate values of m and λ .

Finding λ for given m

Because both (5.7) and (5.9) have d on the right hand side the two integrals are equal. But the path of the integral in (5.9) includes that in (5.7), so that these simplify to give

$$\lambda^{2} = \frac{\int_{1}^{m} \frac{z^{2} \cdot dz}{\left[(z^{2} - m^{2}) \cdot (z^{2} - 1)\right]^{\frac{1}{2}}}}{\int_{1}^{m} \frac{dz}{\left[(z^{2} - m^{2}) \cdot (z^{2} - 1)\right]^{\frac{1}{2}}}}.$$
(5.14)

These two integrals are standard elliptic integrals whose values can be found in tables, e.g. [Abramowitz 1965]. From these it is possible to find the values of the parameters m and λ from c and d. Some values are given below. For certain ranges of values, for example m close to 1 or m large, an approximation to the result in (5.14) can be used.

When m is close to 1 the approximate result is

$$\lambda \simeq 1 + \frac{(m-1)}{2} . \tag{5.15}$$

When m is large

$$\lambda^2 = \frac{m^2}{\ln(4m)} . \tag{5.16}$$

An indication of how these can be derived is given in Appendix B.

m	λ	с	d	a
3	1.991	1	1.8	0.322
2	1.499	1	2.87	0.986
1.5	1.2499	1	4.926	2.654
1.2	1.1	1	10.969	8.328

Table 5.1: Some of the values of the parameters

Using The Transformation Formula

Now we can find all the values we need in the transformation (5.2) we can proceed with using it. Hence we can choose any point z in the z(x, y) plane, and (5.2) gives us the position of the corresponding point in the Z(X, Y) plane. Since the problems are equivalent, we then can say that the pressure at a point Z(X, Y) is the same as that at the corresponding point z(x, y). Thus the problem is essentially solved.

We have the solution for the mixed boundary value problem in the z plane, and used it in chapter 4 to evaluate the result at discrete points in an x, y grid. In using the **r**ansformation (5.2) derived above we find the points in the Z plane to which each point in the grid corresponds. This means we do not get a regular grid of points in the Z plane, but have enough points to find the overall description of the pressure field. By choosing different points in the z plane we can add detail to the most important region in the Z plane.

The integrals in equation (5.2) are difficult because the variable z' is complex. We tackle it by integrating along a convenient path, say (0,0) to (x',0), then to (x',y'). this simplifies the integrals since dz' is either dy or dx.

$$z' = x + i.y$$
$$dz' = dx + i.dy$$

Using this gives two separate integrations to be carried out, in the x and y directions. In this work the real and imaginary parts of the integrals were separated and calculated separately.

It is important to consider the arguments of the terms within the square roots in the terms to be integrated. These are defined as follows

Above the cut line									
$arg(z^{2} - 1)$ $arg(z^{2}/m^{2} - 1)$	= п = п		= 0 = п		= 0 = п		=	0 0	
		1		λ		m			
Deley the sut line									x
Below the cut line									
$arg(z^{2} - 1) =$	= п		= 2n	1	= 2n	1	=	2п	

Figure 5.3: The arguments of the complex terms

In carrying out the integration it is important to be consistent and work either above or below the x axis. This choice affects the arguments resulting from the above diagram, and hence changes the form of the terms to be integrated. However the result is the same in either case, provided the choice is applied consistently.

In order to carry out the numerical integration the standard Simpson's Rule was used for each part of the path required. However the process is not straightforward because the terms on the denominator of equation (5.2) tend to zero at certain points, when z' = 1 or when z' = m. This does not mean the integral diverges, but it does mean that extra care is needed in carrying out the numerical integration.

The value of the denominator tends to 0, but in such a way that the integral of it is finite. In order to evaluate the integral, it is necessary to approach the point in very small steps, but without quite reaching it. This is achieved by reducing the step size progressively as the point is approached, until the contribution is so small it can be ignored.

Pressure Field

A result from the numerical integration of the transformation (5.2) is figure 5.4. We read in the result from the mixed boundary value problem or 'flat' problem of Chapter 4. Then the values of x and y from that problem are transformed to the equivalent points X and Y in the problem with depth, using (5.2). These are then output to a file, along with the corresponding pressure for the point. This information can then be plotted as a contour map of the pressure field.



Figure 5.4: Contours of pressure for the transformed mixed boundary value problem

On a contour plot, flow occurs at right angles to the contour lines of constant pressure. Consider it as a surface plot, and imagine the path a ball-bearing would take on the real surface. This is the path that gas would take down the pressure gradient. The 'wobbly' contours near the wall are a function of the lack of detail in this region, and the dscontinuity occurring there.

Flow rate

The value of the function U is transformed in the same way as the function P by the transformation Z = f(z). In calculating the flow rate in chapter 4 the only values of U which mattered were those where y = 0 and x=-1, and y = 0 and x =+1. These points map onto the points X = -d and X = +d on the X axis of the transformed plane, which are the points which are needed for the flow calculation in the Z plane. Hence, since same values apply at the 'key' points as for the untransformed problem, the flow is again given by:

$$flow = \frac{k}{\mu}.2B \tag{5.17}$$

where B was defined in the chapter 4 equation (4.9). B is a constant for any given problem in the 'flat', untransformed co-ordinates. This flow is the same as for the

corresponding problem and means a given value of 'm' in the 'flat' problem corresponds to a specific ratio of c to d in the second problem. Some examples are given in the table below.

m	d/c	c/d	B/P.
1.001	2001.6 201	0.0005 0.005	2.8612 2.1296
1.02	101	0.010	1.9105
1.05	41	0.024	1.6234
1.1	20.99	0.048	1.4103
1.2	10.97	0.091	1.2039
1.3	7.62	0.131	1.0883
1.5	4.926	0.203	0.9503
1.7	3.759	0.266	0.8655
2	2.87	0.348	0.7817
2.5	2.166	0.462	0.6951
3	1.8	0.556	0.6396
5	1.216	0.822	0.5261
10	0.837	1.195	0.4261
25	0.587	1.704	0.3411
100	0.4	2.500	0.2622
1000	0.26	3.846	0.1894
10000	0.191	5.236	0.1482

Note that when m is close to 1, the ratio c/d is equal to half of m-1. This is encouraging, as it suggests that a shallow cut into the ground has equal effect on the flow rate to a no-flow region on the surface of length equal to the length of both sides of that cut. This seems a reasonable result.

When c/d becomes larger the flow rate reduces significantly, and the relationship between m and c/d changes. In Appendices C and A expressions for c/d and the flow function B are investigated for the cases when m is large and m is close to 1.

Second Conformal Mapping Problem

Problem definition

Now that we have the solution to a problem with thin footing walls it is natural to try to extend it to a more realistic problem, where the walls have thickness and depth. This then corresponds more closely to a real structure.

The problem with thick walls is defined by figure 5.5 below.

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This can be related to the plane problem defined by figure 5.6 below





Figure 5.6: The z plane mapped from

Note that the notation used here is slightly different from that used before, although this is not significant. Here the (a, b, c, d) replace the previous choice of $(1, \lambda, m)$ from figure 5.2. This means that a different choice of arbitrary variables is used to the previous case, and the scale factor 'a' from the first example is chosen as 1.Because of this the point on the z plane nearest the origin, (a, 0), is not necessarily at the point (1, 0) as in the previous case.

The angles α_2 to α_9 are

 $\alpha_2 = \alpha_5 = \alpha_6 = \alpha_9 = \pi/2$

$$\alpha_3 = \alpha_4 = \alpha_7 = \alpha_8 = 3\pi/2$$

so that in the equation for the Schwarz-Christoffel transformation [Carrier 66] we get

$$Z(X,Y) = f(z) = A \int_{z_0}^{z} \frac{\left[(z'+b)(z'+c)(z'-c)(z'-b)\right]^{\frac{1}{2}}}{\left[(z'+a)(z'+d)(z'-d)(z'-a)\right]^{\frac{1}{2}}} dz' + b .$$
(5.18)

Then on setting b and z_0 equal to 0, and A to 1 as the arbitrary constants, then combining the terms we obtain

$$Z = f(z) = \int_{0}^{z} \frac{\left[(z^{\prime 2} - c^{2})(z^{\prime 2} - b^{2})\right]^{\frac{1}{2}}}{\left[(z^{\prime 2} - d^{2})(z^{\prime 2} - a^{2})\right]^{\frac{1}{2}}} dz' .$$
(5.19)

Equation (5.19) defines the transformation. It transforms the points x = a, b, c, d, in the z(x,y) plane, to the points 6, 7, 8, 9 in the Z(X, Y) plane. From symmetry these four points also correspond to the points labelled 5, 4, 3, 2 in the X negative half of the Z plane.

Using the transformation, with the notation

$$f(x_1, y_1) = (X_1, Y_1) = X_1 + iY_1$$

at these known points we have

f(a,0) = (p,0) = p,f(b,0) = (p,r) = p + ir,f(c,0) = (q,r) = q + ir,f(d,0) = (q,s) = q + is,

and hence

$$p = f(a,0) = \int_{0}^{a} \frac{\left[(z^{2}-c^{2})(z^{2}-b^{2})\right]^{\frac{1}{2}}}{\left[(z^{2}-d^{2})(z^{2}-a^{2})\right]^{\frac{1}{2}}} dz'$$
(5.20)

In this case all of the terms in the brackets are purely imaginary, so the 'i' terms cancel and p will be real and positive. Also

$$p + ir = f(b,0) = \int_{0}^{b} \frac{\left[(z^{\prime 2} - c^{2})(z^{\prime 2} - b^{2})\right]^{\frac{1}{2}}}{\left[(z^{\prime 2} - d^{2})(z^{\prime 2} - a^{2})\right]^{\frac{1}{2}}} dz'$$
(5.21)

but p is equal to the part of this integral from 0 to a, so

$$ir = \int_{a}^{b} \frac{\left[(z'^{2} - c^{2})(z'^{2} - b^{2}) \right]^{\frac{1}{2}}}{\left[(z'^{2} - d^{2})(z'^{2} - a^{2}) \right]^{\frac{1}{2}}} dz' .$$
(5.22)

Looking at the terms we see that r is real and negative. Similarly from f(c,0) = q+ir we obtain

$$q = \int_{b}^{c} \frac{\left[(z^{n} - c^{2})(z^{n} - b^{2}) \right]^{\frac{1}{2}}}{\left[(z^{n} - d^{2})(z^{n} - a^{2}) \right]^{\frac{1}{2}}} dz'$$
(5.23)

and q is real and positive; lastly

$$is = \int_{c}^{d} \frac{\left[(z'^2 - c^2)(z'^2 - b^2) \right]^{\frac{1}{2}}}{\left[(z'^2 - d^2)(z'^2 - a^2) \right]^{\frac{1}{2}}} dz' .$$
(5.24)

This gives s as a real value, but with its sign either positive or negative, depending on the sizes of a, b, c, d. Integrating these expressions numerically for a range of values of a, b, c, d gives the results in the table below.

a	b	С	d	p	đ	r	S
1.00	1.200	1.800	2.00	1.323	1.624	-0.282	-0.001
1.00	1.400	1.600	2.00	1.445	1.477	-0.473	-0.003
1.00	1.010	1.990	2.00	1.032	1.963	-0.015	-0.000
1.00	1.0001	1.999	2.00	1.001	1.999	-0.000	-0.000
1.00	1.450	1.550	2.00	1.456	1.464	-0.498	-0.003
1.00	1.490	1.510	2.00	1.459	1.460	-0.508	-0.003
1.00	1.499	1.501	2.00	1.459	1.460	-0.509	-0.003
1.00	2.000	3.000	4.00	2.173	2.451	-1.352	-0.070
1.000	5.240	5.250	10.00	4.252	4.252	-5.083	-0.0066
1.000	4.900	5.600	10.00	4.240	4.286	-5.020	-0.0163
1.000	4.000	6.550	10.00	4.036	4.650	-4.415	0.0036
1.000	2.000	8.900	10.00	2.609	7.359	-1.678	-0.0274
1.000	1.100	9.900	10.00	1.263	9.627	-0.159	-0.0027

Table 5.2: Values of the parameters from the second conformal mapping problem

It doesn't look as if these integral expressions can be simplified further, nor is it possible to choose values of p, q, r and s and find, directly, the values of a, b, c and d which produce them. However it is fairly easy to find particular values of p, q, r and s by experimenting with the values of a, b, c and d within the integration program.

Flow rate

Using exactly the same analysis as for the flow rate of the first conformal mapping solution earlier we know that the flow rate is the same for any problem where the value of 'm' as used in chapter 4, (in this problem d is equivalent to 'm' if 'a' has the value 1).

In this case this means any system which has the same a and d has the same flow rate. The figures 5.7 and 5.8 show equivalent shapes when a = 1 and d = 2 and then d = 10.



Figure 5.7: Examples of equivalent shapes when a = 1 and d = 2



Figure 5.8: Examples of equivalent shapes when a = 1 and d = 10

Case of the radon pit hut

As an example of the application of this flow result, consider the huts on the radon pit at BRE. These are discussed more in Chapter 6. The huts on the site were built on to a concrete ring beam which has dimensions as follows:

Full width of ring beam:	3.3 m,
Horizontal thickness of beam:	0.58 m,
Depth of beam:	0.4 m.

From these the parameters from the Z plane are as follows:

p = 1.07 m, q = 1.65 m and r = -0.4 m.

Hence q/p = 1.54 and r/p = -0.374.

From an iteration process with the numerical integration programme these same ratios of q/p and r/p are found to be given by

a = 1, b = 1.425, c = 3.04 and d = 3.48.

Then to find the flow rate, we use the value of d as the parameter m in equation 4.9 which gives the flow factor B as 0.6014 (from a numerical integration). Hence the flow is given by:

Flow = 2 . k/μ . ΔP . 0.6014 .

So for 50 (Pa) as the pressure difference, and μ for air is 1.83×10^{-5} (Pa.s) this would give

Flow = 3.3×10^{6} .k (m³s⁻¹ per metre of wall).

Since the internal length of the hut built on the ring beam is 2.14 m in each direction this is multiplied by 4.28 to give the total estimated flow rate as

$$Flow = 14x10^{6} \cdot k \quad (m^{3}s^{-1}) = 5x10^{10} \cdot k \quad (m^{3}h^{-1}) \quad .$$
(5.25)

Then with the laboratory measurement of the sand permeability, $1.1 \times 10^{-10} \text{ m}^2$, the predicted flow rate at 50 Pa is

$$Flow_{50} = 5.6$$
 (m³h⁻¹).

The experiment reported on in chapter 6 finds the flow rate at 50 Pa to be 14 (m^3h^{-1}) , so the agreement is quite good. These results and reasons for differences are discussed further in Chapter 7.

Pressure field

By modifying the program used for the thin wall problem, the pressure field for the thick wall problem can be calculated using the transformation equation (5.19). This involves using the calculated values of the parameters a, b, c, d which correspond to the desired values of p, q, r, s.

The results for some of the set of solutions where a is 1 and d is 2 are shown in figures 5.9 to 5.12. The first of these corresponds very closely to that of figure 5.4 of section 2. The others show the result for progressively 'fatter' walls, leading to figure 5.12, which is close to the result for the untransformed problem of figure 5.2.

Note that the model provides no data at all for the region within the wall. The contour lines drawn are 'made up' by the plotting program, and are of no validity. This also causes the 'wobbles' in the lines near the wall, particularly in figure 5.9.



Figure 5.9: Pressure field contours when a = 1, b = 1.45, c = 1.55, d = 2(gives p = 1.456, q = 1.464, r = -.5, s = -0.04)



Figure 5.10: Pressure field contours when a = 1, b = 1.2, c = 1.8, d = 2(gives p = 1.32, q = 1.62, r = -.28, s = -0.001)



Figure 5.11: Pressure field contours when a = 1, b = 1.05, c = 1.95, d = 2(gives p = 1.12, q = 1.86, r = -.08, s = -.001)



Figure 5.12: Pressure field contours when a = 1, b = 1.01, c = 1.99, d = 2(gives p = 1.03, q = 1.96, r = -.016, s = 0)

Conclusion to chapter 5

This chapter has shown that it is possible to transform the result from one soil gas flow problem to another. In this way we obtain the result of a problem which we could not have achieved in a more direct way.

Although the results cannot be found exactly, because of the elliptical integrals within them, they give a number of valuable results. Of most use are the flow rates into a bare soil house predicted for different sized foundation walls. We can also find the pressure field produced for the same problem, which will be useful in the verification of computational models. The approximations to the flow result allow fairly simple expressions for the flow rate predicted to be written down.

The method could be applied to more complex and non-symmetric problems, and to any equivalent problem involving Laplace's equation. An obvious example would be heat loss through a ground floor.

There are also possible extensions to the method; in principle any shape made up of straight lines could be mapped in the same way, although every extra corner makes the calculations longer. Finally it will be necessary to consider the influence of the third dimension, since it will obviously have an impact in most real buildings.

Chapter 6: Experimental results

Introduction

The analytical solutions discussed before all relate to the solution of the pressure and flow equations for a building with a bare soil floor. The purpose of the experiment which is described below was to try to validate the result of that model by taking a measurement of that flow. However this is not straightforward as there is no direct way of measuring the flow through the soil.

The concept of the experiment was to try to measure for soil what a fan pressurisation test measures for a building, that is the overall leakage through all possible flow paths. In a fan pressurisation test, [Stephen 88], a fan is installed in the outer wall of a building, usually in a doorway. The rates of flow required to produce a series of pressure differences between inside and out are measured. These pressures are generally -50 to +50 Pa, with steps of 10 Pa. From the plot of these results the characteristic leakiness of the building is estimated. It is usual to express it as the number of air changes per hour (ach) at 50 Pa pressure, often called n_{50} . The air change rate is the volume flow rate of gas divided by the volume of the building, so it has units s⁻¹ or more usually h⁻¹ or ach.

Typical values of n_{50} for UK houses are in the range 10-15 ach at 50 Pa. This is relatively high on an international comparison, with countries with cold climates, eg Sweden, routinely achieving values as low as 1 ach at 50 Pa, [AIVC 94]. The choice of 50 Pa is essentially arbitrary, but is the value usually used, so I have used it in this work.

In this chapter the experiment carried out by Andrew Cripps and Paul Welsh is described. The following section compares the result to that from the analytical solution.

The BRE Radon Pit

The radon pit consisted of sand about 6 m by 10 m and 4 m deep. The sand contains high levels of radium, which results in high radon levels in soil gas in the sand. On top of the

sand two structures were built. These were essentially identical, so that changes to one structure could be monitored against the other as a control. BRE was using the facility to investigate the methods to remediate houses with timber floors, but this work does not report on these experiments. There were two major problems with the facility which meant that the experiments were not continued in it. One was that the drainage never worked correctly, so that the water level was very high in the sand, greatly effecting the radon level . The other was that the huts were not sufficiently like real houses to make the findings from the main experiments directly applicable to them. The work programme has been transferred to a full size test house.

The purpose of the test was to make measurements to predict the flow through the soil due to an applied pressure below the floor. This then gives data with which to compare the results of the modelling studies into the same problem, and helps us to understand the entry rates of radon into homes constructed with a suspended timber floor.

The main problem in doing this is that if a fan sucks air from the space below the floor, much of the flow will occur through the floor. This is because the floor of the building is much leakier than the sub-floor walls and the soil itself. Hence a method was needed to prevent flow through the floor, or to account for it.

The method we chose to use was to balance the pressure in the 'house', P_m , with that in the under floor area, P_u . The idea is shown in the diagram below.



Figure 6.1: The experimental arrangement

The pressure across the floor is changed using two fans. One fan sucks air from the main

space, at a rate Q_m . A second fan sucks air from the under floor space, at a rate Q_u . All the visible holes in the wall are sealed. The flow Q_u , when the pressures are equal across the floor is the 'leakage' of the soil Q_s and the subfloor walls Q_w combined. The walls here were painted on the inside with a bituminous paint, and the air bricks were carefully sealed. As a result the majority of the flow was probably going through the soil and not through the walls.

The fans used were not very easy to adjust, so it was not practical to obtain zero values for the pressure across the floor. Instead we took a number of readings of pressure difference across the floor for different flow rates for the main space. These then gave a curve from which we could estimate the value of the flow at zero pressure difference.

Basic results

One complete set of these results is given in the table which follows. These data are plotted in figure 6.2 below the table. It shows on the y axis the pressures produced across the floor of the hut, plotted against the hut fan flow which produced it. On the second y axis is the hut to outside pressure difference, plotted at the same under floor fan flow rate. The hut fan was left at the same setting, but the flow rate through it varies with the flow through the under floor fan.

Main - Underfloor	Mean in - out	Hut fan	Under floor fan
pressure P _{floor}	Pressure P _{hut}	flow, Q _m	flow Q _u
Pa	Pa	m ³ h ⁻¹	m ³ h ⁻¹
-0.1	-53.5	46	1.5
-0.06	-63.05	43.5	4.8
-0.05	-72.75	40.5	14.4
0.03	-88.0	36	24.6
0.07	-101	32.5	33
0.16	-112	29	41.4
0.23	-127.5	24	54.6
0.24	-128.5	23	54.6

Table 6.1: Results from test 2



Figure 6.2: Graph of pressures generated against underfloor fan flow

Straight line fits to the two sets of points are also shown on the plot, and for the pressure across the floor the lines showing the 95% confidence limits (assuming normally distributed errors) are also included. These line fits are given by

 $P_{floor} = -0.116 + Q_u \cdot 0.0063$

or with the 95% confidence limits as

 $P_{\text{floor}} = (-0.12 \pm 0.04) + Q_{\text{u}} \cdot (0.0063 \pm 0.0008).$

This gives the flow through the under floor fan at which zero pressure occurs across the floor as

 $Q_0 = 18.3 \text{ m}^3\text{h}^{-1.}$

However within 95% confidence limits this is

 $11 \text{ m}^3\text{h}^{-1} < Q_0 < 28 \text{ m}^3\text{h}^{-1}$

This error is fairly large, in spite of the efforts made to improve the experiment, and indicates the difficulty of accurately measuring the small pressure drop across the floor.

Repeat tests

We carried out three more tests with the underfloor fan at a different setting, and these are summarised below. There is still considerable uncertainty in the final predicted value, but the result is more reliable than the one we made earlier when we attempted to equalise the pressure across the floor.

Set number	Pressure (in-out)	Under floor fan flow
	Pa	m ³ h ⁻¹
2	-79.2	18.3
3	-77.8	24.6
4	-60.7	17.5
5	-34.6	9.7

Table 6.2: Combined results from sets 2-5

These data are plotted in the graph which follows.



Figure 6.3: Flow through the under floor fan against the pressure across the hut

The central line is a straight line fit to the best data as calculated from the 4 points from data sets 2 to 5. These four data points fit to a straight line with an R^2 (root mean square value) of 0.8. Each of these data sets also gave 95% confidence limits, not shown here.

The straight line is given by

$$Q_u = 0.9 - 0.26 P_{hut} m^3 h^{-1}$$

From this we predict the value of the flow at 50 Pa to be

$$Q_{u50} = 14 \text{ m}^3 \text{h}^{-1}$$

with the result being between 8 and 26 $m^{3}h^{-1}$ with a confidence of around 90%.

Measuring the permeability of the radon pit sand

In order to compare the theoretical model to the measured flow rate from the radon pit we needed to know the permeability of the sand. This can be measured either in situ, or using a sample in the laboratory. Stephen Sweeney and Andrew Cripps measured a sample of sand contained within a simple plastic pipe. The air flow was provided by a compressed air cylinder, and the flow rate measured by both a rotameter, a hot wire and a hot bulb anemometer. All three were used partly to check on each other at the low flow rates needed, but also because they were being compared with each other anyway as part of another experiment. These flows were measured in a 100 mm diameter pipe; the sample was in a larger pipe to allow larger flows to be used.

The measured data gave a best line fit as

Velocity = 0.008. Pressure (cms⁻¹).

Hence the flow rate is given by

Flow = $(0.008 / 100) \cdot \pi r^2$. Pressure $(m^3 s^{-1})$

where r is the radius of the small tube. Using this in Darcy's Law, rearranged as

$$k = \mu L Q / (\Delta P. A)$$

where L is the length (m) over which the pressure drop ΔP (Pa) occurs, and the other terms are as defined earlier. The data are as follows

Viscosity of air µ	1.83E-05	Pas
Length of tube L	0.21	m
Radius of small tube, r	0.05	m
Radius of large tube	0.076	m

This gives the result for the permeability as

$$k = 1.3 \times 10^{-10} m^2$$

This result is supported by a better measurement made at the National Radiological Protection Board, which from 4 measurements gave the average answer:

$$k = 0.9 \times 10^{-10} m^2$$

Note that these experiments always contain some error due to the problem of the transfer of the sand from the pit to the laboratory. It is not possible to know if the same conditions of compaction and moisture have been achieved in the laboratory, so there is inevitably some error. It is likely that the NRPB test involved better compaction of the sand, giving the lower permeability result.

Conclusions

We have used two fans in one of the BRE radon pit huts to measure the leakages of the parts of the hut. The floor is a great deal more leaky than the shell of the hut, with the pressure drop across the floor being 150 times smaller than that across the shell of the hut. The upper part of the hut is a little leakier than the lower part, and the overall leakage is about 2.5 air changes per hour at 50 Pa.

The flow through the sand is predicted to be about 14 m³h⁻¹ at 50 Pa.

We also measured the pressure set up in the sand at a number of points near the hut. These show that the pressure is measurable, but not too large to affect the behaviour of the neighbouring hut.

Chapter 7: Comparison of analytical and experimental results

Chapter 5 presents the solution for the flow rate into a 'house' with a bare soil floor and footing walls with width and depth, using the conformal mapping of the solution found in chapter 4. Then chapter 6 discusses an experiment carried out by the author, with colleagues at BRE, on the flow rate into a hut with a suspended timber floor. The geometry corresponded to the previous solution, allowing a comparison of the two; this is the subject of this chapter.

Chapter 5 equation (5.25) gives a theoretical expression for the flow rate into the radon pit as

Flow =
$$14x10^{6}$$
 k (m³s⁻¹)
= $5x10^{10}$ k (m³h⁻¹)

where k is the permeability of the sand in the radon pit.

Since the flow rate was measured (see Chapter 6) as $14 \pm 6 \text{ m}^3\text{h}^{-1}$ using this in equation (5.25) suggests the permeability of the sand to be

$$k_{\text{sand-pit}} \simeq 2.8 \text{ x } 10^{-10} \text{ m}^2$$
,

but with the 90% confidence interval based on the uncertainty in the flow measurement being

$$1.6 \ge 10^{-10} \text{ m}^2 < \text{k}_{\text{sand-pit}} < 5.2 \ge 10^{-10} \text{ m}^2$$
.

This is typical for sands [Mowris 86] and is close to that measured for the sand in the laboratory,

$$k_{sand-lab} \simeq 1 \times 10^{-10} m^2$$

The cause of the difference is likely to be one of the following:

1) Neglecting the corner effects in calculating the theoretical flow rate into the hut.

The fact that the model assumes infinitely long walls means that the flow per metre is under predicted near a corner. This is because it only considered flow in two dimensions, and this is not true near the corner. The effect of this is difficult to calculate, but could be significant.

2) The leakiness of the subfloor walls of the hut has been neglected in the theory.

This means that all of the flow measured going through the fan has not necessarily come through the sand as has been assumed here. Hence the real flow through the sand at a given pressure will be lower than that measured here, although the amount of the difference is hard to estimate.

3) Leaks from the pipes used to measure the sand permeability in the laboratory.

When the permeability of the sand was measured in the laboratory it is possible that some leakage occurred. This would suggest a greater rate of flow through the sand than actually occurs, resulting in a predicted permeability higher than the correct value. This is perhaps indicated by the fact that the NRPB measurement mentioned in chapter 6 gave a result slightly lower than that carried out at BRE. The NRPB test used a metal container that was more air tight than that used in the BRE test.

4) Uncertainty in the compaction and water content of the sand in the laboratory.

The degree of compaction of a material, and its moisture content affect the resulting permeability. In general a more compacted material has less air space within it, and so allows less flow through it for a given pressure. Hence in measuring the permeability of a sample of sand it is necessary to consider the degree of packing, and how this compares to conditions in the ground. It is likely

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that any sample of sand will be less compacted in the laboratory than in the ground, so that the laboratory will give a value of permeability higher than the 'real' one.

However sand is relatively less prone to packing effects because of the small size of the particles, so that the effect due to compaction will be less than in some other materials. This topic is returned to in the part of this thesis on high pressure flows.

The moisture content also has an impact, with a high moisture content expected to reduce permeability by occupying the air space through which gas moves. If the tested sample is too dry it would be expected to give a high result, but if it is too wet it would probably be too low.

Both 1 and 2 would cause the calculated permeability to be reduced from the 3 x 10^{-10} m² predicted, while 3 would reduce the laboratory measured permeability. Point 4 could affect the permeability in either direction, and deserves further investigation. It is likely however that the first two effects will be larger than the others, which would reduce the prediction of permeability to closer to the laboratory result.

Overall the result is clearly very encouraging, and shows good agreement between the two methods of finding the permeability, well within the considerable experimental errors involved in the experiments.

Comparing the two theoretical results

It is also interesting to compare the results of the simple analytical result of chapter 3 and the more advanced solution of chapters 4 and 5. Carrying out the flow rate calculation using equation (3.2), with the factors m and n being 1.07 and 1.65 respectively for the radon pit hut, the flow rate per metre run of wall is given by

Flow $= 1.5 \times 10^{10}$ k (m³h⁻¹ per metre run of wall).

As before there are 4.28 metres of wall, so that the full result is

Flow
$$= 6.4 \times 10^{10}$$
 k (m³h⁻¹).

This result is higher than that predicted for the more advanced theory which with the measured flow rate of 14 (m³h⁻¹) then predicts a slightly lower permeability of

$$k_{\text{sand-pit}} \simeq 2.1 \text{ x } 10^{-10} \text{ m}^2.$$

The simpler theory is expected to produce a higher flow rate because it has neglected the depth of the footing walls, so this difference is as expected. However it is interesting to note that the difference is comparatively small. Further, the simpler theory has given a result closer to the laboratory test for the sand permeability. This is probably due to the reasons given above for the difference between the theory and the experiment, all of which still apply to the simpler theory.

Nevertheless, given the errors in the experimental data, it is not clear that the extra effort involved in the more advanced solution is justified, since the simpler solution has given a similar result. However it is only possible to observe this by calculating both, and extra insight is gained by the process. In addition, as the footing walls become deeper, the difference would become larger.

Chapter 8: Comparing the analytical result with that of Landman and Delsante

The work which is closest to the result in chapter 5 has been done by Landman and Delsante [Landman 86], [Landman 87-1], [Landman 87-2]. They were looking at the heat loss through a ground floor slab, but the solutions they produced, and the flow rate predictions they made can be transferred directly to gas flow. There are some differences in the boundary conditions between the two problems, but the physics is the same.

The technique used by them was to find a Fourier series solution to Laplace's Equation in each of a number of defined regions, and then match these at the boundaries between the regions. In general this results in an infinitely large number of simultaneous equations, to which an approximate solution can be found by assuming the terms beyond a certain number can be ignored. This gives a set of equations which can be solved by matrix inversion techniques.

They used this technique on a number of different problems of heat flow from a concrete floor, and how it is affected by positioning insulation at different places. In their first problem [Landman 86] they considered a thin vertical layer of insulation at the edge of the floor slab. They then considered a thin horizontal layer of insulation [Landman 87-1], and finally looked at a problem close to that of chapter 5 here, with a region of insulation at the edge of the edge of the floor slab with both width and depth [Landman 87-2].

In order to compare the result found here the solutions from the first and third papers need to be combined. In all cases they assumed a linear fall in temperature from the inside of the house to outside, ie the same as looked at in Chapter 3 (but for pressure). In the absence of insulation this assumption has a significant effect on the predicted flow rate. However when there is insulation present the difference caused by the simplifying assumption is not important.

Their geometry is given in figure 8.1 below. Note that δ is the width of the insulation material and d its depth, L the half width of the 'house' and 2ϵ the width of the wall. Region 4 is the insulation material, regions 1, 2 and 3 are the soil, which is considered to be same in each region. The temperature distribution is given as the top half of the plot.



Figure 8.1: Reproduction of Fig 1 of [Landman 87-2]

The most significant difference between the thermal and pressure problem is in the relationship between the conductivities of the different materials (for temperature) and the permeabilities (for pressure). The thermal conductivity of soil and concrete are assumed to be similar by Landman and Delsante, which is a reasonable approximation. However the permeability of concrete is generally many orders of magnitude less than that of soil. Hence it is possible to compare the two modelling results because of the investigation of thermal insulation by Landman and Delsante. The insulation had a very low thermal conductivity (not always treated as zero) which is then equivalent to the no flow assumed for the concrete in this work.

In [Landman 87-2] the rate of flow of heat through the floor is calculated for different parameters, and this is shown in their Fig 2, reproduced here as Figure 8.2.



Figure 8.2: Reproduction of Fig 2 of [Landman 87-2] Graph of normalised flux against the ratio of depth of insulation against building half width,

The parameter γ used here is the ratio of the conductivity of the insulation to that of the soil. We are interested in comparing to their solution with $\gamma = 0$, meaning a perfect insulant, which is what the concrete footing wall in the air flow case approximates too. Taking the case where their d/L is 0.2, their normalised flux ϕ' is close to 0.6. The normalised flux is the ratio of the flux with the insulation to that without it, which is given in [Landman 86 equation (18)] as

$$\Phi_0 = \frac{2}{\pi} \left(\ln \left[\frac{L + \epsilon}{\epsilon} \right] + \frac{L}{\epsilon} \ln \left[\frac{L + \epsilon}{\epsilon} \right] \right)$$
(8.1)

where the parameters have been defined for the figure above. It has already been noted that this solution is the same as that found in chapter 3, with the changes to notation. Using the data given for the calculation by Landman and Delsante, the parameters are given as follows

L = 0.1,

$$d_L = 0.02,$$

 $\epsilon = 0.002,$
 $\delta = 0.0006.$

Using these in equation (8.1) above gives a result for φ_0 as 3.133, so that the result for φ

is

$$\phi = 0.6 \ x \ 3.133 \ = \ 1.88 \tag{8.2}$$

and this is used to define the total flow as

$$Q_T = \phi \ \Delta T \ k_s \tag{8.3}$$

where

 $Q_{\rm T}$ = total heat flux (Watts per metre of wall),

 ΔT = Temperature difference between inside and outside the building (K),

 $k_s =$ thermal conductivity of soil (Wm⁻²K⁻¹).

Hence the variable ϕ is equivalent to twice the variable B found earlier in Chapters 4 and 5, if the pressure difference is factored out of the expression for B (Chapter 4 eq 4.9 for B and Chapter 5 equation 5.17 for the flow rate).

Given the parameters defined above it is possible to use the method of Chapter 5 to find the value of B appropriate for the same geometry. The values of L, d and δ given above produce values of the parameters defined by figure 5.5 of Chapter 5 as

p = 0.1, q = 0.1006, r = -0.02,s = 0.

By iteration, using the integration code developed for chapter 5, these are given by the following set of parameters in the transformed plane:

$$a = 1,$$

 $b = 1.22,$
 $c = 1.29,$
 $d = 1.51.$

This gives the key result that the ratio of d (as defined in Chapter 5 not by Landman) is 1.51 times a, which is equivalent to the parameter 'm' of chapter 4 being 1.51. This gives a result for B as

$$\frac{B}{P_0} = 0.95$$
 : $\frac{2B}{P_0} = 1.9$ (8.4)

The two results (8.2) and (8.4) agree well, indicating that the methods are producing the same result in this case. This gives considerable confidence in the method used here.

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Chapter 9: Using a finite difference model on the natural flow problem

As an alternative to using an analytical model it is possible to use a finite difference technique to find the pressure and flow fields for a given geometry. The theory of this is discussed in the High Pressure Part of this thesis as it has also been used for looking at higher pressure flows. However it can be applied to the same problems discussed in this part.

