#11489

THE IMPLICATIONS OF USING THE STANDARD k-ε TURBULENCE MODEL TO SIMULATE ROOM AIR FLOWS WHICH ARE NOT FULLY TURBULENT

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ABSTRACT

The standard $k-\varepsilon$ model is commonly employed in the numerical simulation of room air flows. Strictly speaking this model is only valid for fully turbulent flow, whereas weakly turbulent, relaminarized, or even stagnant flow can exist in rooms (particularly in regions remote from air-supply systems). Consequently, an important assumption is implicit when the standard $k-\varepsilon$ model is applied to a room: that the flow behaves as though it were fully turbulent. This paper examines the implications of that assumption.

A room exposed to two-dimensional forced flow was simulated using the standard $k-\varepsilon$ model and it was found that the eddy viscosity (and thus the turbulent diffusion of heat and momentum) was over-predicted near the left wall, where flow was almost stagnant. This excessive dose of eddy viscosity had a negligible affect on flow calculations, but had a significant influence on thermal predictions. Heat transfer to the left wall was found to be highly sensitive to the value of the eddy viscosity in the surrounding air. Conclusions are drawn and recommendations made on the use of the standard $k-\varepsilon$ model in whole building/computational fluid dynamics integrated simulators.

KEYWORDS

Turbulence modelling, computational fluid dynamics, surface convection heat transfer.

INTRODUCTION

The integration of CFD and BSim

Computational Fluid Dynamics (CFD) has been used to simulate air flow in and around buildings for a quarter century. In the case of interior air flow modelling—the subject of this paper—analysis is usually restricted to single rooms or spaces within buildings due to high compute requirements. This presents a quandary, in that the boundary conditions of the problem domain (wall-surface temperatures and air flows entering/leaving the room) cannot be determined *a priori*. The room does not exist in isolation: wall temperatures and air flows through openings are dynamic and dependent on the external weather excitations, states prevailing throughout the rest of the building, and the operation of plant equipment, these in turn depending on conditions within the room. Simplifying assumptions are usually made, and the boundary conditions often treated as steady.

Workers have begun to address this issue by integrating dynamic fabric models and inter-surface radiation models into CFD codes (Holmes et al 1990; Chen et al 1995; Moser et al 1995). This allows room air flow to be calculated by prescribing boundary conditions external to the building or in adjoining spaces, rather than within the room.

Negrão (1995) extended this concept by integrating a CFD code into the ESP-r (ESRU 1997) whole-building simulation model (BSim), the two models interacting on a time-step basis, exchanging information at their model boundaries. A thermal and (optionally) a network air flow representation of the whole building and plant is established in the BSim program while a CFD model is created for a single room. BSim then establishes the boundary conditions for the CFD model. Once the CFD solution converges, it passes the thermal or air flow results (room air temperature, surface convection heat transfer, air flows entering/leaving the room) to BSim, which uses the data to calculate the surface temperatures, energy flows, and air flows throughout the building. This process is repeated each time step. The reader is referred to Clarke et al (1995) for details on the ESP-r BSim-CFD conflated simulator.

With these conflated approaches, CFD must calculate room heat transfer as well as room air flow. The quality of the heat transfer predictions not only affects the BSim results, but also influences the air flow predictions within the room, as the boundary conditions for the CFD calculations are affected by the heat transfer predictions at previous time steps.

Turbulence modelling

In essence, CFD involves the solution of a set of non-linear partial differential equations using numerical techniques, the equations expressing fundamental physical laws—the conservation of mass, momentum, and energy. Dealing with turbulence (the presence of random fluctuations which exists in most flows of practical interest) complicates matters considerably. Turbulent fluctuations enhance the transport of momentum, heat, and pollutants, and must be considered in the formulation and solution of the equations of motion.

Techniques of various levels of complexity and computational intensity have been developed to characterize this chaotic motion. Some approaches attempt to model the turbulent fluctuations, necessitating very fine grids and time steps. Applications to room air flow modelling have already been made (Nielsen 1998; Emmerich and McGrattan 1998) but computational costs remain extremely high and further refinement is necessary.

In contrast to these high-resolution techniques, *turbulence models* apply coarser grids and larger time steps and treat the random fluctuations with statistical methods. The equations of motion are filtered with respect to time, so that rather than modelling the *details* of the turbulent motion, these methods account for the *influence* of turbulence on the mean motion.

A plethora of turbulence models have been developed but one, the $k-\varepsilon$ model, has enjoyed the greatest usage by far, not only in the domain of buildings, but in most fields of study (aerodynamics, hydraulics, combustion, etc). The preponderance of $k-\varepsilon$ for modelling room air flows can be seen by reviewing the literature (Whittle 1986; Nielsen 1989; Jones and Whittle 1992; Chen and Jiang 1992; Lemaire et al 1993; Chen 1995).

