GENERALIZED TWO-REGION MODEL FOR INFILTRATION STUDIES

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SUMMARY

In using tracer tests to measure infiltration, it is important to take into account non-ideal mixing in the building. This paper proposes to do this using a two-region model.

air change

A result of non-ideality is that the ~~net infiltration~~ rate, and hence the energy consumption due to infiltration, will usually not be that calculated from the rate of inflow/outflow. The use of the model is illustrated by an example in which the model parameters are deduced both from physical reasoning and from a curve-fitting procedure.

With the current interest in energy conservation, much attention has recently been paid to the monitoring and reduction of air infiltration into buildings. An international agency, the Air Infiltration Centre, has been founded to help co-ordinate world research on this subject. Rapid advances have been made in experimental tracer techniques for measuring infiltration, such as the use of sulphur hexafluoride tracer [1] and the development of automatic tracer injection analysis equipment [2].

Nevertheless the theoretical basis for infiltration studies remains rudimentary. Without exception departures from ideality have been ignored, the experimenters assuming that the buildings concerned behave as ideally well mixed systems (or continuous flow stirred tank reactors, CSTR, in chemical engineering terminology). Jenning and Armstrong [3] introduced a "mixing factor" to take imperfect mixing into account when calculating contaminant dilution. Sinden [4] outlined the general solution for tracer concentrations in multi-chamber systems. These methods have not been applied to experimental infiltration studies. Yet it is known that even small non-idealities such as deadwater zones can have a disproportionate effect on the concentration decay curve [5]. For example, a cupboard in a house may slowly release tracer gas and so slow down the measured tracer decay rate. The CSTR assumption limits the use of tracer gas techniques to systems with low infiltration and little internal partitioning. Efforts to improve internal mixing for test purposes may themselves cause errors as the internal recirculation system may increase infiltration [6]. Even if this does not happen it will be seen that simply changing the mixing characteristics will distort the results.

Many models have been proposed to represent non-ideal mixing in chemical reaction engineering [5, 7]. The results obtained are not always directly applicable to infiltration studies for the following reasons:

1. In chemical reactor engineering, inlet and outlet streams are clearly identifiable, while in infiltration studies they are usually diffuse and often impossible to locate.

2. In chemical reactor engineering, tracers are usually injected into inlet streams and monitored at outlet streams. In infiltration studies tracers are injected directly into the space concerned and also monitored inside that space. Thus, quantities such as residence-time distribution and mean exit-stream concentrations are of little interest to infiltration research, while volume-mean concentrations are of primary importance.

This paper proposes a general model for non-ideal mixing in enclosed spaces, in the context of infiltration studies. To keep the number of model parameters to a minimum, the space is assumed to consist of only two well-mixed regions (or CSTRs). Although models with more than two regions may be more realistic [4], the number of parameters involved would be so large that such models would be useless for practical purposes.

Even with the two-region model the number of parameters usually will be too large for interpreting the available data, and further simplifications will be considered. Such models will help determine the type and degree of non-ideality present. Even when it is not possible to quantify precisely the parameters involved, these models will give experimenters an insight into what is happening and help in the design of their tests.

THE GENERALIZED TWO-REGION MODEL

(MODEL A)

The enclosed space being considered is assumed to consist of two well mixed regions (Fig. 1). In the most general case possible, fluid may flow between the two regions and the environment in any manner, provided steady state and mass balance are maintained. From Fig. 1 we can write for the mass balance:

$$\text{Region 1:}\quad aV\,\frac{dC_1}{dt} = -(b-d+f)vC_1 + (f-d)vC_2 \tag{1}$$

$$\text{Region 2:}\quad (1-a)\,\frac{dC_2}{dt} = f\,v\,C_1 - (1-b+f)vC_2 \tag{2}$$

where v is the net flow rate through the system, V the system volume, t the time, C the concentrations and a, b, d, f are defined in Fig. 1. In dimensionless form:

$$\frac{dc_1}{d\Theta} = -\frac{b-d+f}{a}\,c_1 + \frac{f-d}{a}\,c_2 \tag{3}$$

$$\frac{dc_2}{d\Theta} = \frac{f}{1-a}\,c_1 - \frac{1-b+f}{1-a}\,c_2 \tag{4}$$

where $\Theta = vt/V$ = reduced time

$c = VC/m_o$ = reduced concentration

m_o = Total amount of tracer in system at time 0

This model has four independent parameters, a, b, d and f, whose allowable ranges are:

$a = 0 \text{ to } 1$

$b = 0 \text{ to } 1$

$d = b\text{-}1 \text{ to } b$

$f = 0 \text{ to infinity if } d<0$

$\quad d \text{ to infinity if } d>0$

The general solution to eqns. (3) and (4) is:

$$c_1 = (f-d)\,(k_1\, e^{m_1\Theta} + k_2 e^{m_2\Theta}) \tag{5}$$

$$c_2 = k_1\,(b-d+f+am_1)e^{m_1\Theta} + k_2\,(b-d+f+am_2)e^{m_2\Theta} \tag{6}$$

where k_1, k_2 are constants to be found from the initial conditions and m_1, m_2 are the eigenvalues of the system of equations (3) and (4) and are given by:

$$m_1, m_2 = \frac{a+b-2ab-d+f+ad}{2a(1-a)} \left\{ -1 \pm \sqrt{1 - \frac{4a(1-a)(b-b^2-d+bd+f)}{(a+b-2ab+f-d+ad)^2}} \right\} \tag{7}$$

The reader may check that in the limiting cases

$a = 0$ and $b = 0$

or $a = 1$ and $b = 1$

or $f = \text{infinity}$

we obtain $m_1 = -1$ and $m_2 = -$ infinity, i.e. we obtain a CSTR.

Two useful expressions that may be used in deriving and simplifying subsequent equations are:

$$m_1 + m_2 = \frac{-a-b+2ab+d-f-ad}{a(1-a)} \tag{8}$$

$$m_1 m_2 = \frac{b-b^2-d+bd+f}{a(1-a)} \tag{9}$$

In infiltration studies, a tracer is released into the building. If the inlets and outlets are known (or if at least the main ones, such as open doors, windows, chimneys are known), they may be closed and the tracer is given time to mix to a uniform concentration, then opened again for the test proper. Otherwise good mixing is simply assumed. Samples of the air-tracer mixture are taken either at one particular spot where the air is considered well mixed (e.g., near a fan) or at several places at the same time to give a volume-mean concentration.

In the context of the two-region model there are three types of tests, depending on the mode of tracer release:

1. <u>Type I test:</u> Tracer is released in region 1 at time zero, and immediately reaches a concentration $c_1 = 1/a$ in this region. The concentration in region 2 is $c_2 = 0$ at time zero.

2. <u>Type II test:</u> As in type I except that tracer is released in region 2. At time 0, $c_1 = 0$, $c_2 = 1/(1-a)$.