The most significant results come from the flow rates. Although it depends significantly on the permeability of the soil, for most soils (as against carefully chosen aggregate materials) there is little difference between the flow rates with crack sizes above a relatively small size - about 1 mm for the geometry used to generate the results below.

Crack width (mm)	Flow (m ³ /h)	Effective resistance (Pa per m ³ /h)
10	.16	20.8
5	.155	21.5
1	.143	23.3
0.5	.112	29.8
0.2	.046	72.5

Table 9.1: Flow rate and effective resistance against crack width.

This initially surprising result is very significant, as it explains why it is very difficult to prevent radon entry by the sealing of floors. The resistance of the floor to the flow of gas is generally smaller than the resistance of the soil below the floor. As a result it takes a very large increase in the resistance of the floor (eg sealing up nearly all of the cracks) to have an impact on the flow rates. This work supports this view, but is not the first to observe the result. The use of a resistance model to give an understanding of soil gas flow has been used before and is returned to later in Part 2.

Given the flaws in the modelling techniques used here it is not valid to use this type of result other than in a qualitative way. However the understanding of what is going on in radon entry is still worthwhile.

Chapter 10: Conclusions to natural flow part

In this part the results of analytic studies of the flow of gas into the void below a suspended timber floor have been presented. These give complicated expressions for the pressure fields being produced, but much simpler forms for the flow rates. In each case the flow rate was found to be proportional to the permeability and the internal pressure as expected, but also proportional to a geometrical factor which can be found comparatively easily for any floor geometry.

A method for measuring the flow rate through the soil below a building with no concrete oversite was described, and the initial results presented. The technique will not be generally applicable because of the leakiness of most buildings, but could be of some use in measuring soil leakages.

The results for the two parts of the work have been compared through the permeability they predict for the sand at the BRE radon pit. Considering the considerable variability of permeabilities and the difficulty of measuring them accurately, the two predictions compare well with a direct experimental measurement of permeability.

In addition the overall flow rate has been compared with that predicted by a different theoretical method produced by another worker looking at heat flow. This produces a result in close agreement to that given here, suggesting that both methods are giving good answers.

There is insight gained into the flow processes going on in the soil by carrying out the more complex modelling process. However given the considerable uncertainty in estimating the permeability of the soils involved it is not clear that the more advanced techniques are justified, given that the differences between the two are quite small. Nevertheless it would not be possible to be sure of this without having carried out the calculations.

It would be possible to use the techniques given here to generate an 'atlas' of standard shapes and their corresponding flow rates. This would apply to heat flow problems as

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well. It is left to future workers who might continue work in these areas.

The use of a finite difference model to generate similar results is discussed briefly, but the insight into the flow processes can be gained from either type (analytical or computational) and they each have their benefits.

Appendices to Natural flow part

Introduction

During the development of the analytical solutions presented in chapters 4 and 5 there were a number of results found which can only be expressed in terms of elliptic integrals. These can therefore not be expressed in terms of exact functions, but need to be evaluated by using tables or numerical integration.

However most elliptic integrals can be approximated by simple functions for some range of values. This Appendix gives the methods for deriving these approximate results, and indicates their range of application. In some cases it is preferable to use an approximate result since it allows a result to be expressed in terms of the parameters which define it. Of course the range of application of such a result is important.

Generally these approximate results are only available for the simpler solutions with fewer parameters. In this case only the 'thin wall' problem has been considered, since the thick wall has too many parameters to make progress practical.

The following results are considered:

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Appendix A: Expressions for B as a function of m

In Chapter 4, equation (4.9) gives the flow function B as

$$B = -P_0 \cdot \frac{\int_0^1 \frac{dt}{\sqrt{(1-t^2)(m^2-t^2)}}}{\int_1^m \frac{dt}{\sqrt{(t^2-1)(m^2-t^2)}}}.$$
(A1)

Rearranging the numerator of the expression into the complete elliptic integral of the first kind, usual notation K, gives the numerator as

$$-\frac{P_0}{m} \cdot \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-\frac{t^2}{m^2})}} = -\frac{P_0}{m} \cdot K(\frac{1}{m^2}).$$
(A2)

The integral in the denominator can be also be expressed as the complete elliptical integral K, but with a different parameter. From [Abramowitz 65] p 596, 17.4.43 it is equivalent to $1/m.F(\phi,M)$, where

$$\sin^2(\phi) = m^2(m^2 - 1) / m^2(m^2 - 1),$$

so that

 $\phi = \pi/2$.

Hence the elliptic integral is complete. The parameter M is given by

$$M = (m^2 - 1) / m^2 = 1 - 1/m^2$$

Hence the denominator of (A1) is given by

$$\frac{1}{m} \cdot K \left(1 - \frac{1}{m^2} \right). \tag{A3}$$

and so combining these gives

$$B = -P_0 \cdot \frac{K(1/m^2)}{K(1 - 1/m^2)}$$
 (A4)

Approximate functions for the complete elliptic integral K are given in standard tables, for example [Abramowitz 65] and [Gradstheyn 80]. Using these when m is close to 1 allows (A4) to be rewritten as

$$B \simeq -\frac{P_0}{\pi} \cdot \ln\left(\frac{16}{(1 - 1/m^2)}\right)$$
 (A5)

Expression for B when m is large

A similar process gives the form when m becomes large as

$$B \simeq \frac{-\pi P_0}{\ln(16m^2)} \simeq \frac{-\pi P_0}{2\ln(4m)}$$
(A6)

These approximations allow the value of B to be found very quickly for the extreme values of the parameter m. Since m is often quite close to 1 the form of equation (A6) is often likely to be appropriate.

Appendix B: Finding expressions for λ as a function of m

In chapter 5 the variable λ was found to be given by equation (5.14) as

$$\lambda^{2} = \frac{\int_{1}^{m} \frac{z^{2} \cdot dz}{\left[(z^{2} - m^{2}) \cdot (z^{2} - 1)\right]^{\frac{1}{2}}}}{\int_{1}^{m} \frac{dz}{\left[(z^{2} - m^{2}) \cdot (z^{2} - 1)\right]^{\frac{1}{2}}}}.$$
(B1)

This expression can be simplified when the parameter m is either close to 1 or large. As in Appendix A the method given here uses the fact that the expression for λ involves standard elliptic integrals, for which there are asymptotic results available. In terms of these standard integrals (B1) can be written as

$$\lambda^2 = m^2 \cdot \frac{E(\phi, \alpha)}{F(\phi, \alpha)}$$
(B2)

where

$$\Phi = \sin^{-1}\{1/m^2(1/m^2-1) / \{1/m^2(1/m^2-1)\}\} = \sin^{-1}(1) = \pi/2$$

$$\alpha = \cos^{-1}(1/m)$$

and

E is the elliptical integral of the second kind,

F is the elliptical integral of the first kind.

Because ϕ is $\pi/2$ the elliptical integrals E and F are called complete elliptical integrals. Using the notation for complete elliptical integrals (B2) can be rewritten as

$$\lambda^2 = m^2 \cdot \frac{E(\sin(\alpha))}{K(\sin(\alpha))}$$
(B3)

where

K is the complete elliptical integral of the first kind.

The form of λ when m is close to 1

Equation (B3) together with results from standard tables allow an approximate expression for λ to be found. When m is close to 1 then α is small, and this allows simplification of the expression, according to, eg, [Abramowitz 65] and [Gradstheyn 80].

After some working and a binomial expansion the approximate result is

$$\lambda \approx 1 + \frac{(m-1)}{2} + O((m-1)^4).$$
 (B4)

The form of λ when m is large

When m is large the expression in (B2) gives α as close to $\pi/2$, so that different approximations apply to the previous case. Using these the result is found to be

$$\lambda^2 \simeq \frac{m^2}{\ln(4m)} + O(1)$$
 (B5)

Appendix C: Finding expressions for c/d as a function of m

This Appendix gives the methods for finding approximate results for the ratio of c/d when m is close to 1 and when m is large.

The ratio of c/d is defined by dividing equation (5.11) by (5.7), but using the form of the transformation function f(z) given as the first line in equation (5.2). This defines c/d by

$$\frac{c}{d} = \frac{\int_{1}^{n} \frac{(z^2 - \lambda^2) dz}{[(m^2 - z^2)(z^2 - 1)]^{\frac{1}{2}}}}{\int_{0}^{1} \frac{(z^2 - \lambda^2) dz}{[(m^2 - z^2)(1 - z^2)]^{\frac{1}{2}}}}.$$
(C1)

Finding c/d when m is close to 1

The numerator of the expression for the ratio of c to d can be found for m close to 1 as follows.

Let

$$m = 1 + \partial$$
$$\lambda = 1 + \frac{1}{2}\partial$$
$$z = 1 + \frac{1}{2}u.\partial$$
$$dz = \frac{1}{2}du.\partial$$

where $0 < \partial < 1$. The second of these is suggested by (B4). Then the numerator of (C1) becomes

$$\int_{0}^{1} \frac{(1+u\partial + u^{2}\partial^{2}/4) - (1+\partial + \partial^{2}/4) \cdot \partial/2 \cdot du}{[(1+2\partial + \partial^{2} - (1+u\partial + u^{2}\partial^{2}/4)) \cdot (1+u\partial + u^{2}\partial^{2}/4 - 1)]^{\frac{1}{2}}}.$$
(C2)

This expression can now be put into integrable form by expanding the last two terms in the denominator with the binomial theorem. At this point the number of terms in ∂ needs to be chosen, here we keep terms up to ∂^3 , so that the expansion needs to keep terms to ∂^2 .

$$\frac{\partial}{2} \int_{0}^{1} \frac{(u-1).(1+\frac{\partial(u+1)}{4}) \cdot \left[1-\frac{\partial(2+u)}{8}+\frac{3\partial^{2}}{128}(2+u)^{2}\right] \cdot \left[1-\frac{u\partial}{8}+\frac{3u^{2}\partial^{2}}{128}\right] \cdot du}{\sqrt{2-u} \cdot \sqrt{u}}$$
(C3)

This expression simplifies considerably, to give

$$\frac{\partial}{2} \int_{0}^{1} \frac{(u-1)\left(1+\frac{\partial^{2}}{32}\right) \cdot du}{\sqrt{2-u} \cdot \sqrt{u}}$$
(C4)

The u integrals can be found by substituting $u = s^2$, and then making a second substitution of $s = \sqrt{2}\sin\theta$ leading to the result for the numerator of equation (C1) as

$$\frac{\partial}{m}\left(1 + \frac{\partial^2}{32}\right) \,. \tag{C5}$$

Tackling the denominator involves a different technique, called matched asymptotic expansions. The problem is that the integrand in (C6) below diverges at z=1, even though the presence of the λ^2 in the numerator means that we expect there to be a limit.

denominator =
$$\int_{0}^{1} \frac{(z^2 - \lambda^2) dz}{[(m^2 - z^2)(1 - z^2)]^{\frac{1}{2}}}.$$
 (C6)

Putting the upper limit as Z, denoting the resulting integral by I(Z) and differentiating with respect to Z gives

$$\frac{dI}{dZ} = \frac{(Z^2 - \lambda^2)}{[(m^2 - Z^2)(1 - Z^2)]^{\frac{1}{2}}}.$$
(C7)

Then the denominator required in (C6) is I(1). Making the substitutions for $\lambda = 1 + \partial/2$ and m = 1 + ∂ as before gives

$$\frac{dI}{dZ} = \frac{(Z^2 - (1 + \partial + \partial^2/4))}{[((1 + 2\partial + \partial^2)) - Z^2)(1 - Z^2)]^{\frac{1}{2}}}.$$
 (C8)

Now since the problem with this expression occurs when Z=1, we need to treat it differently near that region. However for other values of Z there is not the same problem, and we can proceed with the solution. Near to Z=1 a different approach is needed to give a different approximation to the solution. Because the solution is needed for all

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values of Z the two regions must match each other; this is the essence of the method of matched asymptotic expansions, see [Nayfeh 73] for more information on the method.

Looking first at the region away from Z=1 and rearranging the denominator of (C8) gives

$$\frac{dI}{dZ} = \frac{(Z^2 - (1 + \partial + \partial^2/4))}{(1 - Z^2) \left(1 + \frac{(2\partial + \partial^2)}{(1 - Z^2)}\right)^{\frac{1}{2}}}.$$
 (C9)

Expanding the root term in the denominator with the binomial theorem and some manipulation produces

$$\frac{dI}{dZ} = -1 + \partial^2 \cdot \left[\frac{1}{4(1-Z^2)} - \frac{1}{2\cdot(1-Z^2)^2} \right] + \partial^3 \cdot \left[\frac{-3}{4(1-Z^2)^2} + \frac{1}{2\cdot(1-Z^2)^3} \right] (C10)$$

The ∂^3 term is not needed and will be dropped in the following. To integrate this the terms in Z² need to be expanded into partial fractions. Using:

$$\frac{1}{(1-Z^2)^2} = \frac{1}{2} \cdot \left[\frac{1-Z/2}{(1-Z)^2} - \frac{1+Z/2}{(1+Z)^2} \right]$$
(C11)

Using this in equation (C10), and integrating gives several terms which cancel, so that I is given by

$$I = -Z + \frac{\partial^2}{8} \left(\frac{1}{1+Z} - \frac{1}{1-Z} \right) + O(\partial^3) + A .$$
 (C12)

But since I = 0 when Z = 0 from equation (C6) that the constant of integration A = 0. To deal with the region near Z=1 define $1-Z = \partial t$ and write the 'outer' solution (C12) in terms of t to give

$$I = \partial t - 1 + \frac{\partial^2}{8} \left(\frac{1}{2 - \partial t} - \frac{1}{\partial t} \right) .$$
(C13)

So to order ∂^2 , and dropping ∂t , which is small compared to 2, the 'outer' solution is

$$I = -1 + \partial \left(t - \frac{1}{8t} \right) + \frac{\partial^2}{16} .$$
 (C14)

To tackle the other region where Z is close to 1 start from equation (C8). When Z is near 1 each term in the expansion (C10) will be of similar size, because the increasing powers of ∂ are matched by increasing powers of $1/(1-Z^2)$. Hence a different expansion is

needed; substitute as in (C13)

 $1-Z = \partial t,$ $dZ = -\partial .dt,$ $Z^{2} = 1-2\partial t + \partial^{2}t^{2}$

then

$$-\frac{1}{\partial} \cdot \frac{dI}{dt} = \frac{(1-2\partial t + \partial^2 t^2 - (1+\partial + \partial^2/4))}{\left[\left((1+2\partial + \partial^2) - (1-2\partial t + \partial^2 t^2)\right) \cdot \left(1 - (1-2\partial t + \partial^2 t^2)\right)\right]^{\frac{1}{2}}}$$
(C15)

Simplifying, using the usual binomial expansion and simplifying to terms of order ∂^3 on the right hand side gives

$$-\frac{dI}{dt} = \frac{\partial(2t+1)}{2[t(1+t)]^{1/2}} \cdot \left(1 + \frac{\partial^2}{32}\right) .$$
(C16)

The integral of this can be written as

$$-I = -1 + \partial f_1 \tag{C17}$$

where f_1 is defined by

$$f_1 = \int_0^t \frac{(2t_1 + 1) dt_1}{2[t_1(1+t_1)]^{\frac{1}{2}}} \cdot \left(1 + \frac{\partial^2}{32}\right) + B_1 .$$
(C18)

And this then defines the inner part of the solution. B_1 is a constant which can be found from matching the two solutions from the Inner and Outer parts. Changing the form of $f_{1,}$ by adding and taking away t, helps with the matching, changing (C18) to become

$$f_1 = \left(1 + \frac{\partial^2}{32}\right) \int_0^t \left(\frac{(2t_1 + 1)}{2[t_1(1 + t_1)]^{\frac{1}{2}}} - 1\right) dt_1 + t + B_1 .$$
(C19)

Now the two parts of the solution must match each other when t is large, i.e. as we move away from the Z=1 region. So equating (C17), (C19) with (C14) gives

$$-1 + \partial \left(1 + \frac{\partial^2}{32}\right) \int_0^t \left(\frac{(2t_1 + 1)}{2[t_1(1 + t_1)]^{\frac{1}{2}}} - 1\right) dt_1 + \partial t + \partial B_1 = \partial t - 1 - \frac{\partial}{8t} + \frac{\partial^2}{16} .$$
(C20)

Now the method requires all of the terms to match between the two sides of this expression, up to the order of the accuracy needed. The terms $-1 + \partial t$ cancel directly, and we drop the ∂^3 term on the left, so (C20) simplifies and rearranges to

$$B_1 = \int_0^t \left(1 - \frac{(2t_1 + 1)}{2[t_1(1+t_1)]^{\frac{1}{2}}} \right) dt_1 - \frac{1}{8t} + \frac{\partial}{16} .$$
 (C21)

This integral cannot be found, but if the upper limit can be changed to infinity then it can be. Although t does not reach infinity we can write the integral to infinity if we add a correction factor. The correction factor matches the 1/8t term in (C21), as the following shows. We write

$$J = \int_{0}^{t} \left(1 - \frac{(2t_1 + 1)}{2[t_1(1+t_1)]^{\frac{1}{2}}} \right) dt_1$$
(C22)

therefore dJ/dt is given by

$$\frac{dJ}{dt} = \left(1 - \frac{(2t+1)}{2[t(1+t)]^{\nu_2}}\right) .$$
(C23)

This expands via the binomial theorem to give, for t >> 1

$$\frac{dJ}{dt} = \frac{-1}{8t^2} + O(t^{-3}) .$$
(C24)

Hence the correction term for changing the integral limit in (C21) from t to ∞ is given by $+1/8t + O(1/t^2)$. Clearly this term will tend to zero as t tends to infinity and it cancels out the term in (C21). The integral in (C21) with t= ∞ can be found by substituting t = $\sinh^2(\theta)$. This leads to the result

$$B_1 = -\frac{1}{2} + \frac{\partial}{16} .$$
 (C25)

The answer we are interested in is the value of I when Z=1, so that from (C17) with t (t = 1-Z) as 0

$$-I(z=1) = -1 + \partial B_1$$
. (C26)

Hence the denominator comes out as

$$1 + \frac{\partial}{2} - \frac{\partial^2}{16}$$
 (C27)

Combining this with the expression for the numerator in (C5), and using the binomial expansion again, simplifies the result for c/d to

$$\frac{c}{d} = \frac{\partial}{2} \left[1 - \frac{\partial}{2} + \frac{11}{32} \partial^2 \right] . \tag{C28}$$

This fits very well with the calculated data from the numerical integration program as shown below. With the 3 term approximation being reasonable even when the ∂ term is 0.7, which is not particularly small compared to 1.



Figure C1: Ratio of c/d as a function of the parameter ∂

The form of c/d as a function of m when m is large

As before the starting point is equation (C1). In this case the denominator is the easier term to find. Because both λ and m are large with respect to 1, the denominator of (C1) simplifies to

denominator
$$\approx \int_{0}^{1} \frac{\lambda^2 \cdot dz}{m[1-z^2]^{\nu_2}}$$
 (C29)

This is a standard integral, $\sin^{-1}(z)$, and hence

denominator
$$\approx \frac{-\lambda^2}{m} \cdot \frac{\pi}{2}$$
 (C30)

Making the substitutions

$$\lambda = 1 + \mu$$
$$m = 1 + \tau$$

where μ and τ are both large then (C30) becomes

$$denominator = \frac{-\mu^2}{\tau} \cdot \frac{\pi}{2} .$$
 (C31)

In Appendix B an expression for λ as a function of m when m is large was given as

$$\lambda^2 = \frac{m^2}{\ln(4m)} \tag{C32}$$

and so, since when m is large τ and m are effectively equal

$$\mu^2 \simeq \frac{\tau^2}{\ln(4\tau)} . \tag{C33}$$

so the leading approximation for the denominator is

$$\frac{-\tau}{\ln(4\tau)} \cdot \frac{\pi}{2}$$
 (C34)

The expression for the numerator is more difficult, because the integrand is complicated near z=1. It can be tackled using the same 'Method of Matched Asymptotic Expansions' used earlier.

Making the substitutions

$$\lambda = 1 + \mu,$$
$$z = 1 + \mu P,$$
$$dz = \mu dP$$

and rearranging the resulting terms gives the numerator as

$$\int_{0}^{1} \frac{\mu^{2} \cdot (P-1) \cdot (2+\mu+\mu P) \cdot dP}{\left[(\mu P(2+\mu P) \cdot (\tau-\mu P) \cdot (2+\tau+\mu P)\right]^{\frac{1}{2}}}.$$
(C35)

To evaluate this it is helpful to replace the upper limit of (C35) by p and differentiate with respect to p. Then, denoting the result by J(p) the required result will be J(1).

$$\frac{dJ}{dp} = \frac{\mu^2 \cdot (p-1) \cdot (2+\mu+\mu p)}{\left[(\mu p(2+\mu p) \cdot (\tau-\mu p) \cdot (2+\tau+\mu p)\right]^{\frac{1}{2}}}$$
(C36)

Then since μ and τ are large compared to 2 this simplifies considerably to

$$\frac{dJ}{dp} = \frac{\mu^3 \cdot (p-1) \cdot (1+p)}{\left[(\mu p \cdot (\tau - \mu p) \cdot (\tau + \mu p)\right]^{\frac{1}{2}}}$$
(C37)

This simplifies further to

$$\frac{dJ}{dp} = \frac{\mu^2 \cdot (p^2 - 1)}{p \cdot [\tau^2 - \mu^2 p^2]^{\frac{1}{2}}}$$
(C38)

This integral is found by splitting the numerator into two, and substituting p = 1/q for the second term. This gives

$$\frac{J(p)}{\mu^2} = -\frac{(\tau^2 - \mu^2 p^2)^{\nu_2}}{\mu^2} + \frac{1}{\tau} \cdot \cosh^{-1}\left(\frac{\tau}{\mu p}\right) + C \quad . \tag{C39}$$

Where C is a constant. This is equivalent to

$$\frac{J(p)}{\mu^2} = -\frac{\left(\tau^2 - \mu^2 p^2\right)^{\nu_2}}{\mu^2} + \frac{1}{\tau} \cdot \ln\left[\frac{\tau}{\mu p} + \frac{\left(\tau^2/\mu^2 - p^2\right)^{\nu_2}}{p}\right] + C \quad .$$
(C40)

At p=1 this is

$$\frac{J(1)}{\mu^2} = -\frac{(\tau^2 - \mu^2)^{\nu_2}}{\mu^2} + \frac{1}{\tau} \cdot \ln \left[\frac{\tau}{\mu} + (\tau^2/\mu^2 - 1)^{\nu_2}\right] + C \quad .$$
(C41)

The constant C needs to be found by matching the solution with that close to the 'difficult' region near p=0. From (C40) with p small,

$$\frac{J(p)}{\mu^2} = -\frac{\tau}{\mu^2} + \frac{1}{\tau} \cdot \ln(p) + \frac{1}{\tau} \cdot \ln\left(\frac{2\tau}{\mu}\right) + C \quad . \tag{C42}$$

For the 'inner solution' for p close to 0, writing $\mu p = t$ in (C36) gives

$$\mu \frac{dJ}{dt} = \frac{\mu^2 \cdot \left(\frac{t}{\mu} - 1\right) \cdot (2 + \mu + t)}{\left[(t(2+t) \cdot (\tau - t) \cdot (2 + \tau + t)\right]^{\frac{1}{2}}}$$
(C43)

This simplifies, since μ and τ are is large, to

$$\mu \frac{dJ}{dt} = \frac{-\mu^3}{\tau \cdot [t (2 + t)]^{\frac{1}{2}}}$$
 (C44)

Hence J can be written as

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$$J(\mu) = \frac{-\mu^2}{\tau} \cdot \left[\int_0^t \left(\frac{dt'}{[t'(2+t')]^{\frac{1}{2}}} - \frac{t'.dt'}{t'^2 + 1} \right) + \frac{1}{2}\ln(t^2 + 1) \right] , \qquad (C45)$$

since J=0 when t=0. Here the 2nd and 3rd terms cancel, but using both helps to simplify the result. The integral part gives the result log(2) when t tends to ∞ , giving the constant of integration, so that J is given by

$$J(\mu) = \frac{-\mu^2}{\tau} \cdot \left[\ln(2) + \ln(t) \right] = \frac{-\mu^2}{\tau} \cdot \left[\ln(2) + \ln(p) + \ln(\mu) \right] \quad . \tag{C46}$$

Now (C46) where $p\mu$ becomes large must match (C42) where p becomes small, and this defines the constant in (C39) by

$$\frac{-(\ln(2) + \ln(p) + \ln(\mu))}{\tau} = -\frac{\tau}{\mu^2} - \frac{\ln(p)}{\tau} + \frac{\ln(2\tau/\mu)}{\tau} + C \quad . \tag{C47}$$

the terms in log(p) cancel, so that the constant is given by

$$C = \frac{\tau}{\mu^2} - \frac{\ln(2\tau/\mu)}{\tau} - \frac{\ln(2)}{\tau} - \frac{\ln(\mu)}{\tau}$$
(C48)

Hence the solution to the numerator of (C1) is given from (C41) and (C48) as

$$\frac{J(1)}{\mu^2} = -\frac{(\tau^2 - \mu^2)^{y_2}}{\mu^2} + \frac{\tau}{\mu^2} - \frac{\ln(2)}{\tau} - \frac{\ln(\mu)}{\tau} + \frac{1}{\tau} \ln \left[\frac{\tau}{\mu} + \left(\frac{\tau^2}{\mu^2} - 1 \right)^{y_2} \right] - \frac{1}{\tau} \ln \left(\frac{2\tau}{\mu} \right) \quad . \tag{C49}$$

Hence the result for c/d is given by dividing (C49) by (C34), or more simply (C31), to give

$$\frac{-\pi c}{2d} = -\frac{\tau (\tau^2 - \mu^2)^{\gamma_2}}{\mu^2} + \frac{\tau^2}{\mu^2} - \ln(2) - \ln(\mu) + \ln\left[\frac{\tau}{\mu} + \left(\frac{\tau^2}{\mu^2} - 1\right)^{\gamma_2}\right] - \ln\left(\frac{2\tau}{\mu}\right) \quad .$$
(C50)

It can be expressed in terms of τ only rather than τ and μ , using (C33), and letting $\log(4\tau) \equiv x$ to simplify the expression to

$$\frac{-\pi c}{2d} = -x(1-1/x)^{\nu_2} + x - \ln(2) - \ln(\frac{\tau}{\sqrt{x}}) + \ln[x^{\nu_2} + (x-1)^{\nu_2}] - \ln(2x^{\nu_2}) \quad . \tag{C51}$$

But the second and fourth terms combine to $\log 4 + \frac{1}{2}\log(x)$, and the first can be expanded with the binomial theorem to give, finally, for the leading terms when x >> 1,

$$\frac{\pi c}{2d} = x \left(1 - \frac{1}{2x} \right) - \ln(2) - \frac{1}{2} \ln(x) \quad . \tag{C52}$$

The result (C52) tends to that found from the full expression (C1) for large values of τ , but it is not a particularly useful approximation. This is because the term $x = \log(4\tau)$ is not very large even for large values of τ .

Appendix D: Finding expressions for B as a function of c/d

The form of B as a function of c/d when m is close to 1

In Appendix A an approximation for the flow parameter B in terms of the length parameter m was found. Then in appendix C an expression for c/d as a function of m was given as well. Here an approximate expression is found which gives B directly as a function of c/d.

Appendix A, equation (A6) gave

$$B = -\frac{P_0}{\pi} \cdot \ln\left(\frac{16}{(1 - 1/m^2)}\right) .$$
(D1)

Writing $m = 1 / (1+\partial)$ where ∂ is small, then neglecting terms of order ∂^2 compared to ∂ , and using a binomial expansion, (D1) simplifies to give

$$B = -\frac{P_0}{\pi} \cdot \ln\left(\frac{8}{\partial}\right) . \tag{D2}$$

In Appendix C, c/d was found to be related to the same ∂ by

$$\frac{c}{d} = \frac{\partial}{2} \cdot \left(1 - \frac{\partial}{2} + \frac{11}{32} \partial^2 \right) . \tag{D3}$$

Again neglecting terms of order ∂^2 compared to ∂ in the simplest approximation this gives

$$\frac{c}{d} \approx \frac{\partial}{2} . \tag{D4}$$

Combining this result with equation (D2) above gives a simple result for B as a function of c/d, given by

$$B = -\frac{P_0}{\pi} \cdot \ln\left(\frac{4d}{c}\right) . \tag{D5}$$

Equation (D5) gives a correlation which is better than might be expected given the approximation made in dropping the ∂^2 term for c/d. The log fit is very good up to c/d equal to about 1.

In appendix C the result for c/d when m becomes large is derived. From (A6) and the first term of (C52), on eliminating x,

$$B \approx -P_0 \frac{d}{c} . \tag{D6}$$

However this result is not particularly useful as it requires an unreasonably large value of the parameter m to be valid. It does however give the end limit of the expression for the relationship between B and c/d.



Part 2: High pressure flows and pressure extension

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Chapter 1: Introduction

In part 1 on natural driven flow the driving forces for the entry of soil gases into buildings were discussed in some detail. Given that pressure driven flow is expected to dominate over diffusive flow in most conditions where risks to health are significant, much attention has been given to preventing this pressure driven flow from occurring.

The main technique used for this is the radon sump [BRE 92], known as a sub-slab ventilation (sometimes depressurisation) system in the USA. In this a small void is formed below the floor of a building, and air is sucked out of it using a fan, see the figure in the introduction at the start of this thesis. If the system works then air is pulled from the building down through the floor and into the sump. This flow is the reverse of the normal situation described in the natural driven flow chapter, and will effectively prevent radon entry. Experience shows it to be effective in most cases [Cliff 91].

The purpose of part 2 of this thesis is to look at the way in which these sump systems behave, and in particular to understand the way the soil or hard core materials in which they operate affect their performance. These experimental and modelling studies serve to support the design of cost effective remedial and protection measures for radon by helping us to understand why the flows and pressures measured in them occur. Through this increased understanding we are able to improve our designs, overcome difficult cases and avoid problems in the future.

Chapter 2 introduces the theory relating to higher pressure flows, and reviews some of the work done elsewhere on these types of problem.

In early work on this subject the Darcy law was always used to describe the flow of gas. Modelling using the Darcy law is discussed in chapter 3 of this part, with a mixture of computational and analytical techniques having been used. This work was carried out by the author, developing from work elsewhere.

However the linear Darcy law is not appropriate in many cases, and a non-linear law is needed instead. This is discussed in more detail in chapter 4, which contains simple solutions to the Darcy-Forcheimer equation found by the author.

In chapter 5 the key variable in soil gas modelling (permeability) is discussed, and the theories used to estimate its value for different materials are presented. These are then compared with BRE measurements of the permeability of some common hard core materials. These experiments were mostly carried out by an undergraduate student working under the author's supervision. Further analysis of these results is then given, and these are related to a number of formulae used by other workers for predicting permeability based on grading curve information.

The issue of pressure field extension is discussed in chapter 6. This is the distance over which the pressure caused by a sump is found to have an influence. If this is too small the sump is less likely to be effective in reducing indoor radon levels. Measurements made by BRE and by contractors on behalf of BRE are considered, and analysed using the models proposed in chapter 4. The impact of the choice of hard core materials is discussed.

Chapter 7 contains the conclusions to this part, which address the implications of the work on non-Darcy modelling and the pressure field extension results. The main choice to be made when designing protection measures is what sort of hard core material to use, and evidence for UK houses presented suggests that it is unlikely to be worthwhile to use the most expensive materials.

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Chapter 2: Theory and literature review

The review of work on low pressure flows covered most of the modelling work carried out into radon movement. There are only a small number of papers which have looked at the non-linear effect of higher pressure and hence higher speed flows. Most of these have come from Lawrence Berkeley Laboratory in the USA.

There has been more work in the other main subject area for this part, that of pressure field extension, so this is discussed in the second half of this chapter.

Darcy-Forcheimer modelling

At low flow speeds the flow through a porous media is generally well described by the linear Darcy Law discussed earlier. At higher flow speeds this starts to break down, and a non-linear description is needed. There are a number of possible choices, [Bear 72], of which the Darcy-Forcheimer law has had the most use. It is given by (2.1)

$$\nabla P = -\frac{\mu}{k} \cdot v \, (1 + c|v|) \tag{2.1}$$

where

v is the velocity of flow (m/s), and v = Q / AQ is the flow rate (m³/s), A is the area of flow (m²), k is the permeability of the soil (m²), μ is the dynamic viscosity of the fluid flowing (Pa.s), P is the excess pressure of the fluid compared to ambient (Pa), x is the length over which flow occurs (m), ∇P is the gradient of P (Pa/m), and c is the Forcheimer term, of order 10, (s/m).

The new 'constant' c is another parameter to describe the material along with the permeability, k. Experiments in the USA [Bonnefous 92-1] and ours at BRE suggest that values of around 10 (s/m) are typical. This suggests that for a material with this c value

and a flow speed above about 0.01 m/s then the non-Darcy flow would be expected to have an influence, and above 0.1 m/s it would start to dominate.

In order to model soil gas flow with the Darcy-Forcheimer equation new techniques are needed which are beyond the scope of this paper. The basic problem is that unlike the Darcy Law it cannot be combined with a continuity equation to give a simple equation for the pressure. In fact the use of the Darcy Law is particularly straightforward because it produces the Laplace Equation when combined with the continuity equation. The Darcy-Forcheimer equation does not have these benefits.

In chapter 4 some particularly simple solutions to the Darcy-Forcheimer equation are presented. However more progress has been made on finding solutions to it numerically at the Lawrence Berkeley Laboratory (LBL) in California. Most of the modelling work was carried out by Yves Bonnefous when studying for his PhD at LBL, but the ideas for the work are shared between the team.

Some of the work on landfill gas modelling has also addressed high pressure flows, particularly when referring to gas extraction from landfill sites. This needs a non-Darcy type law, but is not a problem which has been addressed in this work.

Pressure extension measurements

The idea of measuring the pressure extension of a sump is well established. The basic idea follows from the fact that a sump works by reversing the flow across the floor. In order to do this the pressure below the floor must be lower than that above it at all points on the floor. This can be measured relatively easily, usually by drilling a hole through the floor and inserting a tube, but sometimes by leaving measuring tubes below the floor when it is laid. The pressure extension is then the distance from the suction point at which some given reference pressure is achieved, 5 Pa for example.

Some installers of systems in the USA choose to test the pressure extension achieved by a sump system they are installing to check whether it is going to work. If the pressure extension is not good enough they then install a bigger fan until it is, probably avoiding

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the need to return later.

In the UK we have chosen not to do this test routinely, for a number of reasons. Firstly it does not tell you with certainty if the radon level will be reduced. Sometimes the pressure extension is not perfect, but the main entry route for radon may be stopped so the radon level will be lowered significantly in spite of the incomplete pressure extension. Secondly the people who carry out the radon remedial work in the UK are mostly 'normal' builders, who would not be familiar with the equipment needed, and are not trained to interpret pressure measurements. This could be overcome but it is not clear that it is worth doing so. Finally it is not good practice to drill holes in floors without a very good reason. It is disruptive, noisy and dusty, and provides a potential entry route for radon however carefully it is resealed.

Nevertheless there is a certain amount to be gained from looking at results of pressure extension tests, and those carried out by BRE are addressed in chapter 6.

Much of the work in this subject does not get published as it is done by practitioners. However Chick Craig at the US Environmental Protection Agency (EPA) has published a number of papers on pressure extension [Craig 93]. He was responsible for trying to reduce radon levels in large buildings, schools and hospitals in particular. He was able to achieve very large pressure extensions, up to 50 m in some cases, by using very carefully chosen hard core materials below the floor. He also had to ensure that each sub-floor wall had enough holes within it to allow air flow through it, without weakening the structure.