Nature of room air flow

Despite many successful applications, questions remain regarding the appropriateness of $k-\varepsilon$ for room air flow modelling. Strictly speaking, the model is only valid for fully-developed turbulence (notwithstanding near-wall regions for which adjustments are made to account for viscous effects). Consequently, the application of $k-\varepsilon$ implies a very important assumption: that the flow is fully turbulent or at least behaves like a fully turbulent flow.

But, in general, room air flows are not fully turbulent. Baker et al (1994a) characterize room air motion as typically turbulent, although only weakly so. Baker et al (1994b) state that most room air flows are at least locally turbulent, but flows away from HVAC supply systems and obstructions with edges tend to be subtly turbulent. According to Chen and Jiang (1992), room air flows may be laminar unsteady, locally artificially induced turbulent, transitional, or fully turbulent. Measurements indicate that the flow in the main body of ventilated rooms may be transitional (Jones and Whittle 1992).

According to Chen and Jiang (1992), few results on the subject are available but it is doubtful that a $k-\varepsilon$ model can successfully characterize a partially turbulent flow.

Research objectives and approach

The above discussion raises a number of issues and questions. Given that room air flows are rarely fully turbulent, can the the $k-\varepsilon$ model lead to accurate predictions of air flow and heat transfer? Is $k-\varepsilon$ the appropriate choice for the integrated modelling systems, in which CFD must calculate heat transfer as well as air flow? Can the thermal simulation of whole buildings be enhanced through the integration of CFD?

The first step in addressing these questions is to assess how $k-\varepsilon$ reacts to room air flows which are not fully turbulent and to assess the sensitivities of air flow and heat transfer predictions to uncertainties in turbulence characterization. This is accomplished by performing CFD and BSim-CFD simulations of a room which has a region that can be characterized as weakly turbulent. Modifications to the $k-\varepsilon$ model are made to examine the significance of poor turbulent characterization in this weakly turbulent region.

The $k-\varepsilon$ model is first described. The numerical model is then outlined, the experiments described, results presented, and finally conclusions drawn.

k- ε TURBULENCE MODEL

The equations of motion (conservation of mass, momentum, and energy) relate the *instantaneous* quantities of pressure, velocity, and temperature. These equations are filtered with respect to time, by replacing the instantaneous field quantities with the sum of a time-mean quantity and a fluctuating quantity (eg. $T=\bar{T}+T'$). This process results in the *Reynolds-averaged* form of the equations of motion, which represent time-mean rather than instantaneous quantities.

The form of the equations of motion is not altered by the Reynolds-averaging process, but new terms are added to represent the influence of the turbulent fluctuations on mean-flow quantities. To illustrate, examine the diffusion term in the momentum-conservation equations:

$$\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)-\rho\overline{u'_ju'_i}\right] \qquad (1)$$

The term $\mu(\partial\bar{u}_i/\partial x_j+\partial\bar{u}_j/\partial x_i)$ involves gradients of the time-mean velocities and the molecular viscosity, and represents the diffusion of momentum through molecular motion. The term $-\rho\overline{u'_ju'_i}$ involves the fluctuating velocities, and represents the diffusion of momentum through turbulent motion.

Boussinesq proposed the *eddy-viscosity concept* a century ago, drawing an analogy between molecular and turbulent diffusion. Like the viscous stresses in laminar flow, the turbulent stresses are assumed to be proportional the the mean-velocity gradients:

$$-\overline{u'_iu'_j}=\frac{\mu_t}{\rho}\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)-\frac{2}{3}k\delta_{ij} \qquad (2)$$

where μ_t is the eddy viscosity. The molecular viscosity (μ) is a property of the *fluid*. In contrast μ_t is a property of the *flow*: it can differ significantly from one flow to another and can vary throughout a flow domain. Calculating the μ_t distribution is the function of the turbulence model.