3. <u>Type III test:</u> After tracer release, all inlets and outlets are sealed (assuming they are known), and the test starts when $c_1 = c_2 = 1$.

In each of these tests, samples may be taken in region 1 only, in region 2 only, or in both regions so as to obtain a volume-mean concentration:

$$c_m = ac_1 + (1-a)c_2 \tag{10}$$

Equations for c_1, c_2 and c_m will now be given. Also of interest is how the volume-mean concentration decay curve compares with that for an ideal CSTR (for which $c_m = e^{-\Theta}$).

Type I Test:

At $\Theta = 0$, $c_{I1} = 1/a$ and $c_{I2} = 0$. Substituting into eqns. (5) and (6), one obtains:

$$c_{I1} = \frac{1}{a(1-a)(m_1-m_2)} \left\{ [1-b+f+(1-a)m_1]e^{m_1\Theta} - [1-b+f+(1-a)m_2]e^{m_2\Theta} \right\} \quad (11)$$

$$c_{I2} = \frac{f}{a(1-a)(m_1-m_2)} \quad (e^{m_1\Theta} - e^{m_2\Theta}) \quad (12)$$

$$c_{Im} = \frac{1}{(m_1-m_2)} \left\{ -(\frac{b-d}{a} + m_2)e^{m_1\Theta} + (\frac{b-d}{a} + m_1)e^{m_2\Theta} \right\} \quad (13)$$

Figure 2 gives typical curves on a log-linear scale of c_{I1}, c_{I2}, c_{Im} and also Y_I (where $Y_I = c_{Im}/c_{Im\ (CSTR)}$). ~~The volume-mean concentration, c_{Im}, at first decreases quickly and then tends to a slower exponential decay as the term containing $e^{m_1\Theta}$ takes over. This is a pattern that will recur in other cases.~~ Also plotted, in dashed lines, are the c and Y curves for an ideally mixed system (CSTR).

Type II Test:

At Θ=0, c_{II}=0 and c_{I2}=1/(1-a). Substituting into eqns. (5) and (6) one gets:

$$c_{II1} = \frac{f-d}{a(1-a)(m_1-m_2)} \quad (e^{m_1\Theta} - e^{m_2\Theta}) \quad (14)$$

$$c_{II2} = \frac{1}{a(1-a)(m_1-m_2)} \left\{ (b+f-d+am_1)e^{m_1\Theta} - (b+f-d+am_2)e^{m_2\Theta} \right\} \quad (15)$$

$$c_{IIm} = \frac{1}{m_1-m_2} \left\{ -(\frac{1-b+d}{1-a} + m_2)e^{m_1\Theta} + (\frac{1-b+d}{1-a} + m_1)e^{m_2\Theta} \right\} \quad (16)$$

These curves have the same general shape as in type I tests (Fig. 2), with c_1 and c_2 interchanged.

<u>Type III Test:</u>

At $\Theta = 0$, $c_{III1} = c_{III2} = 1$. Substituting into eqns. (5) and (6) one gets:

$$c_{III1} = \frac{1}{m_1 - m_2} \left\{ -\left(\frac{b}{a} + m_2\right)e^{m_1\Theta} + \left(\frac{b}{a} + m_1\right)e^{m_2\Theta} \right\} \tag{17}$$

$$c_{III2} = \frac{1}{m_1 - m_2} \left\{ -\left(\frac{1-b}{1-a} + m_2\right)e^{m_1\Theta} + \left(\frac{1-b}{1-a} + m_1\right)e^{m_2\Theta} \right\} \tag{18}$$

$$c_{IIIm} = \frac{1}{m_1 - m_2} \left\{ -(1+m_2)e^{m_1\Theta} + (1+m_1)e^{m_2\Theta} \right\} \tag{19}$$

Dividing eqn. (19) by the ideal CSTR response, $e^{-\Theta}$, we obtain:

$$Y_{III} = \frac{1}{(m_1 - m_2)} \left\{ -(1+m_2)e^{(1+m_1)\Theta} + (1+m_1)e^{(1+m_2)\Theta} \right\} \tag{20}$$

Figure 3 shows typical plots for c_{III1}, c_{III2}, c_{IIIm} and Y_{III}. Also plotted, in dashed lines, are c and Y curves for a CSTR.

It would be interesting to know whether the volume-mean concentration c_{IIIm} decays faster or more slowly than it would if the system were ideally mixed; i.e., whether Y_{III} is greater or less than 1. If $Y_{III} > 1$, the air change ~~net infiltration~~ after a time Θ will be less than for a CSTR, even though gross inflows and outflows are equal. The reverse applies if $Y_{III} < 1$.

To find this out we take the derivative of Y_{III} and simplify using eqns. (8) and (9):

$$\frac{dY_{III}}{d\Theta} = \frac{(a-b)(a-b+d)}{a(1-a)} \left(\frac{e^{m_1\Theta} - e^{m_2\Theta}}{m_1 - m_2}\right) e^{\Theta} \tag{21}$$

It can be seen that the sign of $dY/d\Theta$ is that of $(a-b)(a-b+d) = (a-b)[a-(b-d)]$. Thus, if the inflow fraction b and outflow fraction (b-d) are both greater than, or both less than, the volume fraction a, $dY/d\Theta$ will be positive, and Y will be greater than 1 at all non-zero times (since Y=1 at Θ=0). In other words, the ~~net infiltration~~ air change will be less than for an ideally mixed system for the same gross inflow and outflow. If (a-b) and (a-b+d) are of opposite signs, the reverse will apply. Since a, b and d are fixed parameters for a given situation, Y_{III} cannot cross the line $Y_{III} = 1$ (while Y_I and Y_{II} can - see Fig. 2). In practical terms, this means that the energy loss due to infiltration will be always greater or always less than flow measurements indicate, as the case may be.

TWO-REGION MODEL WITH BALANCED INTERCHANGE
(MODEL B)

The generalized two-region model (model A) is too complex for most purposes and it would be useful to simplify it. By putting d=0 in Fig. 1 a three-parameter model is obtained, model B (Fig. 4). Equations for concentrations and Y-ratios in various test conditions are readily obtainable from eqns. (7) to (20) by letting d=0, and will not be presented here. From eqn. (21) it can be seen that $dY_{III}/d\Theta > 0$, and thus $Y > 1$ for all non-zero times. Thus the ~~net infiltration~~ air change will always be less than for an ideal CSTR, except if a = b when it will be the same.

In practice, model B may apply when a building is partitioned into several portions, each with its doors, windows, cracks, etc., and there is limited interchange between the portions.