In the UK, Trevor Gregory from Cornwall County Council and Roger Stephen at BRE have tried to copy the results achieved by Craig, but with varying levels of success [Gregory 93, Stephen 95, Cripps 95-3]. This is discussed further in chapter 6.

Chapter 3: Modelling the Darcy Law

Introduction

This chapter covers attempts to model the behaviour of gas extraction systems using the Darcy Law. It becomes clear in Chapter 4 that this approach is flawed because at the higher flow rates involved in gas extraction the Darcy Law is not valid. However there is still an amount which can be learnt in a qualitative sense from this work.

There are two quite different approaches used in this chapter. The first is a finite difference model, giving an approximate solution to Laplace's Equation in a defined region, by making approximations to the differential equation. The second method assumes that the extraction point behaves as a sink of material, and uses source/sink theory from other Physics disciplines to provide solutions. It might prove useful in making comparisons with some particular types of computational solution, and has been used elsewhere for studying extraction systems from landfill sites. However only the initial steps are reported on here.

The computational solution is more general, but is relatively time consuming to use, in particular when small changes to a parameter are needed. Nevertheless it is the only way to account for all of the details of the geometry of the building.

Finite difference model

The computer model used was based on those developed at LBL [Loureiro 87], [Mowris 86]. It is a finite difference model, which solves Laplace's Equation for the pressure in the soil below a house.

The assumptions made in the computer model are:

- Linear flow according to the Darcy Law (whether this is justified is discussed later),
- No flow through floor or walls, apart from through a smooth edge crack,

• Linear flow through that crack,

- Sump modelled as constant pressure region at centre of floor slab,
- Permeability constant and homogeneous within each of a number of defined regions.

The way in which these are used in the finite difference code is not original, and so is not reproduced here. It has been covered extensively elsewhere, and was based on the work of Mowris [Mowris 86] and Loureiro [Loureiro 87].





Figure 3.1: The modelled house

The model calculates the pressure field within the soil and hard core below the house and the rate of flow through the crack and through the sump. Two contour plots of pressure fields are shown in figures 3.2 and 3.3 below. In figure 3.2 the hard core material (permeability $k = 1e-10 m^2$) is much more permeable than the soil ($k = 1e-12 m^2$). This results in essentially one dimensional flow in the hard core material, from the crack and into the sump. The soil plays little part in the flow, and the fall off in pressure from the

sump is close to linear in the hard core material.



Figure 3.2: Contour map of the pressure field for hard core more permeable than soil

In figure 3.3, however, the soil and hard core permeabilities are equal at $1e-10 \text{ m}^2$, so the flow in the soil is significant, and the pressure decreases from the sump faster than in figure 3.2. The resulting pressure field extension (the pressure field at the edge of the slab compared to that at the sump) is not as great as in figure 3.2.



Figure 3.3: Contour map of the pressure field when hard core and soil are equally permeable

The flow rates predicted vary with the permeability of the soil and hard core as shown in table 3.1 below. The flow rate given assumes a perimeter of 30 m, which is typical for a small UK house. The model ignores edge effects, so we are effectively looking at an infinitely long house, for the purpose of simplifying the problem. This is a serious flaw in the model, particularly when a sump is being considered, which is clearly not 'long'. The model is not as bad when it is used to consider a house without a sump, for which the 'corners' are less important.

The resistance to flow, R, of the combination of house, soil and hard core is defined as the pressure in the sump, P, divided by the flow produced by it, Q, i.e. R = P / Q. When using a radon sump the lower the resistance the better, as this allows the same pressure to be achieved with the minimum fan power. However if no sump is being used the problem is more complicated as it might be better to have more resistence as this helps to keep gas out.

Comparing results 1 and 4 of table 3.1 shows that the difference in the ratio of the permeability of the hard core to that of the soil has a large effect on where the flow occurs.

	Permeability	Permeability	Crack	Sump	Crack	Resistance
	of hard core	of soil	width	flow	flow	(Pa /
	(m ²)	(m ²)	(mm)	(m ³ h ⁻¹)	(m ³ h ⁻¹)	$(m^{3}h^{-1}))$
1	1e-10	1e-12	1	3.9	3.6	23
2	1e-9	1e-12	1	34	30	3.0
3	1e-8	1e-8	1	2200	41	0.045
4	1e-10	1e-10	1	23	2.8	4.4
5	1e-12	1e-12	1	0.24	0.03	420
6	1e-10	le-12	10	4.8	3.7	21
7	1e-10	le-12	0.2	1.4	0.09	72

Table 3.1: Computer predicted flows for different permeabilities and crack widths

In result 1 the hard core is much more permeable than the soil, so that most of the flow goes through the hard core and through the floor crack, and very little (about 10%) goes through the soil. This is a good result from the radon reduction point of view, as is supported by the good pressure extension shown in figure 3.2. In result 4 however the flow through the crack is fairly small compared to that through the soil, as the path through the soil is easier for the gas. This results in decreased pressure extension as seen in figure 3.3.

Comparing results 1 with 2, and 3 with 4 and 5 shows the close to linear relation of flow rate with permeability, as expected from the Darcy Law. When the permeability is high enough the approximation of linear or Darcy flow ceases to be valid [Bonnefous 92]. If we assume a sump surface area of 0.25 m² then a flow rate of 100 m³h⁻¹ represents an average speed of entry to the sump of about 0.1 ms⁻¹. At this speed we are close to the limit for using Darcy's Law.

Comparing cases 1, 6 and 7 shows the predicted relationship between flow rate, resistance to flow and crack width, and these data are plotted in figure 3.4 below, together with some additional data.



Figure 3.4: Graph of predicted resistance to flow against crack width

Until the crack width becomes small, less than 0.5 mm, the width of the crack does not have a significant effect on the resistance to flow, because most of the resistance occurs
in the soil and hard core. For very narrow cracks it becomes significant, but these are unlikely to be found without larger cracks being present as well, so the latter tend to dominate the flow field. This result is significant in the absence of the sump as well, and was discussed in the natural flow part earlier.

Experimental results

During the installation of remedial measures in buildings in the UK, BRE have been able to collect data on the flow rates and pressures produced by sumps. Some results are shown here in table 3.2. Most of these results are unpublished data from visits by BRE staff to high radon houses in Devon and Cornwall, the exact locations being confidential. Those marked GS are also discussed in [Gregory 93].

File number	Sump Pressure (Pa)	Flow rate (m ³ h ⁻¹)	Resistance (Pa / (m ³ h ⁻¹))
57	88	160	0.55
58	105	180	0.58
68	172	120	1.43
134	250	100	2.50
136	100	324	0.31
Pool school GS	334	107	3.12
St Leven School GS	125	158	0.79

Table 3.2: Pressures, flows and resistances measured in UK buildings

The flow rates measured here are similar to those predicted by the computer model for soils with permeability of order 1e-9 to 1e-10 m². From these results we can estimate the permeability of the ground below these buildings, if we make significant approximations about the material below the buildings.

The resistances vary over an order of magnitude, which suggests that the permeabilities will also do so. Given that permeabilities of soils can vary by up to 10 orders of magnitude, it is surprising that there is a comparatively small range of resistances in tests

so far. This probably indicates the leakiness of construction in the UK, as leakage paths other than through the soil or edge crack as modelled disrupt the predictions of the simple model used here.

Usually the make up of the soil below the floor of a building is not known. The modelling results show that a number of different combinations of soil and hard core permeabilities and floor leakages could produce the same overall resistance to flow. For example compare results 2 and 4 of table 3.1, where the flow resistance is similar for different combinations of permeability. The measured flow resistance cannot tell us about this; more pressure measurements would be needed in the hard core or soil to observe the type of pressure distribution.

Cost of running a radon sump

A typical UK sump uses a 75W fan, which at 7.6 p per kW hour would cost £50 per year to run. However the use of the sump results in additional ventilation in the house. The computer model predicts that a proportion of the flow into the sump will come from the house, through cracks in the floor. In fact when the hard core is less permeable than the soil below it, most of the flow comes through the floor. Field evidence of sump flow rates show we can create an additional flow of about 100 m³h⁻¹ through the floor of the house.

Using the BREVENT model of ventilation [Cripps 92] we can estimate that for a typical UK house this would result in an overall increase in ventilation in the house of roughly half of the flow, i.e. $50 \text{ m}^3\text{h}^{-1}$. Then if we assume that the external air is 10 °C colder than the indoor air, we can predict that the fan results in an increase of about 170 W in the heating load. If this is needed throughout a six month heating season we can predict an additional heating cost per year of between £20 for gas and £55 for on-peak electric. This cost may appear small, but will not be negligible for many householders.

Stepped floors

One of the main benefits of using a computational model is that there is no limit, in

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principle, to the complexity of the geometry which can be considered. Therefore as some UK homes are built with a stepped floor, I carried out some modelling work to look at how the location of a radon sump affects its performance.

Given the flaws in the model when it considers radon sumps this work is not considered further here.

Conclusions to finite difference model section

The computer model is able to predict the correct order of magnitude of the flow rates of sumps as measured in the field. The results of computer predictions and field measurements can usefully be expressed via a resistance of the whole system to gas flow.

The relationship between the permeabilities of the soil below the house and any hard core material is a key parameter in the performance of the sump. If the hard core is highly permeable, much of the flow into the sump is likely to come from the house and the pressure extension below the slab is improved. This may lead to higher fuel bills, but is expected to be more effective in reducing the radon levels.

However there are two significant flaws in using the model for this type of work. Firstly is the non-linear nature of the flow, discussed further in chapter 4. It will result in over predictions of the flow rate for a given pressure, because a source of resistance to flow is being ignored. Probably more significant is the fact that a three dimensional problem has been addressed with a two dimensional model. In spite of the fact that buildings are generally rectangular, the sump is better described in cylindrical polar coordinates if only two dimensions can be used on the available computer. This is because the way in which the pressure 'spreads out' from the sump is neglected completely in two dimensions. This was not an issue for the 'no-sump' problem considered in the natural flow part, but cannot be ignored here.

Nevertheless the understanding of how sumps operate, and the significant proportion of the air reaching a sump coming from the house are still valid and useful results from these modelling results.

Source/sink theory

Modelling of sources and sinks

Introduction

In this section the pressure distribution caused by an assumed flow is considered. Often remedial measures to protect against soil gases involve blowing into or sucking gas from the soil. These processes can be represented by sources and sinks of fluid, provided the area of extraction or entry of gas is fairly small, and the detail near that point is not of great importance.

A source, see for example [Batchelor 67], is a point from which fluid is emitted uniformly in all directions and at a constant rate. A sink is the opposite, a point where fluid is removed.

Although not ideal, a two dimensional model of the pressure field around sources and sinks gives some information about how the sink behaves.

Definition of the problem



Figure 3.5: Pressure field problem

In the absence of the source at (a,0) there would be a steady pressure gradient across from x=0 to x=1, with no y variation, i.e.

$$P = P_0 (1 - x) . (3.2)$$

Taking this linear part out of the problem simplifies it slightly, and it can be added in again later.

When this work was first attempted a solution was found using a Fourier Transform technique. However there is a much simpler solution method which uses the method of images. Only the latter is used here.

Ignoring the applied pressure gradient due to P_o the problem can be considered as that in the following diagram.



Figure 3.6: Diagram of problem set out for method of images solution

Putting a source of opposite sign at x=-a cancels the effect of the first source on x=0. It is shown above by the striped box. The pressure field due to both of them will include zero along the line x=0 as required. Similarly another negative source at x=2-a would balance the first source along x=1. However we must also consider the images of these two image sources.

The image source at x=-a has to be balanced by a positive image source at x=2+a in order to keep the zero pressure on x=1. Similarly another positive is needed at x = -2+ato balance the negative at 2-a on the x=0 zero pressure line. This is not the end of course, as each added pair need another pair to balance them and the diagram above gives the first few image sources in either direction, with positive sources as solid square boxes, negative sources as striped boxes. In mathematical notation there are positive sources at x = 2n + a, negatives at x = 2n - a, where n is any integer. The number of image sources is infinite, but since the region of interest is between x=0 and 1 it is clear that the later image sources have less and less effect due to their distance from the origin.

This is the basis of the Method of Images, used widely in physics, and also elsewhere for gas flow [Mutch 90]. A single source of strength m at (r, 0) gives the pressure

$$P = -m \cdot \ln((x - r)^2 + y^2) , \qquad (3.3)$$

and then substituting for each r using the position of the images defined above, and their strengths as +m or -m, the complete solution by the method of images is

$$P = -m \sum_{n=-\infty}^{\infty} \ln \left(\frac{y^2 + (2n + a - x)^2}{y^2 + (2n - a - x)^2} \right) .$$
(3.4)

This result is plotted in figure 3.7, where a source of strength 0.5 has been placed at the point (0.325, 0). On x=0 the pressure is forced to 1, while it is 0 on x=1. All of the boundary conditions are met in this plot, although there is always a small runcation error which can be seen along the x=0 line. There the pressure is very slightly less than 1 for y > 0.6, but not significantly.



Figure 3.7: Pressure field contours for the first source model

By y=1.5 the effect of the source (or sump) has almost gone, and the result is close to that in the absence of it, as we would expect. The pressure at the source is clearly very high, tending to infinity in fact. The value at this point is not significant, because we cannot achieve a point source in practice. What will be of more interest is the pressure a small distance from the source, which could represent the pressure at the edge of a radon sump or landfill bore hole.

There is no reason why a second or third source couldn't be included in the analysis, resulting in more terms of the same form as those in (3.4).

A second problem: No flow on x=1

Another useful problem is defined by having a no-flow boundary at x=1, instead of the zero pressure used earlier. Zero pressure is defined on x=0, and there is no flow across x=1, so that $\partial P/\partial x=0$ there. There is no flow across y=0, which is satisfied by the symmetry of the problem.



Figure 3.8: Diagram for second method of images problem

The source at (a,0) is balanced to give P=0 on x=0 by the negative image source at (-a, 0). Then to force no flow on x=1 due to the original source another positive source is needed the same distance the other side of it, and so on.

The positive sources are found at

... -4+a, -2-a, 0+a, 2-a, 4+a, 6-a ...

or 4n+a, 4n+2-a where n is any integer.

The negative sources are at

... -6+a, 4-a, -2+a, 0-a, 2+a, 4-a

i.e. 4n-2+a, 4n-a where n is any integer.

. .

From these we can write down the solution, in the same way as before.

$$P = -m \sum_{n=-\infty}^{\infty} \ln \left[\left(\frac{y^2 + (4n + a - x)^2}{y^2 + (4n - 2 - a - x)^2} \right) \left(\frac{y^2 + (4n + 2 + a - x)^2}{y^2 + (4n - a - x)^2} \right) \right].$$
 (3.5)

This result is plotted in figure 3.9, with a source of strength -0.5 (i.e. a sink) at x=0.325.



Figure 3.9: Pressure field contours for the second source problem

This shows that the result matches the boundary conditions correctly. The horizontal

lines at the x=1 boundary indicate that no flow is occurring across it, since flow is perpendicular to pressure contours. The zero pressure on x=0 is also clearly satisfied. It is less clear that no flow occurs on y=0, because the pressure lines are not all perpendicular to it. This is probably due to not calculating enough points close to y=0.

Conclusions to source/sink part

In this section analytical solutions have been found for the pressure fields caused by combinations of sources and sinks in two dimensions. Although not corresponding exactly to reality, they give some insight into the way that sumps or other pressure devices behave. In particular it gives the 'shape' of the pressure field generated by an extraction point of a given power, and this corresponds closely to the situation with a sump. The solutions therefore have some value in helping us to visualize the flow.

The solutions generated are better, to some extent, than the finite difference solution, in that a pressure does not need to be defined, just a source strength. This then corresponds quite well to a fan, which has a given power rather than a defined pressure. This analogy fails when the pipe work associated with a fan is included; this cannot be taken into this simple model.

However the method is mainly limited in its application because it can only be used with certain simplified geometries. Because these do not correspond to the 'real' layout of a house, they have a limited application, and so have not been taken further in these studies.

Chapter 4: Modelling using the Darcy-Forcheimer Law

This chapter presents some simple analysis of the Darcy-Forcheimer law, and the solutions to it in cylindrical polar coordinates which are used in the analysis of pressure extension data later on. It also presents a new model within this for considering the pressure within the hard core below a floor slab.

The key equations used are Darcy's Law as before

$$Q = -\frac{Ak}{\mu} \cdot \frac{\partial P}{\partial x} \quad or \quad \nabla P = -\frac{\mu}{k} \cdot v \tag{4.1}$$

where

Q is the flow rate (m^3/s) , k is the permeability of the soil (m^2) , μ is the dynamic viscosity of the fluid flowing (Pa.s), A is the area of flow (m^2) , P is the excess pressure of the fluid compared to ambient (Pa), x is the length over which flow occurs (m), ∇P is the gradient of P (Pa/m), v is the velocity of flow (ms^{-1}) .

and the Darcy Forcheimer Law which is

$$\nabla P = -\frac{\mu}{k} \cdot v (1 + c.v) \tag{4.2}$$

where all the variables are the same apart from the extra constant

c is the Forcheimer term, of order 10, (s/m).

The problem to be considered is described by the figure below, which represents either a sump or a pressure extension test.



Figure 4.1: Diagram of pressure extension test

Assumptions used

- a) The floor of the house and the soil below the hard core are much less permeable than the hard core. Hence it is reasonable to assume no flow through the floor or the soil.
- b) The suction hole extends down to the base of the hard core material, so it is possible to assume no vertical variation in pressure.
- c) Away from the walls there will be radial symmetry, so cylindrical coordinates can be used, with no variation in the ϕ direction.
- d) The effect of the end of the hard core material is ignored, so that the region is effectively infinite.

From a) the total flow rate through a cylinder of any radius will be the same. Hence using c) for either of the two pressure equations, the velocity at any radius is given by

$$v = \frac{Q}{2\pi r.d} \tag{4.3}$$

where

r is the radius from the central suction point (m),

d is the thickness of the hard core layer, assumed constant (m).

Substituting this into the Darcy-Forcheimer Law expressed in cylindrical polar coordinates, and integrating with respect to r, gives the predicted relationship between the pressure difference between any two points and their radii. It is given by

$$P_{1} - P_{2} = -\frac{\mu Q}{k.2\pi d} \left[\log_{e}(\frac{r_{1}}{r_{2}}) - \frac{cQ}{2\pi d} \left(\frac{1}{r_{1}} - \frac{1}{r_{2}} \right) \right]$$
(4.9)

where

 P_1 and P_2 are the pressures (Pa) at radii r_1 and r_2 (m) respectively.

Setting c = 0 simplifies this to the Darcy case, with the non-linear term in Q disappearing. This applies when the velocity of flow is low, or in certain materials, generally with very few 'fines' in them. Fines are the smallest size particles within a hard core material. The term is often used without a particular definition beyond this, but a measurement would need to define the sieve size considered as 'fine', for example 1 mm.

It is interesting to note the fact that in the Darcy case the pressure varies with the log of the radial distance, and hence the assumption of cylindrical symmetry is better than it first appears. This is because the log of the distance to the corner of the floor is not greatly different from the log of the distance to the middle of any wall.

A comparison of this result to experiments is made in chapter 6. In some cases measured by Wimpey Environmental Ltd on behalf of BRE the fit to this model is good. However this is not always the case and another possible model takes account of the resistance to flow of the crack at the join between the wall and the floor.

Resistance model

Other work, for example [Mowris 86], shows that there can be a significant resistance to flow due to the crack between the wall and the floor slab. These cracks are almost impossible to eliminate from the concrete so must be considered in all models. If the hard core material is very permeable it is possible for this to be the dominant resistance to flow, i.e. the element within the path of the flow of gas which has the greatest impact on

the flow. Hence it is useful to consider a resistance model of gas flow to try to include this effect along with that of the hard core. The resistance due to a crack is defined for laminar flow as

$$Q_{crack} = \frac{\Delta P_{crack}}{R_{crack}}$$
(4.5)

where the pressure is the pressure across the crack, and the resistance is given by Mowris [Mowris 86] as

$$R_{crack} = \frac{12\mu L_{slab}}{L_{crack} \cdot t_{crack}}$$
(4.6)

where

 μ is the dynamic viscosity of air (Pa.s),

 L_{slab} is the length of the edge of the crack, ie the perimeter of the floor slab (m),

 L_{crack} is the thickness of the slab, ie the vertical length of the crack (m),

 t_{crack} is the width of the crack (m).

Combining this with the pressure drop for the hard core material by adding the resistances gives the result

$$\Delta P = \alpha Q \left(\ln(r_{edge}) - \ln(r) + \frac{cQ}{2\pi d} \left(\frac{1}{r} - \frac{1}{r_{edge}} \right) \right) + Q \cdot R_{crack}$$
(4.7)

where

 $\alpha = \mu/2\pi dk$,

and the other variables are as defined earlier.

This model contains three parameters to be matched against the experimental data, k, c and R_{crack} , and is helpful in some cases. The main problem is that the width of these shrinkage cracks cannot be measured with great accuracy, and will not be constant anyway. This limits the application of the model to helping to understand the scale of the effects which limit the flow of gas. However as is shown in chapter 6 it is able to give answers to the correct order of magnitude, and is useful in understanding the relative scale of the components of the resistance to flow.

Chapter 5: Permeability theory and measurements

Introduction

This chapter looks at how the permeability and related parameters can be measured, and how the problems in this process were overcome. The data from statistical analysis of pressure and flow rate results are also compared with predictions of permeability based on the measurement of the physical dimensions of the aggregate particles, expressed through the grading curves.

Experimental Method

The basic procedure for these experiments was to pass volume-known flow rates of air through a sample of an aggregate. The pressure drop across the aggregate was measured, and this data was analysed statistically so that the values of c and k could be found. The experiments and the analysis were carried out at BRE by undergraduate students under the close supervision of the author.

The original apparatus is shown below as figure 5.1. The design was based on USA standards [ASTM 90] and UK Department of Transport standards [DoT 90], and from work at LBL [Gadgil 91]. A long wooden box was filled with the aggregate to be tested. Wire mesh was placed at each end of the box to contain the aggregate while allowing air flow through it. A plenum chamber was fixed to both ends of the box to make the pressure measurement easier and to minimise momentum effects which might affect the result. The aggregate was loaded into the box and levelled, but not compacted.

To measure the pressure across the aggregate, pressure tappings were inserted into both 'end boxes', with tubes leading to a micro manometer. A sheet of foam was placed on top of the aggregate to prevent air from flowing between the aggregate and the lid. A fan provided the pressure difference; the rotameter set in series with the fan adjusted the air flow out of the aggregate, and a TSI flowmeter measured how much air was drawn into the aggregate. The rotameters also measure the air flow rate which goes through the fan.

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Figure 5.1: The experimental apparatus

The experiment was carried out as follows. The fan was turned on, and the rotameter valves were opened fully to allow the maximum air flow through the aggregate to allow any settling of particles to occur. Then a series of measurements were taken for different rotameter flow rates, down to the lowest flow possible with that rotameter set, around 5 cubic metres per hour. The results for the *first* experiment undertaken are plotted below.



Figure 5.2: Pressure drop against flow rate for MOT 1

The central line, marked with the squares, is the flow rate measured by the rotameter set. The lower line is the flow measured by the flowmeter at the other end of the box. If the experiment and apparatus were perfect, then these would be the same. However, there is a large difference between them, caused by air leaking into the apparatus. The top line shows the percentage of the rotameter reading that must have been leaked into the box or connecting pipes. A leakage of 10% might be considered acceptable but, since this initial set had an average value of almost 40%, steps were taken to reduce leakage.

Improvements to the method

It seemed that the most likely place for air to enter the apparatus was through the joints and walls of the long aggregate box. The joints were further sealed with silicone sealant and a 1200 gauge polyethylene sheet was laid in the box and the aggregate placed in it, to prevent air from entering the aggregate at any place other than at the ends. Enough plastic sheet was used so that there was an overlap on top.

The long flexible pipe was also seen as a possible way for air to enter the system; it was now under more pressure (a few thousand Pascals) than under normal operation. It was not practical to attempt to seal it, and so a pressurisation test was carried out on it. The pipe was detached from the apparatus and sealed at its open end. A pressure tapping was inserted into the plug. The fan was used to create pressures and the rotameters displayed how much leakage was taking place. The statistics package SPSS fitted a cubic line to this data and this was used to correct the results from the original experiment, reducing the unaccounted for leaks by an average of 33%. When the experiment was repeated with the plastic sheet surrounding the aggregate the unaccounted-for-flow percentage fell to an average value of below 15%.

Another possible source of leakage is the boxes attached to the ends of the test cell. The amount that the end boxes leak will depend on the pressure they contain. Assuming that they are identical, since the box nearest to the fan is at a higher pressure than the other means that it will leak more. This means that the flow rate measured at the lower pressure, 'open to the outside air end' of the system will be closer to the flow through the test box than the flow which reaches the fan. It would therefore be reasonable to take the open end (TSI flowmeter) flow as the 'correct' flow. However in these results a small correction was made to account for the pressure in the outside box.

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Analysing the data

The data were analysed with the SPSS statistics package, the results fitting well to the Darcy-Forcheimer equation (4.2). The results for the first aggregate tested are given below.

Correlation coefficient of SPSS fit, r^2 .	0.99985		
Value of parameter c	13.4 ± 2.0		
Value of permeability k	$2.7 \times 10^{-8} \pm 0.4 \times 10^{-8}$		

Table 5.1: Results of permeability experiments on MOT type 1 sub-base.

The error ranges quoted are 95% confidence limits.

The other aggregates were tested and their values of c and k were calculated in the same manner. Figure 5.3 shows all the Q_a plots for the different aggregates.



Figure 5.3: Pressure against flow results for all five aggregates

These plots were used to calculate c and k. The results are given in the table below.

Aggregate	Forcheimer term, c (sm ⁻¹)	Permeability, k (m ²)
10 mm single sized	4.7 ± 0.7	$(2.1 \pm 0.2) \times 10^{-8}$
20 mm single sized	6.7 ± 0.8	$(9.9 \pm 1.0) \ge 10^{-8}$
20 mm graded	6.0 ± 1.1	$(9.3 \pm 1.4) \ge 10^{-8}$
40 mm graded	14.6 ± 4.6	$(2.7 \pm 0.8) \ge 10^{-7}$
MOT type 1 sub-base	13.4 ± 1.7	$(2.7 \pm 0.3) \ge 10^{-8}$

Table 5.2: Results for permeability and Forcheimer term for each of the five aggregates

Errors quoted are two standard deviations, which represents a 95% confidence interval. Figures 5.4 and 5.5 show both of these sets of results, and the same errors are displayed as bars.



Figure 5.4: Permeabilities of aggregates



Figure 5.5: Forcheimer terms of aggregates

An unknown in these tests is the effect that compacting the aggregates would have. In reality, if the aggregates were being used, they would be compacted, either using a mechanical device, or by the weight of concrete or other materials on top. To investigate this in the laboratory it would be necessary to devise a compaction method that could be quantified. Some aggregates, such as the single sized aggregates, would resist compaction more than aggregates with more widely varied particle sizes. Some tests carried out on a commercial basis, and so not reported on here, indicate that for the MOT type 1 material the effect of compaction is to reduce the permeability considerably.

The MOT type 1 sub-base has a significant amount of fines in it, and the effect that time would have on the settlement of fines is also beyond the scope of these tests. Fine aggregates are defined in BS 882:1983 as *aggregate which mainly passes through a 5.0* mm BS 410 test sieve. Fines is not defined as a term in itself, but is taken to mean the part of the aggregate less than 5 mm in size. It is possible that the region where fines collected would have a different permeability from other parts of the aggregate, and this is certain to affect the performance of real fill materials.

Figure 5.4 above shows the relationships between the permeabilities. The two 20 mm aggregates have, to this level of accuracy, the same permeability. The 40 mm aggregate, as would be expected, has the highest permeability of those tested. It also has the

highest percentage error; this could be due to the box being too small to test particles of this size accurately.

The Forcheimer term represents the way that the resistance to flow of air increases the faster it travels through the aggregate. It has been suggested that a high c value would mean that the aggregate changes, with smaller particles being moved amongst the larger ones as the flow rate increases. Thus it would be expected that the MOT sub-base, with many different sizes of particles, would have a high value of c, which it appears to. The 40 mm graded aggregate also has a high c value, but whether this is due to the same reason is unknown. However, the 20 mm single-sized aggregate has the same value of c as the 20 mm graded, when it would be expected that the single sized aggregate would have fewer small particles and thus a lower c value. These issues are better investigated by looking at the grading curves for the aggregates.

Grading curves

Another test to describe an aggregate is to measure its grading curve. This is carried out with a standard set of sieves, through which the sample is passed, largest sieve first. The weight held by each sieve is recorded, and the combined result plotted as the grading curve. By extrapolating between the measurements, this gives the proportion of each size of material at any given size.

BRE has a concrete laboratory which carried out these tests for this project. The results for the five samples are given as figures 5.6 to 5.10.



Figure 5.6: Grading curve for 10 mm single sized



Figure 5.7: Grading curve for 20 mm single sized



Figure 5.8: Grading curve for 20 mm graded



Figure 5.9: Grading curve for 40 mm graded



Figure 5.10: Grading curve for MOT Type 1

Simplified variables used in the description of aggregates

Rather than using the full grading curve to describe a material, there are a number of different dimensionless grain size coefficients. One is *Hazen's effective grain size* coefficient or the uniformity coefficient [Bear 72], defined as

$$C_u = d_{60} / d_{10}$$

where

 d_{60} is the diameter allowing 60% of the material to pass

 d_{10} is the diameter allowing 10% of the material to pass.

When C_u is small, less than 2 say, the sample is considered to be uniform, and the larger the value, the greater the variation in sizes within the material. The values appropriate for the 5 materials tested have been calculated from the grading curves and are given in table 5.3 below, together with measurements of porosity.

These results help to explain some of the results of the flow experiments. Comparing the

grading curves of the two 20 mm aggregates it is clear that they are extremely similar, so it is no surprise that their permeabilities and Forcheimer terms should be close together.

Both the 40 mm graded and the MOT 1 have a measurable quantity of small sized particles, or fines, whilst the others do not. This probably explains the larger Forcheimer term of these two samples, as this is believed to be a function of the ease of small particles to move within the material, blocking pores. The three smaller sized aggregates are almost without fines, so might be expected to have lower Forcheimer terms.

Another important parameter for an aggregate is the porosity, defined as the proportion of the volume occupied by air. This means it is a dimensionless factor with values between 0 and 1. In general a higher porosity leads to more air flow. It is normally measured by the amount of water a sample can absorb within a known volume, in this case a plastic bucket.

Spheres can have values between 0.26 and 0.48, according to the packing arrangement, so it is clear that the values given here for the un-compacted, single sized aggregates are quite high. The porosity of the MOT is greatly reduced by compaction, as might be expected from the range of sizes within it. However it is difficult to greatly reduce the porosity of single sized materials by compaction, as the 10 mm single sized result shows.

Aggregate	d ₆₀ (mm)	d ₁₀ (mm)	Cu	Porosity ()
10 mm single sized	7.5	3.5	2.1	0.37 uncompacted 0.35 compacted
20 mm single sized	14	9	1.6	0.43 uncompacted
20 mm graded	16.5	10	1.7	0.46 uncompacted
40 mm graded	22	12	1.8	0.45 uncompacted 0.39 compacted
MOT type 1 sub-base	15	0.6	25	0.32 uncompacted 0.22 compacted

Table 5.3: Grain size diameters, uniformity coefficients and porosities for the aggregates

However comparing the grain size coefficients for the two larger sized materials shows there is a considerable difference between them. The MOT type 1 has a very wide spread of sizes, and this causes much less free space in the aggregate for air to flow in, because it packs together so effectively. This causes the significant difference between the permeabilities of the MOT type 1 and the 40 mm graded, which are a factor of ten apart.

Using grading curve information to estimate permeabilities

There are a number of formulae relating the permeability of an aggregate to the grading curve information, and six are given below. In all cases the diameters d are in metres. Hence some of the formulae are not as they appear in the original documents, which used a mixture of m, cm, mm and μ m.

1) Fair & Hatch [Bear 72, p 134]

$$k = \frac{1}{m} \left(\frac{(1-n)^2}{n^3} \left(\frac{\alpha}{100} \sum \frac{p}{d_m} \right)^2 \right)^{-1}$$
(5.1)

where

m = packing factor (around 5),

 α = sand shape factor (6.0 for spheres to 7.7 for angular grains),

n = porosity,

p = percentage of sand held between adjacent sieves,

 d_m = geometric mean sizes of adjacent sieves (m).

2) Kamal et al [Kamal 91],

 $K = -3.46 - 18.5 \times 10^{-3} d_{10} + 17.6 \times 10^{-3} d_{20} + 1.2 \times 10^{-6} d_{10}^{2} + 0.62 \times 10^{-6} d_{20}^{2}$ (5.2) where

K = hydraulic conductivity in ms⁻¹, convert to k by k = K μ / ρ g,

 μ = viscosity (Pa.s),

 ρ = density of air (kgm⁻³),

 $g = acceleration due to gravity (ms^{-2}),$

 d_{10} , d_{20} = diameters of sieves that let 10% and 20% of the material through (m).

3) Sherrard et al [Sherrard 72], taken from [Jones 91] suggest

$$k = 0.0035 d_{15}^{2} . (5.3)$$

$$k = f \times 10^{-3} d_5^2$$
 f in range 0.5 to 1. (5.4)

5) Hazen [Bear 72, p 133]

$$k = 0.617 \times 10^{-3} d_{10}^{-2} . \tag{5.5}$$

6) Kozeny Carman [Bear 72, p 166], for uniform spheres of radius r (m)

$$k = \frac{4r^2}{180} \cdot \frac{n^3}{(1-n)^2}$$
 (5.6)

Each of these has been used to predict the permeability values for each aggregate. The results are shown in figures 5.11 to 5.15, compared with the measured result from the laboratory tests. This shows that there is considerable variation between the different models for permeability based on these types of measured data.



Figure 5.11: Model measurement comparison, 10 mm single sized, porosity 0.36



Figure 5.12: Model measurement comparison uncompacted 20 mm single, porosity 0.46



Figure 5.13: Model measurement comparison for uncompacted 20 mm graded, porosity 0.43



Figure 5.14: Model measurement comparison for 40 mm graded compacted, porosity 0.39



Figure 5.15: Model measurement comparison for MOT type 1 compacted, porosity 0.22

Generally the *Kozeny Carman* equation gives good results for the four cases where the materials were close to single sized. The model is not applicable for the fifth case, the MOT type 1, where the size range is very large, and we cannot assign a radius for the simple 'billiard ball' model. The accuracy of the model requires a good measurement of

the porosity, as this is the dominant part of the equation. For spheres, in the 'billiard ball' model, the calculated porosity varies between 0.26 and 0.48 depending on the way they are arranged [Bear 72]. Hence even in this simplest case it is not possible to know the porosity of a randomly produced sample.

However when the porosity values measured in the laboratory are used the results are reasonably accurate. For the 10 mm aggregate there was little impact on the porosity due to compaction resulting in a small change in the predicted permeability. However for the aggregates which compact more, the MOT 1 and the 40 mm graded, the situation is more complex. For the MOT 1, using the 2 extreme values of porosity in the *Kozeny Carman* equation results in a predicted factor of four difference between the permeability values. The result for the 40 mm graded is closer to the measured value when the compacted value of porosity is used.

The message from these results is that the compaction method for the test cell used for the permeability needs to match that used for the test volume used for the porosity test. We did not give enough attention to this aspect within these tests.

Models 3, 4 and 5 all base their answers on the sieve diameter which allows a particular proportion to pass through them. It is probably because of the small range of sizes present in most of these samples that these methods are unreliable. The *Kenney et al*, and *Hazen* results are generally a little low, but this could also be because of the small degree of compaction in these experiments. This is particularly the case for MOT 1, where the predicted results are more than two orders of magnitude lower than the measured result. We would expect a significant reduction in permeability from compaction for the MOT 1, but we would not expect the same effect with the other, near to single size materials.