The fluctuating quantities are eliminated from the Reynolds-averaged equations of motion with the eddy-viscosity concept, turbulent diffusion now being completely characterized by gradients in the mean quantities and by the eddy viscosity. The conservation of mass, momentum, and energy are expressed as,

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_j}(\rho\bar{u}_j)=0 \qquad (3)$$

$$\frac{\partial}{\partial t}(\rho\bar{u}_i)+\frac{\partial}{\partial x_j}(\rho\bar{u}_j\bar{u}_i)=-\frac{\partial\bar{P}}{\partial x_i}+ \qquad (4)$$

$$\frac{\partial}{\partial x_j}\left[(\mu+\mu_t)\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)\right]-\rho\beta(\bar{T}_\infty-\bar{T})g$$

$$\frac{\partial}{\partial t}(\rho\bar{T}) + \frac{\partial}{\partial x_j}(\rho\bar{u}_j\bar{T}) = \frac{\partial}{\partial x_j}\left[\left(\frac{k}{c_p} + \frac{\mu_t}{\sigma_t}\right)\frac{\partial\bar{T}}{\partial x_j}\right] + \frac{q'''}{c_p} \quad (5)$$

where σ_t is the turbulent Prandtl number, normally treated as a constant.

By calculating the eddy-viscosity distribution, the turbulence model determines the diffusion coefficients for the momentum and energy equations, implicitly establishing the relative strengths of turbulent and molecular diffusion. This can be seen by examining equations 4 and 5. If $\mu_t/\mu \gg 1$ the molecular diffusion terms in the momentum equations (μ) and energy equation (k/c_p) will be dominated by the turbulent terms. Whereas, if $\mu_t/\mu \approx 0$, molecular effects will dominate. Consequently, the ratio μ_t/μ can be thought of as an indicator of "how turbulent" a flow is locally.

Launder and Spalding (1974) related the eddy viscosity at each grid point to local values of the *kinetic energy of turbulence* (k) and the *dissipation rate of turbulence energy* (ε):

$$\mu_t = \frac{C_\mu \rho k^2}{\varepsilon} \quad (6)$$

where C_μ is an empirical constant.

The local distributions of k and ε require the solution of two additional transport equations. The turbulent-kinetic-energy and dissipation-rate transport equations are solved iteratively with the Reynolds-averaged conservation equations for mass, momentum, and energy. At each iteration, the local values of k and ε are calculated, yielding the local values of the μ_t, which are used to update the diffusion coefficients in the momentum and energy equations.

It is important to note that the form of the model under consideration here is the *standard* $k-\varepsilon$ model. Alternate forms of $k-\varepsilon$ are available and have been applied for room air flow prediction (eg. Chen 1995; Nielsen 1998), however the standard form remains the most popular. Although the standard $k-\varepsilon$ model has recognized deficiencies in predicting surface convection (see Chen et al 1990; Yuan et al 1994), this is a separate issue than the one under examination in the current study.

NUMERICAL EXPERIMENTS AND RESULTS

Test case and numerical approach

A well-known benchmark was selected as the object of this study, the International Energy Agency (IEA) Annex 20 two-dimensional test case. Geometry, boundary conditions, and measured data for this configuration are given by Nielsen (1990). The flow, which enters the room at the upper left corner and exits at the lower right (Figure 1), can be treated as two-dimensional and has been successfully simulated by a number of CFD codes.

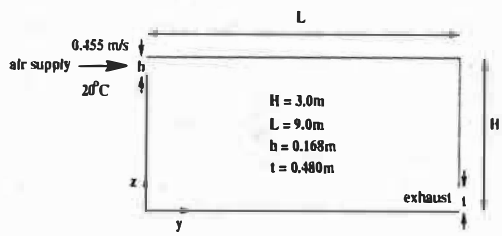

Figure 1: Cross-section of room

ESP-r's *Domain Flow Solver (dfs)* was employed in this analysis. The basis of *dfs* is similar to many codes that have modelled this and other room air flows (see Lemaire et al 1993; Liddament 1991). It is a transient CFD code based on the finite-volume approach. It employs the standard $k-\varepsilon$ model of turbulence and uses the log-law wall functions to account for viscous effects in near-wall regions. The SIMPLE pressure-correction solution approach is used with a combined TDMA/Gauss-Siedel solver.

The *dfs*-predicted flow pattern is illustrated in Figure 2.

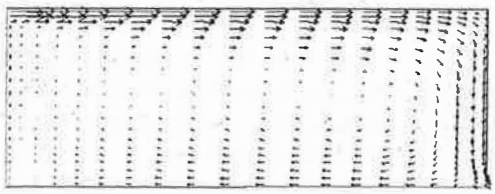

Figure 2: Flow field

The numerical predictions of the horizontal mean velocity and the turbulent fluctuations compared favourably with Nielsen's measured data, although agreement is less good near the floor and ceiling. The flow recirculations in the upper-right and lower-left corners were not predicted. These results are consistent with those of the CFD codes examined in Annex 20 (Lemaire et al 1993).