TWO-REGION FLOWTHROUGH MODEL
(MODEL C)

If b=1 and d=b we get the two-parameter model in Fig. 5, where the air flows into the first region and out of the second region with (unbalanced) interchange between the two. This model has been considered by Levenspiel [5]. In practice such a situation may arise when a building has an open door (or window, chimney, etc.) at one end, another opening at the other end, and the wind's general direction is lined up with the two openings. It will be also common in many old-fashioned multi-storey cold stores, when there is an open door near the top, another at the floor level, and cold air flows out of the bottom door by gravity. A room being warmed by an open fire with accompanying draft is another example. Yet another example is a high-rise cold store with a large open door: warm air will flow in through the upper part of the door and collect near the ceiling (region 1), while cold air flows out through the lower part of the door. If mixing is not thorough, stratification will set in and there will be distinct hot and cold layers.

The eigenvalues m_1, m_2 are found by putting b=d=1 in eqn. (7):

$$m_1, m_2 = \frac{f}{2a(1-a)} \left\{ -1 \pm \sqrt{1 - \frac{4a(1-a)}{f}} \right\} \tag{22}$$

Doing the same to eqns. (11) to (20) will give expressions for c_{I1}, c_{I2}, c_{Im}, c_{II1}, c_{II2}, c_{IIm}, c_{III1}, c_{III2}, c_{IIIm} and Y_{III}.

Eqn. (21) becomes:

$$\frac{dY_{III}}{d\Theta} = -\left(\frac{e^{m_1\Theta} - e^{m_2\Theta}}{m_1-m_2}\right)e^{\Theta} \tag{23}$$

air change

Hence Y_{III} is less than 1 at all non-zero times. In other words, the ~~net infiltration~~ is greater than for an ideal CSTR at a given rate of inflow/outflow.

TWO-REGION MODEL WITH DEADWATER REGION (MODEL D)

If b=1 and d=0, we obtain the two-parameter model shown in Fig. 6, which is a particular case of model B. This model has been considered by Bischoff and Dedrick [8]. Region 2 is now a "deadwater" zone, which is not directly connected with the outside.

This model may apply when the infiltration happens mainly through a single opening or several closely situated openings and there are walls, cupboards, furniture, products, etc. that create dead regions exchanging tracer slowly with the rest of the building.

Equation (7) now becomes:

$$m_1, m_2 = \frac{1-a+f}{2a(1-a)} \left\{ -1 \pm \sqrt{1 - \frac{4af\,(1-a)}{(1-a+f)^2}} \right\} \tag{24}$$

and putting b=1, d=0 in eqns. (11) to (20) will give expressions for c_{I1}, c_{I2}, c_{Im}, c_{II1}, c_{II2}, c_{IIm}, c_{III1}, c_{III2}, c_{IIIm} and Y_{III}.

Since this model is a particular case of model B, Y_{III} will be positive for all non-zero times, and the ~~net infiltration~~ air change will be less than for the ideal CSTR for a given rate of inflow/outflow.

A special situation of interest is when the factor a tends to zero, in which case:

$$c_{II2} = c_{IIm} = c_{III2} = c_{IIIm} = e^{-\frac{f}{1+f}\Theta} \qquad (25)$$

$$Y_{II} = Y_{III} = e^{\frac{\Theta}{1+f}} \qquad (26)$$

for non-zero times.

Physically this means that a small local recirculation zone or "airlock" occurs near the opening (region 1), so that the entering and leaving air streams are mixed together, thus reducing the effective air interchange by a constant factor f/(1+f). This gives a physical explanation for Jenning and Armstrong's "mixing factor" [3]. In tests on cold stores with an open door we have observed reduction factors down to 0.50, corresponding to f=1.

TWO-REGION MODEL WITHOUT INTERCHANGE (MODEL E)

If f=d=0, we obtain the model shown in Fig. 7, which is another particular case of model B. Fluid flows in two separate streams through the two regions, which do not interact. This situation may happen in practice when the building is separated into distinct zones (rooms, levels, etc.,) each with its own openings.

Equations for various concentrations are most easily obtained by direct solution of eqns. (3) and (4) (simplified as required):

Type I Test:

$$c_{I1} = \frac{1}{a} e^{-\frac{b}{a}\Theta} \qquad (27)$$

$$c_{I2} = 0 \qquad (28)$$

$$C_{Im} = e^{-\frac{b}{a}\Theta} \qquad (29)$$

Type II Test:

$$C_{II1} = 0 \qquad (30)$$

$$C_{II2} = \frac{1}{1-a} e^{-\frac{1-b}{1-a}\Theta} \qquad (31)$$

$$C_{IIm} = e^{-\frac{1-b}{1-a}\Theta} \qquad (32)$$

Type III Test:

$$C_{III1} = e^{-\frac{b}{a}\Theta} \qquad (33)$$

$$C_{III2} = e^{-\frac{1-b}{1-a}\Theta} \qquad (34)$$

$$C_{IIIm} = a\,e^{-\frac{b}{a}\Theta} + (1-a)e^{-\frac{1-b}{1-a}\Theta} \qquad (35)$$

Since this model is a particular case of model B, Y is greater than 1 (except when a=b) for all non-zero times, and so the ~~net infiltration~~ air change will be less than for the ideal CSTR when the rate of inflow/outflow is the same.

(Strictly speaking Type III test is meaningless in the context of model E, since no mixing takes place between the two region. However, this model can be used as a limiting case when f is non-zero but much smaller than 1.)

PRACTICAL APPLICATIONS

To apply the foregoing models to a practical case, one should first of all try to understand the physical situation as far as possible, so that a proper model can be selected and perhaps some of the parameters quantified. Only then will it be appropriate to carry out curve fitting procedures, using tracer concentration data, to find the other parameters.

For curve fitting purposes, one must plot the relevant quantity as given by the above equations (C_{I1}, C_{I2}, C_{Im}, Y, etc., as the case may be) against the time Θ. If the rate of inflow/outflow, v, is not known, as is generally the case, then it would be best to plot this quantity for various values of the parameters on a log-log scale. The experimental data (also plotted on a log-log scale) can then be shifted up, down or across until the best fit is obtained [8]. Alternatively, if a computer is at hand, a non-linear regression using hill-climbing techniques can be carried out.

If the rate of inflow/outflow is known, for example when all the inflow (or outflow) happens at one particular opening and can be measured, then a log-linear plot (log of C or Y vs time Θ) will be more convenient, as all curves will then tend to straight lines as Θ tends to larger values.

Since most readers will have access to calculators or computers to help them make up these plots, they will not be presented here (as there are many possible combinations). Instead an actual case will be used to illustrate the technique.

<u>Example</u>: Sulphur hexafluoride tracer was used to measure the ~~net infiltration~~ air change rate into a meat cold store when the door was opened [9]. The store was of the single-storey high-stud type with a gross volume of 4080 m³ and a net volume

(gross volume - product volume) of 2983 m^3. Air flowed in through the top half of the door and out the bottom half, at a rate measured at 3.0 m^3/s (3.6 air changes per hour). The tracer test consisted of releasing the tracer, closing the door until a uniform tracer concentration was obtained, taking samples at various spots in the store, opening the door to start the test, closing it after a given time (5, 10, 15, 20 or 30 min), and then waiting again for equilibrium before sampling. Thus, this was a type III test and the concentration being measured was c_{IIIm}. Table 1 shows the test durations and measured air changes.