The *Fair and Hatch* formula appears the most complete. However it is heavily dependent on the porosity measurement for its result. Hence the extra accuracy implied by the use of all of the grading curve information may be misleading. The *Kamal* formula also gives more terms than seem likely to be relevant, given the considerable uncertainties in the data. The original version gives figures to five significant figures, which cannot possibly be valid.

Hence it is clear that there is no guaranteed method for evaluating permeability without a direct measurement. However for near single sized materials the simplest model works well, and could be used. For materials with a wider distribution of sizes it is necessary to consider the degree of compaction, and prediction of permeability will be very difficult.

Conclusions to chapter 5

In the experiments described in this chapter, the permeability and Forcheimer term for five different aggregates were measured. A long wooden box was used to contain the aggregate, and the pressure gradient through it was measured for different rates of air flow. The data gained was used with the Darcy-Forcheimer law to arrive at values for both the permeabilities and the Forcheimer term.

The grading curves for the aggregates were also measured at BRE, and this information was used to explain the differences between the permeabilities and Forcheimer terms of the different materials. Several different models were compared to see which one best describes the permeability, based on grading curve information. These varied between aggregates, suggesting that using formulae for the permeability is not a reliable method. For the well graded stones the simplest, 'billiard ball' type model gives reasonable results.

The 40 mm graded has the largest permeability by some distance, a factor of three above the two 20 mm ones. These two have very similar results, although they are given different descriptions. In these experiments the difference between the MOT type 1 and the 10 mm single sized is very small. Because the MOT type 1 has a wider spread in material sizes within it, it would be easier to work with in practice, so it would appear to be a better choice. Single sized aggregates generally give better air flow rates, but are not as easy to work with, and are more expensive. Hence their performance must be significantly better to justify their use. In fact only the 40 mm aggregate has a permeability much larger than the MOT type 1, at around nine times greater, so on this evidence the choice would be between these two.

The main concern with this result is over the compaction which would normally be applied to these materials in use, and which would reduce their permeabilities.

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Chapter 6: Pressure extension measurements and analysis

Introduction

This chapter looks at the results of experiments to consider pressure extension, a concept discussed in chapter 4. Pressure extension is the proportion of the pressure generated at the central extraction point which is measured at any other point in the hard core or soil. In general it is good if the pressure extension is large all over the floor area of the dwelling, as this will mean that the effect of the extraction of gas is covering the maximum area possible. It is therefore of interest to see what causes the pressure extension to be good, or similarly why it should be bad.

There are two sets of experimental data, one from a large number of tests carried out by drilling holes after construction, and the second from a smaller set where tubes were laid before the floor slab was poured.

The first experimental programme was undertaken by Wimpey Environmental Ltd, using, where practical, floor slabs constructed by Wimpey Homes Holdings Ltd. They located suitable floor slabs, and before the walls were built, tested the floor for its pressure extension. They also collected a sample of the hard core material used to measure its grading curve.

The second set of tests were carried out by Roger Stephen of BRE and Trevor Gregory of Cornwall County Council. They measured the pressure extension in a number of schools by laying plastic tubing below the floor just before the concrete floor was poured. Then when air was sucked from a central sump after the floor was present they were able to measure the pressures without further disruption to the building. It also allows repeat tests to be carried out to look at the effect of time on the flow of air.

Both of these sets of data give information about the behaviour of different floors and the effect of the hard core used when building them. These results are the subject of the analysis in this chapter. The aim of the work is to support the choices which have to be made in designing houses so that they do not have high radon levels, but without spending more money than is necessary. The concern about hard core materials is that by being too specific about what should be used then builders might be forced to transport material over a long distance, with resulting increased costs and heavy lorry journeys. Therefore it is important to understand what is necessary, and how best to use the materials available.

Set 1: The 'Wimpey' tests

The aim of the pressure extension tests is to measure the pressure produced at the edge of a floor by extracting air from the centre. In these tests this was achieved by drilling a central suction hole, pulling air from it with a vacuum cleaner, and measuring the pressure at a series of smaller holes in the floor slab. Full details of the experiment are in the final contract report, which contains all of the measurements as well [Wimpey 95]. In total 78 tests were carried out in a wide range of locations. This work was also discussed in a paper by Bell and Cripps [Bell 94].

As well as measuring pressures the flow rate through the system was measured, to allow analysis of the overall resistance of the floor to gas flow. A sample of the hard core material from each site was collected, and tested to give the grading curve of each material, using British Standard sieve tests [BSI 90].

It was originally intended to undertake tests on dwellings only in the areas most affected by radon. However this was not possible because of the relatively small number of dwellings being constructed in these areas at present. There is also a general preference for the construction of precast suspended concrete floors in the affected areas, because of the cost and practicalities of building in protective measures for in situ concrete floors. This meant that a number of tests have been undertaken on floors near to, but outside of, the designated affected areas.

Resistance to flow

The simplest way to represent the data is to compare the ratio of the pressure generated

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by the vacuum cleaner to the total flow produced by it. This defines a flow resistance, which describes the performance of the whole system. It can also have value in helping to choose the appropriate size for a fan to suck air from the radon sump. However the size of the drilled hole has an impact on the result, so it would be more usefully done with the full sump system set up.

The results for the flow resistance, $R = \Delta P / Q$ (Pa / m³/h), for all of the Wimpey tests are given in figure 6.1 below. The results show that this floor resistance varies markedly, with variations between 0.15 Pa/(m³/h) to 2500 Pa/(m³/h), although most results were in the region of 100 Pa/(m³/h).



Figure 6.1: Graph of resistance to flow against test number

The first point to note is that the first 10 or so tests carried out were probably flawed as the results are very different from the later ones. There is probably a source of leakage which was later eliminated, as the initial resistance is very low.

The later results are less spread out, but still show a variation over 2 orders of magnitude. This shows the wide range of results occurring, which is typical of tests

involving permeability. However given that permeabilities of soils vary over many orders of magnitude this result suggests that either the permeabilities of hard core materials have a smaller range, or there is some other effect reducing the apparent range.

Pressure field models

The 78 sets of data were analysed using a statistics package to fit the results to equations (4.4 with c = 0), (4.4) and (4.7), as developed in chapter 4. The first ten sets of data were not processed as they were not good enough for the models to be applied to. There were not enough points in some sets and the other results were not smooth enough curves. This may reflect the fact that these were the first tests carried out by Wimpey, and so they were less familiar with the equipment and procedures.

The three models (Darcy-Forcheimer (4.4), Darcy(4.4, c=0), and resistance model(4.7)) were applied to each data group and the resulting estimates for the parameters k, c, and R_{crack} were recorded. These estimates were used with the model equations to plot a line through the data points obtained experimentally. One such set of results is shown below, although in this case the Darcy-Forcheimer model and resistance model coincide, so the resistance line is not shown.



Figure 6.2: Comparing pressure field data with Darcy-Forcheimer and Darcy models

This set of results shows a typical relationship between the Darcy and the Darcy-Forcheimer models. The Darcy fit is quite good but the extra term in the Darcy-Forcheimer model enables a better fit to the data. If another term was added, to consider the effects of the cube of the velocity, the fit could follow the experimental data even more closely under some circumstances. However, it would be hard to justify adding another term from the physics of the system.

Comparing Darcy and Darcy-Forcheimer results

The Darcy model gave reasonable results in all the tested cases. While the fits were not always as close to the data points as the Darcy-Forcheimer lines were, the Darcy values for the permeability were always in the expected orders of magnitude. In contrast, eight data sets gave the Darcy-Forcheimer permeability values to be around six orders of magnitude above the Darcy values from the same data. Examining the data sets revealed that in all these cases, the pressure started high and then dropped off very sharply. The Darcy-Forcheimer fit gave large permeability values and Forcheimer constants with huge errors. An example of one of these data sets is shown below.



Figure 6.3: Graph of pressure against distance where the models fail

All three of the models tested give lines close to the first data point. The models then

cannot produce a curve which turns rapidly enough to fit the lines closely to the next point. Six of the eight data sets that showed this behaviour were the only six where a sandstone aggregate was used under the floor slab.

Examination of the grading curves showed that this material had a much higher proportion of small particles than any of the other aggregates used. This would be expected to give it a low permeability, resulting in the steep pressure gradient near the suction point as the data points show.

To obtain a result from these 6 data sets, the first point was ignored and the models were applied to the remaining points. In some cases this worked, in that the parameters returned by the statistics package were in the expected range. In other cases, this did not work and the next data point had to be removed as well. However, the act of removing these data points to achieve a result may well have altered the results sufficiently to make them meaningless. Certainly in some cases the data had to be cut down so much that the shape of the curve was lost completely. To obtain better results using the sandstone, it would probably be necessary to repeat the experiment, taking more pressure measurements close to the sump.

In the remaining sixty or so cases where the Darcy-Forcheimer model worked, some of the Forcheimer constants were returned as negative values. This would imply that the resistance to flow *decreases* with *increasing* flow rate, which is most unlikely to occur. In some of these cases, the error associated with the Forcheimer constant meant that it could be zero. However, this is not true in all cases and may mean that there is a problem with the model, or that more data points are needed. These results mean that the Darcy-Forcheimer law must be used with care in this type of experiment, and that the Darcy Law may be more reliable.

Resistance model

The resistance model equation (4.7) has produced good results for some cases; however, it returned satisfactory values for k and c in less than half of the cases. In some of the

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others a good fit was found but with values of completely different orders of magnitude to what was produced by the other models. The statistics package does not attribute physical meaning to the values and so produced the best fit it could. Unfortunately, in most cases, the values were unacceptable. Below are some specimen results, case 125 (where the models worked well) and case 120 (where the resistance model returned unacceptable figures).

Experiment	Model used	Permeability K	Forcheimer	Resistance term (Pa
number		(m²)	term c (s/m)	s /m ³)
125	D-F	$(8.6 \pm 1.0) \ge 10^{-11}$	112 ± 30	-
	Darcy	$(5.1 \pm 0.1) \ge 10^{-11}$	-	-
	Resistance	$(8.7 \pm 1.1) \ge 10^{-11}$	118 ± 40	$(5.7 \pm 0.2) \ge 10^{-4}$
120	D-F	$(4.0 \pm 0.4) \ge 10^{-10}$	83 ± 45	-
	Darcy	$(3.2 \pm 0.1) \ge 10^{-10}$	-	-
	Resistance	-7.7 x 10 ⁻³	-8.7 x 10 ⁹	$(6.2 \pm 1.2) \ge 10^4$
		$\pm 2.6 \times 10^{6}$	$\pm 3 \times 10^{18}$	

Table 6.1: Results for a 'good' case and a 'bad' case

Furthermore, the slabs for which the resistance model worked were spread evenly throughout the data. There does not seem to be any common factor in the cases that failed that might suggest a link between them. At least one failure in the resistance model occurred at each site, and the permeability and Forcheimer constants for the failed cases were similar in magnitude to those calculated for the slabs where the model worked. However, where the model worked, it gave permeability and Forcheimer constants very close to those calculated from the Darcy-Forcheimer model. In addition, the 'crack width' values calculated from the resistance model using equation (4.6) were believable in that they were all around 1 mm in magnitude. It is probably the case that the information from the experiment is not sufficient to evaluate the resistance of the crack, which is generally small compared to that in the hard core.

Comparing permeability results for the different sites





Figure 6.4: Graph of Darcy model permeability for each slab.

However each individual slab came from one site, each of which used a similar hard core for each of the slabs laid at it. The average permeability and Forcheimer constant for each site is therefore of interest, and the results are given in the table below.

Aggregate	Slab numbers	D-F model		Darcy model
		Average k (m ²)	Average c (s/m)	Average k (m ²)
MOT 1 #1	116-122	2.3 x 10 ⁻⁹	43.7	1.4 x 10 ⁻¹⁰
MOT 1 #2	152-158	6.4 x 10 ⁻⁹	86.9	1.4 x 10 ⁻¹⁰
MOT 1 #3	159-162	7.8 x 10 ⁻⁹	191.9	1.5 x 10 ⁻¹⁰
2" clean stone	127-128	4.3 x 10 ⁻⁹	-13.9	2.6 x 10 ⁻⁹
2" clean stone #2	147-151	3.5 x 10 ⁻⁹	98.8	1.1 x 10 ⁻¹⁰
2" clean stone #3	130-132	4.5 x 10 ⁻⁹	969.6	8.3 x 10 ⁻¹⁰
40 mm chatter	123-126	2.0 x 10 ⁻¹⁰	784.8	2.8 x 10 ⁻¹²
40 mm chatter #2	140-143	6.9 x 10 ⁻⁹	209.2	3.9 x 10 ⁻¹⁰
Quarry rubble	144-146	1.9 x 10 ⁻⁷	37.7	4.0 x 10 ⁻⁹
3" clean stone	109-115	1.3 x 10 ⁻⁸	-18.9	2.6 x 10 ⁻⁸
Trench fill	129	2.2 x 10 ⁻⁹	66.2	5.6 x 10 ⁻¹¹
Sandstone	133-139	-	-	1.7 x 10 ⁻¹¹
MOT & QR mix	163-169	7.0 x 10 ⁻⁹	234.7	1.2 x 10 ⁻¹⁰
Granular type 2	170	3.0 x 10 ⁻⁹	-40.4	2.0 x 10 ⁻¹⁰
40 mm screened	171-177	1.1 x 10 ⁻⁸	13.6	2.1 x 10 ⁻¹⁰

Table 6.2: Average results for permeability and Forcheimer term by site

Comparing results from Wimpey tests with laboratory experiments.

The average results from table 6.2 can be compared with earlier permeability experiments carried out for the author at BRE in the laboratory and discussed in chapter 5. Because we have the grading curves for both sets of aggregates it is possible to compare the results for similar materials in the two tests. The BRE aggregates were known as 'MOT', 'large', 'medium' and 'small', with the sizes of large being up to 40 mm, of medium up to 20 mm, and small up to 10 mm. There was no Wimpey result corresponding to the 'small' case.

The first graph, figure 6.5, shows the average permeabilities from the Wimpey tests, with the laboratory results overlaid.



Figure 6.5: Comparing results from Wimpey tests to BRE laboratory tests

This shows that the permeabilities measured by Wimpey appear to be about 10 times less than those measured in the laboratory, apart from the MOT type 1. This graph also shows that taking the average over the slabs in the sites reduces the spread in the data significantly. Whereas in figure 6.4 there was a four order of magnitude spread, most of the points in figure 6.5 are within one order of magnitude.

Grading curves for the different aggregates were also drawn up. An example is shown below.

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Figure 6.6: Grading curve for 3" clean stone

We selected the material from the Wimpey tests whose grading curve corresponded most closely to one from the laboratory tests. This was done to compare the results from the two different experiments, as the names of the aggregates alone are not enough: a 40 mm aggregate on one site may have very different constituents to a 40 mm aggregate on another site. The table below shows the results of the comparison.

BRE aggregate	Wimpey aggregates with closest grading curves	Ratio of calculated permeabilities BRE : Wimpey	
MOT type 1 sub-base	MOT type 1 (nos. 1,2,3) 2" clean stone (no. 2) Granular type 2	0.6 : 1 0.97 : 1 1.1 : 1	
Medium	2" clean stone (no. 1)	23 : 1	
Large	3" clean stone 40 mm chatter	13.8 : 1 900 : 1	

Table 6.3: Comparing results of Wimpey and Laboratory tests on similar material

These figures show that the BRE 'medium' and 'large' aggregates, while being similar in composition to some of the Wimpey aggregates, have larger permeabilities than them. This almost certainly reflects the fact that the Wimpey aggregates were more compacted on site than these BRE aggregates could be in the laboratory.

The MOT sub-base 1 tested at BRE might have been compacted a similar amount to that used by Wimpey, as the permeabilities are comparable. This result is unexpected since the MOT is expected to compact the most on site, so a large difference might have been expected, and there is no obvious explanation for this.

Pressure extension

To be effective a sump must overcome the internal pressures within the house generated by wind and stack effects. Typically these are no more than 5 Pa, and so for slabs where the pressure at the edge does not reach more than 5 Pa, there might be problems with remediation later if it proves necessary.

In these tests over half of the slabs did not exceed 5 Pa at the edge. It was hoped that there would be a clear correlation between the aggregate used and whether or not the 5 Pa criterion was met. However, there was no one aggregate that consistently produced the necessary pressure at the slab edge. Most of the aggregates were used in more than one experiment, and of those experiments, some gave 5 Pa and some did not.



Figure 6.7: Comparing pressure extension to permeability

If less permeable aggregates were more likely to give better pressure extension, the

lower part of figure 6.7 should contain squares indicating success, and the upper part should contain circles that show that 5 Pa was not produced at the edge. The points circled with the dotted line are those with the lowest permeabilities, and most of those cases are successes. Of those seven points, the four on the right are slabs where '40 mm chatter' was used and the other three are MOT type 1 sub-base. However, MOT aggregate was used elsewhere with mixed results, as was 40 mm chatter.

The outcome of this is that no aggregate stands out as the best one to use from the 5 Pa test point of view, and it is not clear that the permeability has an impact on the pressure extension. It is not clear why, out of a set of results using one particular aggregate, some satisfy the 5 Pa condition and some do not. There are a number of possible reasons for this:

- 1) The experiments were not identical, so the different geometry could affect the results.
- 2) The permeability of the soil under the aggregate could affect where the air flows from into the suction area, and if it is a high permeability soil the hard core could be largely by-passed. In future work it would be helpful to investigate the permeability of the soil as well as the hard core.
- The size and position of cracks in the floor-slabs and any voids within the hard core could effect the result, by providing a short cut for the air to flow through.

Set 2: Cornwall County Council tests

In each of these tests the floor of a school was due for renewal, and the opportunity was taken to study the way in which the design and installation of the floor affected the pressure extension. In each case a sump was installed in case it was subsequently needed to reduce the radon level if it should be high after the work was carried out. This meant that no extra hole had to be drilled to provide a suction point. In addition a number of plastic tubes were laid below the floor slab to enable a grid of pressure measurements to be taken below the floor, without further disruption of the building. Each of the cases is

considered separately. This work was carried out by Roger Stephen and Trevor Gregory, but the author has analysed the data.

Case studies

The case studies are reported fully in [Gregory 95 and Gregory 93]. A brief description of them is included here to give the background and to allow comparisons between the pressure field extension results obtained.

Case study 1 - St Levan School

In this school of a total floor area of 145 m^2 , an area of floor 87 m² was replaced with concrete on builders rubble as hard core fill. Because of a history of high radon levels, sub-slab depressurisation was provided by incorporating two BRE/CCC "standard sumps" below the floor linked by 110 mm rigid PVCu pipe work to a single extract fan. The pressures measured in the sumps were very different, 125 Pa for the sump with the shortest pipe run and only 56 Pa for the other sump which has a pipe run twice as long and with an extra bend.

A year later the pressures in the sumps were found to have risen considerably to 138 and 71 Pa. The pressures under the floor had also risen, though to varying degrees. A third visit was made to the school another two years later. The sump pressures were found to have fallen to 116 and 61 Pa but the normalised pressure field was almost identical to that found on the second visit. It is not clear why these pressure changes should have occurred. Falling pressure might be expected due to settlement of the hard core making gas flow easier, so this could explain the fall between the second and third results.

The normalised results for the first and third visits are presented in figure 6.8 and show that there is considerable scatter, particularly for the third visit. This may be partly due to interaction between the two sumps and partly due to variations in permeability.



Figure 6.8: Pressure field at St Levan

These data also suggest that the pressure field extension has actually improved slightly over the first year of operation rather than deteriorated. Possible reasons for this improvement include a change in moisture content of hard core/soil and (partial) blockage of air flow paths around the edges of the floor.

Case study 2 - Trannack Primary School

Encouraged by the work at St Levan School, and by work on permeable hard core fill by the EPA in America [Craig 90, Harris 91] a similar exercise was carried out at another small primary school at Trannack. In both size and construction it is very similar to St Levan. A floor replacement exercise was carried out together with the fitting of a single central sump and 51 small bore pressure tubes. The significant difference between this floor and St Levan is that the hard core fill was covered by a 150 mm thick high permeability layer of 25 mm granite chippings. This was an attempt to achieve a particularly good pressure extension in this study.

Unfortunately the chippings delivered to site contained considerable fine material and so the hard core was less permeable than had been intended. The total floor area replaced was 71 m². A fan was fitted to the sump and a full set of sub-floor pressure measurements taken, presented as a contour plot in figure 6.9.



The contour plot shows a computer estimated fit between points at which the same pressure would be measured, equivalent to the height contour lines on a geography map. An interesting feature of this plot is the way in which there is a direction for which the pressure falls away much less than in the other directions. This implies some variation in the hard core in this direction, and it makes the results much harder to interpret. One possible reason for this effect is a drain pipe laid in pea shingle located near this region. It is important to remember that this could occur with any floor slab, so all results where the data were collected in one direction only must be treated with care.

Case study 3 - Pool School

The original floor of one of the classrooms of Pool School was replaced during 1993. Again, a sump with capped off pipe work and small bore test tubing was fitted when the floor was replaced. The new floor was similar to that used at Trannack but this time the permeable hard core layer had little fine material in it and was covered with a 2000 gauge (0.3 mm) polyethylene sheet before blinding with dry lean-mix concrete (coarse sand and cement) instead of sand. Thus the blinding was prevented from entering the permeable layer by the extra membrane. The floor area replaced was 35 m².

A fan fitted to the sump, gave a pressure of 58 Pa, and a full set of sub-floor pressure measurements were taken. The results are presented in figure 6.10. The pressure field extension was good, certainly better than at St Levan or Trannack, although the floor at Pool is significantly smaller than the other two. There was also little scatter in the data.



Figure 6.10: Pressure field extension for Pool School

The pressure measured in the sump at Pool was much lower than that measured at both St Levan and Trannack and with similar pipe work attached to the fan outlet. It was concluded that the good pressure field extension combined with a relatively high air flow rate was due to both high hard core permeability and the small floor area. It was hoped that the next case study, having a much greater floor area, might provide more useful information.

Case study 4 - Launceston College

At Launceston College gymnasium the original 254 m² wooden sprung floor needed to

be replaced. The small bore test tubing and a single central radon sump were fitted across the 16 metre square floor area.

The replacement floor comprised: builders rubble hard core fill up to top of sleeper walls, 300 mm thick layer of high permeability hard core of clean 25 mm granite chippings, 2000 gauge (0.3 mm) polyethylene membrane, blinding layer of dry lean mix concrete, glass fibre reinforced heavy duty polyethylene damp proof membrane with lapped and taped joints followed by the 125 mm thick concrete slab.

A significant difference between Launceston College and the previous case studies was the presence of a concrete oversite below the rubble hard core fill layer. This would be expected to greatly limit, though not eliminate, air flow from the soil below the permeable hard core, which might be expected to help to maintain the pressure field to the edges of the floor.

The results for Launceston College, figure 6.11 show that the pressure field extension achieved was similar to that at Trannack and considerably worse than that achieved at Pool and in an American study [Harris 91]. This result was particularly disappointing because the floor construction was known to have been exactly to specification and should have provided the optimum conditions for excellent pressure field extension.



Figure 6.11: Pressure field from Launceston College

The sump pressure measured at Launceston was only 42 pascals, even without the resistance of a fan outlet pipe and rain cap as was used at Trannack and Pool. This suggests that the air flow rate was indeed higher at Launceston (we estimate an air flow rate of approximately 257 m³/h). It would appear that the concrete oversite, bounding wall and floor slab surrounding the permeable hard core layer were not particularly airtight. Most of the pressure drop appears to occur through the permeable hard core material.

Case study 5 - Stoke Climsland

Stoke Climsland school was under construction when measurements were taken. Changes were made to the original floor specification as follows: top 150 mm of sub slab hard core changed to clean 25 mm granite chippings; use of dry lean-mix concrete as a blinding instead of sand; provision of short lengths of 32 mm diameter plastic pipe at approximately 1800 mm centres to encourage communication through the numerous internal foundation walls below the floor slab. The remaining floor construction remained unaltered. The existence of these internal walls within the foundation region was the main difference from the previous cases, and presents an extra challenge in ensuring good pressure extension.

Results for all three sumps were taken, the actual sump pressures measured being 101, 121 and 151 Pascals and estimated air flow rates 270, 260 and 230 m³/h in sumps 1, 2 and 3 respectively. The pressure results for sump 2 are given in figure 6.12 below.

The pressure field extension at Stoke Climsland, presented in figure 6.12, was similar to that obtained at Launceston, in spite of the numerous internal foundation walls and absence of a membrane under the blinding, but again was not as good as had been hoped for based on American experience [10]; see discussion below.



Figure 6.12: Pressure field at Stoke Climsland, compared to USA result

Although these results show pressure extending a considerable distance from the sump, the results are much less dramatic than those from the USA. It is not clear why this should occur, since a lot of care was taken in the design and installation of this floor. A feature not apparent from the sump 2 data was a definite drop in pressure at the sub floor walls, which the USA study appears to have avoided.

Discussion

The principal problem in building radon protection into floors is to balance the need for air flow through the hard core material to protect against soil gas with the structural role of the hard core in supporting the floor slab and the practicalities of constructing the floor. If the builders are not happy building with a particular hard core they are unlikely to do so, and will build in a different way. In England this has been seen with the move in Devon and Cornwall (as the most affected counties) to building precast suspended concrete floors. This is due in part to problems with the hard core, and also to the requirement to lay the slab on top of the inner leaf of the cavity wall. To reduce the costs to a minimum these may need to be looked at again. This work suggests that some of the materials routinely used on English building sites may not be adequate for ensuring sufficient air flow. The current mention of using 'clean permeable hard core' is not adequate, and it may be necessary to describe the type of material more carefully in future. However the results from the Wimpey tests indicate problems in specifying hard core materials and getting a reliable result, as materials which are officially the same gave very different results.

One problem which it was not possible to address in the Wimpey work is how the layer of sand 'blinding', generally laid on top of the hard core before the floor slab is poured, affects the air flow performance. The use of a membrane over the hard core material seems to have been effective in the Cornwall test, together with lean mix concrete in place of a blinding layer. Whether this could be a routine method will need further investigation.

Other possible solutions to the problem could lie in using mixed layers of hard core, or a ventilation layer of another material. In the former case two layers of hard core would be used, the lower one ideal for compaction, like MOT Sub Base Type 1, and a more permeable upper layer to allow air flow. An alternative is to replace the upper layer with a geo-textile and drainage matting.

An interesting result found in both sets of tests is the significant difference between floors of similar specification. For example at Pool the specification for the floor was accurately followed and the result was excellent pressure field extension but this success was not repeated on the much larger floor at Launceston using an identical floor construction

Another area of concern is why the UK results do not match those found in the USA tests [Craig 90, Harris 91]. This is in spite of the great care taken in some of the tests to achieve good pressure extensions. This deserves further study.

Comparing the test methods

The two different methods of obtaining pressure extension results give different but complementary information on the behaviour of floor slabs. The method used in the Cornwall tests gives a lot more information on each site, but could only be used for selected cases. There would be no benefit in using it in a wider number of cases, and this would be very expensive.

The vacuum cleaner type of test could be used for post-construction testing, in order to help in the sizing of a fan for a particular floor, and if the floor has been laid correctly. However it is unlikely that this will be worthwhile in the UK where the cost of remedial measures has to be kept to a minimum. It also punctures the damp/radon proof membrane in a way which is difficult to repair. The technique does give useful information on how different hard core materials perform, and this information can be used to help to inform future choices.

Conclusions to chapter 6

A total of 78 tests of the air flow through hard core materials were carried out by Wimpey Environmental under contract to BRE, using a post construction technique with a vacuum cleaner. In addition five detailed tests have been carried out by installing plastic tubing below the floor before it is laid. Overall these show that there is a wide variation in the behaviour of the air flow in the hard core materials resulting from sucking from the centre of a floor slab. However careful choice of hard core specification and avoidance of excessive blinding can significantly improve under-floor permeability compared with normal UK floor construction practice, even when not carried out perfectly.

However given that relatively few sump systems fail to reduce radon levels, and most new homes are protected by existing measures [Woolliscroft 94] it would not be appropriate to spend too much money on improving the fill material. It is unlikely to be cost effective. The extent of the pressure extension varies considerably, with more than half of the large group of floor slabs having less than 5 Pa at the edges. In the detailed cases the performance achieved in tests by the EPA in the USA was not reproduced. It is not clear why this occurred.

The Cornwall work indicates that blinding materials should be kept out of the permeable material to improve pressure extension. A polyethylene membrane would achieve this and may allow the usual sand blinding to be used. Alternative systems involving lean mix concrete blinding layers or preformed gas collection layers deserve consideration.

The increased pressure field extension obtained by improved design could result in new-build properties requiring fewer under-floor suction points and/or a reduction in fan power consumption with a greater degree of confidence of success than at present. This will apply particularly to large non-domestic buildings. However, this must be balanced against the extra floor construction costs, practical difficulties and current inconsistency in results.

Chapter 7: Conclusions

In this part the effect on soil gas flow of the pressures caused by fans has been considered. This reflects the widespread use of 'sump' systems for reducing indoor radon levels, and the wish to optimise their design. The way in which these sumps behave has been considered, along with the pressures they achieve in different hard core materials and the soil around a building.

Modelling

The first area of study used a two-dimensional linear finite difference model of the gas flow. This is significantly flawed because neither the 2D nor linear approximations are valid. Hence the results can only be used with caution and in a qualitative way. Nevertheless they do help us to understand where the flow is taking place, and the impact of different layers of hardcore material.

In particular the fact that most of the flow into a sump is coming from the house allows an estimate of the heating cost of the sump system to be made; it is in the range £20-£50 per annum for a typical size house. Another key understanding from this type of modelling, but not original to this work, is the fact that the width of cracks in a floor slab is not very important in soil gas flow. This is because the resistance to flow due to the soil is much greater than that due to the crack (under most circumstances). This therefore explains why it is so difficult to seal a floor to keep out radon.

The possibility of using source and sink theory to examine soil gas flow was also looked into. The lack of flexibility in the geometry meant that this work was not taken very far, but there is some qualitative value in the solutions anyway. They could also serve to give reference solutions for comparing with computational solutions.

The Darcy-Forcheimer Law addresses the non-linear nature of soil gas flow at the pressures occurring when sump systems are used. It requires a second parameter to the Darcy Law, which is known as the Forcheimer term. This work recognises the need for the more complex law, and uses it in some very simple, essentially one dimensional,

Chapter 5: Time-dependent solutions to the landfill gas concentration equation

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Introduction

The steady-state solution Time-dependent solution using Laplace Transform method Time-dependent solution using a numerical method

Introduction

In order to investigate techniques for measuring landfill gas flow, Richard Hartless of BRE had a soil cell built. It has about half a metre depth of sand suspended above a void region, into which gases can be supplied. These gases then travel up through the sand and can be observed leaving the sand at the top surface. Although the soil cell was intended for studying the use of techniques to observe the exit rate of methane and other gases, it has been necessary and interesting to understand the behaviour of the gas *within* the soil cell as well as when it leaves it. This experimental work has not been published formally, so cannot be referenced at this stage.

The problems encountered when using this soil cell are the reason for this modelling work. It was necessary to understand the flow processes going on in the soil cell to interpret the results from it. The experiments were carried out by Richard Hartless in consultation with the author, and by an undergraduate student under the supervision of the author.

Gas concentration equation

Clements and Wilkening [Clements 74] wrote the one-dimensional transport equation for radon, given previously in equation (2.12), as

$$\frac{\partial C}{\partial t} = \frac{D}{\epsilon} \frac{\partial^2 C}{\partial z^2} - \frac{1}{\epsilon} \frac{\partial (\nu C)}{\partial z} - \lambda C + \phi$$
(5.1)

where

C is concentration of radon $(mol m^{-3})$,

D is diffusion coefficient of radon in soil (m^2s^{-1}) ,

 ϵ is the soil porosity (),

v is the velocity of the soil gas, or fluid volume current density (ms⁻¹),

 λ is the decay constant of radon (s⁻¹),

 ϕ is the radon production rate (mol m⁻³ s⁻¹),

z is the spatial direction (m),

t is time (s).

In this form both the diffusion of gas and the pressure driven flow are accounted for. For landfill gas the decay term λC would not be present, since for most situations the gases would not change after production, although the issue of solubility in water could become important. Hence for landfill we would take λ as zero, while the production term ϕ would be hard to define because of the variable nature of landfill gas production. However if our concern is only the transport of methane away from some source, then the production term can also be set to zero, and a concentration and pressure boundary condition assumed instead. This makes for a helpful simplification of the equation to

$$\frac{\partial C}{\partial t} = \frac{D}{\epsilon} \cdot \frac{\partial^2 C}{\partial z^2} - \frac{1}{\epsilon} \cdot \frac{\partial (\nu C)}{\partial z} \qquad (5.2)$$

If the velocity term v is not known it has to be found by solving the pressure equation. This aspect is discussed in chapter 3. In general equation (5.2) will be difficult or impossible to solve exactly if the velocity v changes over time or with depth. It is possible to tackle the general problem with a computer model, but considerable insight can be gained from looking at the cases where the velocity is constrained in some way so as to make the solution much more straightforward.

The simplest case is to set up a constant velocity and wait for a steady-state condition to be reached. This is addressed in the first part of this chapter. From this the natural development is to consider the effect of an imposed velocity on the concentrations. It allows the time scale for changes in measured concentrations to be estimated. Some knowledge comes from an exact solution to this problem, discussed in the second part of the chapter. More detail comes from a simple one-dimensional numerical model, which is

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situations.

Experiments in the laboratory

These results were then used to interpret the results from laboratory and site measurements of pressures and flow rates. The simplest were carried out on a series of samples of hard core in a box in the laboratory. When the leakage of the box itself can be understood these give good results for the permeability and Forcheimer terms of these materials, but of course only for the conditions in the box. The issues of compaction and moisture content were not within the scope of the equipment available, and because of leakage effects not all hard core materials can be measured.

The problem caused by the conditions of the tests can be seen from the comparison of the measured results to those from the various theories (mostly empirical) relating permeability to the grading curve information for a hardcore material. None of the theories gives a result which consistently matches that of the experiments, but they can give a reasonable approximation, and these are much easier to obtain than direct measurements.

Experiments on site

The next set of data presented concern measurements of pressure extension on real sites. Some of these were carried out by colleagues at BRE, others by a contractor. The interpretation of these gives some indication of how different hardcore materials affect gas flow in practice, through the concept known as pressure field extension.

In the tests carried out for BRE by Wimpey Environmental Ltd (now Wimtech) 78 floor slabs were measured after the concrete had set but before the walls were built. The test could be a post construction test for the suitability of the hardcore for use with a sump, but the test is not likely to be cost effective. Combined with measurements of the grading curves of the materials used these results give considerable insight into the variation in air flow between floor slabs. The tests carried out by BRE and Cornwall County Council used tubes inserted before the concrete floor was laid to measure the pressure field. This gives a wider spread of measurements without affecting the floor, but is more expensive. They also show the variation between floors, and in particular the difficulty of achieving really good pressure extension.

The most significant finding is that the differences between the behaviour of different hardcore materials are not as large as expected, and they probably do not justify the cost of using 'better' materials. This financial check is an area of work which needs to be taken further, but it reflects the high cost of using and transporting 'special' hard core materials, and the many complications of what appears to be a simple problem. The cost savings from better pressure extension should come from being able to use a smaller fan to remove radon. Since most new buildings don't need to use the fan anyway, and the cost saving is quite small, it is not worth spending more on the hard core material unless it is needed for some other purpose, drainage being the most likely.

Part 3: Time dependent flows

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Chapter 1: Introduction

Time dependent effects

Most soil gas modelling carried out has concentrated on steady-state problems, principally because of simplicity, but also because some aspects of radon entry can be well described as steady-state. However there is increasing evidence of the need to look at some time-varying effects, as they can be significant in certain circumstances.