Eddy viscosity predictions

Examining Figure 2, it can be seen that there is a strong flow along the ceiling and down the right wall. The rest of the room experiences a recirculating flow, induced by the incoming air stream. This recirculation is strongest in the right and middle portions of the room, but is quite weak in the left of the room, particularly within a 1m distance of the wall (from the floor level to within a few centimetres of the air inlet). Mean velocities in this low-flow region are less than one tenth of those at the air inlet.

The $k-\varepsilon$ model has well characterized the turbulent fluctuations, but this does not necessarily mean that it has well characterized the turbulent diffusion of energy and momentum. An examination of the predicted eddy viscosity field is quite revealing. Recall that the μ_t/μ ratio indicates the relative strength of turbulent to molecular diffusion. Along the flow path of the incoming air μ_t/μ ranges from 25 to 300. These values characterize a fully turbulent flow, or at least a flow which is transitional from weakly to fully turbulent (see Baker et al 1994b). This is in agreement with expectations.

The numerical results do not agree with expectations near the left wall, however, where turbulent diffusion is expected to be weaker. The flow is very low, verging on stagnant. The turbulent kinetic energy should be lower than in the main flow stream, and the predictions are, by an order of magnitude. Despite this, the $k-\varepsilon$ predicted μ_t/μ ratios are as high as those in the main flow stream, ranging from 50 to 300. This is a result of very low turbulent-energy dissipation rates (refer to equation 6), which unfortunately cannot be compared with measurements.

The $k-\varepsilon$ model has predicted μ_t values near the left wall which are consistent with fully turbulent flow. There is some uncertainty in the nature of the flow near the left wall. It could be weakly turbulent and may possibly have relaminarized, but certainly, it is not fully turbulent. Therefore, it can be concluded that the $k-\varepsilon$ model has over-predicted the eddy viscosity near the left wall. This conclusion is corroborated by Baker et al (1994b) who stated that the $k-\varepsilon$ model will produce an "excessive dose of eddy viscosity for the subtly-turbulent flows existing throughout indoor rooms away from HVAC supply systems and obstructions with edges".

Implications on air flow calculations

A series of numerical experiments were then performed to determine the implications of over-predicting μ_t near the left wall. The boundary conditions of the isothermal room were altered so that both heat transfer and air flow could be assessed. The energy equation was added to the solution domain and the temperature of the air flowing into the room set to 25°C, the floor and ceiling to 18°C, the right wall to 10°C, and the left wall to 0°C.

With the standard $k-\varepsilon$ model the μ_t distribution is calculated with equation (6) each iteration, following the solution of the k and ε equations, but before the energy and momentum-equation coefficients are calculated for the next iteration. A simulation with the new boundary conditions was performed with this standard treatment. In the next two simulations, equation (6) was modified to derate μ_t near the left wall ($0m < y < 1m$ and $0m < z < 2.5m$). In one simulation μ_t was halved, reducing turbulent diffusion next to the left wall by 50%. In a second simulation μ_t was set to zero, eliminating turbulent diffusion in this region. Equation (6) was unaltered throughout the rest of the room.

It is not implied that these modifications to the $k-\varepsilon$ model more accurately reflect the reality, but they will give a bound on the errors caused by the excessive dose of eddy viscosity by the left wall. Comparing the results of these three simulations will indicate how sensitive air flow and heat transfer predictions are to μ_t in this region

The mean-velocity predictions were compared by superimposing flow-vectors and by plotting results at two vertical planes ($y=3m$ and $y=6m$) and at two horizontal planes ($z=0.08m$ and $z=2.9m$). The mean-velocity predictions of the three simulations were nearly identical. In addition, the turbulent fluctuations were compared at the four planes, and found to be nearly identical, except at the grid points next to the left wall. Therefore, it can be concluded that the air flow predictions are highly insensitive to μ_t adjacent to the left wall.

This insensitivity can be explained by examining the diffusion terms in the momentum equations (eq. 4). Velocities adjacent to the left wall are very low, as are velocity gradients. Consequently, momentum diffusion is not significant (here) and high relative errors in its estimation can be tolerated. This is a fortuitous feature of the $k-\varepsilon$ model.

Implications on thermal calculations

Very different results are observed in the thermal predictions, however. The simulations with the modified $k-\varepsilon$ models resulted in significantly lower temperature gradients along the left wall. Surface convection at the left wall was highly sensitive to μ_t in the surrounding air, as illustrated in Figure 3.

Halving μ_t reduced heat transfer to the left wall by 17%, while eliminating turbulent diffusion reduced the surface convection by 31%. There are no significant differences in the heat transfer at the right wall, floor, or ceiling.