In Table 1 the reduced time Θ is the same as the air change for CSTR, since for the latter:

$$c_{IIIm} = e^{-\Theta}$$

and air change $= -\ln (c_{IIIm})$.

In Table 1 Y_{III} is simply the difference between the air change for CSTR and the measured air change (column 4 - column 3).

<u>Solution:</u> We will limit ourselves to one of the two-parameter models (C, D or E) in view of the small number of data. Since there is only one main inlet and outlet, model E can be eliminated. Also, since Y_{III} increases with time, model C can be eliminated. We are therefore left with model D (two-region model with deadwater).

Equation (20) for Y_{III} was plotted in the range $\Theta = 0$ to 4 (Fig. 8) with:

$a = 0.2, 0.5, 0.9$

$f = 0.1, 0.3, 1.0$

Since v/V is known, Θ can be calculated for all data, and so we plot ln Y_{III} vs Θ. We then plot the experimental data on the same figure. It can be seen that the data are best fitted by:

a = 0.5

f = 0.1

The physical interpretation of this result is that the product, which is stacked on pallets, tends to partition the rooms and create "dead" pockets of air that exchange tracer very slowly with the main zone.

The curve fitting procedure in this case is quite insensitive to the measured value of the rate of inflow/outflow. An error of 10% in the latter makes hardly any difference in a and f (Fig. 8).

CONCLUSION

Non-ideality in mixing must be taken into account when one measures the infiltration rate. Unfortunately this factor has been ignored by previous workers in this field and this has limited the scope of tracer testing. A good log-linear plot for the tracer concentration is no proof that the system is well mixed: as can be seen from Figs. 2 and 3 and eqns. (11) to (19), all the concentration curves will tend to an exponential decay anyway as the time becomes large, but that final decay (term containing $e^{m_1\Theta}$ where m_1 is the larger, i.e. less negative, eigenvalue) is mainly influenced by the more stagnant region, even though the actual volume of that region may be quite small, e.g., the volume of a closet or cupboard.

Also, from a practical point of view, it is important to know in which part of the building the infiltration is happening. For example, a large air interchange in the attic is clearly less important than one in the living area. Our simplified models and curve-fitting procedures will help in the determination of these relative infiltrations. The tracer-response approach can be complementary to recent attempts to model multi-zone situations with fluid mechanics, using computers [10].

Finally, an interesting and potentially important result of this paper is that the ~~net infiltration~~ air change after a period of time may be quite different from what could be expected from the values of inflow/outflow: it may be larger (model C), in which case more energy may be required than expected, or smaller (models B, D and E), in which case less may be required than expected. In general, the energy (heat or refrigeration) load will be intermediate between that estimated by a tracer test and that deduced from direct inflow/outflow measurements, since heat may be transferred to or from the infiltration air both by mixing with the air inside and by contact with walls, furniture, people, products, and so forth.

NOTATION

a	volume of region 1 as a fraction of total volume V
b	flow into region 1 as a fraction of total inflow v
c	dimensionless tracer concentration (VC/m_o)
C	tracer concentration, kg/m^3
CSTR	ideally well mixed system (short for continuous-flow stirred-tank reactor)
d	difference between inflow into and outflow from region 1, as a fraction of total inflow v
f	flow from region 1 to region 2, as a fraction of total inflow v
k_1, k_2	constants
m_o	mass of tracer in system at time 0, kg
m_1, m_2	eigenvalues of the system of equations (3) and (4)
t	time after test starts, s
v	total flow into or out of system, m^3/s
V	total volume of system, m^3
Y	ratio of volume-mean concentration of system to that of ideal CSTR
Θ	dimensionless time (vt/V)

Subscripts

1	concentration in region 1
2	concentration in region 2
m	volume-mean concentration of system
I	type I test (response to pulse in region 1)
II	type II test (response to pulse in region 2)
III	type III test (response to initially well-mixed tracer injection)

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List of Figures

List of Tables

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Generalized Two-region Model for Infiltration Studies

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NOMENCLATURE

- a volume of region 1 as a fraction of total volume V
- b flow into region 1 as a fraction of total inflow v
- c dimensionless tracer concentration (VC/m_0)
- C tracer concentration, kg/m^3
- CSTR ideally well-mixed system (short for continuous-flow stirred-tank reactor)
- d difference between inflow into and outflow from region 1, as a fraction of total inflow v
- f flow from region 1 to region 2, as a fraction of total inflow v
- k_1, k_2 constants
- m_0 mass of tracer in system at time 0, kg
- m_1, m_2 eigenvalues of the system of equations (3) and (4)
- t time after test starts, s
- v total flow into or out of system, m^3/s
- V total volume of system, m^3
- Y ratio of volume-mean concentration of system to that of ideal CSTR
- θ dimensionless time (vt/V)

Subscripts

- 1 concentration in region 1
- 2 concentration in region 2
- m volume-mean concentration of system
- I type I test (response to pulse in region 1)
- II type II test (response to pulse in region 2)
- III type III test (response to initially well-mixed tracer injection)

INTRODUCTION

WITH the current interest in energy conservation, much attention has recently been paid to the monitoring and reduction of air infiltration into buildings. An international agency, the Air Infiltration Centre, has been founded to help coordinate world research on this subject. Rapid advances have been made in experimental tracer techniques for measuring infiltration, such as the use of sulphur hexafluoride tracer [1] and the development of automatic tracer injection and analysis equipment [2].

Nevertheless the theoretical basis for infiltration studies remains rudimentary. Without exception departures from ideality have been ignored, the experimenters assuming that the buildings concerned behave as ideally well-mixed systems (or continuous-flow stirred-tank reactors, CSTR, in chemical engineering terminology). Jenning and Armstrong [3] introduced a 'mixing factor' to take imperfect mixing into account when calculating contaminant dilution. Sinden [4] outlined the general solution for tracer concentrations in multi-chamber systems. These methods have not been applied to experimental infiltration studies. Yet it is known that even small non-idealities such as deadwater zones can have a disproportionate effect on the concentration decay curve [5]. For example, a cupboard in a house may slowly release tracer gas and so slow down the measured tracer decay rate. The CSTR assumption limits the use of tracer gas techniques to systems with low infiltration and little internal partitioning. Efforts to improve internal mixing for test purposes may themselves cause errors as the internal recirculation system may increase infiltration [6]. Even if this does not happen it will be seen that simply changing the mixing characteristics will distort the results.