There are many possible time-dependent effects, the main ones of which are briefly discussed below.

1) Changing atmospheric pressure

As the pressure in the atmosphere changes, due to the movement of weather systems, the pressure of the air at the surface of the soil is also changed. This pressure is transmitted into the soil, and generates flows of gas. Because the soil does not allow very free flow of gas it can take a considerable time for these flows to take place, so time scales of order days will need to be considered.

2) Wind induced pressure fluctuations

The wind speed changes continually, over time scales of seconds for gusts, and hours for average speeds. The pressures induced by the wind at the surface of the ground or in buildings can cause gas flow in the ground, particularly for more permeable soils.

3) Atmospheric tides

The effect of the gravity of the moon causes the familiar water tides that affect the oceans. A much smaller effect happens to the air as well, and results in a 12 hour cycle of changing pressure. Under some conditions the flow of air in and out of the soil is enough to cause significant soil-gas entry into a building.

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4) Changes in water table

The level of the water table can affect gases in the ground, in a number of different ways. Radon is soluble in water, so more radon will escape from the ground if the water table falls. Methane is fairly insoluble, so that a rising water table causes more methane to leave the ground, in particular as compared to carbon dioxide which is water soluble. The level of the water table tends to change slowly, but could still be significant under some circumstances.

5) Human behaviour

There are many ways in which the way in which people behave can affect gases in the ground. Changes in ventilation systems, opening windows, levels of heating and others can all affect the flows of gases. This behaviour is generally time-dependent but also unpredictable, so it is usual to assume a worst case behaviour.

6) Changes in landfill site behaviour

A landfill site is usually filled over many years, and the chemistry and biology going on inside it develop as the amount of air, water and nutrients change. This results in significant changes in the gas flow rates caused, although this usually is relatively slow, with time scale of years.

This work on time-dependent flows

The work presented in this part concentrates on the first of the effects given above (atmospheric pressure effects) since it is known to be able to have a significant impact. All of the others could have impacts over a wide range of time scales, and might need to be addressed under certain circumstances.

There has been less work in the area of time-dependent modelling than in the steadystate; the main work that has been carried out is discussed in the literature review presented as chapter 2. Because the subject is less advanced it has been necessary to carry out studies at a more fundamental level than in the steady-state case. Therefore the main aim of the work presented here was to understand the processes going on in order to decide on the key factors likely to affect the levels of hazardous gases observed in measurement studies.

In chapter 3 solutions to a time-dependent equation for the pressure field are presented and discussed. These give insight into the flow of soil gas in general, without considering the nature of the gases themselves. It shows the time scales appropriate to particular types of problems, and the considerable impact that the soil permeability has on the response time of gases in the ground.

In chapter 4 some specific problems to do with radon gas are considered, and the way in which the concentration develops in response to changing atmospheric pressure is modelled. This again shows the time taken for changes to develop, and the impact of falling pressure in raising the gas concentration. The main problem with the approach taken in this chapter is that it cannot account for all of the factors which take place in reality.

In chapter 5 problems of landfill gas flow are looked at. This subject is less advanced than that for radon, and there are extra problems with measurement techniques. Hence the work presented here was carried out to make better use of measurement data being collected by my colleagues at BRE. The main findings relate to making sense of the problems encountered in laboratory tests, but also the way in which both diffusive and pressure driven flow need to be considered in landfill gas studies.

The two modelling studies of chapters 4 and 5 are then compared with experimental data in chapter 6. There are two major long term experiments being carried out by my colleagues at BRE, and they provide data to make simple comparison with the modelling predictions. These are only able to give qualitative support to the predictions as the problems modelled are not of the same complexity as the 'real' ones. However they indicate that the modelling is giving the right trends. The most useful experiment is the 'sand box' one also reported on in chapter 6, which corresponds closely to the modelling result found in chapter 5. Here it has been possible to understand the considerable problems found with the experiments, as the time-dependent effects were very significant.

These studies of time-dependent flows are at a less advanced stage than the steady-state ones. Hence it has not been possible to design any new or improved solutions to the problems of protecting buildings against hazardous gases. However we have been able to reach a number of conclusions on the significant factors and the time scales which matter to the entry rates. In order to take this further, models will need to be developed which include the more complex interactions between the pressure inside a building and the weather outside, and the way in which both affect the gases in the ground.

For the subject of radon the long term aim from this work is to understand the short term variations of gas level, with a view to predicting the long term average values. This work only represents a step in this direction, and much more work needs to be done.

For landfill gases the main problems to be worked on are understanding the measurements made on site, and then predicting the worst case effects. The former is important to cost-effective decision making on what to do with contaminated sites, the latter to protecting buildings against possible explosions. This work has made some progress in part of the measurement problem, but further work is needed.

Chapter 2: Theory and literature review

This chapter gives the background to other work which has looked at time-dependent effects on soil gas, and an outline of the theory. It is an extract from a published BRE report, [Cripps 95-2] and reviews relevant work done so far in the field. The papers reviewed are divided into

- Analytical studies,
- Computational studies,
- Those consisting only of experimental results.

Of course many papers contain a mixture of the types of study. Papers considered come from a variety of different fields, principally radon and landfill gas migration, but including heat flow and soil clean-up by air extraction. Those considered here are those of most relevance to the work which follows in this part of the thesis, and these are therefore mostly analytical studies. The purpose is to show where other workers have reached, and what this thesis has developed. It should also be useful to others taking the subject on further.

The analytical studies mostly use the same equation for soil gas pressure, but then produce different solutions according to the boundary conditions applied. The computational work is quite varied in its coverage, including studies of methane generation in landfill sites, migration from sites and radon entry. The experimental studies cover both landfill site behaviour and work on the release of radon from soil.

Analytical Studies

Effect of changing atmospheric pressure

One of the earliest papers to consider time-dependent pressure driven gas flow in soil was by Fukuda [Fukuda 55]. He derived the one dimensional equation for pressure variation in the soil with time as

$$\frac{\partial^2 p}{\partial z^2} = \alpha^2 \cdot \frac{\partial p}{\partial t}$$
(2.1)

where

$$\alpha^2 = \mu \epsilon / k P_o$$

and

P(z, t) is the pressure in the soil, taken as a difference compared to P_o (Pa), P_o is the mean atmospheric pressure (Pa), z is distance into soil (m), t is time (s), μ is viscosity of soil gas (Pa.s), ϵ is porosity of soil (), k is permeability of soil (m²).

This equation (the diffusion equation) in similar form to (2.1) is used to describe the pressure field in soil in almost all of the papers. In some it is generalised to three dimensions, [Kimball 71]. To proceed with a solution to (2.1) the boundary conditions for the problem need to defined.

Sinusoidal pressure variation at the surface

Fukuda solved for the surface pressure varying as $e^{i\omega t}$ on the surface z=0, as an approximation of the type of pressure variation seen due to the wind. This gave a result for the pressure transmitted into the soil which varied with soil permeability and porosity:

$$P = P_0 + P_t \cdot real \left[e^{i\omega t} \cdot \exp\left(-\sqrt{\frac{\omega}{2}}\alpha z\right) \cdot \exp\left(-i\sqrt{\frac{\omega}{2}}\alpha z\right) \right]$$

= $P_0 + P_t \cdot \exp\left(-\sqrt{\frac{\omega}{2}}\alpha z\right) \cdot \cos(\omega t - \sqrt{\frac{\omega}{2}}\alpha z) \cdot (2.2)$

He concluded that wind gustiness only transmits a very small distance into a sandy soil, and this has little impact on gas movement. However he did not consider other possible time scales nor a range of soil permeabilities. Kimball and Lemon [Kimball 71] extended the method to three dimensions. They modelled the way a general sinusoidal pressure variation affects the pressure in the soil. Any effect can then be found from the superposition of waves of different frequency. The purpose of the work was to look at evaporation from soil and aeration of the soil, but the results are still of interest.

The equation they modelled was

$$\frac{\partial p}{\partial t} = -\frac{k P_0}{\mu \epsilon} \cdot \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right) .$$
(2.3)

This is the three dimensional version of (2.1). The solution they derived is too long to reproduce here, but gives the pressure field in the soil due to a two-dimensional pressure wave caused by wind fluctuations on the soil surface. They first gave the general solution, which assumes waves of different frequencies, and then looked at single frequencies and how they vary in their transmission into the soil. The results were discussed in terms of the root-mean-square velocities and displacements, and showed that the frequency has an impact on both. A major trend reported was that low frequencies result in the largest displacements.

Delsante, Stokes and Walsh [Delsante 83] also tackled the sinusoidally varying pressure problem, but used a Fourier Transform approach. Delsante co-authored a number of papers which are relevant to soil gas modelling, although written in heat flow notation, but the other papers were steady-state. In this paper they used Fourier Transforms to tackle the general problem of heat flow in three dimensions and with time variation, although to obtain a manageable result some simplifications were needed. Most of the expressions for the answers to problems are cumbersome, but the results are useful.

The temperature at the surface was assumed to vary as

$$T(x, y, z, t) = F(x, y, z) \cdot e^{i\Omega t}$$
 (2.4)

The diffusion equation for temperature is

$$\nabla^2 T(x, y, z, t) = \frac{1}{\kappa} \cdot \frac{\delta T(x, y, z, t)}{\delta t}$$
(2.5)

Note that this is exactly the same as the pressure equation used by Kimball and Lemon [Kimball 71], apart from the change from P to T and the constant being written in a different way. Hence the results of this paper can easily be transferred to the pressure field problem.

They gave the solution to the time-dependent problem by using Fourier Transforms as

$$F(x, y, z) = \frac{1}{2\pi^2} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\omega_1 x - i\omega_2 y} \cdot e^{-z(\omega_1^2 + \omega_2^2 + a^2)^{\frac{1}{2}}} \cdot g(\omega_1, \omega_2) \ d\omega_1 d\omega_2$$
(2.6)

where $a = (i\Omega/\kappa)^{\frac{1}{2}}$, κ is the diffusivity and g is given by

$$g(\omega_1, \omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\omega_1 x + i\omega_2 y} \cdot F(x, y, 0) \, dx \, dy \, .$$
 (2.7)

Then a particular problem can be tackled by including a chosen form of the T distribution on the soil surface. However the mathematics of processing the results is not trivial, and they only presented results for simplified cases. These include a single inclined step and a double inclined step, both in only two dimensions, and the equivalent in three dimensions. These step functions mean assuming that the temperature falls linearly across the walls of a house, which is often a good approximation. An indication of complexity of the result is that in the last case the flux is given by a function with 16 different terms added together.

Linear pressure increase at the surface

Clements and Wilkening, and Clements [Clements 74-1], [Clements 74-2] chose to look at a steadily increasing pressure on the surface. They used the same pressure equation, (2.1), as Fukuda, with slightly different notation.

With the linear pressure change at the boundary this gave the general solution:

$$p = -\frac{\alpha z}{2\sqrt{\pi}} \cdot \int_{0}^{t} p^{*}(t - \tau) \cdot \tau^{-3/2} \cdot \exp\left(-\frac{\alpha^{2} z^{2}}{4\tau}\right) d\tau$$
(2.8)

where

 $p^*(t)$ is the atmospheric pressure variation at the surface z=0.

If the function $p^*(t)$ is chosen as *at* where a is a constant and t is time then (2.8) leads to

$$p = (at + a\alpha^2 z^2/2) \ erfc\left(\frac{-\alpha z}{2\sqrt{t}}\right) + a\alpha z\sqrt{t/\pi} \ . \ \exp\left(-\frac{\alpha^2 z^2}{4t}\right) \ . \tag{2.9}$$

Putting this pressure result into Darcy's Law gives the vertical flow velocity v at any point as

$$v = -2a\alpha \left(\frac{k}{\pi}\right) \sqrt{\left(\frac{t}{\mu}\right)} \cdot \left(\frac{1}{2}\alpha z \frac{\sqrt{\pi}}{\sqrt{t}} \operatorname{erfc}\left(\frac{-\alpha z}{2\sqrt{t}}\right) + \exp\left(-\frac{\alpha^2 z^2}{4t}\right)\right) .$$
(2.10)

At z=0, the soil surface, this simplifies to

$$v = -2a \sqrt{\frac{\epsilon k}{\mu P_0 \pi}} \cdot \sqrt{t} . \qquad (2.11)$$

This function v gives the velocity of gas flow out of the surface when the atmospheric pressure is changing at the given rate, *at*. Hence this should be measurable with a flux box technique, Hartless [Hartless 95].

The authors also tried to solve the radon concentration equation. They wrote the one dimensional transport equation for radon as

$$\frac{\partial C}{\partial t} = \frac{D}{\epsilon} \frac{\partial^2 C}{\partial z^2} - \frac{1}{\epsilon} \frac{\partial (\nu C)}{\partial z} - \lambda C + \Phi$$
(2.12)

where

C is concentration of radon in the soil (Bqm⁻³),

D is diffusion coefficient of radon in soil (m^2s^{-1}) ,

 ϵ is the soil porosity (),

v is the velocity of the soil gas, or fluid volume current density (ms⁻¹),

 λ is the decay constant of radon (s⁻¹), ϕ is the radon production rate (Bqm⁻³s⁻¹), z is the spatial direction (m), t is time (s).

The steady-state solution of (2.12) when the velocity v = 0 and the surface concentration is constrained to be zero is then

$$C = \frac{\Phi}{\lambda} \cdot \left(1 - \exp\left(z \cdot \sqrt{\frac{\epsilon \cdot \lambda}{D}}\right) \right) .$$
 (2.13)

They stated that the radon concentration equation (2.12) cannot be solved analytically other than for the case with v = 0. They proposed to use a finite difference technique to tackle the full problem.

In the last part of the paper the authors compared the theory to simple soil cell results, and data from a site in New Mexico. They were able to monitor radon flux from the ground as well as changing atmospheric pressure, and these gave good agreement on the key prediction, i.e. the change in flux due to a change in atmospheric pressure.

Radon

Hintenlang and Al-Ahmady [Hintenlang 92] and [Al-Ahmady 93] did some of the earliest work to address the time-varying aspects of radon entry into homes. Experimental data from one particular Florida house showed that there is an unexpected peak in radon entry into a dwelling when the internal/external pressure difference is close to zero. They proposed that this was due to atmospheric tidal effects over a 12 hour period. A fairly simple model was used to look at this effect, with the soil pressure as a decaying exponential, following the air pressure. The time-lag between the two pressures was held to be the cause of the radon entry. This gave a method of understanding the experimental result, rather than a solution to the fundamental equations.

The expression they proposed was a decaying exponential

$$P_{ss}(t+\Delta t) = P_b(t+\Delta t) - [P_b(t+\Delta t) - P_{ss}(t)] \cdot \exp\left(\frac{-\Delta t}{T_R}\right)$$
(2.14)

where

 P_{ss} is the sub-slab pressure (Pa), P_{b} is the basement pressure (Pa), t is the time (s) and T_{R} is a characteristic time for the soil to adjust to the pressure change (s).

This model then allows for the fact that it takes a certain time for any change in pressure to be transmitted through the soil, and excess pressure is therefore 'stored up' and released during times of falling atmospheric pressure. This is then proposed as the possible mechanism for why the radon levels are high when the average indoor-outdoor pressure difference is small.

Recent work by Robinson and Sextro [Robinson 95] at Lawrence Berkeley Laboratory involves examining the effect of changing atmospheric pressure on the entry rate of radon into a test basement. They conclude that a significant proportion of radon entry can be due to atmospheric fluctuations, equivalent in their case to the entry rate caused by a steady pressure of 0.5 Pa, although this depends on the soil conditions. They used continuous monitoring of radon levels and pressures, together with a one dimensional time-varying model.

Other analytical studies

Alan Young [Young 89] derived an analytical solution to the problem of the rate at which methane could be extracted from a landfill site, with the intention of burning it for electricity generation. He modelled a number of extraction pipes at different points in the landfill site as sinks of gas, and was able to predict the effect of their failure. The modelling was all steady-state, and assumed a constant and simple production rate of methane.

Computer Modelling Studies

The many computational studies of soil gas flow cover a variety of different types of problem. Some concentrate on the landfill site and gas generation, whilst others assume some generation rate of gas and look at the way in which gas migrates from the site. This is our principal concern, since it affects the measures needed for protecting buildings against landfill gases. Other work has looked at the effect of varying atmospheric pressure on radon movement, and some studies consider vapour stripping wells. Only the most relevant ones are presented here.

Migration from landfill sites

Moore [Moore 79-1] started with a model of flow in capillary tubes and generalised this to a porous medium. The computer predictions from this model were then compared with measured flows in test cells, containing different sands.

Developing from this he applied the same methods to a landfill site, and the study of boreholes. The finite element method was used in the study of the landfill site, and methane levels predicted over many years. These predictions are compared with data from a site in California. In the last part the code was used to evaluate protective strategies against the migration of methane.

Moody, Rodwell and Ghabaee [Moody 92] is a major report which builds on a previous literature review by Ghabaee and Rodwell [Ghabaee 89], and is focused on landfill gas modelling. They assumed a constant gas generation rate, but looked at the transport of gas in the gaseous form, and dissolved into water. They included the different behaviour of different gases within and around a landfill site.

The model used a finite difference method to predict the development of concentrations of different gases over a long period of time. The predictions are compared with

simplified analytical solutions. They briefly considered changing atmospheric pressure, and saw the expected increase in gas flow out of the ground when the atmospheric pressure drops.

Movement within a landfill site

Mohsen, Farquhar and Kouwen [Mohsen 80] modelled the movement of gases in a landfill site, using a time-dependent finite element computer model. They gave a good description of the theory of the problem, and showed that the method agrees well with a simple analytical solution. The model assumed a source region with a constant concentration of landfill gases and a constant pressure gradient. Although they described the pressure field equations for soil gas flow, these were not used in the paper. The predicted results were used to compare with data from a landfill site over a number of years. They used a time-varying boundary condition to model freeze/thaw of the soil, which is unusual.

The authors presented the equations in axi-symmetric form, which is an improvement on the one-dimensional model. The resulting equations were then solved using a finite element method, which is explained in reasonable detail. However in their analysis the authors assume a flow velocity, and only solve for the diffusion of the gas concentration.

They compared the result of their finite element method solution with a solution to a simplified problem. Using only 12 domains (or elements) they obtained a fair agreement.

Next they looked at data from a landfill site and compared it to their modelled predictions of concentrations of methane. These were good considering the difficulty of the subject, generally under-predicting, but always within a factor of two.

In the last sections they looked at the effect of freezing the soil surface, installing venting trenches, and using the model as a design tool. The effect of freezing and thawing the soil surface was to increase the gas concentrations in the soil during the winter months, and then have them falling in the summer. In their case the effect was not large, but this

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could have a more significant effect in more permeable soils, or with higher gas production rates. Also because of the modelling exercise being limited to the landfill site itself, they did not address the fact that the soil under a house would not be frozen while that around it would be. This could be significant where freezing is a common effect.

Metcalfe and Farquhar [Metcalfe 87] gave results from a time-dependent finite element computer model of gas migration from a landfill site, and the corresponding field evidence to complement it. The mathematical model is presented briefly but adequately; the computer code used was taken from an existing ground water flow model.

The model included diffusion of the different components of the flow through each other, to give a concentration equation as shown below. They modelled the concentration equation as

$$\epsilon \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left(D_{xx} \frac{\partial C}{\partial x} + D_{xz} \frac{\partial C}{\partial z} \right) +$$

$$+ \frac{\partial}{\partial z} \left(D_{zx} \frac{\partial C}{\partial x} + D_{zz} \frac{\partial C}{\partial z} \right) - v_x \frac{\partial C}{\partial x} - v_z \frac{\partial C}{\partial z}$$
(2.15)

where

 D_{xx} , D_{xz} and D_{zz} are components of the hydrodynamic dispersion tensor, v_x , v_z are the velocity components calculated from the Darcy Law, C is the concentration at any point, ϵ is the porosity of the soil.

They used a finite element technique, first to solve the pressure field, and hence find the velocities, and then to calculate the concentration field from those. The code used was originally developed for ground water flow, and was adapted for this use by the authors. To run the model they assumed a fixed concentration of methane in the centre of the landfill site, which they justified from experimental data.

Atmospheric pressure changes - Landfill sites

Several papers by Alan Young, from Oxford University Mathematics Department, report on computational studies of how landfill sites respond to atmospheric pressure changes.

[Young 90] is a short paper giving an outline of the methods being used. The model had only one spatial dimension but is time-dependent, and allowed the atmospheric pressure to change. This was seen to be an important factor in the rate of exit from the landfill site. The equations were solved with the finite element method.

The equations used were the usual Darcy's Law and mass conservation, but were written in different units from those used in the other papers. He predicted the response of the system to sharp changes in atmospheric pressure.

[Young 92] is an extension of [Young 90]. Here the author looked at the differences in behaviour of the different gases in the mixture of gases that makes up landfill gas. The results predicted that the measured proportions of methane and carbon dioxide will vary with time because of atmospheric pressure fluctuations rather than changes in generation rates. Hence it is important to look at the changes in pressure over time when considering measured data, and he recommended that measurements should be taken during periods of falling atmospheric pressure.

He predicted that this variation in concentrations was a result of the different solubilities in water of the principal landfill gases, methane and carbon dioxide. If the atmospheric pressure falls then soil gas will rise up through the soil. Because carbon dioxide is more soluble than methane more of the carbon dioxide will dissolve into the water in the upper levels of soil, and the methane concentration in the soil gas which reaches the surface will be higher than for steady-state conditions.

In the reverse condition, when atmospheric pressure is rising, the soil gas is pushed down into the soil by air entering from outside. As a result, carbon dioxide dissolved in water will come out again, and the relative proportion of carbon dioxide to methane will increase. Since some of the gases still diffuse out of the soil surface, a measurement

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Radon

Narasimham et al [Narasimham 90] used a fluid-flow model developed for water flows, but adapted for gas flow. By running the model with varying time periods for the oscillation of the surface pressure they achieved different flow rates. They concluded that the effect can be significant at low permeabilities with higher frequencies. In these cases the radon entry rate can be enhanced considerably. The concept clearly deserves more study.

Another paper by the same authors [Tsang 92] extended [Narasimham 90]. Here the authors looked at the way in which changing atmospheric pressure affects radon movement, using a time-dependent finite difference model. The model was two-dimensional in space, and was used in two configurations, one a solid floor with a crack in it, the other a bare soil floor.

They were able to predict the increase in radon entry over that due to a steady-state pressure difference, for different permeabilities and atmospheric pressure fluctuations. For example, for permeability $k = 1 \times 10^{-12} \text{ m}^2$, and a 250 Pa pressure change amplitude, a time of fluctuation of 24 hours gave the largest increase in entry rate, of 120%. They also found that small fluctuation effects can increase flow above diffusion levels, even in the absence of a fixed pressure driving force, explaining the continued presence of radon in summer conditions when the stack effect (internal-external pressure difference due to temperature) is reduced.

Purely Experimental Studies

This section contains a description of a number of papers which consist almost entirely of the results from experimental work. These provide possible sources of data to use to compare with models, although the applicability of the results varies considerably.

Radon flux with atmospheric pressure changes

Kraner et al [Kraner 64] reported on the successful measurement of radon fluxes from the ground and concentrations in the soil as long ago as 1964. They included significant observations on the effect of atmospheric variables on radon. As well as the usual increased flux for falling atmospheric pressure, and the consequent changes in concentration with depth, they saw an increase in soil gas radon following heavy rain. This was thought to be due to the soil being effectively capped by the rain water.

Schery and Gaeddert [Schery 82] and Schery et al [Schery 84] gave experimental data for the variation in radon flux with atmospheric pressure. No modelling was attempted at this stage. They used a 'flux box' technique with a continuous radon monitor to measure the radon flux from the ground. They predicted an enhancement of around 10% in the flux as a result of atmospheric pressure cycling.

More recently advances in technology have lead to an increase in the number of people able to measure radon levels continuously, and there is a growing amount of data available. Examples include [Welsh 95], [Kies 95], [Genrich 95] and [Hubbard 95]. This group of papers were all presented at the NRE VI conference at Montreal in June 1995.

Behaviour of landfill sites

CH2M Hill Engineering Ltd [CH2M 89 and 90] is a detailed report on a major study of how to protect buildings against landfill gas. The reports start with a lot of data from site investigations. From these remedial actions were designed and then installed, and the success of these measures was then assessed with more measurements. The protective technique used was subfloor de-pressurisation, as in many radon-affected buildings, and this was used in several different buildings. Overall the work was found to be successful, since the methane levels near the houses were reduced considerably.

The second phase work involved further monitoring on the same site. Overall the work

gives valuable data on the development of gas concentrations and pressures in soil near a landfill site, and the effectiveness of the chosen remedial works. No modelling work was included in the study.

Hartless [Hartless 95] at BRE has been developing methods for monitoring landfill gas on sites. He covers the techniques which can be used, and many of the problems with them. The techniques considered address pressure and flow measurements, with flow measurements being the main problem. The methods available include direct ones, such as rotameters and hot wire probes, and indirect ones, of which the flux box is the most important.

Chapter 3: Time-dependent solutions to the pressure equation

Introduction

In this chapter particular problems of time-dependent flow which can be tackled analytically are considered, to see what insight they can give to problems of soil gas flow. In particular the effect of changing atmospheric pressure on gas flow is examined.

As discussed in chapter 2, some other work has been done, particularly on the pressure field equation. The main results are extended and applied in this chapter to the solutions noted below. Studying the pressure equation allows the flow rates produced by changing pressures to be predicted, but not the concentrations of the gases involved. In some situations this may be all that is needed, and a `worst case' result can always be found from assuming the maximum gas concentration applies throughout the soil.

In this chapter there are solutions given for three different problems:

- 1) Steadily changing surface pressure
- 2) A sudden jump in surface pressure
- 3) A sinusoidal variation in surface pressure

The first and third solutions are reproduced from papers by other workers, whilst the second one uses the method of the first on a new problem. In each case the significance of these to soil gas flow is discussed, and this chapter concentrates on application rather than attempting new solutions. The principle issue is how these affect the measurements made during the investigation of soil gas flows. In particular the flux from landfill gas sites is seen to vary with atmospheric pressure changes, and this affects the interpretation of any particular reading of gas flow.

The solutions for the first problem are not new. The second solution uses the same method as the first but has not been published before, to my knowledge. The third solution has been looked at in a different way by other workers. This work extends that presented in [Cripps 95-1].

The pressure equation

Starting from the paper by Fukuda [Fukuda 55] or Clements and Wilkening [Clements 74-1], also given as equation (2.1) in chapter 2, the pressure equation can be taken to be

$$\frac{\partial^2 p}{\partial z^2} = \alpha^2 \cdot \frac{\partial p}{\partial t}$$
(3.1)

where

 $\alpha^2 = \epsilon \mu / k P_0$ (sm⁻²)

 ϵ is the porosity of the soil, the proportion of free space within it (), μ is viscosity of soil gas (Pa.s),

 \mathbf{P}_{o} is the mean atmospheric pressure (Pa),

k is the permeability (m^2) .

This equation c an be derived by assuming a flow into the ground, and considering the flow into and out of an element of thickness dz. For fairly small flows the pressure does not change significantly, and the perfect gas law can be taken to apply, which all leads to equation (3.1). A derivation of it is given by Fukuda [Fukuda 55].

Equation (3.1) is the standard diffusion equation, for which much work has been done, especially in the field of heat transfer. To tackle it the boundary conditions must be specified, since they affect the method which is best suited to solving it. The choice of boundary condition defines the three solutions presented here.

Problem 1: Steady increase in surface pressure starting from time zero

In this case the initial pressure distribution is defined as zero for the whole region, and then an applied change in surface pressure starts at time t=0. The change in pressure will be taken as linear with time, since this is simplest, but also represents the effect of a moving low pressure atmospheric (cold) front reasonably well. Equation (3.1) is solved using a Laplace Transform method. This method reproduces the work of Clements and Wilkening [Clements 74-1], but extends it a little.

The Laplace Transform is defined by

$$\overline{p}(z, s) = \int_{0}^{\infty} p(z, t) \cdot e^{-st} \cdot dt$$
 (3.2)

Applying this to both sides of equation (3.1) means that the left-hand side does not change, but the right hand side simplifies to

$$\frac{\partial^2 \overline{p}}{\partial z^2} = \alpha^2 (s\overline{p} - p(0)) \quad . \tag{3.3}$$

Since p(0) = 0 this can be solved easily giving

$$\overline{p} = A(s) \cdot \exp(\alpha z \sqrt{s}) + B(s) \cdot \exp(-\alpha z \sqrt{s})$$
 (3.4)

Now [Clements 74-1] worked with conventional axes, so that as z tended to $-\infty$ the solution p is required to be finite. Hence the term B(s) must be zero or the inverse transform and the solution p will not be finite. Thus

$$\overline{p} = A(s) \cdot \exp(\alpha z \sqrt{s})$$
 (3.5)

The function A is an arbitrary function of s, which can be found from the boundary condition on z=0. There $p = p^*(t)$ so that in (3.2)

$$\overline{p}(0, s) = A(s) = \int_{0}^{\infty} p^{*}(t) \cdot e^{-st} \cdot dt$$
 (3.6)

Hence A is the Laplace Transform of p^* . The solution for p is the Inverse Laplace Transform of (3.4). To obtain p from (3.5) and (3.6) the Convolution Theorem must be used, see for example Arfken [Arfken 85], p 849, equation (15.196). It states that if the inverse of p is equal to the product of two Laplace Transforms, $A^*(s) \cdot q^*(s)$, then p can be written as

$$p(t) = \int_{0}^{t} A(t-\tau) \cdot q(\tau) \cdot d\tau \quad .$$
(3.7)

In this case A is then p*, and q is found from the inverse transform of the exponential term in (3.4). This is given in tables, for example [Abramowitz 65] p 1026 no 29.3.82. This gives the general solution as

$$p = -\frac{\alpha z}{2\sqrt{\pi}} \cdot \int_{0}^{t} p^{*}(t - \tau) \cdot \tau^{-3/2} \cdot \exp\left(-\frac{\alpha^{2} z^{2}}{4\tau}\right) d\tau \quad . \tag{3.8}$$

If $p^{*}(t)$ is defined as *at* so that the surface pressure increases linearly with time, then

(3.8) can be re-written to give the general solution ([Clements 74-1] equation 17). Note that since z is defined to always be negative it can be simpler to write z = -y. Then using

$$K = \alpha^2 y^2 / 4$$

gives (3.8) as

$$p = \frac{a\sqrt{K}}{\sqrt{\pi}} \cdot \int_{0}^{t} (t - \tau) \cdot \tau^{-3/2} \cdot \exp\left(-\frac{K}{\tau}\right) d\tau \quad . \tag{3.9}$$

Then substituting $K/u^2 = \tau$ gives

$$p = \frac{a\sqrt{K}}{\sqrt{\pi}} \cdot \int_{(K/t)^{1/2}}^{\infty} (t - \frac{K}{u^2}) \cdot \exp(-u^2) du \quad .$$
(3.10)

Integrating the second term by parts gives two terms still to be integrated with factors multiplying $\exp(-u^2)$, and one other term. The two terms cannot be integrated exactly, but are in a standard form called the complementary error function, erfc, which is given in tables. Writing those integrals as erfc gives the final pressure solution as

$$p = (at + a\alpha^2 y^2/2) \ erfc\left(\frac{\alpha y}{2\sqrt{t}}\right) - a\alpha y\sqrt{t/\pi} \ . \ \exp\left(-\frac{\alpha^2 y^2}{4t}\right) \ . \tag{3.11}$$

Then substituting back to z gives the solution as derived by Clements and Wilkening, their equation 17 [Clements 74-1].

$$p = (at + a\alpha^2 z^2/2) \operatorname{erfc}\left(\frac{-\alpha z}{2\sqrt{t}}\right) + a\alpha z\sqrt{t/\pi} \cdot \exp\left(-\frac{\alpha^2 z^2}{4t}\right) \quad . \tag{3.12}$$

An example of this is given as figure 3.1 below, where the pressure at different depths is shown as a function of time. In this case and the velocity plot later the following data were used

a = -1 (Pa/s)
k = 1e⁻¹¹ (m²⁾

$$\epsilon = 0.5$$
 ()
 $\mu = 1.83e^{-5}$ (Pa.s)

In the figure the surface pressure is falling at 1 Pascal per second, while the pressure at

different depths follows behind, with little effect at a depth of 0.7 metres, in this time scale.



Figure 3.1: Graph of pressure against time for the sloping step problem

Finally by putting this pressure result into Darcy's Law the flow velocity at any point can be found. Darcy's Law is

$$v = -\frac{k}{\mu} \cdot \nabla P \quad . \tag{3.13}$$

The differentiation of (3.12) is not trivial, initially giving 4 terms. The derivative of the erfc term is given by

$$\frac{d}{dx} \operatorname{erfc}(y) = -\frac{2}{\sqrt{\pi}} \exp\left(-y^2\right) \cdot \frac{dy}{dx} \quad . \tag{3.14}$$

However two of the resulting terms cancel out, and the others combine, giving the velocity as equation (3.15). Equation (3.15) is slightly different from [Clements 74-1] equation 18, which is probably a typographical error since it disagrees with their result for v at z=0. The expression for the velocity is

$$v = -2a\alpha \left(\frac{k}{\mu}\right) \sqrt{\left(\frac{t}{\pi}\right)} \cdot \left(\frac{1}{2}\alpha z \frac{\sqrt{\pi}}{\sqrt{t}} \operatorname{erfc}\left(\frac{-\alpha z}{2\sqrt{t}}\right) + \exp\left(-\frac{\alpha^2 z^2}{4t}\right)\right) \quad . \tag{3.15}$$

At z=0, and after substituting for α this simplifies and agrees with Clements and Wilkening [Clements 74-1] equation 19.

$$v = -2a \sqrt{\frac{\epsilon k}{\mu P_0 \pi}} \cdot \sqrt{t} \quad . \tag{3.16}$$

Figure 3.2 is a plot of equation (3.15), showing the development of the velocity with time, and hence with the changing atmospheric pressure. Because the surface pressure is falling, the velocity is upwards out of the ground. The highest value occurs at the surface, but the flow starts to be noticeable at a depth of about 1 metre after 10 seconds.



Figure 3.2: Graph of velocity against time for the sloping step problem

From this expression we can calculate a rate of flux out of a unit area of ground, if we know the appropriate values of the parameters a, ϵ and k. The fact that the rate is dependent on the square root of the time t, and also the permeability k is significant. It reflects the fact that the pressure in the soil builds up gradually with time, and faster for higher permeability soils. This increase in the sub-surface pressure then restricts the rate of increase in velocity at the surface.

Examples

An active landfill site might develop an overpressure of 100 Pa within it, which would generate a steady-state flow through the surface layer. Suppose that this layer has the following properties:

permeability	$k = 1e^{-12} (m^2)$
depth	L = 1 (m)
soil gas viscosity	$\mu = 1.83e^{-5}$.

Then the background flow velocity can be estimated from Darcy's Law (3.13) as

 $v = 1e^{-12} * 100 / 1.83e^{-5} = 5.5e^{-6} m/s.$

This gives a flux rate per square metre of 0.002 m³/h or 330 cc/min.

Now consider the effect of falling atmospheric pressure. If a low pressure front passes the landfill site, the pressure could fall of the order of 50 mBar over 5 hours. This is the same as 1000 Pa per hour, or 0.28Pa/s. Using this in equation (3.16) for the surface velocity gives a flow velocity of

$$v = 2 * 0.28 * (2.95e^{-7}) * t^{\frac{1}{2}}$$

Hence after 1 hour the surface velocity would have reached 9.9e⁻⁶ m/s, while by 5 hours it would be 2.2e⁻⁵ m/s. These are a factor of 2 and 4 larger than the expected steady-state velocity, although there are a large number of approximations contained in the calculation. However the main result is that the effect of the changing atmospheric pressure can easily be of the same order as typical steady-state flows. This means that the flux rate measured at this time will be significantly different from the long term average.