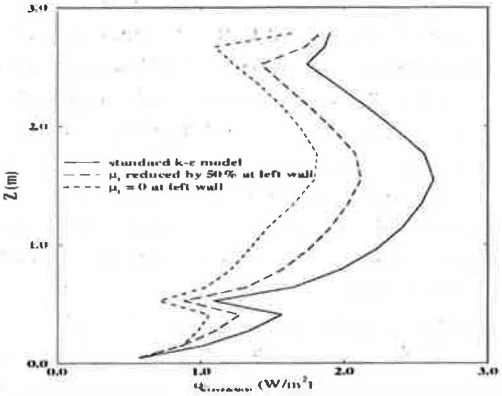

Figure 3: Surface convection to left wall

This sensitivity can be explained by examining the diffusion terms in the energy equation. When large temperature gradients exist in the vicinity of poor turbulence characterization, significant errors in the estimation of thermal diffusion result. Further, as there is little flow adjacent to the left wall thermal convection is relatively weak, so diffusion plays a significant role in the energy equation. The standard $k-\varepsilon$ model produces Peclet numbers (ratio of strengths of convection and diffusion) in the order of 10^0-10^1 near the left wall, whereas when μ_t is set to zero, the Peclet numbers are in the order of 10^2. With Peclet numbers this low, an accurate calculation of turbulent thermal diffusion (hence μ_t) is very important.

Significance in BSim-CFD integrated simulators

As mentioned, with the BSim-CFD integrated simulators the quality of heat transfe: predctions will affect boundary conditions for furture time-row CFD calculations and will affect the quality of the thermal simulations on the BSim side. To assess the implications of μ_t over-prediction next to the left wall, the CFD model of the room was conflated with the ESP-r whole-building simulator.

The floor, ceiling, and right wall were simulated as opaque insulated surfaces, while the left wall was simulated as a window. A constant flow of air at 25°C was introduced to the room to provide base heating, while an electric reheater was controlled to maintain the room at 20°C. The simulation was run for a one-week period in January using Ottawa weather data. The simulation time-step was one-hour.

Two one-week simulations were performed, the first with the standard $k-\varepsilon$ model and the second with μ_t set to zero next to the left wall. Figure 4 contrasts the power required by the electric reheater for the two simulations. As can be seen, greater power consumption was predicted with the standard $k-\varepsilon$ model. Over the one-week period, the difference in energy consumption between the two simulations was 15%.

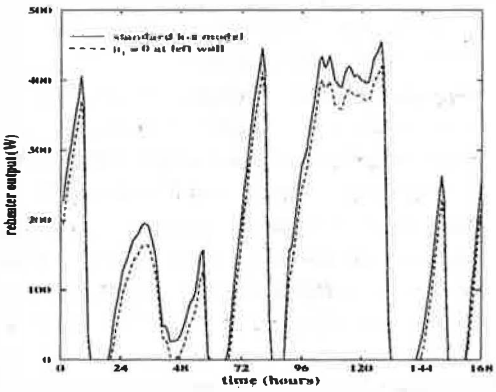

Figure 4: Reheat predicted with BSim-CFD simulations

CONCLUSIONS

The standard $k-\varepsilon$ model tends to over-predict the eddy viscosity—and thus the turbulent diffusion of heat and momentum—in regions which are subtly turbulent. This has a negligible impact on mean flow calculations, due to the low diffusion of momentum in these regions. However, this can have a significant influence on thermal predictions, due to the importance of thermal diffusion in regions of low flow. Substantial errors in the prediction of surface convection can result. This influence was seen to be local, indicating that the over-prediction of eddy viscosity in low-turbulence regions has a negligible influence on thermal predictions in high-turbulence regions of the flow.

In BSim-CFD integrated simulators in which CFD is used to calculate surface convection, the over-prediction of eddy viscosity in low-turbulence regions can result in significant errors in energy calculations. It is recommended that BSim appraise the results returned by CFD, perhaps accepting convection estimates for some surfaces while rejecting those for surfaces adjacent to weakly turbulent regions. As the mean flow field is unaffected by the eddy viscosity over-prediction, BSim may be able to predict the surface convection in the weakly turbulent regions using the CFD-predicted flow field as input to empirical correlations (empirical correlations are relied on at present to calculate surface convection using room averaged conditions).

A modified $k-\varepsilon$ model utilizing damping functions to reduce the eddy viscosity in areas with low velocity and relaminarization (as discussed by Chikamoto et al 1992 and Nielsen 1998) may address the eddy viscosity over-prediction in the low-turbulence regions, and should be investigated for use in BSim-CFD simulators. Low-Reynolds-number $k-\varepsilon$ models (eg. Launder and Spalding 1974; Lam and Bremhorst 1981) may also address the problem, but it is felt that these methods remain too computationally intense for use in BSim-CFD simulators at the present time.

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