Many models have been proposed to represent non-ideal mixing in chemical reaction engineering [5, 7]. The results obtained are not always directly applicable to infiltration studies for the following reasons:

(1) In chemical reactor engineering, inlet and outlet streams are clearly identifiable, while in infiltration studies they are usually diffuse and often impossible to locate.

(2) In chemical reactor engineering, tracers are usually injected into inlet streams and monitored at outlet streams. In infiltration studies tracers are injected directly into the space concerned and also monitored inside that space. Thus, quantities such as residence-time distribution and mean exit-stream concentrations are of little interest to infiltration research, while volume-mean concentrations are of primary importance.

This paper proposes a general model for non-ideal mixing in enclosed spaces, in the context of infiltration studies. To keep the number of model parameters to a minimum, the space is assumed to consist of only two well-mixed regions (or CSTRs). Although models with more than two regions may be more realistic [4], the number of parameters involved would be so large that such models would be useless for practical purposes.

Even with the two-region model the number of parameters usually will be too large for interpreting the available data, and further simplifications will be

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considered. Such models will help determine the type and degree of non-ideality present. Even when it is not possible to quantify precisely the parameters involved, these models will give experimenters an insight into what is happening and help in the design of their tests.

THE GENERALIZED TWO-REGION MODEL (MODEL A)

The enclosed space being considered is assumed to consist of two well-mixed regions (Fig. 1). In the most general case possible, fluid may flow between the two regions and the environment in any manner, provided steady state and mass balance are maintained. From Fig. 1 one can write for the mass balance

$$\text{Region 1:}\quad aV\frac{dC_1}{dt} = -(b-d+f)vC_1+(f-d)vC_2 \tag{1}$$

$$\text{Region 2:}\quad (1-a)\frac{dC_2}{dt} = fvC_1-(1-b+f)vC_2 \tag{2}$$

where v is the net flow rate through the system, V the system volume, t the time, C the concentrations and a, b, d, f are defined in Fig. 1. In dimensionless form

$$\frac{dc_1}{d\theta} = -\frac{b-d+f}{a}c_1 + \frac{f-d}{a}c_2 \tag{3}$$

$$\frac{dc_2}{d\theta} = \frac{f}{1-a}c_1 - \frac{1-b+f}{1-a}c_2 \tag{4}$$

where $\theta = vt/V$ = reduced time, $c = VC/m_0$ = reduced concentration, and m_0 = total amount of tracer in system at time 0.

This model has four independent parameters, a, b, d and f, whose allowable ranges are

$$a = 0 \text{ to } 1$$
$$b = 0 \text{ to } 1$$
$$d = b-1 \text{ to } b$$
$$f = 0 \text{ to } \infty \quad \text{if } d < 0$$
$$d \text{ to } \infty \quad \text{if } d > 0.$$

The general solution to equations (3) and (4) is

$$c_1 = (f-d)(k_1 e^{m_1\theta} + k_2 e^{m_2\theta}) \tag{5}$$

$$c_2 = k_1(b-d+f+am_1)e^{m_1\theta}+k_2(b-d+f+am_2)e^{m_2\theta} \tag{6}$$

where k_1, k_2 are constants to be found from the initial conditions and m_1, m_2 are the eigenvalues of the system of equations (3) and (4) and are given by

$$m_1, m_2 = \frac{a+b-2ab-d+f+ad}{2a(1-a)} \times \left\{-1 \pm \sqrt{\left[1-\frac{4a(1-a)(b-b^2-d+bd+f)}{(a+b-2ab+f-d+ad)^2}\right]}\right\}. \tag{7}$$

The reader may check that in the limiting cases

$a = 0$ and $b = 0$ or $a = 1$ and $b = 1$ or f = infinity

one obtains $m_1 = -1$ and $m_2 = -\infty$, i.e. a CSTR is obtained.

Two useful expressions that may be used in deriving and simplifying subsequent equations are

$$m_1 + m_2 = \frac{-a-b+2ab+d-f-ad}{a(1-a)} \tag{8}$$

$$m_1 m_2 = \frac{b-b^2-d+bd+f}{a(1-a)}. \tag{9}$$

In infiltration studies, a tracer is released into the building. If the inlets and outlets are known (or if at least the main ones, such as open doors, windows, chimneys are known), they may be closed and the tracer is given time to mix to a uniform concentration, then opened again for the test proper. Otherwise good mixing is simply assumed. Samples of the air–tracer mixture are taken either at one particular spot where the air is considered well-mixed (e.g. near a fan) or at several places at the same time to give a volume-mean concentration.

In the context of the two-region model there are three types of tests, depending on the mode of tracer release:

(1) *Type I test*: Tracer is released in region 1 at time zero, and immediately reaches a concentration $c_1 = 1/a$ in this region. The concentration in region 2 is $c_2 = 0$ at time zero.

(2) *Type II test*: As in type I except that tracer is released in region 2. At time 0, $c_1 = 0$, $c_2 = 1/(1-a)$.

(3) *Type III test*: After tracer release, all inlets and outlets are sealed (assuming they are known), and the test starts when $c_1 = c_2 = 1$.

In each of these tests, samples may be taken in region 1 only, in region 2 only, or in both regions so as to obtain a volume-mean concentration

$$c_m = ac_1 + (1-a)c_2. \tag{10}$$

Equations for c_1, c_2 and c_m will now be given. Also of interest is how the volume-mean concentration decay curve compares with that for an ideal CSTR (for which $c_m = e^{-\theta}$).

Type I test

At $\theta = 0$, $c_{11} = 1/a$ and $c_{12} = 0$. Substituting into equations (5) and (6), one obtains

$$c_{11} = \frac{1}{a(1-a)(m_1-m_2)}\{[1-b+f+(1-a)m_1]e^{m_1\theta} - [1-b+f+(1-a)m_2]e^{m_2\theta}\} \tag{11}$$

$$c_{12} = \frac{f}{a(1-a)(m_1-m_2)}(e^{m_1\theta}-e^{m_2\theta}) \tag{12}$$

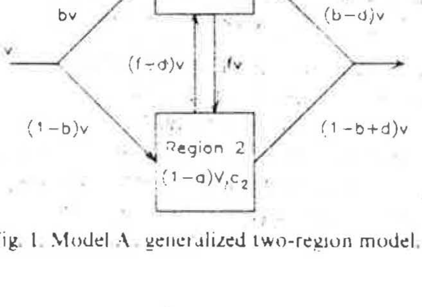

Fig. 1. Model A: generalized two-region model.

$$c_{\mathrm{Im}} = \frac{1}{(m_1 - m_2)}\left\{-\left(\frac{b-d}{a} + m_2\right)e^{m_1\theta} + \left(\frac{b-d}{a} + m_1\right)e^{m_2\theta}\right\}. \quad (13)$$

Figure 2 gives typical curves on a log–linear scale of c_{I1}, c_{I2}, c_{Im} and also Y_I (where $Y_I = c_{\mathrm{Im}}/c_{\mathrm{Im\,(CSTR)}}$). Also plotted, in dashed lines, are the c and Y curves for an ideally mixed system (CSTR).