Reversing the change in pressure, ie rising pressure, means that the flow from the ground will be significantly reduced from the average level. This is probably the worst condition for making a measurement, because the level measured will be significantly lower than the 'real' values, and a site might be considered 'non-gassing' when it is.

This type of analysis has been developed by Young [Young 90] who used a numerical model to predict the flow of gas out of the ground for varying pressures. He also considered the mixture of gases in the ground, and their varying solubilities. In particular because Carbon Dioxide, CO_2 , is soluble in water the proportion of methane in soil gas

will rise because of CO_2 being dissolved into water within the soil. This will have its strongest effect when atmospheric pressure is falling, so that gas is rising up through the soil and CO_2 is dissolved into water with low CO_2 levels.

The main conclusion is that the atmospheric pressure can have a significant impact on gas flows, and must be taken into account when taking readings of flux rates and gas concentrations.

Problem 2: Instant change in pressure

If instead of the gradual increase in pressure of the above example the pressure is taken to jump suddenly to some fixed surface pressure at time zero, the same method can produce a result. It gives a simple one-dimensional solution to the problem of how a fan system affects the soil below it when it is turned on. The analysis begins at equation (3.8) above.

The pressure is defined to be zero everywhere up until time t=0. At that moment the surface pressure is then assumed to jump instantly to P_T and our interest is in the way in which the soil below responds to the sudden change. In equation (3.8) the function $p^*(t)$ is simply the constant P_T so that the solution is just

$$p = -\frac{\alpha z}{2\sqrt{\pi}} \cdot P_T \int_0^t \tau^{-3/2} \cdot \exp\left(-\frac{\alpha^2 z^2}{4\tau}\right) d\tau \quad . \tag{3.17}$$

Making the same substitutions as before, y = -z, $K = \alpha^2 y^2/4$ and $\tau = K/u^2$ makes the integration the standard erfc form again. Terms cancel leaving the result just as

$$p = P_T \cdot \operatorname{erfc}\left(-\frac{\alpha z}{2\sqrt{t}}\right) \quad . \tag{3.18}$$

This development in pressure is plotted in figure 3.3. It shows the initial surface pressure being maintained, and the pressures at greater depths gradually rising towards the surface pressure.



Figure 3.3: Graph of pressure against time for the instant pressure problem

The flow velocity can also be calculated easily from this result, using equation (3.14) for the differential of the erfc term. This gives the result for the velocity as

$$\nu = P_T \sqrt{\frac{\epsilon k}{\mu P_0 \pi}} \cdot \frac{1}{\sqrt{t}} \cdot \exp\left(-\frac{\alpha^2 z^2}{4t}\right) \quad . \tag{3.19}$$

The similarity with the earlier result is clear from the constant terms. The terms in t and z show the expected behaviour. The flow rate is initially small apart from at the surface itself, then builds up in the lower regions of the soil, before reducing again as the whole soil region reaches the same pressure P_{T} . This is shown in figure 3.4, where for small time the velocities at depth are increasing, before falling away as the effect of the pressure pulse dies out.



Figure 3.4: Graph of velocity against time for the instant pressure problem

It is also useful to find the maxima and minima of this result. By differentiating (3.19) with respect to t the minimum velocity is found to be as t tends to infinity, as expected. The maximum velocity occurs when

$$t = \frac{\epsilon \mu}{kP_0} \cdot \frac{z^2}{2} \quad . \tag{3.20}$$

This gives insight into the time which it takes for the signal from a pressure change to travel into the soil. Of course it would not occur exactly like this in a multi-dimensional situation, so the result has only limited application, but it does give a useful time constant for soil flow rates.

Given the following typical values of these parameters, the typical time scales for different permeabilities can be estimated.

Porosity ϵ	0.5 ()
Viscosity µ	1.83 e ⁻⁵ (Pa.s)
Atmospheric pressure P _o	1 e ⁺⁵ (Pa)

$$t = \frac{4.6e - 11}{k} \cdot z^2 \quad . \tag{3.21}$$

Hence at a depth of 1 metre, the maximum velocity for different soil types can be estimated to occur as shown in table 3.1 below. This shows the considerable variation in time scales associated with the different types of soil. In the case of the clay it would only require a depth of 4 metres before the maximum velocity would take a whole day to occur, while the high permeability fills respond almost instantly to the imposed pressure change. This is significant because a similar process must occur in the real threedimensional situation, and this must be considered when using a pump at the top of a bore hole to extract gas from a site for measurement. The gas reaching the surface will not be representative of the deep soil gas for quite some time after turning on a pump.

Soil type	Typical soil permeability m ²	Characteristic Time
Clay	1e-14	1.3 hours
Sand	1e-12	46 seconds
Gravel	1e-10	0.46 seconds
Special graded fill	1e-8	0.0046 seconds

Table 3.1: Time scales for velocity at 1 metre to reach its maximum for different soil types

In these cases the characteristic times are inversely proportional to the permeability of the soil.

Problem 3: Cyclical (sinusoidal) variations in pressure

Another reasonable assumption is to consider the change in atmospheric pressure to be sinusoidal. Actual weather systems do give a fair approximation to this, with the change from high to low pressure and back again occurring typically with a time scale of around five days. In addition wind effects can be represented by the superposition of different sinusoidal changes in pressure with time scales of order seconds, and this was considered by Kimball and Lemon [Kimball 71]. Some work by Hintenlang and Al-Ahmady

[Hintenlang 93] suggests that atmospheric tides, with a 12 hour cycle like ocean tides also affect soil gas flow.

Hence we see that there is the possibility of atmospheric or surface pressure cycling over a whole range of time scales, making a general result of considerable value.

Although we wish to represent a sinusoidal variation, it is most easily done in the complex exponential form. We assume that the surface pressure varies as $p(t) = e^{i\omega t}$, then we expect a solution in the form $P = P_0 + Q(z) \cdot e^{i\omega t}$. In equation (3.1) this gives for Q

$$e^{i\omega t} \cdot \frac{\partial^2 Q}{\partial z^2} = \alpha^2 \cdot i\omega \cdot e^{i\omega t} \cdot Q(z)$$
 (3.22)

This gives the general solution for Q as

$$Q = A.\exp(\alpha z \sqrt{i\omega}) + B.\exp(-\alpha z \sqrt{i\omega}) \quad . \tag{3.23}$$

But the root of i is $(1+i)/\sqrt{2}$, and the solution for Q must be finite as z tends to infinity, so that A must be 0. This gives Q, and hence P, as:

$$P = P_0 + P_1 \cdot e^{i\omega t} \cdot \exp(-\sqrt{\frac{\omega}{2}}\alpha z) \cdot \exp(-i\sqrt{\frac{\omega}{2}}\alpha z) = P_0 + P_1 \cdot \exp(-\sqrt{\frac{\omega}{2}}\alpha z) \cdot \cos(\omega t - \sqrt{\frac{\omega}{2}}\alpha z) , \qquad (3.24)$$

where P_t is the amplitude of the surface pressure fluctuation. Generally the complex exponential would be replaced by $\cos(\omega t - \alpha z \sqrt{(\omega/2)})$, ie the real part of the result. This result reproduces that of Fukuda. It can then be differentiated to find the velocity at any point from Darcy's Law.

$$\frac{\partial P}{\partial z} = P_t \cdot -(1+i)\alpha \sqrt{\frac{\omega}{2}} \cdot \exp(-\sqrt{\frac{\omega}{2}\alpha z}) \cdot e^{i\omega t} \cdot \exp(-i\sqrt{\frac{\omega}{2}\alpha z}) \quad .$$
(3.25)

Taking the real part of this and using it in Darcy's Law, and substituting C for $\alpha\sqrt{(\omega/2)}$, and then taking the real part of the expanded expression (3.25) gives

$$V = \frac{kCP_{t} e^{-Cz}}{\mu} \cdot [\cos(\omega t - Cz) - \sin(\omega t - Cz)] . \qquad (3.26)$$

When z=0 this simplifies to

$$V(0) = \frac{kCP_t}{\mu} \cdot \left[\cos\omega t - \sin\omega t\right] \quad . \tag{3.27}$$

But C = $\alpha \sqrt{(\omega/2)}$, and $\alpha^2 = \epsilon \mu/kP_o$ so that C = $\sqrt{(\omega \epsilon \mu / 2kP_o)}$ and the permeability k and viscosity μ appear in the full expression as root terms, equation (3.27) becoming

$$V(0) = P_t \cdot \sqrt{\frac{\epsilon k\omega}{2\mu P_0}} \cdot [\cos\omega t - \sin\omega t] \quad . \tag{3.28}$$

This result means that the permeability has a square root impact, ie less than linear, on the velocity of soil gas flow from the soil surface. This is of essentially the same form as that for the linear pressure rise considered earlier. The finding that the permeability has only a square root impact on the velocity in equation (3.28) is interesting, since it is not the same as in the Darcy Law (3.13). It reflects the fact that the soil gas pressure follows the surface pressure, and more permeable soils follow more closely than less permeable ones.

The pressure field decays with depth exponentially, with the decay constant as C. Hence, as expected, the pressure field extends further for larger permeabilities. It is also predicting that the pressure extends further for lower frequencies of atmospheric pressure fluctuation.

Examples

As with the other two problems the analytical result to this problem lends itself to a visual representation of the pressure developing with time and depth. Figure 3.5 shows the pressure against time at different depths. The pressure at any depth lags behind that at the surface, and is damped so that its absolute value is lower. Figure 3.6 shows the velocity developing with time with the same parameters as Figure 3.5.



Figure 3.5: Graph of pressure against time for the sinusoidal pressure problem



Figure 3.6: Graph of velocity against time for the sinusoidal pressure problem

An interesting feature of the plots against time is that there is generally a depth in the soil where the pressure is in the opposite phase to that at the surface. This is because of the time lag for the changing pressure signal to reach the lower levels of the soil. The radon modelling work by Hintenlang and Al-Ahmady in Florida [Hintenlang 93] is based on this concept. The atmospheric pressure fluctuations they took to occur due to Atmospheric Tides, with a time cycle of around 12 hours. The fact that the inside of the basement of a house was in step with the external pressure then allowed radon-laden air to enter from the soil next to it, which is at a higher pressure.

To examine how significant this effect can be, substitute for α in (3.24) and look at the phase effects on the pressure:

$$P = P_0 + P_t \cdot \exp\left(-z\sqrt{\frac{\omega\,\epsilon\mu}{2kP_o}}\right) \cdot \cos\left(\omega\,t - z\sqrt{\frac{\omega\,\epsilon\mu}{2kP_o}}\right) \quad . \tag{3.29}$$

The exponential term indicates the damping effect of the soil on the overall size of the pressure, and so has no impact on the phase of the pressure wave. However if ω is large and k small then the wave will be damped fairly quickly.

The expression in the cosine term defines the phase of the pressure at any point. If the second part, which includes the z term, is equal to π then the pressure at that depth z will have the opposite phase to that at the surface. Using the same values for the parameters in the expression that were used before:

Porosity ϵ	0.5 (),
Viscosity µ	1.83 e-5 (Pa.s)
Atmospheric pressure P _o	1 e+5 (Pa),

the expression for the first depth, $z_{opposite}$ where the soil pressure is first of opposite sign to the surface pressure is

$$z_{opposite} = 4.64e5 \ (k/\omega)^{\frac{1}{2}} = 1.85e5 \ (k\tau)^{\frac{1}{2}} , \qquad (3.30)$$

where

 ω defines the rate of change in surface pressure, and is given by $2\pi / \tau$,

 τ is the time for one complete cycle.

Thus to find a typical value of the depth at which the pressure is first in the opposite phase we need to choose an appropriate value for ω . Wind gustiness can be considered to be a change with time scales of order 10 seconds, while the atmospheric tides have a cycle of 12 hours. These give values for ω of 0.63 and 1.45x10⁻⁴ s⁻¹ respectively.

Substituting in (3.30) gives simple expressions for the depth value as a function of the permeability as follows

 $10 \ second \qquad 12 \ hour$ $z_{opposite} = 5.8x10^5 \ k^{\frac{1}{2}} \qquad 3.8x10^7 \ k^{\frac{1}{2}} \ .$ (3.31)

Hence if we are most interested in a depth of 1 metre, the permeability at which it is first exactly out of phase is

$$10 \ second \qquad 12 \ hour \\ k_{opposite} = 3x10^{-12} \qquad 7x10^{-16} \ . \tag{3.32}$$

These results only indicate a feature of a soil type but help to explain the atmospheric pumping effect over a half day time scale, which can result in increased entry rates. When the atmospheric pressure is lower than its mean value, flow could occur into a basement. The expression here for phase would only matter when greater accuracy was needed, or in very permeable soil.

Conclusions to chapter 3

The results in this chapter show how soil gas pressure responds to atmospheric pressure changes, and allows us to understand some aspects of the gas flow produced by these atmospheric driving forces.

If there is a steady change in atmospheric pressure, due to a passing weather front, it can produce a flow as large as that due to the pressure which has built up in a landfill site. As a result the weather conditions need to be taken into account when considering the measurements taken on site.

When a pressure is switched on instantly, for example with a fan, then there is a time lag before the effect reaches soil at greater depth. This would be seen on site with a delay before the concentration of gas changed significantly.

When a sinusoidal variation of pressure is assumed, different depths in the soil will be at different phases of the cycle. This could help to cause the entry of soil gas into houses, because the soil at depth is out of phase with that in the house.

Chapter 4: Time dependent solution to the radon concentration equation

Introduction

In this chapter the radon concentration equation is studied, and steady-state and timevarying solutions to it are produced and their implications considered. These solutions involve using two analytical techniques, Matched Asymptotic Expansions and Laplace Transforms, and a simple numerical model. The Expansion results are not easily obtained, and serve mainly to validate the numerical predictions. The Laplace Transform Solution gives a useful general result for the flux of radon from the soil surface. The numerical solution allows any general problem to be solved, and has been seen to compare well with the exact solutions found analytically. Considerable knowledge of the time it takes for pressure changes to result in concentration changes comes from analysing these results.

The solution for the pressure field problem from the previous chapter is used here again as a necessary part of solving the radon equation. Clements and Wilkening [Clements 74-1] wrote the one-dimensional transport equation (2.12) for radon as

$$\frac{\partial C}{\partial t} = \frac{D}{\epsilon} \frac{\partial^2 C}{\partial z^2} - \frac{1}{\epsilon} \frac{\partial (\nu C)}{\partial z} - \lambda C + \phi$$
(4.1)

where

C is concentration of radon $(mol m^{-3})$,

D is diffusion coefficient of radon in soil (m^2s^{-1}) ,

 ϵ is the soil porosity (),

v is the velocity of the soil gas, or fluid volume current density (ms⁻¹), defined as positive upwards,

 λ is the decay constant of radon (s⁻¹),

 ϕ is the radon production rate (mol m⁻³ s⁻¹),

z is the spatial direction (m) and

t is time (s).

In this form both the diffusion of gas and the pressure-driven flow are accounted for. It is much simpler to solve this in steady-state, and the results are useful when considering time varying problems, so the steady-state solutions are presented first.

Steady-state Solutions

Assuming that the surface has zero concentration, and that the soil is of infinite depth, the steady-state solution of (4.1) when the velocity v = 0 can be calculated as

$$C = \frac{\Phi}{\lambda} \cdot \left(1 - \exp\left(z \cdot \sqrt{\frac{\epsilon \cdot \lambda}{D}}\right) \right) \quad . \tag{4.2}$$

Note that z has to be negative or this solution diverges. Here the decay term λ cannot be set to zero since this gives the correct but not helpful result of infinite concentration if there is production and no decay. If we impose a constant velocity onto the system, but still allow it to reach steady-state then the result is a little more complex. If the imposed velocity is V_o then equation (4.1) becomes

$$0 = \frac{D}{\epsilon} \frac{\partial^2 C}{\partial z^2} - \frac{V_o}{\epsilon} \frac{\partial C}{\partial z} - \lambda C + \Phi \quad .$$
(4.3)

Solving first for the equation without the constant ϕ by assuming a solution of the form $C = De^{fz} + Ee^{-gz}$

gives a quadratic equation in f, since E must be zero to satisfy the condition as z tends to $-\infty$. Combining with a constant term to match the condition at z=0 this gives the result for the concentration to be

$$C = \frac{\Phi}{\lambda} \cdot \left(1 - \exp z \cdot \left(\frac{V_o + \sqrt{V_o^2 + 4D\epsilon\lambda}}{2D} \right) \right) \quad . \tag{4.4}$$

This result simplifies to that of (4.2) when $V_o = 0$. It is also interesting to look at the result when the velocity is large compared with the diffusion term D, so that for V_o positive (4.4) can be simplified to

$$C = \frac{\Phi}{\lambda} \cdot \left(1 - \exp \frac{zV_o}{D}\right) \quad . \tag{4.5}$$

Hence if the velocity is large and positive (i.e. towards the soil surface) then the exponential term tends to zero, and nearly all of the soil has the same concentration as the deep soil. This assumes that there is no depletion of the radon or other gas being created, which may not be the case.

If the velocity is negative (i.e. into the soil) then (4.4) simplifies differently for large velocity, and needs a binomial expansion of the root term, which gives

$$C = \frac{\Phi}{\lambda} \cdot \left(1 - \exp \frac{z \epsilon \lambda}{|V_o|} \right) \quad . \tag{4.6}$$

Because z is always negative, and V_0 taken to be large, this gives zero except when $z \in \lambda$ becomes large enough to match V_0 . The results for the radon concentration at different applied velocities in steady-state conditions are shown in figure 4.1. It shows the zero velocity result, with the lines either side for positive and negative velocities. When the flow is down into the ground the radon concentration is lower at all depths. As well as the exact result from equation (4.4) the approximate results from equation (4.5) and (4.6) are also shown in figure 4.2. This shows that a velocity of 10^{-5} (m/s) is large for a soil gas flow, because the exact and approximate results are so close together.



Figure 4.1: Graph of radon concentration against depth, exact results.



Figure 4.2: Graph of radon concentration against depth, showing approximations

Non steady-state problems

As soon as the velocity within the concentration equation (4.1) is not a constant the solution becomes a lot more difficult, and an exact solution is unlikely to possible. However some progress can be possible, after making approximations.

In this part of the chapter the same general type of problem has been tackled in a number of different ways. First the limit of analytical results is explored, taking an approximation to the full problem for small time. This method is applied to two different flow situations, a gradually increasing surface pressure, and a suddenly applied velocity.

Next the development of concentration is considered with a Laplace Transform method, which can give an exact solution under certain conditions.

The final solution type considered here is numerical, because the governing equations can be solved to some degree of accuracy for any situation with a numerical method. It has been used to compare to the earlier analytical results, and to the general cases which the analytical methods cannot solve. The comparisons are generally good, and show that the numerical method is valid, if slower to use, than an exact solution.

Linear increase in surface pressure

We first consider the problem where the pressure is gradually increased at the surface, which was discussed earlier for the pressure alone. This can result from the changing atmospheric pressure, perhaps as the result of a cold front passing the point. This leads to a fall in pressure of about 50 mBar in a period of hours, so a maximum of around 10 mBar per hour. This is equivalent to 1000 Pa per hour or 2.8 Pa per second.

The velocity of the gas varies according to equation (3.15), as found in chapter 3. This shows that the velocity is initially very small, and the effect will take some time to reach the deeper parts of the soil. Hence it is interesting to look separately at the two zones, one near the surface where the effect of the pressure change will have reached, and the other deeper in the soil, where no change can occur for some time. The solution for the two regions must join together in the central region, but the solution will be hardest to obtain there.

This problem can be tackled using the method of matched asymptotic expansions. The workings are rather long, so they are presented in Appendix A. The result is reproduced here. An approximation to the solution for small time and $z = O(t^{1/2})$ is assumed to be in the form of an expansion in powers of time T (T = t) and functions of the variable, ζ , equal to the depth z over the square root of the time, so $\zeta = z / t^{1/2}$:

$$C = C_0(\zeta) \cdot T^{\frac{1}{2}} + C_1(\zeta) \cdot T + C_2(\zeta) \cdot T^{\frac{3}{2}}$$
(4.7)

After separating the terms in the radon equation by powers of T each term C_0 , C_1 and C_2 can be found in turn

$$C_0 \simeq \frac{\varphi \zeta p}{\lambda}$$
, (4.8)

$$C_1 = \frac{-\Phi}{2A} \zeta^2 = \frac{-\Phi}{2D/\epsilon} \frac{z^2}{T} , \qquad (4.9)$$

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$$C_{2} = B_{1}\left(\frac{\zeta}{2} + \frac{\zeta^{3}}{12A}\right) \cdot \sqrt{\pi A} erf\left(\frac{-\zeta}{2\sqrt{A}}\right) + \frac{B_{1}}{12}(2\zeta^{2} + 8A)e^{\frac{-\zeta^{2}}{4A}} - B_{2}\varphi(\frac{\zeta^{3}}{6A} + \zeta) + \frac{\gamma f'(0)}{\epsilon}\left[-\frac{1}{2}\exp\left(-\frac{\alpha^{2}\zeta^{2}}{4}\right) - \alpha\zeta\sqrt{\pi} erf\left(\frac{-\alpha\zeta}{2}\right)\right] + \frac{\zeta^{3}}{6}\frac{\epsilon}{D}f'(0)\left[\lambda - \frac{\gamma\alpha\sqrt{\pi}}{\epsilon}\right].$$
(4.10)

where

 B_1 and B_2 are found from the boundary conditions, and are given in Appendix A, $A = D/\epsilon$, $p = (\epsilon \lambda/D)^{\frac{1}{2}}$, f is the solution to the steady-state solution, ie equation (4.2),

 $\gamma = 2a\alpha k/(\mu \pi^{4})$ from equation (3.15) for the velocity of flow written as v =

$$\gamma t^n g(\zeta),$$

$$\alpha = \epsilon \mu / k P_0$$

and the other variables are as defined before.

Application and interpretation of the solution

In principle it would be possible to calculate values of C resulting from this analysis using a spreadsheet, and approximations to the erf terms. However some of the terms in the expression for C2 give problems due to their extremely small size, so it was better to write a short QuickBASIC program to sum the terms. Some results are shown in figures 4.3 and 4.4.



Figure 4.3: Graph of radon concentration against depth



Figure 4.4: Graph of radon concentration against depth, greater depth

These graphs show that the boundary conditions have been met, but also that the range of application of the result is very limited. In figure 4.3 the development with time can be clearly seen, with air moving upwards carrying higher concentrations of radon up with it. That the solution matches at large ζ rather than diverging is shown in figure 4.4, which extends the result of 3 to larger z. It is a limitation of the method that for z more than about 0.6 the approximation to the initial condition as an expansion in powers of z is no longer valid. However for this region the initial condition will be the solution for small time, as the effect of the change at the surface will not have reached that depth.

Comparing the sizes of the respective parts of the solution gives an indication of the range of application of the solution. For small values of the time, the terms in the expansion (4.7) differ by an order of magnitude. However as the time progresses the third term increases in size, so that it becomes equal in order to the second and then the first. At this point the assumption in the method is not valid, and it cannot be applied. Hence the solution can only be used for the initial period of time, while the third term component is small. In the case shown in figure 4.5 below, beyond about 200 seconds the result is not valid, and since the rate of increase in pressure was 0.1 Pa/s this implies the solution would not be valid for a change in pressure of more than 20 Pa.





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It is not easy to apply the result of this section directly. However later in this chapter it is used to verify the small time accuracy of a numerical solution which can be extended to much larger values of time. The two agree well.

Instant velocity problem

Although the problem is not strictly physically realistic, the application of an instant velocity V_0 at time zero is not too far from the reality of some situations. When a pressure is imposed the velocity will often reach steady-state quite quickly, while the concentration takes much longer to adjust, because it requires the bulk movement of gas, not just a pressure wave to transmit it.

There are two methods to attack this problem, the method used in Appendix A for small time, and the Laplace Transform method used for the pressure problem earlier. It can also be tackled numerically, and this is covered later.

Matched asymptotic expansions

The main part of this approach is reproduced as Appendix B. Because the velocity is constant with time the procedure is simpler than for the earlier case in Appendix A. The result only needs to use two terms to involve the velocity, and is therefore simpler to use. It is limited in application, however, because the expansion of the initial condition into powers of z is poorer in two terms than in three. This further reduces the length scale over which the solution is valid. The result of this calculation can be seen in figure 4.6, where the result is plotted against the numerical result discussed later.

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Figure 4.6 Graph of radon concentration against depth

For small values of z and t the concentration results are very close to each other, but as z increases the analytical method tends to give higher concentrations than the numerical calculation. This can be seen in the way the solid lines separate from the points as the depth increases. This is because the solution given is the inner solution, which matches to the steady-state solution at greater depth. It is not able to give the solution at larger depths or times. As a result of this the analytical solution is mainly of use to check that the numerical solution is giving the correct initial solution.

Laplace Transform method

An alternative to the matched expansions technique for this problem is to use a Laplace Transform to solve it. Laplace Transforms are a powerful technique for solving differential equations, particularly for those where an initial condition is given, and the time development from that is needed. The standard work on this topic relates to heat conduction [Carslaw 59], and has been used in work at the laboratories of KVI in the Netherlands for radon [Van der Spoel 93]. Again the working of this is quite complicated, so it is presented in Appendix C. A summary of results are given below. It is not possible to evaluate the concentration without doing a numerical integration of the following expression

$$C = \frac{K_1(z)}{\pi} \int_p^{\infty} \frac{\sin\left(K_2(z).(u-p)^{0.5}\right) e^{-ut} du}{u(\lambda - u)} + \frac{\Phi}{\lambda} \left[1 - \exp\left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon\lambda}{D}}\right)\right] - \frac{\Phi}{\lambda} \left[1 - \exp\left(\frac{zV_0}{D}\right)\right] e^{-\lambda t} , \qquad (4.11)$$

where

 K_1 and K_2 are functions of z defined in Appendix C,

$$p = V_0^2/4D\epsilon + \lambda$$

and the other variables and constants are as defined earlier.

However more progress is possible on the flux rate, which comes from the gradient of the concentration. The total flux at any point is a combination of diffusion and pressure driven flow. This can be written for a general flow velocity V as

$$Flux J = -D \cdot \frac{\partial C}{\partial z} + V \cdot C \quad . \tag{4.12}$$

However we are most concerned with the flux at the surface, which is simpler because we generally define the surface concentration as 0, so only the diffusive flux is needed. The value of $\partial C/\partial z$ at z=0 can also be evaluated as (Appendix C)

$$\frac{\partial C}{\partial z} = \left(\frac{\epsilon}{D}\right)^{0.5} \frac{\Phi}{\lambda \pi} \left[\sqrt{\frac{\pi}{t}} \cdot e^{-\left(\frac{V_0^2}{4D\epsilon} + \lambda\right)t} - \pi \sqrt{\left(\frac{V_0^2}{4D\epsilon} + \lambda\right)} \cdot \left(1 - erf\left(\sqrt{t\left(\frac{V_0^2}{4D\epsilon} + \lambda\right)}\right)\right) \right] - \left(\frac{\epsilon}{D}\right)^{0.5} \frac{\Phi}{\lambda \pi} e^{-\lambda t} \left[\sqrt{\frac{\pi}{t}} e^{\frac{-V_0^2 t}{4D\epsilon}} - \pi \sqrt{\frac{V_0^2}{4D\epsilon}} \cdot \left(1 - erf\left(\sqrt{\frac{V_0^2}{4D\epsilon}t}\right)\right) \right] + \frac{\Phi}{\lambda} \left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon\lambda}{D}} - \frac{V_0}{D} e^{-\lambda t}\right) .$$
(4.13)

From (4.13) we observe that, at time t = 0, the gradient is zero as all the terms cancel, as

expected. As t tends to infinity, only terms from the third line remain as the error function terms tend to one and therefore disappear, as do all of the exponential terms. These then match the steady-state solution given by differentiating (4.4) and setting z=0.

Applying this solution

This solution allows the calculation of the development of radon concentration with time, using a numerical integration, but more usefully the development of the radon flux at the surface can be found. Figure 4.7 shows the radon concentration gradient at the surface developing with time. In this case it takes a considerable time to reach the steady-state value, but it is important to note that there is zero initial radon in the ground, so much of the time is spent building up the radon level in the soil.



Figure 4.7: Graph of the gradient of the radon concentration against time

This type of solution will be more useful for methane (discussed in chapter 5) where BRE have a test cell where the knowledge of the time taken to reach near steady-state will be of value. The KVI work [Van der Spoel 93] relates better to this radon solution, but they also had a finite length scale, which this does not.

Numerical solution

In general a problem cannot always be solved analytically, and as we have seen, even simplified problems are hard to solve, and limited in their range. As a result it is often necessary to tackle the problem with a numerical or computational method. The easiest way is to use an *explicit* method to solve the concentration equation.

The equation to solve is (4.1) again, and to make the computer solution possible the space is divided up into a grid, with steps dz and time is advanced in steps dt. Then each of the differential terms is approximated from the values of the concentration at neighbouring grid points as follows

$$\frac{\partial C}{\partial t} = \frac{C_{r+1} - C_r}{t}$$
(4.14)

where C_{r+1} , C_r are the concentrations at time step r+1 and r respectively

For the z differential there is a choice to be made between the three possible ways to discretise the equation, the choice depending on the direction of any flow in the system. In order they are known as upwind, downwind and central differences.

$$\frac{\partial C}{\partial z} = \frac{C_{n+1} - C_n}{dz} \text{ or } \frac{C_n - C_{n-1}}{dz} \text{ or } \frac{C_{n+1} - C_{n-1}}{2.dz}$$
(4.15)

where C_{n+1} , C_n , C_{n-1} are the concentrations at grid positions n+1, n and n-1 respectively. For the second differential the first two forms for dC/dz are combined to give

$$\frac{\partial^2 C}{\partial z^2} = \frac{C_{n+1} + C_{n-1} - 2C_n}{(dz)^2} \quad . \tag{4.16}$$

These can then be combined into equation (4.1) to give the expression for the concentration at time step r+1 from the concentrations at time step r at the point n itself and the two neighbouring points. This defines an explicit method, where the calculation can be carried out in one step, because it does not involve the concentrations at the new time step of the values at neighbouring points. Methods which do use that information are called implicit, are more stable (ie not prone to diverge and give completely wrong answers), but are harder to use.

Constant velocities

As a first step, setting the velocity equal to zero allows the method to be checked when there is an exact solution to the steady-state result - equation (4.4). The full expression from combining equations (4.14), (4.15) and (4.16) into (4.1) is

$$C_{r+1} = C_r + dt \left[\left(\frac{D}{\epsilon} \frac{(C_{n+1} + C_{n-1} - 2C_n)}{(dz)^2} \right) - \frac{v}{\epsilon} \left(\frac{C_{n+1} - C_n}{dz} \right) - \lambda C_n + \phi \right] .$$
(4.17)

The first test of this was with no velocity, and C=0 everywhere initially, and the second with different values of v. Both compare well with the exact results, and the numerical result also indicates how long it takes for the result to reach the steady-state solution. In both cases it is necessary to have a large enough maximum z value for the numerical solution, or an error is introduced.

The boundary conditions used in the z direction were C=0 on z=0, and $\partial C/\partial z = 0$ at the bottom boundary of the region. This is why it had to be set deep enough to match the exact solution. The exact solution with a finite boundary condition can be found but is a little more complicated.

Time varying velocities

Because we have the exact solutions for velocity developing as the surface pressure develops under some conditions we can insert the velocity into the computational solution without the need to calculate it numerically. This concentrates on the rising surface pressure problem, as it is the case where most progress was possible analytically.

The velocity comes from equation (3.15) of chapter 3 and needs to be differentiated with respect to z to give $\partial V/\partial z$. These are then used in the modified concentration equation as

$$C_{r+1} = C_r + dt \left[\left(\frac{D}{\epsilon} \frac{(C_{n+1} + C_{n-1} - 2C_n)}{(dz)^2} \right) - \frac{V(z,t)}{\epsilon} \left(\frac{C_{n+1} - C_n}{dz} \right) - \frac{C_n}{\epsilon} \frac{\partial V(z,t)}{\partial z} - \lambda C_n + \phi \right].$$
(4.18)

These are then used to calculate the velocity and gradient of velocity at each time and depth in turn. A result for the linear increase in surface pressure was shown in figure 4.3.

This plot indicates that there is very good correlation between the analytical and numerical solutions in this case. The analytical solution found in Appendix A starts to become too high for larger z, because the approximations made in obtaining it depend on the expansion of an exponential in powers of z. The analytical solution is only really valid close to z and t=0.

This can also be seen from figure 4.6 earlier which is for the case where a velocity is switched on at time t=0 and maintained after that. This solution has been given in Appendix B as an analytical solution, using only two terms. This results in the two solutions diverging earlier on, because the two term expansion is not as good as the three term one.

Conclusions to chapter 4

In this chapter several different problems of the time varying radon equation have been tackled, using a mixture of different methods.

An analytical method (matched asymptotic expansions) has given a solution for small time to the problem of a linear increase in surface pressure, and an instantly applied velocity. The same problems have been addressed with the Laplace Transform method, which gives an exact definition of the result, but because this involves an integral it is not easy to use. Then all the solutions have been compared with that from a one-dimensional numerical model, which produced results very close to the analytical methods.

These results show that there is a considerable time between a change to the velocity of flow of soil gas and a significant change in the radon concentration. This means that care must be taken when collecting data on concentrations as to what the flow conditions are at the time of any measurement.

Depending on the conditions there will be either an analytical or numerical solution available for the radon concentration. Comparison of the modelled solutions is discussed in chapter 6.

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discussed in the third part of the chapter.

The conditions modelled here correspond most closely to the soil cell set up in the laboratory at BRE. This allows us to make a direct comparison between the measured and predicted behaviour, and improves our understanding of the results from the soil cell, an outline description of which is shown in the figure below.



Figure 5.1: Diagram of soil cell

Steady-state solutions

In steady-state, with a constant velocity V_0 , the concentration equation simplifies to

$$0 = D \cdot \frac{\partial^2 C}{\partial z^2} - V_0 \cdot \frac{\partial C}{\partial z} \quad .$$
 (5.3)

For the controlled experiment in the soil cell, the boundary conditions can be defined by

$$z = 0$$
 $C = C_0$
 $z = a$ $C = C_3$

This corresponds reasonably well to the arrangement in the laboratory soil cell, once it has reached steady-state. Then in the simplest case, $V_0=0$, the concentration is given by

$$C(z) = C_0 + (C_a - C_0) \cdot \frac{z}{a}$$
(5.4)

and the concentration gradient, which is needed to find the flux at any point is

$$\frac{\partial C}{\partial z} = \frac{C_a - C_0}{a} \quad . \tag{5.5}$$

If the velocity is non zero, the solution is slightly more complex, but the solution is of the form $A + Be^{pz}$, and after applying the boundary conditions it gives

$$C(z) = \frac{C_a - C_0 e^{\frac{V_0 a}{D}} + (C_0 - C_a) e^{\frac{V_0 z}{D}}}{1 - e^{\frac{V_0 a}{D}}}$$
(5.6)

The gradient of this is given by

$$\frac{\partial C(z)}{\partial z} = \frac{V_0}{D} \cdot \frac{(C_0 - C_a)e^{\frac{V_0 z}{D}}}{1 - e^{\frac{V_0 a}{D}}} \quad .$$
(5.7)

Although it is not immediately obvious, these results are the same as V_0 tends to zero as those for $V_0 = 0$. This can be shown by expanding the exponentials in powers of V_0/D .

The final limit to consider is that of large velocity, or equivalently small diffusion coefficient. When $D/V_0 \ll 1$ (5.6) implies that $C(z) \approx C_0$ except where $z \cdot a = O(D/V_0)$. Then it becomes

$$C(z) \simeq C_0 + (C_0 - C_a)e^{\frac{V_0(z-a)}{D}}$$
 (5.8)

Thus the concentration is at its 'floor' value except in a narrow region near the top where it adjusts to the boundary layer of thickness $O(D/V_0)$. This boundary layer is evident in Figures 5.2 and 5.3 below.

Figure 5.2 shows the concentration as a function of depth across the soil cell for different flow velocities. Similarly figure 5.3 shows the gradients of concentration varying with flow velocity. Note that the flux depends on the gradient of the concentration as well as the velocity.

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Figure 5.2: Graph of methane concentration against depth in steady-state



Figure 5.3: Graph of concentration gradient against depth in steady-state

However in the flux box tests using the soil cell the flux out of the soil cell was found to take an extremely long time to reach a steady-state value. Hence it is useful to look at the expected time for the concentration to reach steady-state, and the expected

behaviour in between. This is particularly important for 'real' site conditions, where steady-state is unlikely to occur.

Laplace Transform solution for time dependent flow

The Laplace Transform method was used in chapters 3 and 4 on pressure and radon. The method follows that of the radon case quite closely, and has strong similarities, but the different boundary conditions cause the result to be quite different.

We are trying to calculate the resulting development in the concentration, due to a constant velocity V_0 imposed at time 0. Taking a Laplace Transform of equation (5.2), with respect to time, gives

$$\frac{D}{\epsilon} \frac{\partial^2 \overline{C}}{\partial z^2} - \frac{V_0}{\epsilon} \frac{\partial \overline{C}}{\partial z} - s\overline{C} = -F(0) \quad .$$
(5.9)

Here F(0) is the concentration function before the velocity V_0 is 'switched on'; we will generally use zero for simplicity. It is possible to obtain a solution with other expressions for F(0), for example the solution for one steady-state velocity. However the more complicated expressions are unlikely to lead to results of greater practical importance. The solution for the complementary function is of the form $e^{\alpha z}$ where α is given by

$$\alpha = \frac{V_0}{2D} \pm \sqrt{\left(\frac{V_0^2}{4D^2} + \frac{\epsilon s}{D}\right)} \quad . \tag{5.10}$$

At this point the boundary conditions are applied. These again correspond to the laboratory experiment. This requires the concentration to be forced to a certain value C_a at the top and C_o at the bottom of a box, (usually 100% at the bottom, and 0% at the top), with the top of the box *a* metres above the bottom. Using these the transformed boundary conditions are C_0 / s at the bottom, z = 0, and C_a / s at the top, z = a. These can then be used to find the form of the transform of C when $C_a = 0$ as

$$\overline{C} = +\frac{C_0}{s} \exp\left(\frac{V_0 z}{2D}\right) \left[\frac{\exp\left((z-a)\sqrt{k_s}\right) - \exp\left((a-z)\sqrt{k_s}\right)}{\exp\left(-a\sqrt{k_s}\right) - \exp\left(a\sqrt{k_s}\right)}\right]$$

$$where \quad k_s = \frac{V_0^2}{4D^2} + \frac{\epsilon s}{D} \quad .$$
(5.11)

Now as before we have to invert this to give the result for the concentration C. The

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result is found using the inverse Laplace Transform integral

$$C = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \overline{C}(s,z) \ e^{st} \ ds \qquad (5.12)$$

As for the radon case in Appendix C qnd chapter 4, using this involves finding a residue, at s = 0, and handling the point where $k_s = 0$. The residue at s = 0 is found from (5.11) to give part of the result as

$$C(z) = \frac{C_a - C_0 e^{\frac{V_0 a}{D}} + (C_0 - C_a) e^{\frac{V_0 z}{D}}}{1 - e^{\frac{V_0 a}{D}}} = C_0 \frac{\left(\frac{e^{\frac{V_0 z}{D}} - e^{\frac{V_0 a}{D}}}{1 - e^{\frac{V_0 a}{D}}}\right)}{1 - e^{\frac{V_0 a}{D}}},$$
(5.13)

the second result applying when $C_a=0$. This is exactly the same as the steady-state solution, which is encouraging. However since it has no time dependence, the residue at the other point must give the rest of the result.

It simplifies the working to make substitutions at this point

$$k_{s} = \frac{\epsilon}{D} \left(\frac{V_{0}^{2}}{4D\epsilon} + s \right) = q^{2} (p + s) \quad and \quad K_{3}(z) = C_{0} \exp\left(\frac{V_{0}z}{2D}\right)$$

$$where \ q = \sqrt{\frac{\epsilon}{D}} \quad and \quad p = \frac{V_{0}^{2}}{4D\epsilon} \quad .$$
(5.14)

Then (5.10) becomes

$$\overline{C} = \frac{K_3(z)}{s} \left[\frac{\exp((z-a)q\sqrt{p+s}) - \exp((a-z)q\sqrt{p+s})}{\exp(-aq\sqrt{p+s}) - \exp(aq\sqrt{p+s})} \right] .$$
(5.15)

This is expressed more simply in terms of hyperbolic sines

$$\overline{C} = \frac{K_3(z)}{s} \left[\frac{\sinh((a-z)q\sqrt{p+s})}{\sinh(aq\sqrt{p+s})} \right] .$$
(5.16)

Now this expression *might* contain a branch point when p+s = 0, as occurred in Appendix C. However because both numerator and denominator have a sinh term this does not occur. However there is a series of poles resulting from the denominator, which can be seen by rewriting the expression again in terms of sines, since sinh x = sin(ix) / i

$$\overline{C} = \frac{K_3(z)}{s} \left[\frac{\sin(i(a-z)q\sqrt{p+s})}{\sin(iaq\sqrt{p+s})} \right] .$$
(5.17)

This expression has poles whenever the sine in the denominator is equal to zero, that is when

$$iaq\sqrt{p + s} = \pm n\pi \quad . \tag{5.18}$$

Hence the poles are whenever s is given by

$$s_n = -\left(\frac{n\pi}{aq}\right)^2 - p \quad . \tag{5.19}$$

This result means that there is an infinite series of points where the expression has a pole, and each will contribute to the total through the residue at a given point. To find the residue from (5.17) we use the numerator over the differential of the denominator, evaluated at s_n , see eg [Boas 83] p 599. This gives

$$R(s_n) = \frac{K_3(z)}{s_n} \left[\frac{\sin\left(i(a-z)q\sqrt{p+s_n}\right)}{\frac{iaq}{2\sqrt{p+s_n}} \cos\left(iaq\sqrt{p+s_n}\right)} \right] \cdot e^{s_n t} \quad .$$
(5.20)

But the expression (5.18) allows this to be simplified considerably, since it gives a value for $(p + s_n)^{\nu_2}$ in terms of n, and this makes the *i* terms cancel. The cos term on the bottom is either 1 or -1 depending on the value of n. Hence it gives a term $(-1)^n$. The final form for the residue due to the point where $k_s = 0$ is then

$$R(s_n) = \frac{2n\pi K_3(z)}{(-1)^n s_n (aq)^2} \left[\sin\left(n\pi \frac{(z-a)}{a}\right) \right] \cdot e^{s_n t} \quad .$$
(5.21)

The index n has integral values from 0 to infinity. The summation over n of the expression in equation (5.21) matches the steady-state solution at time t=0. However it takes a large number of terms to reach convergence of the series when the time is small. A value of n of around 100 or more is needed, because the result is of order 1/n only. Once the time increases the exponential term greatly reduces the size of each term and the convergence is quicker. In summing the terms the sine terms and the -1^n simplify.

Hence the full solution for the concentration at any given time is given by

$$C(z,t) = C_0 \frac{\left(e^{\frac{\nu z}{D}} - e^{\frac{\nu a}{D}}\right)}{1 - e^{\frac{\nu a}{D}}} + \sum_{1}^{n} \frac{2n\pi K_3(z) \sin\left(\frac{n\pi z}{a}\right)}{s_n (aq)^2} \cdot e^{s_n t} .$$
(5.22)

This method produced the plot shown in figure 5.4. It shows the methane concentration as a function of depth at different times before steady-state has been reached. It also shows the result of a numerical method for solving exactly the same problem, and the two solutions match extremely well. The method used for the numerical model was a simplified version of that discussed in the next section of this report.



Figure 5.4: Graph of concentration against depth for different times

The gradient of the concentration at any point can be found directly from (5.22) by differentiation. It is also possible to use the Laplace Transform method directly to obtain it, but there is no benefit from doing this.

Numerical solution - taking account of a void

In chapter 4 a simple numerical model was introduced to solve the radon concentration problem in the many conditions where it cannot be solved exactly. The same process can

be used in the case of landfill gas. As a first step the method has been applied to the arrangement of the soil cell discussed earlier. However the void at the bottom of the sand has been introduced as an extra feature of the model.

An important feature of the soil cell is the considerable time it takes for the methane concentration to reach a significant level, or for the sand to become saturated. This reflects the fairly low rate of input needed to keep the pressures down and to avoid the risk of explosion! This time scale has already been understood from the Laplace Transform solution given previously.

It is also significant that the methane is supplied through a void space at the base of the sand. This region is initially full of air, and it takes some time to fill with methane. In addition it is possible for this void region to retain some air, which can diffuse down through the sand just as methane diffuses up. Hence the void is included as a part of the numerical model. It is one of the advantages of a numerical model that this sort of feature can be added quite easily to the model. It would be difficult to include this in an analytical model as it involves linking two differential equations through a solution at the boundary of the regions they describe.

There are two main disadvantages compared to an analytical solution. One is that it takes longer to produce a new solution with a slightly changed parameter. The other is that there is some uncertainty about the accuracy of the solution because of the approximations involved in the method. However the fact that the numerical method is possible for most situations where an analytical solution cannot be found makes it very useful.

It is possible to use a numerical method to calculate the pressure field caused by introducing gas into the bottom of the soil cell. However common sense, and approximate calculations indicate that the time taken for the pressure and velocity to stabilise is very small compared to the time it takes for the concentrations to change. Hence at present the model assumes that the pressure, and hence the velocity, can be assumed constant. Then the equation for the sand region is, from (5.2).

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$$\frac{\partial C}{\partial t} = \frac{D}{\epsilon} \frac{\partial^2 C}{\partial z^2} - \frac{V}{\epsilon} \frac{\partial C}{\partial z}$$
(5.23)

The basis of the numerical model is that the equation can be expressed in terms of approximations to the differential terms. These approximations are based on the values of the concentration at points on a 'grid' of depth and time. As in chapter 4 the key approximations are

$$\frac{\partial C}{\partial t} = \frac{C_{r+1} - C_r}{dt}$$
(5.24)

where C_{r+1} , C_r are the concentrations at time step r+1 and r respectively.

For the z differential there is a choice to be made between the three possible ways to discretise the equation, the choice depending on the direction of any flow in the system. In order they are known as upwind, downwind and central differences.

$$\frac{\partial C}{\partial z} = \frac{C_{n+1} - C_n}{dz} \text{ or } \frac{C_n - C_{n-1}}{dz} \text{ or } \frac{C_{n+1} - C_{n-1}}{2.dz} .$$
(5.25)

Here C_{n+1} , C_n , C_{n-1} are the concentrations at grid positions n+1, n and n-1 respectively. The third form of (5.25) is appropriate for most cases, and was used here. A form which gives greater accuracy can be used for the end element, ie when n=0. It is

$$\frac{\partial C_0}{\partial z} = \frac{-3C_0 + 4C_1 - C_2}{2 \cdot dz} \quad . \tag{5.26}$$

This form gives a better result when the gradient at the boundary is needed. This does not have a particularly large effect in most cases, but helps to account for differences in flux rates at higher velocities. For the second differential the first two forms for $\partial C/\partial z$ are combined to give

$$\frac{\partial^2 C}{\partial z^2} = \frac{C_{n+1} + C_{n-1} - 2C_n}{(dz)^2} \quad .$$
(5.27)

These expressions are combined into equation (5.23) to give the solution for any point at time r+1 based on the values for the concentration at time r. A significant limitation of the method is that the time step used cannot be chosen freely. To prevent the solution diverging, the time step must be less than a value dependent on the spatial step length dz, defined by

$$dt \le \frac{\epsilon}{D} \cdot \frac{dz^2}{2} \quad . \tag{5.28}$$

This limits the speed of the run, because the time step is of order ten seconds to one minute, but with modern computers it is not a significant problem.

The model for the concentration in the void is a simple mass balance one. It assumes that the mixing is good between the air and methane, so that a single zone can be used. The mass of methane in the void below the soil cell is then governed by

$$d \cdot A \cdot \frac{\partial C}{\partial t} = Q_{in} \cdot C_{in} - V \cdot A \cdot C + D \cdot A \cdot \frac{\partial C}{\partial z} , \qquad (5.29)$$

where

d is the depth of the void region (m),

A is the area of the sand, (m^2) and also of the void, so d.A is the void volume,

C is the methane concentration in the void (%),

 Q_{in} is the volume flow rate into the void, controlled by the user (m³/s),

 C_{in} is the concentration of the gas flowing into the void, chosen by the user (%),

V is the velocity of the flow in the sand (m/s),

D is the diffusion coefficient of methane in the sand (m^2/s) ,

 $\partial C/\partial z$ is the concentration gradient at the bottom of the sand (%/m).

In the method the void concentration is recalculated each time step, with the last value of $\partial C/\partial z$ providing a link with the condition of the sand. The new value of C is taken as the lower boundary condition for the sand region, so that C is continuous across the join between the two regions. The concentration in the void is a function of time only, i.e. there is no spatial variation considered. This method is acceptable provided the changes are not too fast, but can lead to instabilities if the flow rate into the void is large compared to the volume of the void.

A detail of the method concerns the velocity and the porosity ϵ . The velocity of flow out of the soil cell, assuming no leakage, is given by $V = Q_m / A$. However the velocity of flow in the sand must be greater than this because a proportion of the space is filled with sand particles. Hence the velocity of flow in the sand is $V = Q_m / (A\epsilon)$. It is important to account for this when considering the time it takes for the methane to reach the top of

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the soil cell. If there is no diffusion it must take a time equal to the depth of the sand divided by the velocity for any methane to arrive. This will be affected by the choice of which velocity to use. This time factor could give an alternative method to measure porosity. Although it is not likely to be more accurate than the conventional methods it does allow for the measurement to be taken without disturbing the sand in the soil cell.

The final part of the theory of the model is the flux equation. The key parameter of concern is the amount of methane which leaves the surface of the ground, since this is what can be measured and is needed for risk assessment. This is given by the flux rate, J, defined as

$$J = -D \cdot \frac{\partial C}{\partial z} + v \cdot C \quad . \tag{5.30}$$

This can be found at any point, since it should be the same in steady-state for all values of z. It is generally easiest to evaluate at the top surface where C is zero, and so only the gradient term matters.

It is because both the flux and the void equation use the concentration gradient of the sand at the end of the box that the more advanced form given in (5.26) had to be used. It was not used for the radon case, but might have helped with the radon flux predictions.

Details of the programme are not given here. However it was programmed in Microsoft Quickbasic, and runs easily on a 486 IBM compatible computer. The run time depends on the grid sizing and time duration required, so varied from a few seconds to 20 minutes.

Application of the model

Figures 5.5 to 5.8 show typical outputs from the model. The values chosen for the key parameters were permeability, $k = 1e-10 \text{ m}^2$, and diffusion coefficient $D = 6.3e-7 \text{ m}^2/\text{s}$. The permeability was that found earlier for the sand in unpublished work by Richard Hartless. The diffusion coefficient needs more work on it, and so this value may not be appropriate. Figure 5.5 shows the development of concentration with time for a very

small flow rate into the void region, 0.01 litres/min. The concentration takes around 200 hours to approach steady-state, as is shown by figure 5.6, giving the surface flux rate changing with time. Because there is very little pressure-driven flow in figure 5.5 the result is nearly a straight line, which would be the result for diffusion only. Notice also that the concentration at depth zero, the bottom of the sand, is predicted to reach only about 0.13, because of air diffusing down through the sand to the void region.

In figure 5.7 the concentrations for a higher input flow rate, 0.5 litres/min are given. Here the curve is strongly affected by the pressure-driven flow, with concentrations close to 100% occurring for most of the sand depth. In addition the system reaches equilibrium more quickly, as the flow carries methane across the sand more quickly than diffusion alone can. This is seen in figure 5.8, where the flux rate at the surface reaches the steady-state result by 40 hours. Nevertheless this process is still quite slow to reach equilibrium, given that at this flow velocity the methane might be expected to take around 17 hours to travel the 0.5 metres up the soil cell.



Figure 5.5: Graph of normalised concentration against time, flow = 0.01 litres/min

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Figure 5.6: Graph of variation of surface flux against time, and input flux



Figure 5.7 Graph of normalised concentration against time, flow = 0.01 litres/min



Figure 5.8 Graph of variation of surface flux against time, and input flux

Changing velocity

The model can also be used to look at how the flux and concentration respond to changes in applied flow rates. This proved too time consuming with the Laplace Transform method. Two typical results are given as figures 5.9 and 5.10. In figure 5.9 the input flux was halved after 60 hours, and the flux out of the soil then falls off quite rapidly, but still take many hours to approach the new steady-state value. In figure 5.10 the input flux is increased after 30 hours, before the first steady-state level has been reached. It then takes another 20 hours to approach the new steady-state value. The way in which these results compare with experiments is discussed in the next chapter.



Figure 5.9: Graph of gas flux against time for the soil cell: step decrease in input flux



Figure 5.10: Graph of gas flux against time for the soil cell: step increase in input flux

Chapter 6: Comparison of results with experimental data

There are three sources of data from BRE experiments which can be used to make comparison with the results from the modelling studies given in chapters 3, 4 and 5. The most direct comparison comes with the methane work from chapter 5 and soil cell tests in the laboratory. This is the first area discussed in this chapter and was carried out by the author and a student working under direct supervision. In addition BRE have two full-scale tests going on, one in a house affected by radon, and another in a test structure built on a landfill site. In both of these cases data for gas levels in the structures, and atmospheric pressure and other weather data are being measured continuously over extended periods of time. This allows the detailed examination of changes over time, and also a statistical based approach to longer term trends and averages. This chapter looks more over the shorter term. These two experiments are being carried out by colleagues at BRE, and the author has no direct involvement in them.

Laboratory tests

The soil cell was discussed in chapter 5, so the description will not be reproduced here.

One of the main observations from the soil cell tests described earlier is the very large time that it takes for the soil cell to respond to changes in the input flux rates. This reflects the significant part that diffusion has to play in the movement of gas for the flow rates being applied. In one particular test of interest the input flow rate was reduced from 370 to 72 ml/min, having been held at the higher value for a considerable time. The outgoing flux was then measured at subsequent times as indicated by the discrete points on figure 6.1. This flux was found by measuring the build up of gas in a flux box on top of the sand.



Figure 6.1: Graph of flux rate against time, comparing model to experiment

In the flux box measuring technique used here, gas reaching one quarter of the top of the sand surface is collected in the flux box. Hence the total flow will be close to four times that given here, assuming that each quarter of the box behaves in the same way. This is a reasonable assumption, although the presence of the flux box must have some impact. Multiplying the flow results by 4 implies that more methane leaves the soil cell than is supplied to it. This means there must be some problems remaining with the experiment, some of which are addressed below.

The main problems with the experiment are:

- a) Input flow rates are difficult to measure because they are small, and they fluctuate. This fluctuation is due to the difficulty in maintaining a steady flow rate from a gas bottle which gradually discharges the gas in it.
- b) The gas detector used (Flame Ionization Detector) is very sensitive and has a limited range: 0-1% by volume. Hence the input flow rates had to be small to

ensure the concentrations in the flux box were not off scale. It is possible to use an Infra Red detector with the flux box, and this has a wider range, so this might be used in future work.

c) There remains an uncertain amount of leakage through the walls of the soil cell. In spite of considerable efforts to seal the sides of the box there is still a percentage of the flow which escapes through the wooden sides of the box. It is hard to estimate the scale or effect of this, but it is probably around 10% of the total flow, meaning that a 10% error can be anticipated in many results.

In figure 6.1 the experimental result is compared to that produced by the computer model, although the considerable uncertainty of the experimental data makes a quantitative comparison difficult. In order to carry out the calculation the diffusion coefficient for the sand had to be estimated. The value of the diffusion coefficient of methane in air is 1.5e-5 m²/s [Hooker 93], and an estimate of the value in a porous material can be made as [Penman 40]

$$D = 0.66 \cdot \epsilon \cdot D_{air} \tag{6.1}$$

In this case the value was estimated as $4e-6 \text{ m}^2$, and the correlation is fairly close.

Methane test site

At the gassing landfill site the levels of methane and carbon monoxide are being monitored every two hours underneath the cap of the landfill site. Because little is known about the horizontal variations this gives a condition which is close to one-dimensional. A typical result is shown as figure 6.2, which comes from the work by my colleagues Richard Hartless and Louise Collins. Their work will be published elsewhere in the future, so this study has not taken the analysis of their data very far.



Figure 6.2: Graph of landfill gas concentrations and atmospheric pressure against time

It is clear from this plot that, in qualitative terms, when the atmospheric pressure falls the methane and carbon dioxide levels at the surface rise, and vice versa. Although the analysis is slightly different for landfill gas and radon, this result strongly supports the results of the modelling work in chapters 3 and 4.

However there is not a direct correspondence with the modelling results from earlier, as the physical arrangements are different. At the landfill gas test site the data are collected for the concentration just below a solid floor slab, but this is not the situation modelled, and adds complications. This is because the flow path from a point below the floor slab is complicated to calculate, and will not be a one-dimensional problem. Hence no further analysis has been made to try to find a quantitative fit to the experimental data.

Further BRE work by Richard Hartless and Louise Collins is expected to address the correlations between weather variables and landfill gas levels.

The radon house

The BRE test house is at Okehampton in Devon, within a "radon affected area", and is being used to test out remedial measures. It is set up to monitor radon concentrations above and below the ground floor, and all of the relevant weather and internal conditions. The radon test house is discussed more fully in the paper by Welsh [Welsh 95], who is responsible for running this experiment and provided the data for the graph shown below.

The radon test results do not agree as well with such a simple model as those from the gassing landfill site. For the radon case the radon is measured in a void below the floor of the house. This void is linked to the ground across a soil surface, but also to outside air through ventilation openings, and to the house through holes and cracks in the floor. Hence it is affected by other factors, of which wind speed seems to be the most important. Figure 6.3 shows a comparison between radon level, atmospheric pressure and wind speed for different times. Note that the wind is quoted in (mm/s) so the values are of a similar order of magnitude to the radon concentration values. The wind seems to have a direct effect on radon levels, probably because higher wind speeds lead to greater depressurization of the house, causing more radon to be sucked up from the soil. This effect appears to be large enough to cover any atmospheric pressure effect. This is complicated by the existence of a relationship between pressure changes and wind speeds.

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Figure 6.3: Radon concentration, wind speed and atmospheric pressure against time

The monitoring work continues at both sites, so that more data will be available in the future. For the radon case it will be vital to model the more complicated interactions of the house, subfloor void and soil with the changing weather conditions, in order to understand fully the processes taking place.

It would in principle be possible to extend the existing one-dimensional model of radon concentration to account for the way in which the house interacts with the weather. The basis of models to consider the ventilation aspects are well known [Cripps 92] and progress in this area is being made by other workers, including Lynn Hubbard in Sweden. This work will help the use of shorter term measurements in detecting high radon houses. However within the time scale of this work it has not been possible to address this next step.

Chapter 7: Conclusion to time dependent flow part

In this part the way in which soil gas levels change over time has been investigated, using a mixture of analytical and computational techniques. These models have been compared with some experimental results, which indicate that the general results are correct, but that in several cases there is much more to be included in a complete model.

Pressure problems

In chapter 3 the solutions to the pressure equation were considered. The particular cases considered were a steady increase in surface pressure, a suddenly applied pressure change and a sinusoidal variation in pressure. These allow us to understand the way in which bulk flows of gas respond to external pressure changes, caused by the weather or mechanical systems. Since these can have a significant effect, taking account of the time was seen to be important under some conditions.

Radon problems

In chapter 4 several different problems of the time varying radon equation were tackled, again using a mixture of different methods.

An analytical method gave a solution for small time to the problem of a linear increase in surface pressure, and an instantly applied velocity. The same problems were tried with the Laplace Transform method, which gives an exact definition of the result, but because this involves an integral which cannot be evaluated exactly it is not easy to use. Then all the solutions were compared with that from a simple one-dimensional numerical model, which produced results very close to the analytical methods.

These results show that there is a considerable time between a change to the velocity of flow of soil gas and a significant change in the radon concentration. This means that care must be taken when collecting data on concentrations as to what the flow conditions are at the time of any measurement.

Landfill gas problems

In chapter 5 several different problems relating to landfill gas were tackled, using a mixture of different methods. They concentrated on problems related to the BRE laboratory soil cell. In this a flow of pure methane is introduced into the base of column of sand, and the flux of methane emerging from the top of the sand is measured.

The problem has been tried with the Laplace Transform method, which gives an exact definition of the result for a simplified geometry. In order to model the effect of a void region below the sand a simple one-dimensional numerical model was used. This produced results which match the simplified analytical solution very closely.

These results show that there is a considerable time between a change to the velocity of flow of soil gas and the resulting change in the gas concentration. This means that care must be taken when collecting data on concentrations as to what the flow conditions are at the time of any measurement, and what they have been in the past. As well as predicting the result following the imposition of a flow of gas onto the sand, the model can predict the effect of a change in the input flow rate on concentrations and the outgoing flux.

Comparison with experiments

In chapter 6 the results from three different BRE experiments were compared with the modelling results given earlier.

The laboratory results give good qualitative support to the landfill modelling work, but there remain problems with the accuracy of the measurements. It is interesting that the tests which were supposed to test out the performance of the computer model against a known data set ended up showing the problems with the experiment. These are still under examination and have still to be resolved.

The landfill gas test site gives very good qualitative support for the modelling results, as

the concentrations of gas below the floor of the structure respond as expected to changes in atmospheric pressure.

However at the radon house the situation is more complex, and the effect of the wind on the radon level in the void below the floor means that a simple model based on atmospheric pressure is not enough, and a more complete model to include the ventilation of the house is needed.

Overall it is clear that there are significant time-dependent effects occurring which affect the rates of gas entry into homes. Modelling these allows us to understand the processes going on, and therefore have guidance on how to prevent problems occurring. There are also applications in making more use of short-term measurements of radon levels in houses, by using other data about the house and weather conditions to interpret a smaller amount of time-varying data.

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Appendices to chapter 4 of time dependent part

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Appendix A: Solution to radon equation with rising surface pressure

This appendix gives the detail of the solution method for the radon concentration in soil when the surface pressure is changing linearly. The method used is that of matched asymptotic expansions. Because of the nature of the solution method, the solution produced is limited in its range of application to small time and small depth of soil.

The principle of the method is that for most problems there are two distinct regions where different approximations can be made. For both of these regions a solution can be found, subject to the validity of the approximations being made. Then a good approximation to the complete solution can be found by matching the two solutions across the region which separates them.

At large depth, where the effect of the surface pressure changing has not yet reached, the result for the concentration is that given by (4.1) for zero velocity. Along the z=0 boundary the concentration will still be zero. It is not a condition that is used, but at large time the velocity will be very large, and the concentration will be the same for all depths, either C=0 for flow into the soil, or C= Φ/λ if the flow is upwards, as in (4.4) and (4.5).

Because, from (3.15), the velocity is proportional to t^{ν_2} it is reasonable to assume that the concentration will be a function expanded in powers of t^{ν_2} also. Also the form of the velocity equation (3.15) suggests that the new variable $\zeta = z / t^{\nu_2}$ will be useful. Note that because the flow is below ground level, z and hence ζ are both negative. Using this the velocity can be written as

$$v = \gamma t^{\frac{1}{2}} g(\zeta) \quad where \quad g(\zeta) = \left(\frac{1}{2}\alpha\zeta\sqrt{\pi} \ erfc\left(\frac{-\alpha\zeta}{2}\right) + \exp\left(-\frac{\alpha^2\zeta^2}{4}\right)\right)$$
(A1)

and

$$\gamma = 2a\alpha k / (\mu \pi^{1/2})$$

Using the new variable, together with T for t means that the equation for concentration also has to be changed. Using the chain rule the transformation gives

$$\frac{\partial}{\partial z} = \frac{1}{T^{\frac{1}{2}}} \frac{\partial}{\partial \zeta} \quad and \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial T} - \frac{1}{2} \frac{\zeta}{T} \cdot \frac{\partial}{\partial \zeta} \quad . \tag{A2}$$

Using these in the concentration equation gives

$$\frac{\partial C}{\partial T} - \frac{\frac{1}{2}\zeta}{T} \frac{\partial C}{\partial \zeta} = \frac{D}{T\epsilon} \frac{\partial^2 C}{\partial \zeta^2} - \frac{1}{T^{\frac{1}{2}\epsilon}} \frac{\partial (\nu C)}{\partial \zeta} - \lambda C + \phi \quad . \tag{A3}$$

Near the surface where C=0, the solution is assumed to be of the form

$$C = C_0(\zeta) \cdot T^{\frac{1}{2}} + C_1(\zeta) \cdot T + C_2(\zeta) \cdot T^{\frac{3}{2}} + O(T^2) \quad .$$
 (A4)

Substituting this into equation (A3) gives an expansion of the terms of the equation in powers of T, namely

$$\frac{1}{2}C_{0}T^{-\frac{1}{2}} + C_{1} + \frac{3}{2}C_{2}T^{\frac{1}{2}} - \frac{\zeta}{2} \cdot \frac{\partial}{\partial\zeta} (C_{0}T^{-\frac{1}{2}} + C_{1} + C_{2}T^{\frac{1}{2}}) = \frac{D}{\epsilon} \cdot \frac{\partial^{2}}{\partial\zeta^{2}} (C_{0}T^{-\frac{1}{2}} + C_{1} + C_{2}T^{\frac{1}{2}}) - \frac{\gamma}{\epsilon} \cdot \frac{\partial}{\partial\zeta} (gC_{0}T^{\frac{1}{2}}) - \lambda(C_{0}T^{\frac{1}{2}}) + \phi ,$$
(A5)

where terms up to and including those of order $T^{\frac{1}{2}}$ have been displayed. In order to proceed with it T must be assumed to be small, so that the different powers of T can be taken separately. Doing this gives equations for each of C₀, C₁ and C₂. Consider first the three terms in $T^{\frac{1}{2}}$.

First term of the inner solution

From (A5) it follows that the equation for C_0 is

$$\frac{1}{2}C_0 - \frac{\zeta}{2} \cdot \frac{\partial C_0}{\partial \zeta} = \frac{D}{\epsilon} \cdot \frac{\partial^2 C_0}{\partial \zeta^2} \quad . \tag{A6}$$

One way to tackle this equation is to differentiate again with respect to ζ , giving

$$\frac{\zeta}{2} \cdot \frac{\partial^2 C_0}{\partial \zeta^2} + \frac{D}{\epsilon} \cdot \frac{\partial^3 C_0}{\partial \zeta^3} = 0 \quad . \tag{A7}$$

This gives the second derivative of C_0 as

$$\frac{\partial^2 C_0}{\partial \zeta^2} = A \cdot e^{(-B\zeta^2)} \quad \text{where } B = \frac{\epsilon}{4D}$$
(A8)

and A is a constant to be found. It is easily given by considering (A6) when $\zeta = 0$. Since the boundary condition at the surface implies that $C_0(0) = 0$, it follows from (A6) that $C_0''(0) = 0$ also. Hence A = 0, and a double integration of (A8) leads to

$$C_0 = C'\zeta + D' \tag{A9}$$

where D' = 0 and C' is a constant to be determined by matching with the outer solution, where $\zeta = O(1)$.

The outer solution is dominated by the condition at t=0. This is given by a function f(z) which in most cases will determined by equation (4.2), the steady-state solution for no applied velocity. Since $g(\zeta)$ is exponentially small as t tends to 0 when (- ζ) is O(1) it follows that, in the outer region, C is unaffected by the term in v in (A3). Thus C may be written as

$$C = f(z) + t \cdot F_1(z) + O(t^2)$$
(A10)

where

$$F_1(z) = \frac{D}{\epsilon} f''(z) - f(z) + \phi$$
(A11)

and is zero if f(z) happens to be the steady-state solution. It follows from (A10) that at the inner edge of the outer region

$$C \simeq f(0) + zf'(0) + \frac{z^2}{2}f''(0) + \frac{z^3}{6}f'''(0) + \frac{z^3}{6}f'''(0) + t\left[\frac{D}{\epsilon}(f''(0) + zf''(0) + ...) - \lambda[f(0) + zf'(0) + \frac{z^2}{2}f''(0)] + \phi\right] .$$
(A12)

Now in our case f(0) = 0, and then substituting for ζ in place of z gives

$$C \simeq \zeta \sqrt{t} f'(0) + \frac{\zeta^2 t}{2} f''(0) + \frac{\zeta^3 t^2}{6} f'''(0) + \frac{\zeta^2 t}{6} f'''(0) + \frac{\zeta^2 t}{2} f''(0) + \frac{\zeta^2$$

Note that the second line is absent if f(z) is the steady-state solution (4.2). For simplicity henceforth we shall assume this is so and hence for small z

$$f(z) \simeq \frac{\Phi}{\lambda} (-zp - \frac{z^2 p^2}{2} - \frac{z^3 p^3}{6})$$
 (A14)

where the constant p is $(\epsilon \lambda/D)^{\frac{1}{2}}$. Matching the leading term of (A12) determines the constant C' in (A9) as $-\Phi/\lambda$ so that C_o is given by

$$C_0 \simeq -\frac{\Phi \zeta p}{\lambda}$$
 (A15)

Here we recall that z, and hence ζ , is negative so that the match is effected as z tends to 0 from below and as ζ tends to minus infinity. Hence to this approximation the first term in the expansion for the concentration C is just an approximation to the steady state solution, $-\phi zp/\lambda$, as it must be. The 1 / t^{1/2} contained in the ζ term of (A15) cancels with the t^{1/4} in equation (A4), so there is no time dependence in this term.

Second term of the inner solution

The terms in T^0 from equation (A5) give an equation in C_1 similar to that in C_0 , but differing by the constant multiplying the first order term in C. It is

$$\frac{D}{\epsilon} \cdot \frac{\partial^2 C_1}{\partial \zeta^2} + \frac{\zeta}{2} \cdot \frac{\partial C_1}{\partial \zeta} - C_1 = -\phi \quad . \tag{A16}$$

To make the notation easier, let D / ϵ be A. Then a complementary function comes from trying $C_1 = \zeta^2 + b$, which gives b = 2A. Progress can then be made by trying a solution of the form $y(\zeta)$. ($\zeta^2 + 2A$) in (A16) with the right-hand side set equal to zero. This gives an equation for the new function y which can be simplified to

$$\frac{\partial^2 y}{\partial \zeta^2} + \frac{\partial y}{\partial \zeta} \left(\frac{\zeta}{2A} + \frac{4\zeta}{\zeta^2 + 2A} \right) = 0 \quad . \tag{A17}$$

The integral of this is given by

$$\frac{\partial y}{\partial \zeta} = B \cdot \frac{e^{\frac{-\zeta^2}{4A}}}{(\zeta^2 + 2A)^2} \quad . \tag{A18}$$

The integral is not obvious, but can be checked by differentiation. This enables a part of the solution to be evaluated, as the integral of (A18), but the exponential term can be put into a simpler form. If we satisfy the boundary condition that $C_1(0) = 0$ and take the solution

$$C = A_{1} \left(\frac{1}{2} + \frac{\zeta^{2}}{4A} \right) \cdot \int_{0}^{\zeta} e^{\frac{-\zeta'^{2}}{4A}} d\zeta' + \frac{1}{2} A_{1} \zeta e^{\frac{-\zeta^{2}}{4A}} - \frac{\Phi}{2A} \zeta^{2} , \qquad (A19)$$

then differentiation and substitution in equation (A16) shows that this satisfies the equation correctly, including the right-hand side. Now for large negative values of ζ this must match with the second term of the expansion of the boundary condition at time t = 0, equation (A14), and this allows the determination of the arbitrary constant A₁. Hence $C_1 = f'(0) \cdot \zeta^2$ which for the case defined here gives simply $-\phi/A \cdot \zeta^2$. This means that A₁ must be zero for this initial distribution. This simplifies the result significantly, so that

$$C_1 = \frac{-\phi}{2A} \zeta^2 = \frac{-\phi}{2D/\epsilon} \frac{z^2}{T} \quad . \tag{A20}$$

Putting this into equation (A4) for C shows that the T term cancels, and we are left with the second term in the expansion of the function f(z). Whilst this result is to be expected in retrospect, the solution (A19) would apply to other forms of the function f, and leads to equation (A23) below. This would allow the calculation of the development in C from some initial condition with or without an imposed velocity.