Type II test

At $\theta = 0$, $c_{I1} = 0$ and $c_{I2} = 1/(1-a)$. Substituting into equations (5) and (6) one gets

$$c_{II1} = \frac{f-d}{a(1-a)(m_1-m_2)}\left(e^{m_1\theta} - e^{m_2\theta}\right) \quad (14)$$

$$c_{II2} = \frac{1}{a(1-a)(m_1-m_2)} \times \{(b+f-d+am_1)e^{m_1\theta} - (b+f-d+am_2)e^{m_2\theta}\} \quad (15)$$

$$c_{\mathrm{IIm}} = \frac{1}{m_1-m_2}\left\{-\left(\frac{1-b+d}{1-a} + m_2\right)e^{m_1\theta} + \left(\frac{1-b+d}{1-a} + m_1\right)e^{m_2\theta}\right\}. \quad (16)$$

These curves have the same general shape as in type I tests (Fig. 2), with c_1 and c_2 interchanged.

Type III test

At $\theta = 0$, $c_{III1} = c_{III2} = 1$. Substituting into equations (5) and (6) one gets

$$c_{III1} = \frac{1}{m_1-m_2}\left\{-\left(\frac{b}{a} + m_2\right)e^{m_1\theta} + \left(\frac{b}{a} + m_1\right)e^{m_2\theta}\right\} \quad (17)$$

$$c_{III2} = \frac{1}{m_1-m_2}\left\{-\left(\frac{1-b}{1-a} + m_2\right)e^{m_1\theta} + \left(\frac{1-b}{1-a} + m_1\right)e^{m_2\theta}\right\} \quad (18)$$

$$c_{\mathrm{IIIm}} = \frac{1}{m_1-m_2}\{-(1+m_2)\,e^{m_1\theta} + (1+m_1)\,e^{m_2\theta}\}. \quad (19)$$

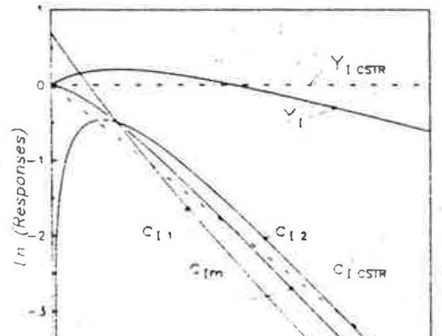

Fig. 2. Concentration variations in type I test. ($a = 0.5$, $b = 0.8$, $d = 0.7$, $f = 0.8$.)

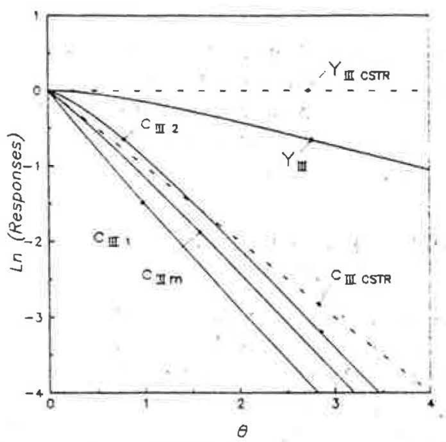

Fig. 3. Concentration variations in type III test. ($a = 0.5$, $b = 0.8$, $d = 0.7$, $f = 0.8$.)

Dividing equation (19) by the ideal CSTR response, $e^{-\theta}$, one obtains

$$Y_{III} = \frac{1}{(m_1-m_2)}\{-(1+m_2)\,e^{(1+m_1)\theta} + (1+m_1)\,e^{(1+m_2)\theta}\}. \quad (20)$$

Figure 3 shows typical plots for c_{III1}, c_{III2}, c_{IIIm} and Y_{III}. Also plotted, in dashed lines, are c and Y curves for a CSTR.

It would be interesting to know whether the volume-mean concentration c_{IIIm} decays faster or more slowly than it would if the system were ideally mixed; i.e. whether Y_{III} is greater or less than 1. If $Y_{III} > 1$, the air change after a time θ will be less than for a CSTR, even though gross inflows and outflows are equal. The reverse applies if $Y_{III} < 1$.

To find this out we take the derivative of Y_{III} and simplify using equations (8) and (9)

$$\frac{dY_{III}}{d\theta} = \frac{(a-b)(a-b+d)}{a(1-a)}\left(\frac{e^{m_1\theta} - e^{m_2\theta}}{m_1-m_2}\right)e^{\theta}. \quad (21)$$

It can be seen that the sign of $dY/d\theta$ is that of

$$(a-b)(a-b+d) = (a-b)[a-(b-d)].$$

Thus, if the inflow fraction b and outflow fraction $(b-d)$ are both greater than, or both less than, the volume fraction a, $dY/d\theta$ will be positive, and Y will be greater than 1 at all non-zero times (since $Y = 1$ at $\theta = 0$). In other words, the air change will be less than for an ideally mixed system for the same gross inflow and outflow. If $(a-b)$ and $(a-b+d)$ are of opposite signs, the reverse will apply. Since a, b and d are fixed parameters for a given situation, Y_{III} cannot cross the line $Y_{III} = 1$ (while Y_I and Y_{II} can—see Fig. 2). In practical terms, this means that the energy loss due to infiltration will be always greater or always less than flow measurements indicate, as the case may be.

TWO-REGION MODEL WITH BALANCED INTERCHANGE (MODEL B)

The generalized two-region model (model A) is too complex for most purposes and it would be useful to simplify it. By putting $d = 0$ in Fig. 1 a three-parameter model is obtained, model B (Fig. 4). Equations for concentrations and Y-ratios in various test conditions are readily obtainable from equations (7)–(20) by letting $d = 0$.

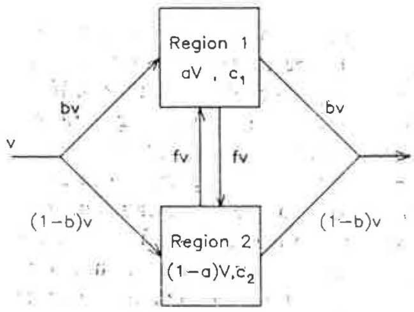

Fig. 4. Model B: two regions with balanced interchange.

and will not be presented here. From equation (21) it can be seen that $\mathrm{d}Y_{\rm III}/\mathrm{d}\theta > 0$, and thus $Y > 1$ for all non-zero times. *Thus the air change will always be less than for an ideal CSTR*, except if $a = b$ when it will be the same.

In practice, model B may apply when a building is partitioned into several portions, each with its doors, windows, cracks, etc., and there is limited interchange between the portions.

TWO-REGION FLOWTHROUGH MODEL (MODEL C)

If $b = 1$ and $d = b$ the two-parameter model in Fig. 5 is obtained, where the air flows into the first region and out of the second region with (unbalanced) interchange between the two. This model has been considered by Levenspiel [5]. In practice such a situation may arise when a building has an open door (or window, chimney, etc.) at one end, another opening at the other end, and the wind's general direction is lined up with the two openings. It will be also common in many old-fashioned multi-storey cold stores, when there is an open door near the top, another at the floor level, and cold air flows out of the bottom door by gravity. A room being warmed by an open fire with accompanying draught is another example. Yet another example is a high-rise cold store with a large open door: warm air will flow in through the upper part of the door and collect near the ceiling (region 1), while cold air flows out through the lower part of the door. If mixing is not thorough, stratification will set in and there will be distinct hot and cold layers.

The eigenvalues m_1, m_2 are found by putting $b = d = 1$ in equation (7)

$$m_1, m_2 = \frac{f}{2a(1-a)}\left\{-1 \pm \sqrt{\left[1 - \frac{4a(1-a)}{f}\right]}\right\}. \tag{22}$$

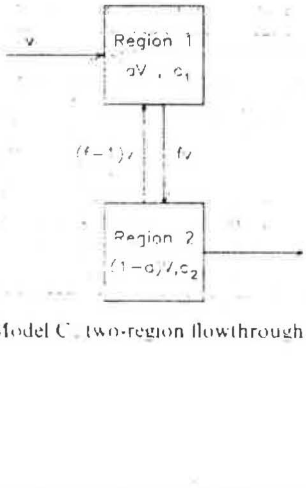

Fig. 5. Model C: two-region flowthrough model.

Doing the same to equations (11)–(20) will give expressions for c_{11}, c_{12}, $c_{\rm Im}$, $c_{\rm II1}$, $c_{\rm II2}$, $c_{\rm IIm}$, $c_{\rm III1}$, $c_{\rm III2}$, $c_{\rm IIIm}$ and $Y_{\rm III}$. Equation (21) becomes

$$\frac{\mathrm{d}Y_{\rm III}}{\mathrm{d}\theta} = -\left(\frac{e^{m_1\theta} - e^{m_2\theta}}{m_1 - m_2}\right)e^{\theta}. \tag{23}$$

Hence $Y_{\rm III}$ is less than 1 at all non-zero times. In other words, the air change is greater than for an ideal CSTR at a given rate of inflow outflow.

TWO-REGION MODEL WITH DEADWATER REGION (MODEL D)

If $b = 1$ and $d = 0$, one obtains the two-parameter model shown in Fig. 6, which is a particular case of model B. This model has been considered by Bischoff and Dedrick [8]. Region 2 is now a 'deadwater' zone, which is not directly connected with the outside.

This model may apply when the infiltration happens mainly through a single opening or several closely situated openings and there are walls, cupboards, furniture, products, etc. that create dead regions exchanging tracer slowly with the rest of the building.

Equation (7) now becomes

$$m_1, m_2 = \frac{1-a+f}{2a(1-a)}\left\{-1 \pm \sqrt{\left[1 - \frac{4af(1-a)}{(1-a+f)^2}\right]}\right\} \tag{24}$$

and putting $b = 1$, $d = 0$ in equations (11)–(20) will give expressions for c_{11}, c_{12}, $c_{\rm Im}$, $c_{\rm II1}$, $c_{\rm II2}$, $c_{\rm IIm}$, $c_{\rm III1}$, $c_{\rm III2}$, $c_{\rm IIIm}$ and $Y_{\rm III}$.

Since this model is a particular case of model B, $Y_{\rm III}$ will be positive for all non-zero times, and the air change will be less than for the ideal CSTR for a given rate of inflow outflow.

A special situation of interest is when the factor a tends to zero, in which case

$$c_{\rm II2} = c_{\rm IIm} = c_{\rm III2} = c_{\rm IIIm} = e^{-[1/(1+f)]\theta} \tag{25}$$

$$Y_{\rm II} = Y_{\rm III} = e^{\theta/(1+f)} \tag{26}$$

for non-zero times.

Physically this means that a small local recirculation zone or 'airlock' occurs near the opening (region 1), so that the entering and leaving air streams are mixed together, thus reducing the effective air interchange by a constant factor $f/(1+f)$. This gives a physical explanation for Jenning and Armstrong's 'mixing factor' [3]. In tests on cold stores with an open door we have observed reduction factors down to 0.50, corresponding to $f = 1$.

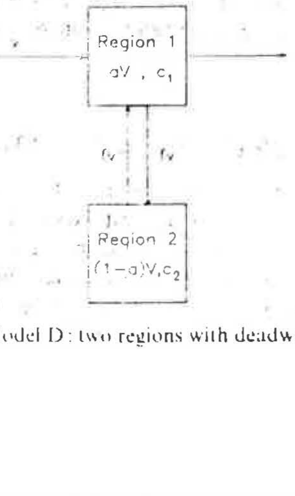

Fig. 6. Model D: two regions with deadwater.

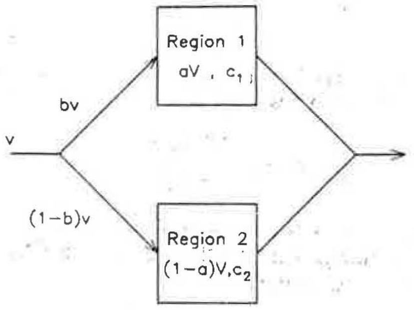

Fig. 7. Model E: two regions without interchange.

TWO-REGION MODEL WITHOUT INTERCHANGE (MODEL E)

If $f = d = 0$, the model shown in Fig. 7, which is another particular case of model B, is obtained. Fluid flows in two separate streams through the two regions, which do not interact. This situation may happen in practice when the building is separated into distinct zones (rooms, levels, etc.) each with its own openings.

Equations for various concentrations are most easily obtained by direct solution of equations (3) and (4) (simplified as required)

Type I test

$$c_{11} = \frac{1}{a} e^{-(b/a)\theta} \quad (27)$$

$$c_{12} = 0 \quad (28)$$

$$c_{1m} = e^{-(b/a)\theta} \quad (29)$$

Type II test

$$c_{II1} = 0 \quad (30)$$

$$c_{II2} = \frac{1}{1-a} e^{-[(1-b)/(1-a)]\theta} \quad (31)$$

$$c_{IIm} = e^{-[(1-b)/(1-a)]\theta} \quad (32)$$

Type III test

$$C_{III1} = e^{-(b/a)\theta} \quad (33)$$

$$c_{III2} = e^{-[(1-b)/(1-a)]\theta} \quad (34)$$

$$c_{IIIm} = a\, e^{-(b/a)\theta} + (1-a)\, e^{-[(1-b)/(1-a)]\theta} \quad (35)$$

Since this model is a particular case of model B, Y is greater than 1 (except when $a = b$) for all non-zero times, and so the air change will be less than for the ideal CSTR when the rate of inflow/outflow is the same.