Third Term of the inner solution

The term of (A5) in T^{4} is more important, because it brings in the velocity term for the first time. The terms are

$$\frac{D}{\epsilon} \cdot \frac{\partial^2 C_2}{\partial \zeta^2} + \frac{\zeta}{2} \cdot \frac{\partial C_2}{\partial \zeta} - \frac{3}{2} C_2 = + \frac{\gamma}{\epsilon} \cdot \frac{\partial}{\partial \zeta} (gC_0) + \lambda C_0 \quad . \tag{A21}$$

It is significant that the last two terms contain C_0 as a forcing term. The terms in C_2 are similar to those which have occurred before for C_0 and C_1 the complementary problem is given by

$$A \cdot \frac{\partial^2 C_2}{\partial \zeta^2} + \frac{\zeta}{2} \cdot \frac{\partial C_2}{\partial \zeta} - \frac{3}{2} C_2 = 0 \quad . \tag{A22}$$

This can be solved by differentiating (A22), and then observing that the equation in $\partial C_2 / \partial \zeta$ is the same as that for the complementary function for C_1 earlier. This means that the solution to (A22) is

$$\frac{\partial C_2}{\partial \zeta} = B_1 \left(\frac{1}{2} + \frac{\zeta^2}{4A} \right) \cdot \int_0^{\zeta} e^{\frac{-\zeta'^2}{4A}} d\zeta' + \frac{1}{2} B_1 \zeta e^{\frac{-\zeta^2}{4A}} - B_2 \frac{\Phi}{2A} (\zeta^2 + 2A) \quad , \tag{A23}$$

where B_1 and B_2 are arbitrary constants. This can be integrated by parts to give the complementary function for C_2 as

$$C_{2} = B_{1}\left(\frac{\zeta}{2} + \frac{\zeta^{3}}{12A}\right) \cdot \int_{0}^{\zeta} e^{\frac{-\zeta'^{2}}{4A}} d\zeta' + \frac{B_{1}}{12}(2\zeta^{2} + 8A)e^{\frac{-\zeta^{2}}{4A}} - B_{2}\phi(\frac{\zeta^{3}}{6A} + \zeta) \quad .$$
(A24)

There could have been an additional constant in this expression, but it turns out to be zero for (A22) to be satisfied.

Moving on to the particular integral, using equation (A15) for C_0 as f (0). ζ and expanding the right hand side of (A21) gives

$$\frac{D}{\epsilon} \cdot \frac{\partial^2 C_2}{\partial \zeta^2} + \frac{\zeta}{2} \cdot \frac{\partial C_2}{\partial \zeta} - \frac{3}{2} C_2 = + \frac{\gamma f'(0)}{\epsilon} (g + \zeta g') + \lambda f'(0) \zeta \quad .$$
(A25)

Then using equation (3.15) for the velocity term g gives,

$$g = -\left(\frac{\alpha\zeta\sqrt{\pi}}{2} \operatorname{erfc}\left(\frac{-\alpha\zeta}{2}\right) + \exp\left(-\frac{\alpha^2\zeta^2}{4}\right)\right)$$
(A26)

so that, writing erfc as 1 - erf, we obtain

$$g + \zeta g' = \left(-\exp\left(-\frac{\alpha^2 \zeta^2}{4}\right) \right) - \alpha \zeta \sqrt{\pi} \left(1 - erf\left(\frac{-\alpha \zeta}{2}\right) \right) \quad . \tag{A27}$$

Substituting this into equation (A25) gives the right-hand side as

$$RHS = + \frac{\gamma f'(0)}{\epsilon} \left[-\exp\left(-\frac{\alpha^2 \zeta^2}{4}\right) + \alpha \zeta \sqrt{\pi} \ erf\left(\frac{-\alpha \zeta}{2}\right) \right] + \zeta f'(0) \left[\lambda - \frac{\gamma \alpha \sqrt{\pi}}{\epsilon}\right] .$$
(A28)

There are then three parts which need to be tackled separately. Starting with the linear terms in ζ , trying $C_2 = \zeta^3$ in the LHS (left-hand side of (A25)) produces a linear term,

$$LHS(\zeta^3) = 6\zeta D/\epsilon \quad . \tag{A29}$$

The exponential term also satisfies the equation itself, provided that the value of α is limited, as the following shows. Trying $C_2 = \exp(-\alpha^2 \zeta^2/4)$ in the LHS of (A25) gives:

Let
$$e^* = \exp\left(-\frac{\alpha^2\zeta^2}{4}\right)$$
 then

$$LHS(e^*) = \left[-\frac{D}{\epsilon} \cdot \frac{\alpha^2}{2} \left(1 - \frac{\alpha^2\zeta^2}{2}\right) + \frac{-\alpha^2\zeta^2}{4} - \frac{3}{2}\right]e^* \quad .$$
(A30)

This rearranges to give the condition as

$$LHS(e^*) = \left[\frac{-\zeta^2 \alpha^2}{4} \left(\alpha^2 \cdot \frac{D}{\epsilon} - 1\right) - \alpha^2 \cdot \frac{D}{2\epsilon} - \frac{3}{2}\right] e^* \quad . \tag{A31}$$

Hence we are able to continue only if $\alpha^2 D/\epsilon = 1$. This restriction is rather significant and limits the range of application of the results produced by this method. There is a more general solution without this extra condition, but it significantly complicates the method. It is given at the end of this appendix, but here we proceed with the simpler solution. Since at this stage the purpose of this solution is to verify a numerical method under restricted conditions, and at small time, then the restriction is not so important. If the condition is met then the left hand side simplifies to

$$LHS(e^*) = \left[-\frac{1}{2} - \frac{3}{2} \right] e^* = -2e^* .$$
 (A32)

Moving on to the erf term, trying it multiplied by ζ in the left hand side gives

$$LHS(\zeta erf) = \frac{D}{\epsilon} \cdot \left(\frac{-2\alpha e^*}{\sqrt{\pi}} + \frac{\alpha^3 \zeta^2 e^*}{2\sqrt{\pi}}\right) + \frac{\zeta}{2} \cdot \left(erf - \frac{\alpha \zeta e^*}{\sqrt{\pi}}\right) - \frac{3}{2} \zeta erf \quad (A33)$$

Where e* is the same exponential term appearing in equation (A30) and erf is the term erf($-\zeta \alpha/2$) term being considered. Again this simplifies if $\alpha^2 \cdot D/\epsilon = 1$, as the terms in ζ^2 will then cancel out leaving only the erf terms, which combine, and one exponential term.

$$LHS(\zeta erf) = \frac{-2e^*}{\alpha\sqrt{\pi}} - \zeta erf \quad . \tag{A34}$$

From the three equations, (A29), (A32) and (A34) the complete Particular Integral can be built up, combining terms from the e* and erf parts to cancel each other out and to match the right hand side as given in equation (A25). This gives the particular integral as

$$PI = +\frac{\gamma f'(0)}{\epsilon} \left[+\frac{3}{2} \exp\left(-\frac{\alpha^2 \zeta^2}{4}\right) - \alpha \zeta \sqrt{\pi} \ erf\left(\frac{-\alpha \zeta}{2}\right) \right] + \frac{\zeta^3}{6} \frac{\epsilon}{D} f'(0) \left[\lambda - \frac{\gamma \alpha \sqrt{\pi}}{\epsilon}\right] .$$
(A35)

Full solution

Combining this with the solution to the complementary equation, (A24), gives the full solution. This contains two constants to be established from the boundary conditions. Thus altogether

$$C_{2} = -B_{1}\left(\frac{\zeta}{2} + \frac{\zeta^{3}}{12A}\right) \cdot \sqrt{\pi A} \operatorname{erf}\left(\frac{-\zeta}{2\sqrt{A}}\right) + \frac{B_{1}}{12}(2\zeta^{2} + 8A)e^{\frac{-\zeta^{2}}{4A}} - B_{2}\varphi(\frac{\zeta^{3}}{6A} + \zeta)$$

$$+ \frac{\gamma f'(0)}{\epsilon} \left[+ \frac{3}{2}\exp\left(-\frac{\alpha^{2}\zeta^{2}}{4}\right) - \alpha\zeta\sqrt{\pi} \operatorname{erf}\left(\frac{-\alpha\zeta}{2}\right) \right] + \frac{\zeta^{3}}{6} \frac{\epsilon}{D}f'(0)\left[\lambda - \frac{\gamma\alpha\sqrt{\pi}}{\epsilon}\right] .$$
(A36)

Now on $\zeta = 0$, $C_2 = 0$, and using this in (A36) gives B_1 as

$$B_1 = \frac{-9}{4A} \cdot \frac{\gamma f'(0)}{\epsilon} = \frac{9\gamma \phi}{4D} \cdot \sqrt{\frac{\epsilon}{D\lambda}} \quad . \tag{A37}$$

The terms in ζ^3 must match those in the solution for small z, ie the initial condition f(z) as in (A14). Hence

$$-\frac{\Phi}{\lambda} \cdot \frac{p^3}{6} = -\frac{B_1}{12A} \cdot \int_0^\infty e^{\frac{-\zeta'^2}{4A}} d\zeta' - \frac{B_2 \Phi}{6A} + \frac{1}{6} \frac{\epsilon}{D} f'(0) \left[\lambda - \frac{\gamma \alpha \sqrt{\pi}}{\epsilon}\right] \quad . \tag{A38}$$

Note that the sign of the B_1 term has changed because the integral is now to infinity, which is positive, rather than ζ which is negative and was present in (A24). Substituting for the form of f ' (0) means the left-hand side cancels with the term in λ on the right. The integral multiplying B_1 is equal to $(\pi A)^{\frac{1}{2}}$, so that the constant B_2 can be evaluated as

$$B_2 = -B_1 \frac{\sqrt{\pi A}}{2\phi} + \left(\frac{\epsilon \lambda}{D}\right)^{\frac{1}{2}} \left(\frac{\gamma \alpha \sqrt{\pi}}{\lambda \epsilon}\right) \quad . \tag{A39}$$

Inserting the expression for B_1 brings (A39) to

$$B_2 = -\frac{9}{8} \frac{\gamma}{D} \sqrt{\frac{\pi}{\lambda}} + \left(\frac{\epsilon \lambda}{D}\right)^{\frac{1}{2}} \left(\frac{\gamma \alpha \sqrt{\pi}}{\lambda \epsilon}\right) , \qquad (A40)$$

which simplifies to give

$$B_2 = +\gamma \sqrt{\frac{\pi}{\lambda}} \left(-\frac{9}{8D} + \sqrt{\frac{\mu}{DkP_0}} \right) \quad . \tag{A41}$$

These can then be combined to give the complete solution. As a check it is worth noting that at large enough negative ζ the terms in ζ all cancel out so that only the term in ζ^3 remains, and this is required to match the solution in the outer region.

In fact this matching defines the range in the z direction for which the method is valid. We have matched the solution to the solution in the absence of velocity expanded in powers of z. This is only valid for values of |zp| very much less than 1. This can be seen from the graphs discussed in the main part of the report, which show the solution to be converging to the dominant term of the approximation, rather than to the steady state solution.

Solution without the condition on α

If α is allowed any value then a particular integral to equation (A25) can be found using the method of variation of parameters. With $D/\epsilon = A$ set equal to 1 it is found to be

$$PI = \frac{\sqrt{\pi} erf\left(\frac{\zeta\alpha}{2}\right)}{6(\alpha^2 - 1)^2} \cdot (\alpha^3 \zeta^3 (\alpha^2 - 2) - 6\alpha \zeta) + \frac{e^{\frac{-\alpha^2 \zeta^2}{4}}}{6(\alpha^2 - 1)^2} \cdot (2\alpha^2 \zeta^2 (\alpha^2 - 2) - 4(\alpha^2 + 1)) \quad .$$
(A42)

This solution can be checked by differentiating it twice, and inserting the results into (A25) with A = 1. In addition it would need to be scaled to account for A taking values other than 1. However the special case was sufficient to check on the numerical results.
Appendix B: Concentration equation with instantaneously applied velocity

If the velocity V_0 is switched on at time t=0 then the solution method is made easier because the velocity has no ζ dependence. Making the same change of variables to ζ and T, and the same assumed expansion in powers of T^{1/2} as before equation (A5) of Appendix A becomes

$$\frac{1}{2}C_{0}T^{-\frac{1}{2}} + C_{1} + \frac{3}{2}C_{2}T^{\frac{1}{2}} - \frac{\zeta}{2} \cdot \frac{\partial}{\partial\zeta} (C_{0}T^{-\frac{1}{2}} + C_{1} + C_{2}T^{\frac{1}{2}}) =$$

$$\frac{D}{\epsilon} \cdot \frac{\partial^{2}}{\partial\zeta^{2}} (C_{0}T^{-\frac{1}{2}} + C_{1} + C_{2}T^{\frac{1}{2}}) - \frac{V_{0}}{\epsilon} \cdot \frac{\partial}{\partial\zeta} (C_{0} + C_{1}T^{\frac{1}{2}} + C_{2}T) \qquad (B1)$$

$$-\lambda (C_{0}T^{\frac{1}{2}} + C_{1}T + C_{2}T^{\frac{3}{2}}) + \phi$$

Then equating terms in each power of T gives equations in C_0 , C_1 and C_2 as before. In fact the equation involving C_0 alone, in powers of $T^{\frac{14}{5}}$ is exactly the same as in the earlier case, equation (A6). Hence the result for C_0 is the same as that given in equation (A15),

$$C_0 \simeq -\frac{\varphi \zeta p}{\lambda}$$
 (B2)

The terms in T⁰ (i.e. no T dependence) are

$$\frac{D}{\epsilon} \frac{\partial^2 C_1}{\partial \zeta^2} + \frac{\zeta}{2} \frac{\partial C_1}{\partial \zeta} - C_1 = -\frac{V_0 \Phi p}{\epsilon \lambda} - \Phi \quad . \tag{B3}$$

This is very similar to (A16), but with an extra constant term on the right hand side. In Appendix A the full solution for equation (A16) was found as (A19), from which

$$C = A_{1} \left(\frac{1}{2} + \frac{\zeta^{2}}{4A} \right) \cdot \int_{0}^{\zeta} e^{\frac{-\zeta'^{2}}{4A}} d\zeta' + \frac{1}{2} A_{1} \zeta e^{\frac{-\zeta^{2}}{4A}} - \frac{\Phi}{2A} \zeta^{2} \left(1 + \frac{V_{0}p}{\epsilon\lambda} \right) , \qquad (B4)$$

since C(0) = 0. If, again, the initial condition is taken to be the steady-state solution (4.2) in the absence of V_0 , then the large negative ζ behaviour of (B4) must be $-\zeta^2 \phi/2A$ as for C_1 in (A20). This determines the constant A_1 as

$$A_1 = -\frac{2V_0 \Phi p}{\lambda \sqrt{\epsilon D \pi}} \quad . \tag{B5}$$

The first two terms are

$$C = C_0(\zeta) \cdot T^{\frac{1}{2}} + C_1(\zeta) \cdot T$$
(B6)

and were plotted earlier as figure 4.6. The last two terms of (B4) are time independent, but the first two terms illustrate the early time dependence. Further terms are required to improve the approximation for larger times and depths, but it is unlikely to offer any additional advantage to the numerical solutions of the full equation.

Appendix C: The Laplace Transform method for solving a time dependent differential equation

The problem is to calculate the resulting development in the concentration, due to a constant velocity V_0 imposed at time t=0. The equation under consideration is again the radon concentration equation (4.1), namely

$$\frac{D}{\epsilon} \cdot \frac{\partial^2 C}{\partial z^2} - \frac{1}{\epsilon} \cdot \frac{\partial (\nu C)}{\partial z} - \lambda C + \phi = \frac{\partial C}{\partial t} \quad . \tag{C1}$$

Taking a Laplace Transform of this, with respect to time and with parameter s, gives

$$\frac{D}{\epsilon} \frac{\partial^2 \overline{C}}{\partial z^2} - \frac{V_0}{\epsilon} \frac{\partial \overline{C}}{\partial z} - (\lambda + s)\overline{C} = -\frac{\Phi}{s} - F(0)$$
(C2)

where F(0) is the solution to the radon concentration before the velocity V_0 is 'switched on'. Trying $e^{\alpha z}$ as the complementary function implies that α is given by

$$\alpha = \pm \frac{V_0}{2D} \pm \sqrt{\left(\frac{V_0^2}{4D^2} + \frac{\epsilon}{D}(\lambda + s)\right)} \quad . \tag{C3}$$

Since in these problems z is always taken to be negative then the negative sign leads to divergence as z tends to minus infinity, so that only the positive sign can apply. The particular integral to (C2) depends on the form of F(0) assumed. In the simplest case it is zero, and then a particular integral is

$$\overline{C} = \frac{\Phi}{s(\lambda + s)} \quad . \tag{C4}$$

The solution to the transformed concentration is found by combining the two parts of the solution, and using the fact that C and its transform are both 0 at z=0, to give

$$\overline{C} = \frac{\Phi}{s(\lambda + s)} \left(1 - e^{\alpha z}\right)$$
(C5)

where z is always negative and α positive as defined above.

In the more realistic case F(0) will be given by the solution for the steady state conditions as found earlier, eq (4.2). Then the equation becomes

$$\frac{D}{\epsilon} \frac{\partial^2 \overline{C}}{\partial z^2} - \frac{V_0}{\epsilon} \frac{\partial \overline{C}}{\partial z} - (\lambda + s)\overline{C} = -\frac{\Phi}{s} - \frac{\Phi}{\lambda} \left(1 - \exp\left(z\sqrt{\frac{\epsilon\lambda}{D}}\right)\right) \quad . \tag{C6}$$

The complementary function is the same as before. The first two parts of the right handside are matched by $\Phi / \lambda s$ leaving just the last term with the exponential. Trying a multiple of the exponential as the particular integral satisfies the equation when it is multiplied by a term which simplifies to

$$\overline{C} = B' \exp\left(z\sqrt{\frac{\epsilon\lambda}{D}}\right) \quad \text{where} \quad B' = -\frac{\Phi}{\lambda}\left(\frac{1}{V_0\sqrt{\lambda / D\epsilon} + s}\right) \quad . \tag{C7}$$

Hence combining this with the complementary function gives

$$\overline{C} = E \exp(\alpha_z) + \frac{\Phi}{\lambda s} - \frac{\Phi}{\lambda} \left(\frac{\exp(z\sqrt{\epsilon\lambda / D})}{V_0\sqrt{\lambda / D\epsilon} + s} \right) , \qquad (C8)$$

where α is given by (C3) with the positive sign. Because C=0 on z=0, the transform is also zero for z=0, giving us the constant E, so that the transformed concentration is

$$\overline{C} = \frac{\Phi}{\lambda} \left[\exp(\alpha z) \cdot \left(\frac{-V_0 \sqrt{\lambda / D\epsilon}}{s (V_0 \sqrt{\lambda / D\epsilon} + s)} \right) + \frac{1}{s} - \left(\frac{\exp(z \sqrt{\epsilon \lambda / D})}{V_0 \sqrt{\lambda / D\epsilon} + s} \right) \right] \quad . \tag{C9}$$

Writing J for V_0 . $(\lambda/D\epsilon)^{4}$ and grouping the terms on the right brings this to a form which is essentially very close to that for the simpler case, i.e.

$$\overline{C} = \frac{\Phi}{\lambda} \left[\exp(\alpha z) \cdot \left(\frac{-J}{s \ (J+s)} \right) + \left(\frac{J + s(1 - \exp(z\sqrt{\epsilon\lambda / D}))}{s \ (J+s)} \right) \right] \quad . \tag{C10}$$

The problem now is how to transform these back to give the required solution C. For both the simpler solution with F(0) = 0, and the more realistic form given as (C10), the chief problem is the inverse transform of the term $exp(\alpha z)$ because it involves the exponential of a root of s within the term α .

The inverse Laplace Transform integral is defined as

$$C = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \overline{C}(s,z) \ e^{st} \ ds \quad .$$
 (C11)

The γ is chosen so that all of the "singularities" of the integrand are to its left. To find an integral of this form usually involves finding the sum of the residues, using the Cauchy Integral Theorem, eg [Boas 83], p590, and integrating around a path which takes account of any branch point.

The Residue Theorem states that the integral around a closed loop is [Carslaw 59]

$$C = 2\pi i \Sigma$$
 residues within curve , (C12)

provided that the only singularities of the function within the region C are poles.

Initial condition no radon

Now we can use (C11) in our cases, first for the simpler case where the radon concentration is initially zero. The solution will be the sum of the residues coming from the terms from (C5) above, multiplied by the e^{st} term. These must be found at the two poles, s = 0, and $s = -\lambda$. Using [Boas 83] p 598, the residue is given by

$$R(z_0) = f(z_0) \cdot (s - z_0)$$
 (C13)

Here the function is given by (C5), expanded as

$$\overline{C} = \frac{\Phi}{s(\lambda + s)} \left(1 - e^{\left(\frac{zv_0}{2D}\right)} \cdot \exp\left(z\sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon}{D}(\lambda + s)}\right) \right) \quad . \tag{C14}$$

This then gives the residue at s = 0 as

$$R(0) = \frac{\Phi}{\lambda} \left[1 - \exp z \left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon\lambda}{D}} \right) \right]$$
(C15)

while for $s = -\lambda$ it is

$$R(-\lambda) = -\frac{\Phi}{\lambda} \left[1 - \exp z \left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2}} \right) \right] e^{-\lambda t} \quad .$$
(C16)

This simplifies further, to

$$R(-\lambda) = -\frac{\Phi}{\lambda} \left[1 - \exp\left(\frac{zV_0}{D}\right) \right] e^{-\lambda t} \quad . \tag{C17}$$

Hence in the simpler case with F(0) = 0 we can find the sum of the residues, and hence the result for the closed curve. However in both cases there is also a branch point to take account of, which makes the solution more difficult.

Because of the form of α in the expression for the inverse of C there is a point, p, where (s+p) changes sign. Because the root of this term is taken there is a branch point at s = -p and account must be taken of this in the integration. The problem can be avoided by integrating along the path shown in figure C1, where the branch line is avoided in the integration, which follows the path abcdefga.



Figure C1: Argand diagram with cut line

The part of the integration path needed is from g to b, and is referred to as I. The integrals from b to c and f to g will go to zero as the points are taken towards infinity. The integral round the circle d to e goes to zero as the radius tends to zero. This leaves the remaining parts from c to d and e to f along with the residual within the closed curve. Hence I is defined by

$$I + \int_{-\infty}^{-p} \overline{C}e^{st}ds + \int_{-p}^{-\infty} \overline{C}e^{st}ds = 2\pi i \Sigma res .$$
(C18)

The process of tackling the integral is almost identical for both forms of the inverse of C, as defined by (C5) and (C10). This is because they both contain the same term α . Here we will continue with the simpler form, but it is no different to use the other form. The terms not including α are not included, as they cannot contribute to this integral. The full expression (C5) can be made shorter using temporary groups of parameters

$$\overline{C} = \frac{-\phi e^{\frac{V_{02}}{2D}}}{s(\lambda + s)} e^{z\left(\frac{\epsilon}{D}\right)^{V_{1}} \cdot \left(\frac{V_{0}^{2}}{4D\epsilon} + \lambda + s\right)^{V_{2}}} = K_{1}(z) \cdot \frac{e^{K_{2}(z) \cdot (p+s)^{V_{1}}}}{s(\lambda + s)}$$
where $K_{1}(z) = -\phi e^{\frac{V_{0}z}{2D}}$, (C19)
 $K_{2}(z) = z\left(\frac{\epsilon}{D}\right)^{V_{2}}$ and
 $p = \frac{V_{0}^{2}}{4D\epsilon} + \lambda$.

The expression for the case where F(0) is not 0 is of exactly the same form, but the constant terms change. The important point is the argument of the exponential of $(p+s)^{44}$. Just above the negative x axis of the argand diagram, figure C1, we define it as $+\pi$ while just below it is $-\pi$. Then when the square root is taken the two points come out with opposite sign, because $e^{i\pi/2}$ and $e^{-i\pi/2}$ give i and -i respectively. If we insert the expression from (C19) into (C18) and use the square root results we have

$$I + \int_{-\infty}^{-p} K_{1}(z) \cdot \frac{e^{iK_{2}(z) \cdot |p+s|^{\frac{1}{2}}}}{s(\lambda + s)} e^{st} ds + \int_{-p}^{-\infty} K_{1}(z) \cdot \frac{e^{-iK_{2}(z) \cdot |p+s|^{\frac{1}{2}}}}{s(\lambda + s)} e^{st} ds = 2\pi i \Sigma res \quad . \quad (C20)$$

This simplifies further to

$$I + K_{1}(z) \int_{-\infty}^{-p} \frac{2i \sin \left(K_{2}(z) \cdot |p+s|^{0.5}\right) e^{st} ds}{s(\lambda + s)} = 2\pi i \Sigma res .$$
(C21)

Substituting s for -u makes the result look better formulated, dividing through by 2i, and

removing the modulus signs as they are no longer needed, and inserting the results for the residues

$$\frac{I}{2\pi i} = C = \frac{K_1(z)}{\pi} \int_p^{\infty} \frac{\sin\left(K_2(z).(u-p)^{0.5}\right) e^{-ut} du}{u(\lambda - u)} + \frac{\Phi}{\lambda} \left[1 - \exp\left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon\lambda}{D}}\right)\right] - \frac{\Phi}{\lambda} e^{-\lambda t} \left[1 - \exp\left(\frac{zV_0}{D}\right)\right] .$$
(C22)

This fully defines the result for the concentration C. Unfortunately although the integral is well defined, it cannot be integrated exactly. It could be found numerically but this is not particularly useful.

Flux

However more progress is possible on the flux, which comes from the gradient of the concentration. The total flux at any point is a combination of diffusion and pressure driven flow. This can be written as

$$Flux \ J = -D \ . \ \frac{\partial C}{\partial z} + v \ . \ C \quad . \tag{C23}$$

However we are most concerned with the flux at the surface, which is simpler because we generally define the surface concentration as 0, so only the diffusive flux is needed.

The z differentials of the two residual terms give

$$\frac{\partial}{\partial z}R(0) = -\frac{\Phi}{\lambda} \left[\left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon\lambda}{D}} \right) \exp z \left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon\lambda}{D}} \right) \right]$$
(C24)

and

$$\frac{\partial}{\partial z}R(-\lambda) = \frac{\Phi}{\lambda} \frac{V_0}{D} e^{\frac{zV_0}{D}} e^{-\lambda t} .$$
(C25)

At z=0 they are

$$\frac{\partial}{\partial z} \Sigma R(z=0) = -\frac{\Phi}{\lambda} \left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon\lambda}{D}} - \frac{V_0}{D} e^{-\lambda t} \right) .$$
(C26)

The integral term is slightly more complicated, but because sin(z) is 0 at z=0, the term which includes dK_1 / dz from the differentiation disappears, and the cos term is 1, so from equation (C22) the differential becomes

$$\frac{\partial C}{\partial z} = \frac{K_1(z)}{\pi} \int_p^{\infty} \frac{\frac{dK_2(z)}{dz} \cdot (u-p)^{0.5} e^{-ut} du}{u(\lambda - u)} .$$
(C27)

Substituting for K_1 and K_2 at z = 0 gives

$$\frac{\partial C}{\partial z}\Big|_{z=0} = -\left(\frac{\epsilon}{D}\right)^{0.5} \frac{\Phi}{\pi} \int_{p}^{\infty} \frac{(u-p)^{0.5} e^{-ut} du}{u(\lambda - u)}$$
(C28)

Then splitting the integral into two parts using partial fractions

$$\frac{\partial C}{\partial z}\Big|_{z=0} = -\left(\frac{\epsilon}{D}\right)^{0.5} \frac{\Phi}{\lambda \pi} \left[\int_{p}^{\infty} \frac{(u-p)^{0.5} e^{-ut} du}{u} + \int_{p}^{\infty} \frac{(u-p)^{0.5} e^{-ut} du}{(\lambda - u)}\right] .$$
(C29)

The first integral is in a form given in tables [Gradstheyn 80] page 315 no 3.363 1, while the second one is of the same form after substituting $u = v + \lambda$.

$$\frac{\partial C}{\partial z}\Big|_{z=0} = -\left(\frac{\epsilon}{D}\right)^{0.5} \frac{\phi}{\lambda \pi} \left[\int_{p}^{\infty} \frac{(u-p)^{0.5} e^{-ut} du}{u} + \int_{p-\lambda}^{\infty} \frac{(v-(p-\lambda))^{0.5} e^{-vt} e^{-\lambda t} dv}{-v} \right] \quad . (C30)$$

The integral is given as

$$\int_{p}^{\infty} \frac{(u-p)^{0.5} e^{-ut} du}{u} = \sqrt{\frac{\pi}{t}} \cdot e^{-pt} - \pi \sqrt{p} \cdot (1 - erf(\sqrt{pt})) \quad .$$
(C31)

While for the second integral p must be replaced by $p-\lambda$, which is $V_0^2/4D\epsilon$. Using this we can write the complete solution for the concentration gradient at z=0 as

$$\frac{\partial C}{\partial z}|_{z=0} = -\left(\frac{\epsilon}{D}\right)^{0.5} \frac{\Phi}{\lambda \pi} \left[\sqrt{\frac{\pi}{t}} e^{-\left(\frac{V_0^2}{4D\epsilon} + \lambda\right)t} - \pi \sqrt{\left(\frac{V_0^2}{4D\epsilon} + \lambda\right)} \left[1 - erf\left(\sqrt{\left(\frac{V_0^2}{4D\epsilon} + \lambda\right)t}\right) \right] \right] \\ + \left(\frac{\epsilon}{D}\right)^{0.5} \frac{\Phi}{\lambda \pi} e^{-\lambda t} \left[\sqrt{\frac{\pi}{t}} e^{\frac{-V_0^2 t}{4D\epsilon}} - \pi \sqrt{\frac{V_0^2}{4D\epsilon}} \cdot \left(1 - erf\left(\sqrt{\frac{V_0^2}{4D\epsilon}t}\right) \right) \right] \\ - \frac{\Phi}{\lambda} \left(\frac{V_0}{2D} + \sqrt{\frac{V_0^2}{4D^2} + \frac{\epsilon\lambda}{D}} - \frac{V_0}{D} e^{-\lambda t} \right] C_{32}$$

We see on looking at the limiting cases of this, that at time t = 0 the gradient is zero as all the terms cancel, as expected. As t tends to infinity, only terms from the third line remain as the erf terms tend to one and therefore disappear, as do all of the exponential terms. These then match the steady state solution given by differentiating (4.4) and setting z=0.

Applying this solution

This solution allows the calculation of the development of radon concentration with time, using a numerical integration, but more usefully the development of the radon flux at the surface can be found. Figure 4.7 shows the radon concentration gradient at the surface developing with time. In this case it takes a considerable time to reach the steady state value.

Conclusions

The work presented in this thesis covers several different modelling and experimental studies carried out by the author as part of the team at the Building Research Establishment investigating how to protect buildings against soil gases. It has addressed the issues under three separate headings:

- Flow due to natural driving forces,
- High pressure flows, and
- Time dependent effects.

Each of these separate parts has its own conclusions, so this more specific material is not repeated here.

For each part a combination of modelling techniques have been applied to the problems considered and experiments analysed. The modelling techniques used varied from simple analytical models through more advanced analytical techniques to numerical solutions. Some of these developed directly from the work of others, but many are new to the soilgas field. Most of the experiments were carried out by the author or under his direct supervision, but others were being carried out by colleagues at BRE for other reasons, and proved useful here as well. Further, some work has been developed directly from the work of other researchers elsewhere.

It is difficult to assess the real level of contribution of this type of work. This is because the direct applications are generated by the team as a whole, and the key question is, "How much benefit is derived from the process of modelling and experimental investigation?"

This is hard to quantify, as it comes in the form of increased understanding of the processes involved in soil gas flow, and explanations of the phenomena which are observed on site. It would be good to be able to say that a new technique for removing radon from homes developed from this work, but it would not be true! However we as a team have continued to improve our advice to the Government and the public, and have

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published reports on many aspects of how to protect buildings from soil gases.

Particular areas developed in this work were:

- The flow rates generated by 'sumps'.
- Where the flow from a sump comes from, and hence their associated energy costs.
- A technique for measuring the leakage of the substructure of a house (under some conditions).
- The ease of air flow through different hard core materials.
- How pressure extension tests can be used in testing floors for air flow.
- The way in which changing atmospheric pressure affects soil gas.
- The techniques used for monitoring the flow of gas from soil.

In all of these areas there have been some new developments presented here, sometimes in the form of results specific to the UK, i.e. our floor type or our hard core materials, but in general these represent a small step forward in some part of the overall understanding of soil gas flow.

The future

In the UK at least the main issues relating to radon gas seem to be understood, and we have a good record of reducing radon levels in houses. This means the main steps need to address locating houses with high levels and ensuring their cost effective treatment. This has been reflected in the change of the BRE work programme, with more bias towards advice work, and less on the fundamentals. Remaining areas of interest will mainly concern reducing the cost of remedial actions, and the use of lower power fans in sump systems is a particular interest.

In the European context however there is a lot more need for progress. The UK, along with Sweden, are well ahead of the rest of Europe in carrying out radon work on houses. This means we have a lot of experience which we can share with our partners, as they address the particular issues in their countries. These are caused by the significantly different building styles in different countries, and the climate differences too.

The landfill gas side is less advanced, but as the pressures on building land continue into the next century it is likely to become more of an issue. At present housing developments are not allowed on landfill sites because it cannot be guaranteed that the householders will look after the measures needed to stop any hazard. Hence only certain types of controlled developments are allowed on these sites. Although this restriction is unlikely to change there is the possibility of building near to landfill sites, if good, cheap measures can be designed.

However the main area for current progress is that of continuing to understand what the measurements made on landfill sites mean. There is considerable variation in what is meant by particular measurements made on a landfill site, and the way in which these are interpreted varies considerable. More modelling studies will help in the interpretation of measurements, and can hopefully help towards better and cheaper information.

This point leads on to the need for more work on time dependent modelling. At present radon measurements need to be taken over a long time, usually three months, to obtain a satisfactory average value. This leads to considerable delay between the decision to investigate for radon and any action being taken. It is a particular problem because it cannot be incorporated into the house buying process as it would cause an unreasonable delay. However it is likely that a better understanding of time dependent effects and the way that these interact with the weather should lead to the option to use shorter term measurements combined with weather data and a model. This could be a useful tool to encourage wider take up of measurements, especially if it can be made cheap enough to be within the house buying chain.

Another area for consideration is the use of preformed plastic 'void formers' just below the floor of a building to improve the ventilation there. These have been used successfully for water drainage, but have not been studied as thoroughly for radon or landfill gas.

It is interesting that the experiments on hard core materials did not show a particular

preference for any material, suggesting that it is not the crucial factor in sump performance. This deserves further study to try to come to a full understanding of what the main factors are which cause this, and whether these can be used to improve the protection of new buildings against soil gas.

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