(Strictly speaking type III test is meaningless in the context of model E, since no mixing takes place between the two regions. However, in practice, this model can be used as a limiting case when f is non-zero but much smaller than 1.)

PRACTICAL APPLICATIONS

To apply the foregoing models to a practical case, one should first of all try to understand the physical situation as far as possible, so that a proper model can be selected and perhaps some of the parameters quantified. Only then will it be appropriate to carry out curve-fitting procedures, using tracer concentration data, to find the other parameters.

For curve-fitting purposes, one must plot the relevant quantity as given by the above equations (C_{11}, C_{12}, C_{1m}, Y, etc., as the case may be) against the time θ. If the rate of inflow/outflow, v, is not known, as is generally the case, then it would be best to plot this quantity for various values of the parameters on a log–log scale. The experimental data (also plotted on a log–log scale) can then be shifted up, down or across until the best fit is obtained [8]. Alternatively, if a computer is at hand, a nonlinear regression using hill-climbing techniques can be carried out.

If the rate of inflow/outflow is known, for example when all the inflow (or outflow) happens at one particular opening and can be measured, then a log–linear plot (log of C or Y vs time θ) will be more convenient, as all curves will then tend to straight lines as θ tends to larger values.

Since most readers will have access to calculators or computers to help them make up these plots, they will not be presented here (as there are many possible combinations). Instead an actual case will be used to illustrate the technique.

Example

Sulphur hexafluoride tracer was used to measure the air change rate into a meat cold store when the door was opened [9]. The store was of the single-storey high-stud type with a gross volume of 4080 m^3 and a net volume (gross volume − product volume) of 2983 m^3. Air flowed in through the top half of the door and out the bottom half, at a rate measured at 3.0 m^3/s (3.6 air changes per hour). The tracer test consisted of releasing the tracer, closing the door until a uniform tracer concentration was obtained, taking samples at various spots in the store, opening the door to start the test, closing it after a given time (5, 10, 15, 20 or 30 min), and then waiting again for equilibrium before sampling. Thus, this was a type III test and the concentration being measured was c_{IIIm}. Table 1 shows the test durations and measured air changes.

In Table 1 the reduced time θ is the same as the air change for CSTR, since for the latter

$$c_{IIIm} = e^{-\theta}$$

and air change $= -\ln(c_{IIIm})$.

In Table 1 Y_{III} is simply the difference between the air change for CSTR and the measured air change (column 4 − column 3).

Solution

We will limit ourselves to one of the two-parameter models (C, D or E) in view of the small number of data.

Table 1. Data from tracer test for worked example

Test duration, s	Dimensionless time θ	Measured air change ($-\ln c_{IIIm}$)	Air change for CSTR	Y_{III}
300	0.30	0.32	0.30	−0.02
600	0.60	0.49	0.60	0.11
900	0.90	0.54	0.90	0.36
1200	1.20	0.69	1.20	0.51
1800	1.80	0.86	1.80	0.94

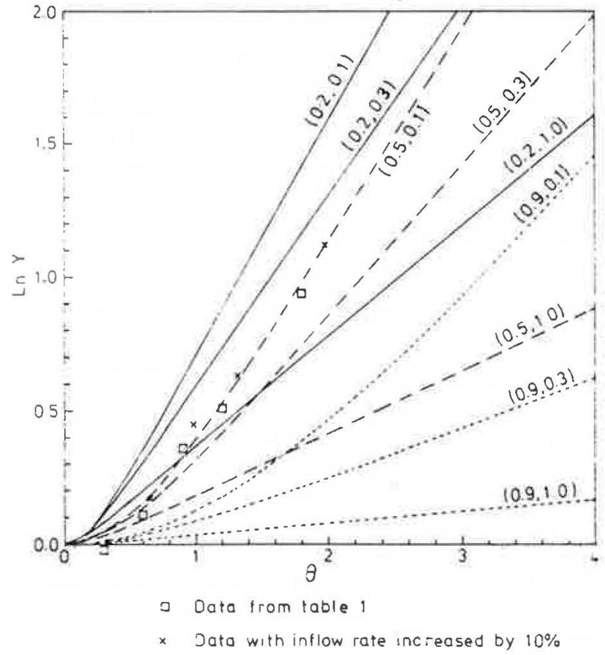

Fig. 8. Curve-fitting model D for cold store example. Parameters for each curve are (a, f).

Since there is only one main inlet and outlet, model E can be eliminated. Also, since Y_{III} increases with time, model C can be eliminated. We are therefore left with model D (two-region model with deadwater).

Equation (20) for Y_{III} was plotted in the range $\theta = 0$–4 (Fig. 8) with

$$a = 0.2, 0.5, 0.9 \quad \text{and} \quad f - 0.1, 0.3, 1.0$$

Since v/V is known, θ can be calculated for all data, and so $\ln Y_{III}$ vs θ is plotted. Then the experimental data is plotted on the same figure. It can be seen that the data are best fitted by

$$a = 0.5 \quad \text{and} \quad f = 0.1.$$

The physical interpretation of this result is that the product, which is stacked on pallets, tends to partition the rooms and create 'dead' pockets of air that exchange tracer very slowly with the main zone.

The curve-fitting procedure in this case is quite insensitive to the measured value of the rate of inflow/outflow. An error of 10% in the latter makes hardly any difference in a and f (Fig. 8).

CONCLUSION

Non-ideality in mixing must be taken into account when one measures the infiltration rate. Unfortunately this factor has been ignored by previous workers in this field and this has limited the scope of tracer testing. A good log-linear plot for the tracer concentration is no proof that the system is well-mixed: as can be seen from Figs 2 and 3 and equations (11)–(19), all the concentration curves will tend to an exponential decay anyway as the time becomes large, but that final decay (term containing $e^{m_1\theta}$ where m_1 is the larger, i.e. less negative, eigenvalue) is mainly influenced by the more stagnant region, even though the actual volume of that region may be quite small, e.g. the volume of a closet or cupboard.

Also, from a practical point of view, it is important to know in which part of the building the infiltration is happening. For example, a large air interchange in the attic is clearly less important than one in the living area. The simplified models and curve-fitting procedures will help in the determination of these relative infiltrations. The tracer-response approach can be complementary to recent attempts to model multi-zone situations with fluid mechanics, using computers [10].

Finally, an interesting and potentially important result of this paper is that the air change after a period of time may be quite different from what could be expected from the values of inflow outflow: it may be larger (model C), in which case more energy may be required than expected, or smaller (models B, D and E), in which case less may be required than expected. In general, the energy (heat or refrigeration) load will be intermediate between that estimated by a tracer test and that deduced from direct inflow/outflow measurements, since heat may be transferred to or from the infiltration air both by mixing with the air inside and by contact with walls, furniture, people, products, and so forth.

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